1 一致收敛定义

$(i)\forall\varepsilon>0,\exists N>0,\forall n>N,\forall x:|f_n(x)-f(x)|<\varepsilon$

$(ii)\forall\varepsilon>0,\exists N>0,\forall n>N,\forall x:|\sum_{n}^{\infty}u_n(x)-S(x)|<\varepsilon$

$(iii)F(x,A)=\int_a^A f(x,y)dy,\forall\varepsilon>0,\exists G>0,\forall A>G,\forall x:\left|\int_a^A f(x,y)dy-I(x)\right|<\varepsilon$

$(iv)F(x,t)=\int_a^t f(x,y)dy,\forall\varepsilon>0,\exists\delta>0,\forall t\in U_-^o(b,\delta),\forall t,\left|\int_a^t f(x,y)dy-I(x)\right|<\varepsilon$

2 一致收敛的优美表达形式

$(i)\lim\limits_{n\to\infty}\sup\limits_{\forall x}|f_n(x)-f(x)|=0$

$(ii)\lim\limits_{n\to\infty}\sup\limits_{\forall x}\left|\sum_1^n a_n(x)-\sum a_n(x)\right|=0$

$(iii)\lim\limits_{A\to\infty}\sup\limits_{\forall x}\left|\int_a^A f(x,y)dy-\int_a^{+\infty}f(x,y)dy\right|=0$

$(iv)\lim\limits_{a\to A}\sup\limits_{\forall x}\left|\int_a^B f(x,y)dy-\int_A^B f(x,y)dy\right|=0$

3 一致收敛的魏尔斯特拉斯判则

(i)无

(ii)若$\exists M_n,\forall x,|u_n(x)|\le M_n,\sum M_n$收敛，则：$\sum u_n$一致收敛

(iii)若$\exists g(y),\forall x,|f(x,y)|\le g(y),\int_a^{+\infty}g(y)dy$收敛，则：$\int_a^{+\infty}f(x,y)dy$一致收敛

(iv)若$\exists g(y),\forall x,|f(x,y)|\le g(y),\int_a^{b}g(y)dy$收敛，则：$\int_a^{b}f(x,y)dy$一致收敛

4 A-D判则

(i)无

(ii)若$\forall x,u_n(x)$单调一致趋于0,$\sum_1^n v_i(x)$一致有界，则：$\sum u_n(x)v_n(x)$一致收敛

若$\forall x,u_n(x)$单调且一致有界,$\sum_1^n v_i(x)$一致收敛，则：$\sum u_n(x)v_n(x)$一致收敛

(iii)若$\forall x,g(x,y)$关于y单调一致趋于0,$\int_a^A f(x,y)dy$关于A一致有界，则：$\int_a^{+\infty}f(x,y)dy$一致收敛

若$\forall x,g(x,y)$关于y单调且一致有界,$\int_a^{+\infty}f(x,y)dy$一致收敛,则：$\int_a^{+\infty}f(x,y)dy$一致收敛

(iv)若$\forall x,g(x,y)$关于y单调一致趋于0,$\int_a^t f(x,y)dy,t\in[a,b)$关于t一致有界，则：$\int_a^b f(x,y)dy$一致收敛

若$\forall x,g(x,y)$关于y单调且一致有界,$\int_a^b f(x,y)dy$一致收敛，则：$\int_a^b f(x,y)dy$一致收敛

5 连续、可积、可导性

(i)连续性

(i)若$f_n(x) \rightrightarrows f(x), f_n(x)$连续，则：$f(x)$连续

(ii)若$S_n(x) \rightrightarrows S(x), a_n(x)$连续，则：$S(x)$连续

(iii)记$F(x,A) = \int_a^A f(x,y)dy$若$F(x,A) \rightrightarrows F(x,+\infty) = I(x), F(x,A)$关于x连续,则：$I(x)$连续

(iv)记$F(x,t) = \int_a^t f(x,y)dy$若$F(x,t) \rightrightarrows F(x,b) = I(x), F(x,t)$关于x连续,则：$I(x)$连续

