1、(3) $\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}(x^2+y^2)^{-\frac{1}{2}}=-\frac{1}{2}(x^2+y^2)^{-\frac{3}{2}}\cdot 2x=-\frac{x}{(x^2+y^2)^{\frac{3}{2}}}$

同理 $\frac{\partial z}{\partial y}=\frac{-y}{(x^2+y^2)^{\frac{3}{2}}}$.

(6) $\frac{\partial z}{\partial x}=\frac{\partial}{\partial x}(\arctan\frac{y}{x})=\frac{1}{1+(\frac{y}{x})^2}(-\frac{y}{x^2})=\frac{-y}{x^2+y^2}$

$\frac{\partial z}{\partial y}=\frac{1}{1+(\frac{y}{x})^2}\cdot\frac{1}{x}=\frac{x}{x^2+y^2}$

(9) $\frac{\partial u}{\partial x}=z(xy)^{z-1}$ $\quad\frac{\partial u}{\partial y}=z(xy)^{z-1}$

$\frac{\partial u}{\partial z}=(xy)^z\ln(xy)$.

2、 $f(x,1)=x$ $\quad f_x(x,1)=1$

3、 $\frac{\partial f}{\partial x}\Big|_{(0,0)}=\lim\limits_{x\to 0}\frac{f(x,0)-f(0,0)}{x-0}=\lim\limits_{x\to 0}\frac{0}{x}=0$

4、 $\lim\limits_{(x,y)\to(0,0)}\sqrt{x^2+y^2}=0=f(0,0)$ $\quad\therefore f$在$(0,0)$连续;

$\frac{\partial f}{\partial x}\Big|_{(0,0)}=\lim\limits_{x\to 0}\frac{f(x,0)-f(0,0)}{x-0}\ \lim\limits_{x\to 0}\frac{\sqrt{x^2}}{x}=\lim\limits_{x\to 0}\frac{|x|}{x}$ 不存在

$\frac{\partial f}{\partial y}\Big|_{(0,0)}=\lim\limits_{y\to 0}\frac{f(0,y)-f(0,0)}{y-0}=\lim\limits_{y\to 0}\frac{|y|}{y}$ 不存在.

6、证明：令 $x=r\cos\theta,\ y=r\sin\theta$.

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{r\to 0}\frac{r^3\cos^2\theta\sin\theta}{r^2}=\lim_{r\to 0} r\sin\theta\cos^2\theta=0=f(0,0)$$

$\therefore f(x,y)$ 在 $(0,0)$ 连续.

$$f_x(0,0)=\lim_{x\to 0}\frac{1}{x}\cdot\frac{x^2}{x^2+0^2}=0=A$$

$$f_y(0,0)=\lim_{y\to 0}\frac{1}{y}\frac{0^2\cdot y}{0^2+y^2}=0=B$$

$\therefore f$ 在点 $(0,0)$ 的偏导数存在；

不妨假设 f 在 $(0,0)$ 处可微，

$$\Delta z=f(0+\Delta x,0+\Delta y)-f(0,0)=A\Delta x+B\Delta y+o(\rho)$$

即 $\dfrac{(\Delta x)^2(\Delta y)}{(\Delta x)^2+(\Delta y)^2}=o(\rho)$，$\rho=\sqrt{(\Delta x)^2+(\Delta y)^2}$

由于对 $\forall\Delta x,\Delta y$ 成立，不妨令 $\Delta x=\Delta y$，

$$\frac{(\Delta x)^3}{2(\Delta x)^2}=\frac{1}{2}\Delta x=o(\sqrt{2}|\Delta x|)$$ 显然不可能

即 f 在 $(0,0)$ 不可微.

9.(1) $\dfrac{\partial z}{\partial x}=y\cos(y+x)$ $\dfrac{\partial z}{\partial y}=\sin(x+y)+y\cos(x+y)$.

$\therefore dz=y\cos(y+x)dx+(\sin(x+y)+y\cos(x+y))dy$.

