Theory of Computation, Fall 2021
Assignment 3 Solutions

Exercises

Q1. (a) True.

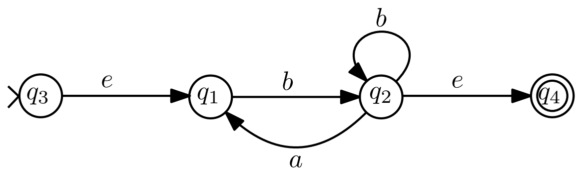
(b) False. $R\emptyset = \emptyset$.

(c) False. $L(R)$ may not contain the empty string e. However, $L(R \cup \emptyset^*)$ contains e.

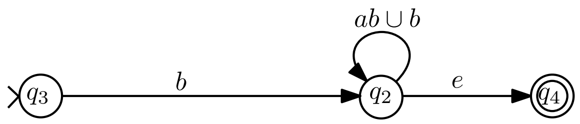
(d) True.

Q2. $(a^*ba^*ba^*ba^*)^* \cup a^*$

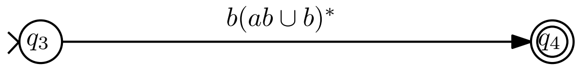
Q3. $b(ab \cup b)^*$



(a) Step 1



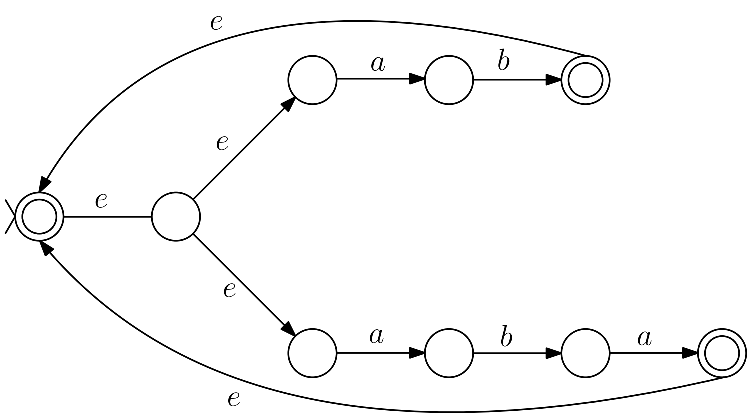
(b) Step 2



(c) Step 3

Q4. It is easy to see that ba is in $L(N')$ but is not in $(L(N))^*$.

Q5. The NFA is as follows.



Q6. (a) False. The non-regular language $\{a^i b^i : i \geq 0\}$ is a subset of the regular language $(a \cup b)^*$.

(b) True. It can be proved by recursively applying the theorem that the union of two regular languages are regular.

(c) True. A language that contains a finite number of strings can be seen as a union of a finite number of languages that contain only one string. By (b), such a language must be regular.

(d) False. For every $i \geq 0$, define $L_i = \{a^i b^i\}$. Clearly every L_i is regular. But their union

$$\cup_{i=0}^{\infty} L_i = \{a^i b^i : i \geq 0\}$$

is not regular.

(e) False. Consider any union of infinite number of regular languages $\cup_{i=0}^{\infty} L_i$. By De Morgan's Laws, we have

$$\cup_{i=0}^{\infty} L_i = \overline{\cap_{i=0}^{\infty} \overline{L_i}}.$$

We already know that the complement of a regular language is regular. If statement (e) is true, then by above formula, we can conclude that $\cup_{i=0}^{\infty} L_i$ is regular. But in (d), we know that $\cup_{i=0}^{\infty} L_i$ is not necessarily regular. Contradiction.