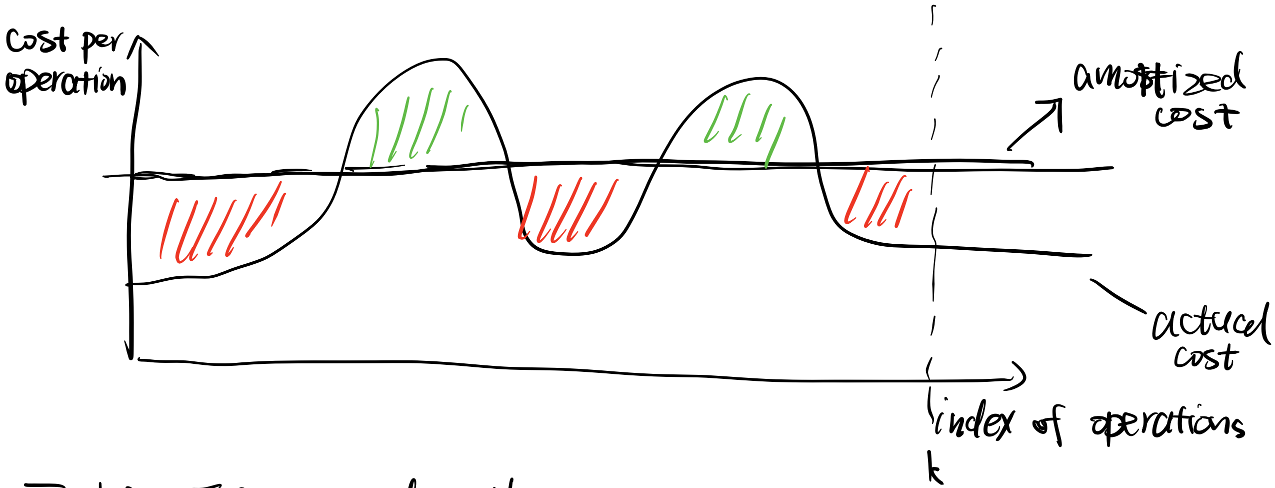
amortized cost $T(k)/k$ worst case for sequences of k operations



$T(k)$ = area under the curve

red area $\geq$ green area

Potential method

$\Phi(i)$ = red − green after the ith operation

$\hat{c}_i = c_i + \Phi(i) - \Phi(i-1)$

red $\geq$ green $\Longleftrightarrow$ $\Phi(k) \geq \Phi(0)$

$\forall k \geq 0,$
$\Phi(k) \geq \Phi(0)$

1. $\Phi(k/2) < \Phi(0)$? ×

2. $\Phi(k) - \Phi(0) = -O(k)$ ✓

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splay tree

findkey
ins
del

$O(\lg n)$ sizeof(D)

$A(D) = \log(\text{sizeof}(D))$

Given m types of operations $t_1, \dots, t_m$ with

actual cost $T_{t_1}(D) \quad \dots \quad T_{t_m}(D)$

We say that they have amortized cost $A_{t_1}(D), \quad \dots, A_{t_m}(D)$

if for any sequence of k operations $o_1, o_2, \dots, o_k$

$D_0 \quad D_1 \quad D_2 \quad \dots \quad D_k$

$\sum$ amortized cost $\geq \sum$ actual cost

$$\sum_{i=1}^{k} A_{type(o_i)}(D_{i-1}) \geq \sum_{i=1}^{k} T_{type(o_i)}(D_{i-1})$$

Potential method

$\Phi(D)$: potential of D $\quad D' = t_i(D)$

$$A_{t_i}(D) = T_{t_i}(D) + \Phi(D') - \Phi(D)$$

$$\sum_{i=1}^{k} A_{type(o_i)}(D_{i-1}) \geq \sum_{i=1}^{k} T_{type(o_i)}(D_{i-1})$$

$$\sum_{i=1}^{k} \left(A_{type(o_i)}(D_{i-1}) - T_{type(o_i)}(D_{i-1}) \right) \geq 0$$

$$\sum_{i=1}^{k} \left(\Phi(D_i) - \Phi(D_{i-1}) \right) \geq 0$$

$$\Phi(D_k) \geq \Phi(D_0)$$

$$\begin{cases} \Phi(D_0) = 0 \\ \Phi(D_k) \geq 0 \quad \forall k \geq 0 \end{cases}$$