Problems & Solutions of Quiz 1

> Theory of Computation, Fall 2022

Q1. (15 pts) Determine the language accepted by each of the following NFA.

M_1 M_2 M_3

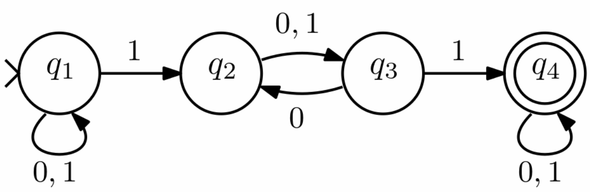
Sol:

$$\begin{aligned} L(M_1) &= \emptyset \\ L(M_2) &= \{e\} \\ L(M_3) &= \{w \mid w \in \{0,1\}^*\} \\ &\text{or} \\ L(M_3) &= \{0,1\}^* \end{aligned} \tag{1}$$

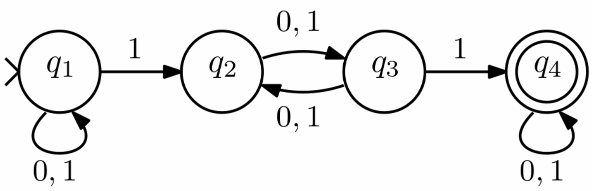
Q2. (10 pts) Design a NFA to accept the following languages. Your NFA should have at most 4 states.

$$\{w \in \{0,1\}^* : w \text{ has a pair of } 1'\text{s that are separated by odd number of symbols}\} \tag{2}$$

Sol:



or



Q3. (15 pts) Are the following statements true or false? No explanation is required.

(a) A finite automaton may have no final states.

(b) Let L be a regular language. Then the following language is also regular.

$$L^+ = \{w_1 \cdots w_k : w_i \in L \wedge k \geq 1\} \tag{3}$$

(c) Let A and B be regular languages. Then the following language is also regular.

$$A \oplus B = \{w : w \in A \cup B \wedge w \notin A \cap B\} \tag{4}$$

(d) Every language that satisfies the pumping theorem is regular.

(e) Let N be a NFA with k states. There must exist a DFA M with at most 2^k states that is equivalent to N
(That is, $L(M) = L(N)$).

Ind.	**Key**
(a)	True
(b)	True
(c)	True
(d)	False
(e)	True

Q4. (10 pts) Consider the following language over $\{0, 1, \#\}$. Show that it is not regular.

$$L = \{w\#u : w, u \in \{0,1\}^* \text{ and } w \text{ has strictly less 1's than } u \text{ does.}\} \tag{5}$$

Proof:

Assume L is regular. Let $p \geq 1$ denote its pumping length.

Take string $s = 1^p\#1^{p+1} \in L$. According to the pumping theorem, s can be expressed as $s = xyz$ subject to

1.

$$\forall i \geq 0, xy^i z \in L \tag{6}$$

2.

$$|y| > 0 \tag{7}$$

3.

$$|xy| \leq p \tag{8}$$

For 2. and 3., x and y can expressed as $x = 1^\alpha$ and $y = 1^\beta$ that $\alpha + \beta \leq p, \beta > 0$.

Take $i = 2$, then $xy^i z = w\#u = 1^{p+\beta}\#1^{p+1}$. Here $w = 1^{p+\beta}, u = 1^{p+1}$.

Since $p + \beta \geq p + 1$, there is $|w| \geq |u|$, and w contains no strictly less 1's than u does.

Therefore, $xy^2 z \notin L$, and L fails to meet the requirements of the pumping theorem, hence L is not regular. End of proof.