(ii)可积性

(i)若$f_n(x) \rightrightarrows f(x), f_n(x)$可积，则：$f(x)$可积

(ii)若$S_n(x) \rightrightarrows S(x), a_n(x)$可积，则：$S(x)$可积

(iii)记$F(x,A) = \int_a^A f(x,y)dy$若$F(x,A) \rightrightarrows F(x,+\infty) = I(x), F(x,A)$关于x可积,则：$I(x)$可积

(iv)记$F(x,t) = \int_a^t f(x,y)dy$若$F(x,t) \rightrightarrows F(x,b) = I(x), F(x,t)$关于x可积,则：$I(x)$可积

(iii)可导性

(i)若$(I)f_n(x)$逐点收敛于$f(x)(II)f_n(x)$可导$(III)f_n'(x) \rightrightarrows g(x)$则:

$(I)f(x)$可导$, f'(x) = g(x)(II)f_n(x) \rightrightarrows f(x)$

(ii)若$(I)\sum_1^\infty a_n(x)$逐点收敛于$S(x)(II)a_n(x)$可导$(III)\sum_1^\infty a_n'(x) \rightrightarrows g(x)$则:

$(I)S(x)$可导$, S'(x) = g(x)(II)S_n(x) \rightrightarrows S(x)$

$$(iii)\text{记}F(x,A) = \int_a^A f(x,y)dy\text{若}\begin{cases}(I)F(x,A)\text{逐点收敛于}I(x)(A \to +\infty)\\(II)\forall x, \forall A, F(x,A)\text{对x可导}\\(III)\frac{\partial}{\partial x}F(x,A) \rightrightarrows g(x)(A \to +\infty)\end{cases}$$

$$\text{则：}\begin{cases}(I)I(x)\text{可导}, I'(x) = g(x)\\(II)F(x,A) \rightrightarrows I(x)(A \to +\infty)\end{cases}$$

$$(iv)\text{记}F(x,t) = \int_a^t f(x,y)dy(t \in [a,b))\text{若}\begin{cases}(I)F(x,t)\text{逐点收敛于}I(x)(t \to b)\\(II)\forall x, \forall t, F(x,t)\text{对x可导}\\(III)\frac{\partial}{\partial x}F(x,t) \rightrightarrows g(x)(t \to b)\end{cases}$$

$$\text{则：}\begin{cases}(I)I(x)\text{可导}, I'(x) = g(x)\\(II)F(x,A) \rightrightarrows I(x)(t \to b)\end{cases}$$

6 对上述含参积分的部分条件的进一步研究

(i)命题:$I(x,A)=\int_a^A f(x,y)dy$关于x连续

充分条件：$f(x,y)\in C([c,d]\times[a,A])$

且: $\lim\limits_{x\to x_0}\int_a^b f(x,y)dy=\int_a^b \lim\limits_{x\to x_0} f(x,y)dy=\int_a^b f(x_0,y)dy$

变限形式: $\int_{c(x)}^{d(x)} f(x,y)dy$ 连续,当 $c(x),d(x),f(x,y),$关于x,y全连续时

(ii)命题:$I(x,A)=\int_a^A f(x,y)dy$关于x可导

充分条件：$f(x,y),f_x(x,y)\in C([c,d]\times[a,A])$

且:$\frac{d}{dx}\int_a^b f(x,y),dy=\int_a^b \frac{\partial}{\partial x}f(x,y)dy$

变限形式:若$c(x),d(x)$可导,$f(x,y),f_x(x,y)\in C([c,d]\times[a,A])$则:

$$\frac{d}{dx}\int_{c(x)}^{d(x)} f(x,y)dy=\int_{c(x)}^{d(x)}\frac{\partial}{\partial x}f\left(x,y\right)dy+d'(x)f(d,y)-c'(x)f(c,y)$$

(iii)命题:$I(x,A)=\int_a^A f(x,y)dy$关于x可积

充分条件：$f(x,y)\in C([c,d]\times[a,A])$

且: $\int_a^b dx\int_c^d F(x,y)dy=\int_c^d dy\int_a^b f(x,y)dx$