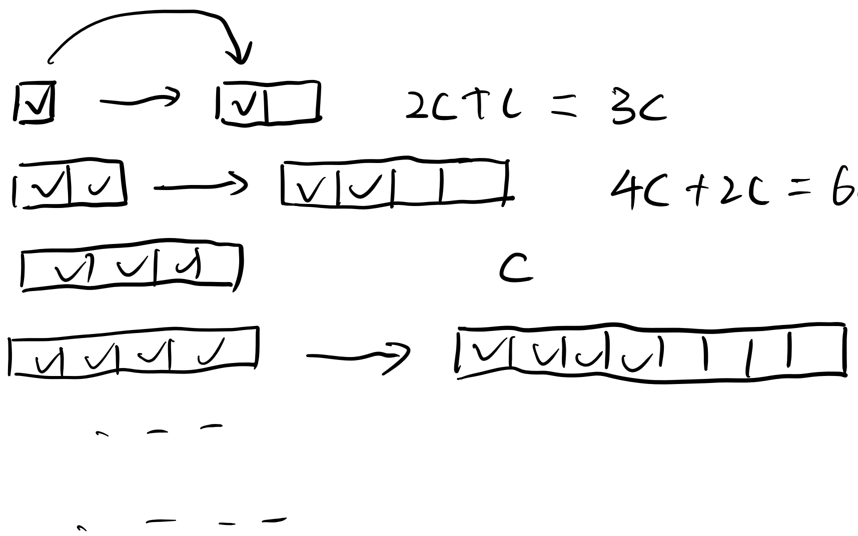
dynamic array

- A[i]
- Insertion
- O(n) space



$n=1$ $\quad 2c+c = 3c$

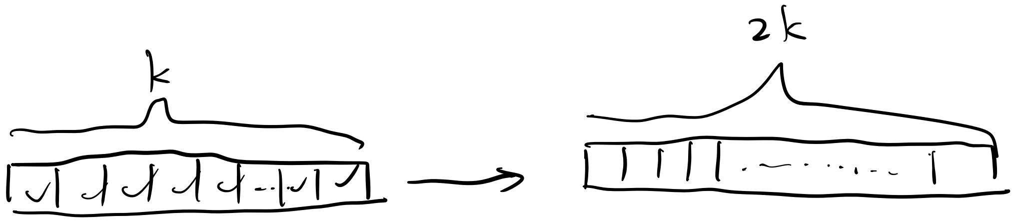
$n=2$ $\quad 4c+2c = 6c$

$n=3$ $\quad c$

$n=4$ $\quad 8c+4c = 12c$

$n=5$ - - - $\quad c$

$n=6$ - - - $\quad c$



$n=k$ $\quad 3k\cdot c \quad O(k)$

Insertion $\quad O(n)$

$T(n)$: cost of a sequence of n insertions (from empty array) in worst case.

$$T(n) \le cn + 2^0\cdot 3c + 2^1 \cdot 3c + \cdots + 2^{k}\cdot 3c$$

largest k $\quad 2^k \le n$

$$\le cn + (2^{k+1}-1)\cdot 3c$$

$$\le cn + (2\cdot n - 1)\cdot 3c$$

$$\le 7c\cdot n \qquad O(n)$$

$$T(n)/n = O(1) \longrightarrow \text{amortized cost}$$

Amortized cost $A(n) = \frac{T(n)}{n}$ where $T(n)$ is the worst-case

Amortized cost $A(n) = \frac{\quad}{n}$ where

cost of all possible sequences of n operations.

similar

When there are multiple types of operations.

