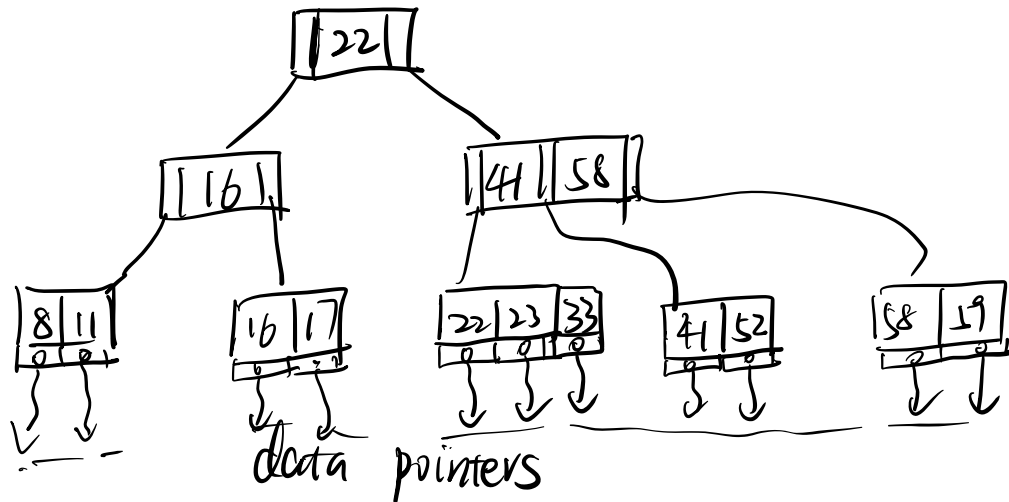BST

$h \geq \lg_2 n$

B+ tree

$h = O(\lg_M n)$

Computation model

RAM model
CPU time

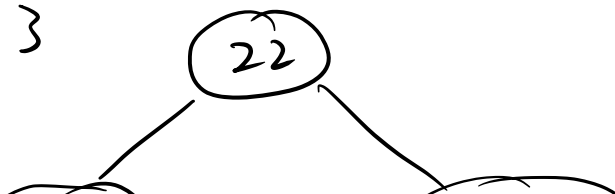external memory model
I/O costs
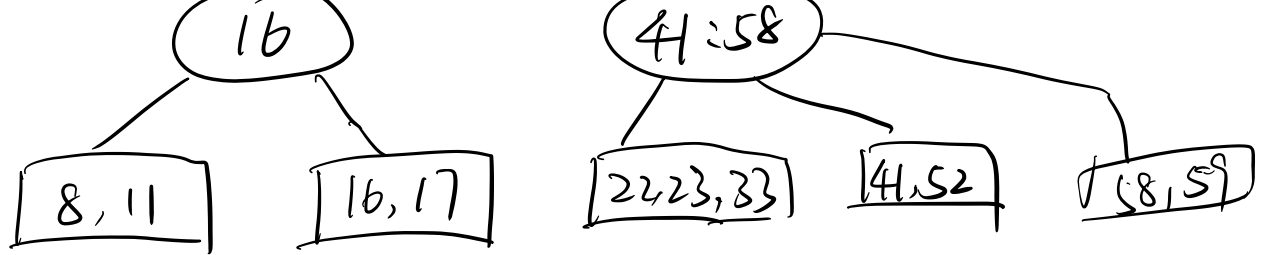

data pointer
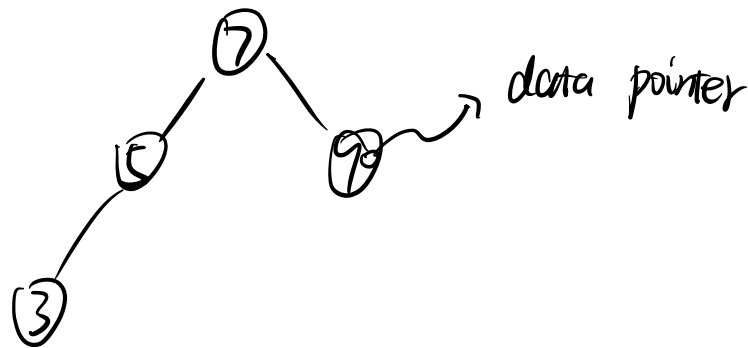
data pointers

1. more than 2 children
2. data pointers are stored in leaves.

