

Problems & Solutions of Quiz 2

Theory of Computation, Fall 2022

Q1. (20 pts) Are the following statements true or false? No explanation is required.

- (a) Every context-free grammar generates at least one string.
- (b) A context-free grammar in Chomsky norm form must be unambiguous.
- (c) A context-free grammar may have an infinite number of rules.
- (d) Non-deterministic PDA are equivalent to deterministic PDA.
- (e) Let L be a language over Σ . If L is context-free, so is the following language:

$$Pref(L) = \{u \in \Sigma^* : uv \in L \text{ for some } v \in \Sigma^*\} \quad (1)$$

Ind.	Key
(a)	False
(b)	False
(c)	False
(d)	False
(e)	True

Q2. (15 pts) Construct a context-free grammar that generates the following language.

$$\{xy \in \{0,1\}^* : x \text{ has equal number of 0's and 1's, and } y = y^R\} \quad (2)$$

Sol:

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow AA \mid 0A1 \mid 1A0 \mid e \\ B &\rightarrow 0B0 \mid 1B1 \mid 0 \mid 1 \mid e \end{aligned} \quad (3)$$

Q3. (15 pts) Construct a pushdown automaton that accepts the following language.

$$\{w \in \{a, b\}^* : \text{in } w, 2 \cdot \#a = \#b + 3\} \quad (4)$$

Sol:

$$\begin{aligned} P &= (K, \Sigma, \Gamma, \Delta, s, F) \\ K &= \{s, f\} \\ \Sigma &= \{a, b\} \\ \Gamma &= \{A, B\} \\ s &= s \\ F &= \{f\} \end{aligned} \tag{5}$$

The list of $((q, a, \beta), (p, \gamma))$ shown below contains all elements in Δ .

(q, a, β)	(p, γ)
(s, e, e)	(f, BBB)
(f, a, e)	(f, AA)
(f, a, B)	(f, A)
(f, a, BB)	(f, e)
(f, b, e)	(f, B)
(f, b, A)	(f, e)