Stack

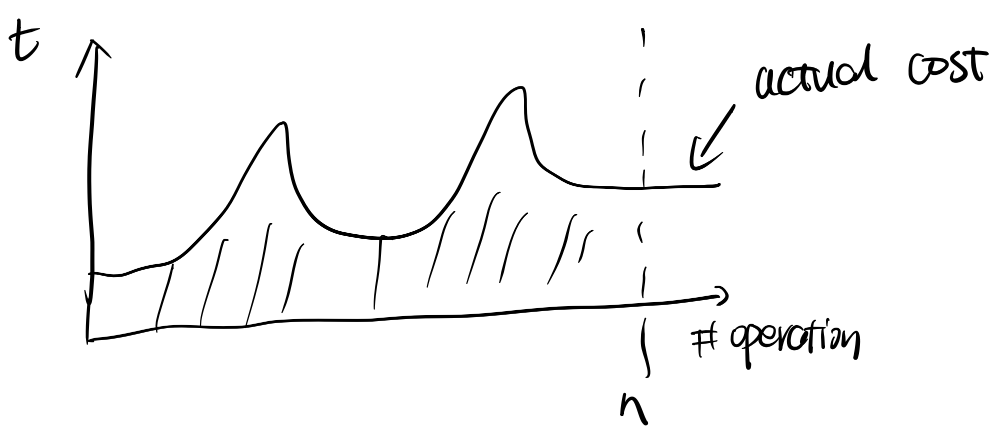
	Worst-case	Amortized
- PUSH(S, x)	$O(1)$	$O(1)$
- POP(S) if S is empty error	$O(1)$	$O(1)$
- MULTIPOP(S, k)	$O(k)$	$O(1)$

- While S is not empty and $k>0$
  - POP(S)
  - $k = k-1$

$$T(n) \le O(n) + O(n) = O(n)$$

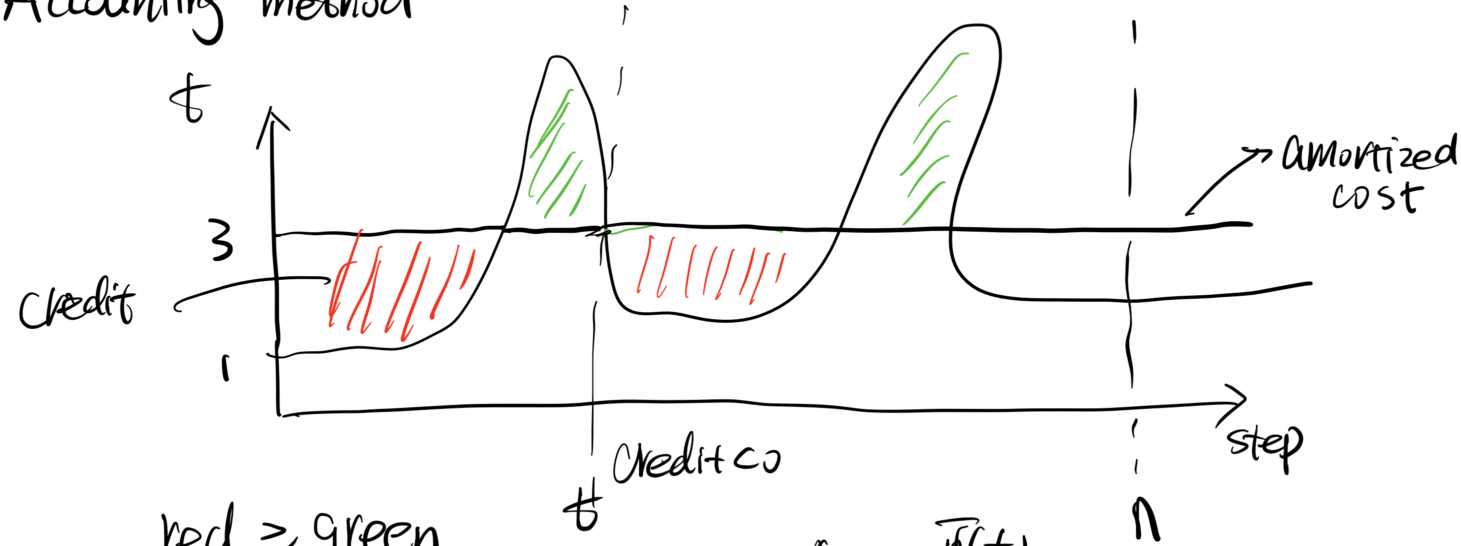
$$T(n)/n = O(1)$$

Aggregate analysis



T(n)

