

Theory of Computation, Fall 2022  
Assignment 6 (Due October 31 Monday 10:00 am)

Q1. Show that if  $L$  is context-free, so is

$$L^R = \{w^R : w \in L\}.$$

Q2. Let  $a, b, c$  be three distinct symbols. Use pumping theorem to show that  $A = \{wcw : w \in \{a, b\}^*\}$  is not context-free.

Q3. Let  $A$  be a context-free language. Let  $B$  be a regular language. Prove that  $A \cap B$  is context-free. You may assume that  $A$  and  $B$  are defined over the same alphabet  $\Sigma$ . (Hint: let  $P_A$  be a PDA accepting  $A$ . Let  $M_B$  be an NFA accepting  $B$ . Construct a PDA  $P_\cap$  that conceptually runs  $P_A$  and  $M_B$  in parallel.)

Q4. Let  $A = \{w \in \{a, b, c\}^* : w \text{ has same number of } a\text{'s, } b\text{'s, and } c\text{'s}\}$ .

- (a) Prove that  $A$  is not context-free. (Hint: It is not necessary to use pumping theorem. You may try the conclusion of Q3.)
- (b) Show that  $\overline{A}$  is context-free. (Hint: it suffices to show that  $\overline{A}$  is a union of several context-free languages.)
- (c) What conclusion can you draw from (a) and (b)?

Q5. Suppose that  $L$  is context-free and  $R$  is regular? Is  $L - R$  necessarily context-free? What about  $R - L$ ? Justify your answer.