(2). $\dfrac{\partial u}{\partial x}=e^{yz}$ $\dfrac{\partial u}{\partial y}=xze^{yz}+1$ $\dfrac{\partial u}{\partial z}=xye^{yz}-e^{-z}$

$du=e^{yz}dx+(1+xze^{yz})dy+(xye^{yz}-e^{-z})dz$

P_{111}

15、证明：先证 f 在 $P(x_0, y_0)$ 连续，$\exists M>0$，使 $|f_x(x,y)| \le M$，$|f_y(x,y)| \le M$，$\forall (x,y) \in U(P)$

$$|f(x,y)-f(x_0,y_0)| \le |f(x,y)-f(x,y_0)| + |f(x,y_0)-f(x_0,y_0)|.$$

由拉格朗日中值定理 $\le |f_y(x, y_0+\theta_1 \Delta y)|\,|\Delta y| + |f_x(x_0+\theta_2 \Delta x, y_0)|\,|\Delta x|.$

$\le M(|\Delta x|+|\Delta y|)$，$\Delta x = x-x_0$，$\Delta y = y-y_0$，$0<\theta_1, \theta_2<1$.

显然 $\lim\limits_{(x,y)\to(0,1)} f(x,y) = f(x_0,y_0)$，$f$ 在 $P(x_0,y_0)$ 连续.

对 $\forall P_1(x_1,y_1) \in U(P)$，$P_1$ 为 $U(P)$ 的内点，故存在 P_1 的某邻域 $U(P_1) \subset U(P)$.

且 f_x f_y 在 $U(P_1)$ 有界，则由上面证明知 f 在 P_1 处也连续，

由 $P_1 \in U(P)$ 的任意性知 f 在 $U(P)$ 内连续，

P_{117} 1.(1) $\dfrac{dz}{dx} = \dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dx} = \dfrac{y}{1+x^2y^2} + \dfrac{x}{1+x^2y^2}e^x = \dfrac{e^x(1+x)}{1+x^2e^{2x}}$

(3) $\dfrac{dz}{dt} = \dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt} + \dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt} = (2x+y)\cdot 2t + (x+2y)\cdot 1 = 4t^3+3t^2+2t$

(6) 令 $s=\dfrac{x}{y}$，$t=\dfrac{y}{z}$，$f_1' = \dfrac{\partial u}{\partial s}$ $f_2' = \dfrac{\partial u}{\partial t}$.

$\dfrac{\partial u}{\partial x} = \dfrac{\partial f}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial f}{\partial t}\dfrac{\partial t}{\partial x} = f_1'\cdot\dfrac{1}{y} + f_2'\cdot 0 = \dfrac{1}{y}f_1'$

$\dfrac{\partial u}{\partial y} = \dfrac{\partial f}{\partial s}\cdot\dfrac{\partial s}{\partial y} + \dfrac{\partial f}{\partial t}\cdot\dfrac{\partial t}{\partial y} = f_1'\cdot\left(-\dfrac{x}{y^2}\right) + f_2'\cdot\dfrac{1}{z} = -\dfrac{x}{y^2}f_1' + \dfrac{1}{z}f_2'$

$\dfrac{\partial u}{\partial z} = \dfrac{\partial f}{\partial s}\cdot\dfrac{\partial s}{\partial z} + \dfrac{\partial f}{\partial t}\dfrac{\partial t}{\partial z} = f_1'\cdot 0 + f_2'\cdot\left(-\dfrac{y}{z^2}\right) = -\dfrac{y}{z^2}f_2'$

2、解: $\frac{\partial z}{\partial x}=(x+y)^{xy}\left[\frac{xy}{x+y}+y\ln(x+y)\right]$

$\frac{\partial z}{\partial y}=(x+y)^{xy}\left[\frac{xy}{x+y}+x\ln(x+y)\right]$

$dz=(x+y)^{xy}\left[\frac{xy}{x+y}+y\ln(x+y)\right]dx+(x+y)^{xy}\left[\frac{xy}{x+y}+x\ln(x+y)\right]dz$