B+ tree of order 3

22

```
        ( 16 )                    ( 41:58 )
       /      \               /    |        \
 [ 8,11 ]   [ 16,17 ]   [ 22,23,33 ]  [ 41,52 ]  [ 58,59 ]
```

fanout of a node u = # children of u

A B+ tree of order M is a tree with following properties.

(1) $2 \leq$ fanout of the root $\leq M$ (if the root is not leaf)

(2) $\lceil M/2 \rceil \leq$ fanout of a internal node (non-root) $\leq M$

(3) $\lceil M/2 \rceil \leq$ # <u>keys</u> (data pointers) in a leaf $\leq M$ (if leaf is not a root)

  $\searrow$ in sorted order

(4) all leaves are at the same level.

(5) for each internal node u with children $v_1, v_2, \ldots, v_f$

   (a) all keys in leaves of $\underline{T_{v_{i+1}}} >$ those of T_{v_i}

   $\downarrow$
   the subtree
   root at v_{i+1}

   (b) u stores $f-1$ key values $k_1, \ldots, k_{f-1}$ where
   k_i is the smallest key in leaves of $T_{v_{i+1}}$

(6) If a root is also a leaf,
   $1 \leq$ # keys in it $\leq M$
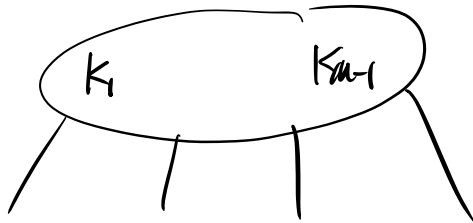

```
        [ 1, 2, 3 ]
```

leaves # $\lceil M/2 \rceil$

$$height = O\left(lg_{\lceil M/2 \rceil} \frac{n}{\lceil M/2 \rceil}\right) = O(lg_M N)$$

$$findkey = O(lg_2 M) \cdot O(lg_M N) = O(lg N)$$
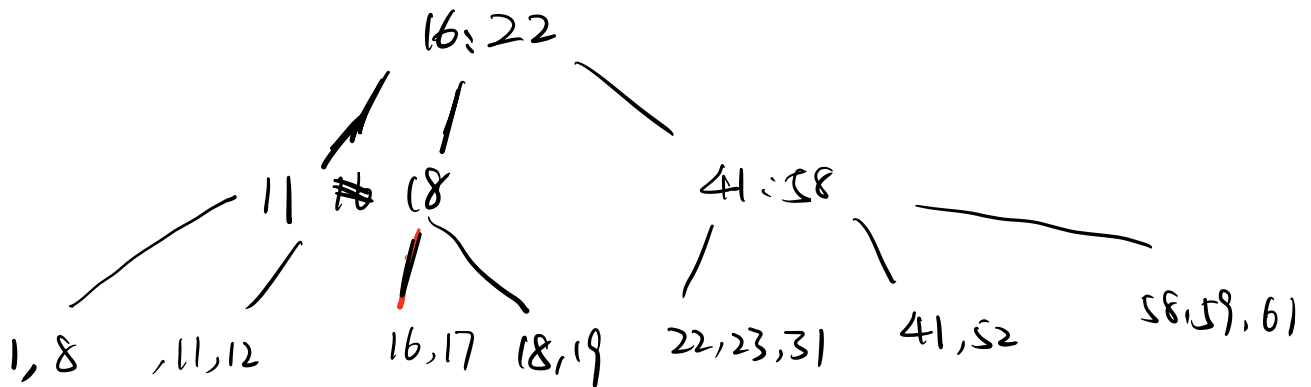
I/O

$$\frac{lg_2 N}{lg_2 M}$$

M children

Insertion

B+ tree of order 3


```
                16:22
         11  #  18        41:58
      1,8  ,11,12  16,17  18,19  22,23,31  41,52  58,59,61
```

Ins(18)   Ins(1)   Ins(19)

