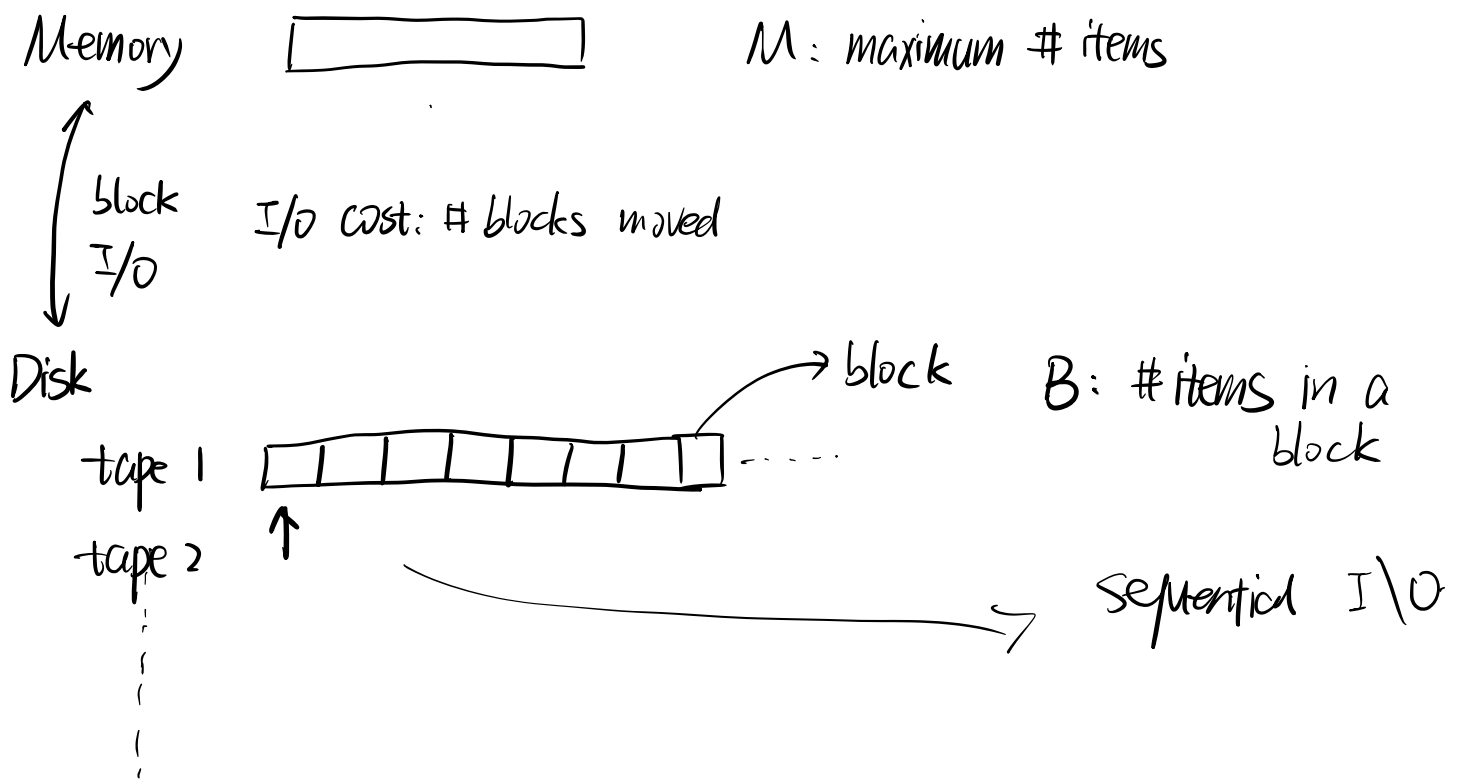


CPU < external memory model.



N : # items in the problem instance.

