OUTLINE OF TUTORIAL 1 (new)

> Theory of Computation, 2022
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TOPICS

- Outlines of Regular Languages (RL)
  - Preliminary
  - Finite Automata (FA)
    - Deterministic Finite Automata (DFA)
    - Computation
    - Nondeterministic Finite Automata (NFA)
  - Regular Expressions (REX)
    - Regular Operations
    - Equivalence with FA
  - Nonregular Languages **[HARD]**
    - Pumping Theorem (Pumping Lemma)
- Developments and a problem set

Regular Languages

Preliminary

- Alphabet
  - Set of **symbols** (characters)
- String
  - **Finite** sequence of symbols
  - empty string: e or ε
  - string operations
- Language
  - Set of **strings**

Finite Automata

- Deterministic Finite Automata (DFA)
  - Formal definition
  - *Issue: How to construct a FA accurately, efficiently and quickly*
    - *Formal check: does it make a DFA*
    - *Check using test cases*
    - *Dead states*
- Computation
  - *Issue: What does "accept" or "recognize" means*
- Nondeterministic Finite Automata (NFA)
  - Equivalence with DFA
    - *Development: State Minimization*
  - Better proof on closure under regular operations using NFA

Regular Expressions

- Regular Operations
- Equivalence with FA
  - From Atomic to Composite
  - *Development: Generalized NFA (GNFA)*
- *FAQ: $L(M)$? $L(R)$? What can be placed after L*
  - $L(e)$ *and* $L(\emptyset)$
- *FAQ: More on regular operations*
  - $L(R) \cup L(\emptyset)$ *vs* $R \cup \emptyset$

- *Kleene star on emptyset: $\emptyset^*$*

Nonregular Languages

- Pumping Theorem (Pumping Lemma)
- *FAQ: What FINITE means*
  - *No counting up, but counting down*
    - $\{0^n1^n\}$
  - *No memory*
    - $\{ww\}$
  - *Closure under finite number of operations*
    - $\{0 \circ 1 \circ \cdots \circ n \circ \ldots\}$
- *Issue: How to describe the proof in formal languages*
- *Development: More on nonregular languages and pumping lemma*
- *Development: Why we need RL*

Developments and Problems

Developments

- In the Courses
- In the Tutorials
- In the Reference Books
  - [1] Sipser M.. Introduction to the Theory of Computation. CENGAGE Learning (2013)
  - [2] Lewis H., Papadimitriou C.. Elements of the Theory of Computation. Prentice-Gall (1998)

A Friendly Problem Set

1. Given an alphabet $\Sigma^* = \{a, b, c\}$ and 2 regular languages over Σ:

   -
   $$A = \{w \in \Sigma^* | w \text{ begins with } ab \text{ and end with } ba\} \tag{1}$$

   -
   $$B = \{w \in \Sigma^* | w \text{ begins with } aa\} \tag{2}$$

   a) Construct a regular expression of language A;

   b) Construct a DFA that accepts $(A \cup B)^*$.

2. Determine whether the following languages are regular. Give your judgement for each language, and construct a DFA that accepts the language if you think it is regular, or show it is not regular using pumping theorem.

   a)
   $$A = \{w | w \text{ has an equal number of } 0'\text{s and } 1'\text{s}\} \tag{3}$$

   b)
   $$B = \{w | w \text{ has an equal number of occurrences of } 00 \text{ and } 11 \text{ assubstrings}\} \tag{4}$$

   c)
   $$C = \{w | w \text{ has an equal number of occurrences of } 01 \text{ and } 10 \text{ assubstrings}\} \tag{5}$$

   d)
   $$D = \{0^k u 0^k | k \geq 1 \text{ and } u \in \{0, 1\}^*\} \tag{6}$$

   e)
   $$E = \{0^k 1 u 0^k | k \geq 1 \text{ and } u \in \{0, 1\}^*\} \tag{7}$$

   f)
   $$F = \{abc | a, b, c \in \{0, 1\}^*, a, b, c \text{ are binary numbers and } c \text{ is the sum of } a \text{ and } b\} \tag{8}$$

3. Determine whether the following proof is valid. Briefly explain why.
   To show language $L = \{w \in \{a, b\} | a^n b^m b^m a^n, m > 0, n \geq 0\}$ is not regular, we can choose $s = abba$ under pumping length $p = 3$, and split s into 3 pieces xyz. Since none of the following 6 possible cases guarantees that $xy^0z \in L$, we show that L in not regular according to the pumping theorem.

   - case 1: $x = e, y = a, z = bba, xy^0z = bba \notin L$
   - case 2: $x = e, y = ab, z = ba, xy^0z = ba \notin L$
   - case 3: $x = e, y = abb, z = a, xy^0z = a \notin L$
   - case 4: $x = a, y = b, z = ba, xy^0z = aba \notin L$
   - case 5: $x = a, y = bb, z = a, xy^0z = aa \notin L$
   - case 6: $x = ab, y = b, z = a, xy^0z = aba \notin L$

4. **[HARD]** Give a proof on these regular languages.

a) Let $A/B = \{w | wx \in A \text{ for some } x \in B\}$. Show that if A is regular and B is any language, then A/B is regular.

b) If A is any language, let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that

$$A_{\frac{1}{2}-} = \{x | \text{ for some } y, |x| = |y| \text{ and } xy \in A\} \tag{9}$$

Show that if A is regular, then so is $A_{\frac{1}{2}-}$.

Correction

On the example showing that $F = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$ is not regular, I have mentioned that:

- If A is not regular, and B is regular, then $A \cup B$ is not regular.

This is incorrect. Another condition $A \cap B = \emptyset$ is required.

You may refer to *A Non-Regular Language that Satisfies the Conditions of the Pumping Theorem* by Prof. Mao.