Theory of Computation, Fall 2021
Assignment 4 Solutions

Exercises

Q1. Let $L = \{ww^R : w \in \{a, b\}^*\}$. Suppose, for the sake of contradiction, that this language is regular. Let p be the pumping length given by the pumping theorem. Consider the string $w = a^p bba^p \in L$. By pumping theorem, w can be written as $w = xyz$ such that

(i) $xy^i z \in L$ for any $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

(ii) and (iii) imply that $y = a^k$ for some $k > 0$. Consider the string $xy^0 z = a^{p-k} bba^p$. Clearly, this string does not belong to L. This contradicts with (i). Therefore, L cannot be regular.

Q2. True. Following the proof of pumping theorem, we know that w can be written as $w = xyz$ such that $xy^i z \in L$ for any $i \geq 0$. This implies that L contains an infinite number of string.

Q3. (a) $S \to aSa|bSb|a|b|e$

(b) $S \to aSb|aS|e$

Q4. We have that

- $K' = K$,
- $\Gamma = \emptyset$,
- $s' = s$,
- $F' = F$, and
- $\Delta' = \{((p, a, e), (q, e)) : (p, a, q) \in \Delta\}$.

Q5. The key idea is that when we read a a, we behave as if we read two a's: (i) we push two A's onto the stack, or (ii) pop two B's, or (iii) pop a B and push an A. The PDA $P = (K, \Sigma, \Gamma, \Delta, s, F)$ is as follows.

- $K = \{q\}$,
- $\Sigma = \{a, b\}$,
- $\Gamma = \{A, B\}$,
- $s = q$,
- $F = \{q\}$, and
- Δ contains the following transitions.

$$
\begin{gathered}
((q, a, e), (q, AA)) \\
((q, a, B), (q, A)) \\
((q, a, BB), (q, e)) \\
((q, b, e), (q, B)) \\
((q, b, A)), (q, e))
\end{gathered}
$$

Note that the above PDA uses the feature of non-determinism.

Q6. The PDA $P = (K, \Sigma, \Gamma, \Delta, s, F)$ is as follows.

- $K = \{p, q\}$,
- $\Sigma = \{a, b\}$,
- $\Gamma = \{a, b, S\}$,
- $s = p$,
- $F = \{q\}$, and
- Δ contains the following transitions.

$$
\begin{array}{l}
((p, e, e), (q, S)) \\
((q, e, S), (q, aSa)) \\
((q, e, S), (q, bSb)) \\
((q, e, S), (q, a)) \\
((q, e, S), (q, b)) \\
((q, e, S), (q, e)) \\
((q, a, a), (q, e)) \\
((q, b, b), (q, e))
\end{array}
$$