Theory of Computation, Fall 2021
Assignment 3 (Due October 16 Saturday 9:35am)

Q1. [1, Chapter 1.3] For any two regular expressions R_1 and R_2, we say $R_1 = R_2$ if $L(R_1) = L(R_2)$. Let R be a regular expression. Are the following statements true or false? Provide counterexamples for false statements.

(a) $R \cup \emptyset = R$

(b) $R\emptyset = R$

(c) $R \cup \emptyset^* = R$

(d) $R\emptyset^* = R$

Q2. Write a regular expression for the language

$$\{w \in \{a, b\}^* : \text{the number of } b\text{'s in } w \text{ is divisible by } 3\}.$$

Q3. Consider the NFA N in Figure 1. Construct a regular expression R such that $L(R) = L(N)$. You should strictly follow the algorithm we used in the class, and show all the intermediate steps. More precisely, you should first convert N into an equivalent NFA that satisfies certain conditions, and then eliminate state q_1 and q_2 in order.

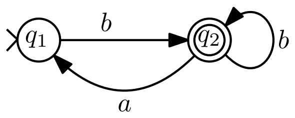

Figure 1: N

Q4. Let N be the NFA in Figure 1. Someone constructs another NFA N' as in Figure 2 and claims that $L(N') = (L(N))^*$. Prove that he\she is wrong. (Hint: It suffices to find a string that is in $L(N')$ but not in $(L(N))^*$).

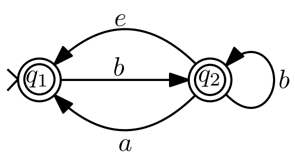

Figure 2: N'

Q5. Construct a NFA that accepts $(ab \cup aba)^*$.

Q6. Are the following statement true of false? Explain your answer. You may use the fact that the language $\{a^i b^i : i \geq 0\}$ is not regular (we will prove this using pumping theorem in next class).

(a) Every subset of a regular language is regular.

(b) The union of a finite number of regular languages must be regular.

(c) Languages that contain only a finite number of strings must be regular.

(d) The union of an infinite number of regular languages must be regular.

(e) The intersection of an infinite number of regular languages must be regular.
(Hint: Think about the De Morgan's Laws)

References

[1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)

[2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Gall (1998)