	Undergraduate Course
ELEMENTS OF COMPUTATION THEORY	College of Computer Science
Chapter 1	ZHEJIANG UNIVERSITY
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1.7.4 Show each of the following.

(c) If a and b are distinct symbols, then $\{a,b\}^* = \{a\}^*(\{b\}\{a\}^*)^*$.

(d) If Σ is an alphabet, $e \in L_1 \subseteq \Sigma^*$ and $e \in L_2 \subseteq \Sigma^*$, then $(L_1\Sigma^*L_2) = \Sigma^*$.

Solution:

(c) It is obvious that $\{a\}^*(\{b\}\{a\}^*)^* \subseteq \{a,b\}^*$.

On the other hand, suppose that $w \in \{a,b\}^*$, then $w = a^*$ or $w = a^*ba^*ba^* \cdots ba^*$ $\in \{a\}^*(\{b\}\{a\}^*)^*$.

(d) Suppose $w \in (L_1\Sigma^*L_2)$. Then $w = xyz$, where $x \in L_1 \subseteq \Sigma^*, y \in \Sigma^*, z \in L_2 \subseteq \Sigma^*$. Thus $xyz \in (\Sigma^*)^* = \Sigma^*$.

On the other hand, suppose $w \in \Sigma^*$. Then because $e \in L_1$ and $e \in L_2$, $w = ewe \in (L_1\Sigma^*L_2)$.

1.7.6 Under what circumstances is $L^+ = L^* - \{e\}$?

Solution: $L^+ = L^* - \{e\}$ *exactly when* $e \notin L$.

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1.8.3 Let $\Sigma = \{a,b\}$. Write regular expressions for the following sets:

(c) All strings in Σ^* with exactly one occurrence of the substring aaa .

Solution: $((a \cup aa \cup b^*)b)^*aaa(bb^*(a \cup aa \cup b^*))^*$.

1.8.5 Which of the following are true? Explain.

(a) $baa \in a^*b^*a^*b^*$

(b) $b^*a^* \cap a^*b^* = a^* \cup b^*$

(c) $a^*b^* \cap b^*a^* = \emptyset$

(d) $abcd \in (a(cd)^*b)^*$

Solution:

(a) **true.** *$baaa$ consists of zero repetitions of a, followed by one repetition of b, then two repetitions of a, and finally aero repetitions of b.*

(b) **true.** *Any string described by a^*b^* consists of s string of as followed by a string bs. If b^*a^* also describes this string, then there cannot be any as followed by a b in the string, so either there are zero as or zero bs, making it into a string of any num -ber of as or a string of any number of bs, by taking zero repetitions of b or a, res -pectively.*

(c) **false.** *Any string consisting only of bs is described both by a^*b^* and by b^*c^*, so that their intersection is not the empty set, but rather b^*.*

(d) **false.** *If d appears in a string described by $(a(cd)^*b)^*$, it must be immediately followed by a c or a b.*
But this is not the case in $abcd$.