---

split leaf

$K_1 : \cdots\cdots$

$\lfloor K_1, \cdots\cdots K_{\lceil M/2 \rceil} \cdots K_M, K_{M+1} \rfloor$

$K_1 : K_{\lceil M/2 \rceil +1} : \cdots$

$K_1, \ldots, K_{\lceil M/2 \rceil}$

$K_{\lceil M/2 \rceil +1}, \ldots, K_{M+1}$
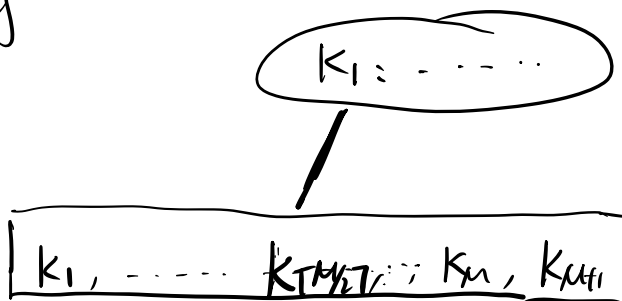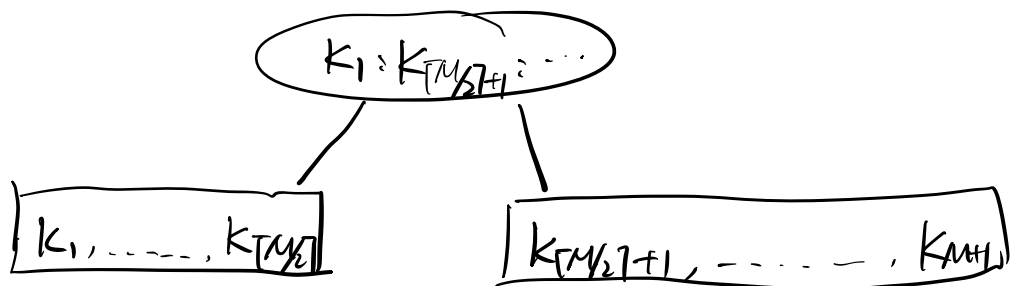
Split internal nodes

$K^x : \cdots$

$K_1, \quad K_{\lceil M/2 \rceil -1}, \ K_{\lceil M/2 \rceil}, \ K_{\lceil M/2 \rceil +1}, \quad K_M$
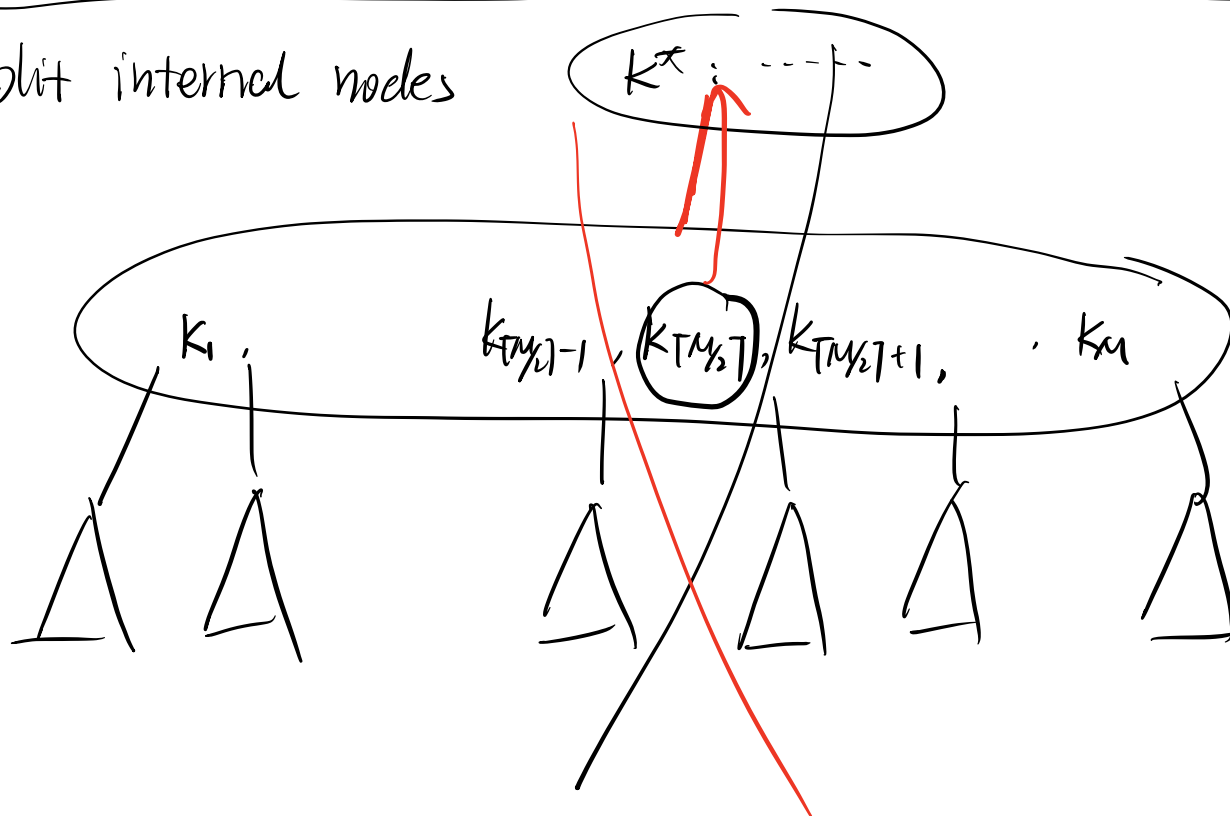
Ins: $O(M) \cdot O(\log_M N) = O(M \log_M N)$