3、证明. 令 $s=x^2-y^2$ $\therefore z=\frac{x}{f(s)}$ $f=\frac{y}{z}$

$\frac{\partial z}{\partial x}=\frac{-yf'(s)\frac{\partial s}{\partial x}}{f^2}=\frac{2xyf'(s)}{f^2}$

$\frac{\partial z}{\partial y}=\frac{f-yf'(s)\frac{\partial s}{\partial y}}{f^2}=\frac{f+2y^2f'(s)}{f^2}$

$\frac{1}{x}\frac{\partial z}{\partial x}+\frac{1}{y}\frac{\partial z}{\partial y}=-\frac{2yf'(s)}{f^2}+\frac{f+2y^2f'(s)}{f^2 y}=\frac{1}{yf}=\frac{z}{y^2}$

5、证明: $g_u=\frac{\partial g}{\partial u}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial u}=f_x\cos\theta+f_y\sin\theta$

$g_v=\frac{\partial g}{\partial v}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial v}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial v}=f_x(-\sin\theta)+f_y\cos\theta$

$\therefore (g_u)^2+(g_v)^2=(f_x)^2(\sin^2\theta+\cos^2\theta)+(f_y)^2(\sin^2\theta+\cos^2\theta)$
$=(f_x)^2+(f_y)^2$

P120 1.解：$f_x(1,1,2)=-1$ $f_y(1,1,2)=0$ $f_z(1,1,2)=11$

l的方向余弦 $\cos\frac{\pi}{3}=\frac{1}{2}$，$\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$$\frac{\partial u}{\partial l}\Big|_{(1,1,2)}=f_x(1,1,2)\cos\frac{\pi}{3}+f_y(1,1,2)\cos\frac{\pi}{4}+f_z(1,1,2)\cos\frac{\pi}{3}$$

$$=5.$$

2.解 $\overrightarrow{AB}=(4,3,12)$

方向余弦 $\cos\alpha=\frac{4}{13}$ $\cos\beta=\frac{3}{13}$ $\cos\gamma=\frac{12}{13}$

$f_x(5,1,2)=2$ $f_y(5,1,2)=10$ $f_z(5,1,2)=5$

$$\frac{\partial u}{\partial \overrightarrow{AB}}\Big|_{(5,1,2)}=\frac{4}{13}\times2+\frac{3}{13}\times10+\frac{12}{13}\times5$$

$$=\frac{98}{13}$$

4.解 $\frac{\partial u}{\partial x}=\frac{du}{dr}\frac{\partial r}{\partial x}=-\frac{1}{r}\frac{(x-a)}{\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}}=-\frac{x-a}{r^2}$

$\frac{\partial u}{\partial y}=-\frac{y-b}{r^2}$ $\frac{\partial u}{\partial z}=-\frac{z-c}{r^2}$

$\therefore \text{grad}\, u=-\frac{1}{r^2}(x-a,y-b,z-c).$

$|\text{grad}\, u|=\frac{1}{r^2}\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}=\frac{1}{r}$

令 $|\text{grad}\, u|=\frac{1}{r}=1$ $\therefore (x-a)^2+(y-b)^2+(z-c)^2=1$ 上的点使 $|\text{grad}\, u|=1$ 成立

2、(1) $f'(x_1, x_2) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \sin x_2 & x_1 \cos x_2 \\ 2(x_1 - x_2) & -2(x_1 - x_2) \\ 0 & 4x_2 \end{bmatrix}$

$f'(0, \frac{\pi}{2}) = \begin{bmatrix} \sin\frac{\pi}{2} & 0 \\ -2 \cdot \frac{\pi}{2} & 2 \cdot \frac{\pi}{2} \\ 0 & 4 \times \frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\pi & \pi \\ 0 & 2\pi \end{bmatrix}$

(2) $f'(x_1, x_2, x_3) = \begin{bmatrix} 2x_1 & 1 & 0 \\ x_2 e^{x_1+x_3} & e^{x_1+x_3} & x_2 e^{x_1+x_3} \end{bmatrix}$

$f'(1, 0, 1) = \begin{bmatrix} 2 & 1 & 0 \\ 0 & e^2 & 0 \end{bmatrix}$