Accounting method



red ≥ green

⇒ rect ≥ curve

⇒ Σ amortized cost ≥ Σ actual cost

$$A(t) = \frac{\pi(t)}{t}$$

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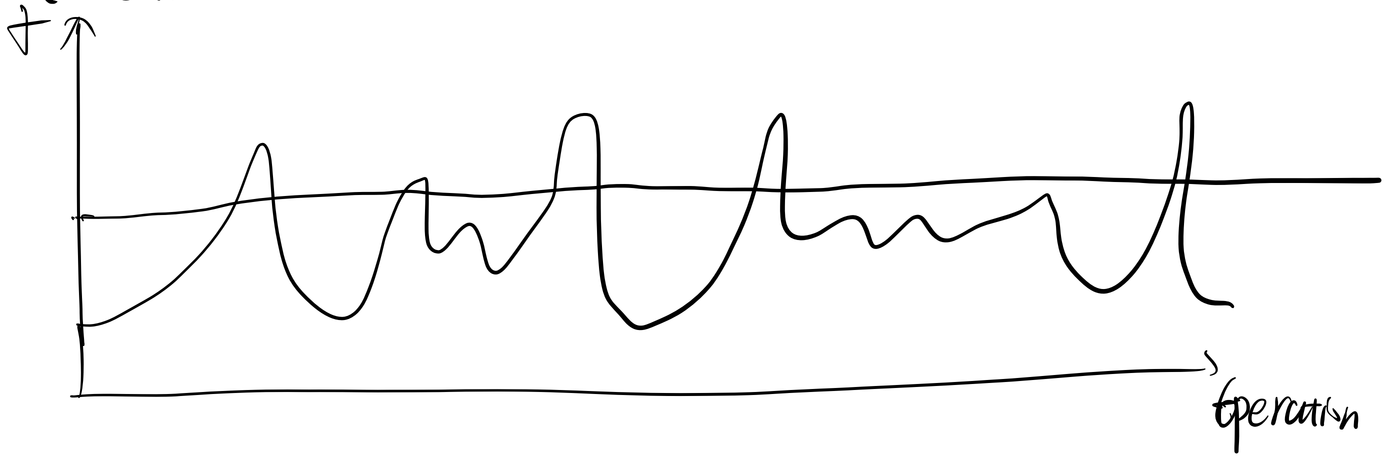
	actual cost	amortized cost
PUSH(S,x)	c	2c (associate the extra c with x)
POP(S)	c	0
MultiPOP(S,k)	c · min(|S|, k)	0

Potential method.



hard to analyze directly that red ≥ green

$\Phi(i)$ = # credit left after the ith operation $\Phi(0)$ = # credits at the begining

potential function   energy

② $\hat{c}_i = c_i + \Phi(i) - \Phi(i-1)$

$\hat{c}_i$ ↓ amortized c_i ↓ actual cost.

① $\Phi(n) \geq \Phi(0) \Rightarrow$ red $\geq$ green $\Rightarrow \sum_i \hat{c}_i \geq \sum_i c_i$

$\Rightarrow$

$$\sum_i \hat{c}_i = \sum_i c_i + \Phi(n) - \Phi(0)$$

$\Phi(i) = c \cdot$ # objects in the stack after ith operation

$\Phi(0) = 0$ $\Phi(n) \geq 0$

PUSH:

$$\hat{c}_i = c_i + \Phi(i) - \Phi(i-1) = c + c = 2c$$

POP

$$\hat{c}_i = c_i + \Phi(i) - \Phi(i-1) = c - c = 0$$

MULTIPOP

$$\cdots = c \cdot \#\text{POP} - c \cdot \#\text{POP} = 0$$