$$= O\left(\frac{M}{\lg M} \cdot \lg N\right)$$

order 3

$\lceil M/2 \rceil = 2 \sim 3$

```
            23
       12        41 : 58
     /   \      /    \      \
  8,⫶    12,17  23,31  41,52  58, 59, 61
```
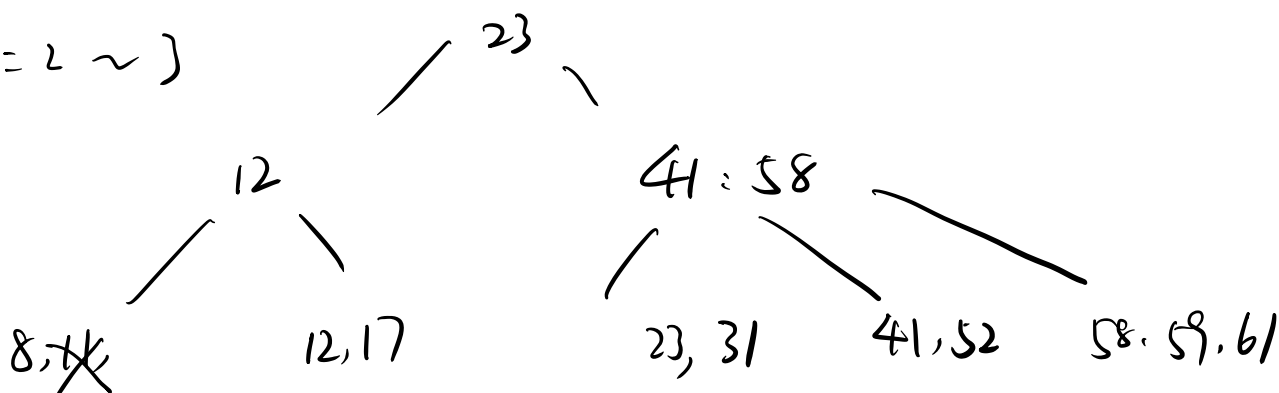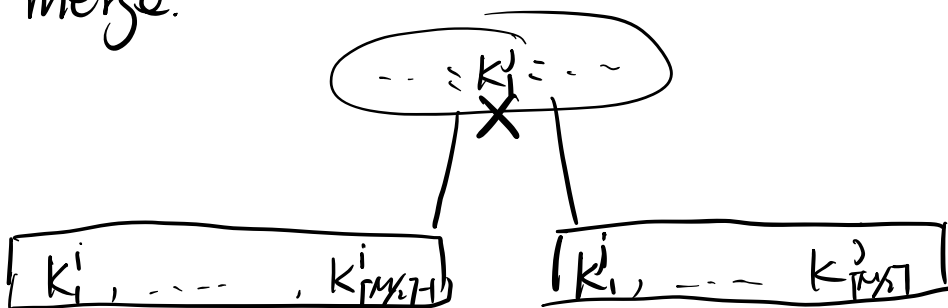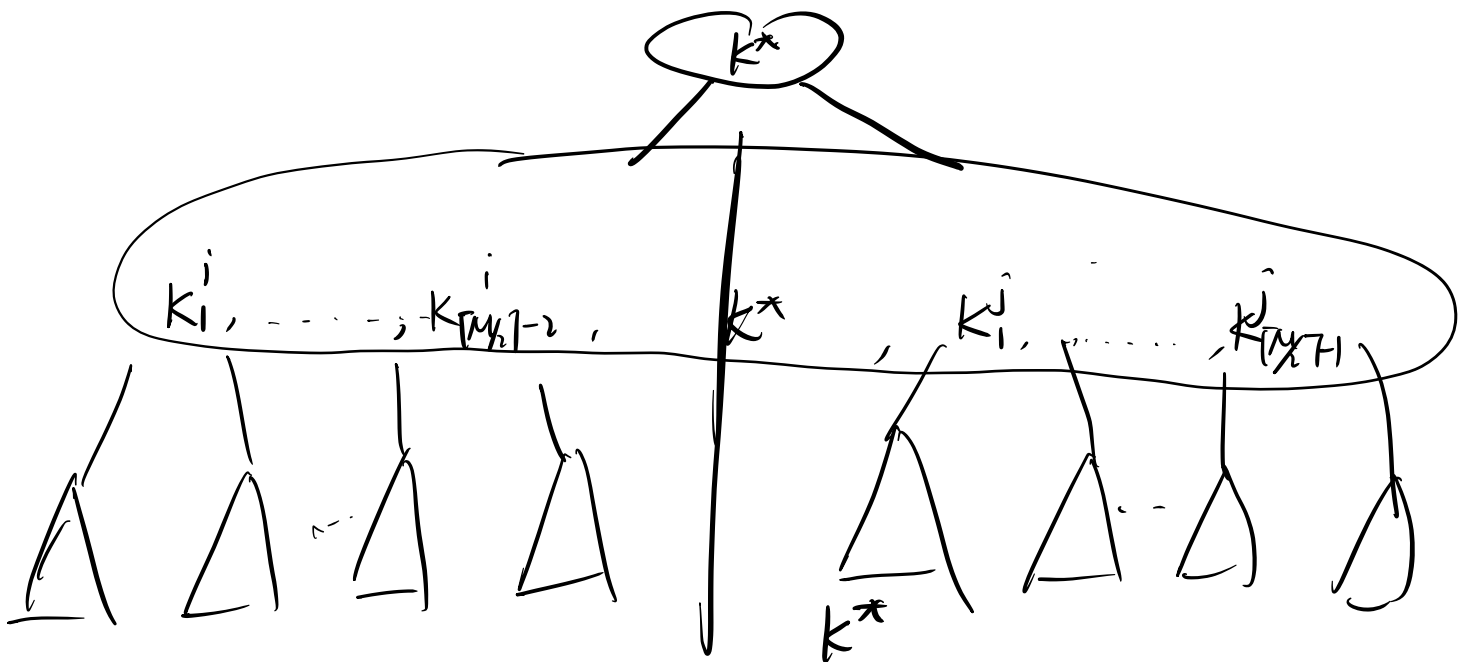
Del(22)   Del(16)   Del(11)

If siblings next to it has more than $\lceil M/2 \rceil$ keys,
　　take 1 from the sibling
　　update the keys in its ancestors.
If siblings next to it ~~has~~ each has only $\lceil M/2 \rceil$ keys,
　　merge.

$$\lceil M/2 \rceil - 1 + \lceil M/2 \rceil \leq M$$

$$Del = O(M) \cdot O(\log_M N) = O(M/\log M \cdot \lg N)$$

	AVL	RB	B+
find key	$O(\lg n)$	$O(\lg n)$	$O(\lg n)$
Ins	$O(\lg n)$	$O(\lg n)$	$O\left(\frac{M}{\lg M} \cdot \lg N\right)$
Del	$O(\lg n)$	$O(\lg n)$	$O\left(\frac{M}{\lg M} \cdot \lg N\right)$