Theory of Computation, Fall 2021
Assignment 7 (Due December 17 Friday 9:35am)

Q1. Prove that every language is countable. You can use any theorem that we've proved in class.

Q2. Prove that there is an undecidable subset of $\{1\}^*$.

Q3. We have already proved that the following language is not recursive. Show that it is recursively enumerable.

$$A = \{\text{“}M\text{”} : M \text{ is a Turing machine that halts on some string}\}$$

(Hint: You may use the fact that Turing machines are able to enumerate all the strings in Σ^* as $s_1, s_2, s_3, \ldots$)

Q4. Let A and B be two languages. Let f be a reduction from A to B.

(a) Suppose that you have a Turing machine M_B that semidecides B. Try to construct a Turing machine M_A that semidecides A.

(b) What conclusion can you draw from (a)?

Q5. Consider the following language. Show that the following language is not recursive by showing a reduction from H to L.

$A = \{\text{“}M\text{”} : M \text{ is a TM such that for any string } w, M \text{ halts on a string } w^R \text{ whenever it halts on } w\}$.

(a) Show that the A is not recursive by showing a reduction from H to A.

(b) Use Rice's theorem to prove that A is not recursive.

References

[1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)

[2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Gall (1998)