Theory of Computation, Fall 2022
Assignment 4 Solutions

Q1. (25 pts)
$S \to 0|1|0S0|1S1|0S1|1S0$

Q2. (25 pts)
$S \to e|1S0|0S1$

Q3. (30 pts)
$S \to S_1S_2$
$S_1 \to 0S_10|1S_11|\#S_2$
$S_2 \to 0S_2|1S_2|e$

Q4. (20 pts)
For every $w \in L(G)$, we need exactly $2|w|-1$ steps of derivations since G is in Chomsky norm form. And in every step, there are at most $|R|$ choices, so the number of distinct derivations from S to w is no more than $|R|^{2|w|-1}$, which is finite.

*Note: Since G is some context-free grammar in Chomsky norm form, its every rule is in the form
(i) $S \to e$
(ii) $A \to BC$ where $B, C \in V - \Sigma - \{S\}$
(iii) $A \to a$, $a \in \Sigma$
The number of symbols changes from 1 to $|w|$ need exactly $|w|-1$ steps of derivations using rules in form (ii). And we need $|w|$ steps of of derivations using rules in form (iii) to change each non-terminals to terminals. To sum up, we need exactly $2|w|-1$ steps of derivations.