Scan



I/O cost: $O(N/B)$

one pass

Sorting (Merge Sort)

Assume $B=1$

internal sort 1 pass $\rightarrow N/2$

$$1 + \lceil \log_2 \frac{N}{3} \rceil \text{ passes}$$

N/m

$$1 + \lceil \log_2 \frac{N}{m} \rceil \text{ passes}$$

4 tapes

$$\text{I/O cost: } \frac{N}{B} (1 + \lceil \log_2 \frac{N}{m} \rceil)$$

k-way merge

k. m. B

$$1 + \lceil \log_k \frac{N}{m} \rceil \text{ passes}$$

Polyphase Merge.

k-way merge using k+1 tapes

k=2

0 1 1 2 3 5 8 ...

$$\bar{F}_i = \bar{F}_{i-1} + \bar{F}_{i-2}$$

T1

$$T_1 \quad \bar{F}_{i-2} (\bar{F}_{i-3} + \bar{F}_{i-4})$$

$$T_2 \quad \bar{F}_{i-1} (\bar{F}_{i-2} + \bar{F}_{i-3}) \rightarrow$$

$$T_2 \quad \bar{F}_{i-3}$$

$$T_3 \quad \bar{F}_{i-2}$$

$$T_3 \quad 0$$

↓

$$\begin{array}{l} T_1 \quad \bar{f}_{i-4} \\ T_2 \quad 0 \\ T_3 \quad \bar{f}_{i-3} \end{array}$$

General k .

Fabonacci of order k .

$$\bar{f}_i = 0 \quad \text{for } i \leq k-2$$

$$\bar{f}_{k-1} = 1$$

$$\bar{f}_i = \bar{f}_{i-1} + \bar{f}_{i-2} + \dots + \bar{f}_{i-k} \quad \text{for } i \geq k$$

$$\begin{array}{l} T_1 \quad \bar{f}_{i-1} + \dots + \bar{f}_{i-k} \\ T_2 \quad \bar{f}_{i-1} + \dots + \bar{f}_{i-k+1} \\ T_3 \quad \bar{f}_{i-1} + \dots + \bar{f}_{i-k+2} \\ \vdots \\ T_{k-1} \quad \bar{f}_{i-1} + \bar{f}_{i-2} \\ T_k \quad \bar{f}_{i-1} \\ T_{k+1} \quad 0 \end{array}$$

→

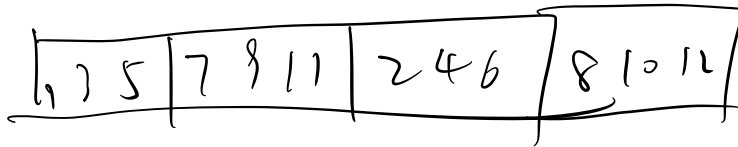
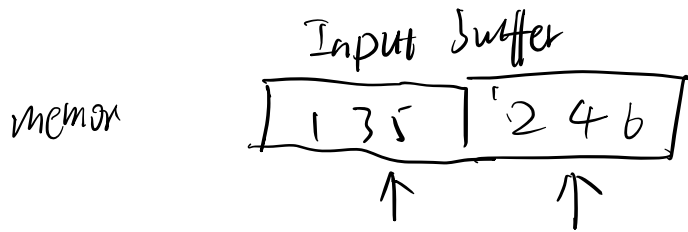
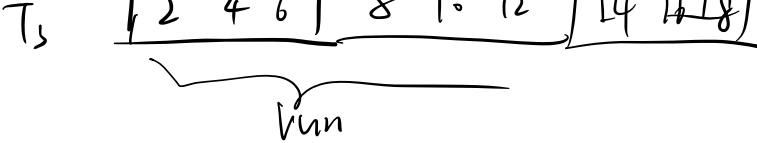
$$\begin{array}{l} T_1 \quad \bar{f}_{i-2} + \dots + \bar{f}_{i-k} \\ T_2 \quad \bar{f}_{i-2} + \dots + \bar{f}_{i-k+1} \\ \vdots \\ T_{k-1} \quad \bar{f}_{i-2} \\ T_k \quad 0 \\ T_{k+1} \quad \bar{f}_{i-1} \end{array}$$

$$\bar{f}_{i-2} + \dots + \bar{f}_{i-k-1}$$

ham

$$T_2 \quad \boxed{1 \quad 3 \quad 5} \quad \boxed{7 \quad 9 \quad 11} \quad \boxed{13 \quad 15 \quad 17} \quad \dots$$

$$\boxed{2 \quad 4 \quad 6} \quad \boxed{8 \quad 10 \quad 12} \quad \boxed{14 \quad 16 \quad 18}$$



Output buffer : 2

input buffer : $2k$ for k -way merge

$$M = (2k+2)B$$

$$\frac{N}{B} (1 + \lceil \log_k \frac{N}{M} \rceil)$$