P_{142}

2.3 解：令 $F(x,y,z)=xy+z\ln y+e^{xz}-1$

$F(0,1,1)=0$ $F_x=y+ze^{xz}$ $F_y=x+\frac{z}{y}$ ， $F_z=\ln y+xe^{xz}$.

$F_x(0,1,1)=2\neq 0$ $F_y(0,1,1)=1\neq 0$ $F_z(0,1,1)=0$

$\because F$、F_x, F_y, F_z 在包含 $(0,1,1)$ 的某邻域 $D=\{(x,y,z)|x,z\in R, y>0\}$.

一定连续

方程 $xy+z\ln y+e^{xz}=1$ 在 $(0,1,1)$ 的某邻域可确定隐函数.

$x=x(y,z)$ $y=y(x,z)$.

3.(2) 令 $F(x,y)=\ln\sqrt{x^2+y^2}-\arctan\frac{y}{x}$

$F_x(x,y)=\frac{x+y}{x^2+y^2}$ $F_y(x,y)=\frac{y-x}{x^2+y^2}$

$$\frac{dy}{dx}=-\frac{F_x}{F_y}=\frac{x+y}{x-y}.$$

(4) 令 $F(x,y)=a+\sqrt{a^2-y^2}-ye^{\frac{x+\sqrt{a^2-y^2}}{a}}$ ， $u=\frac{x+\sqrt{a^2-y^2}}{a}$.

$F_x(x,y)=-\frac{y}{a}e^u$ $F_y(x,y)=-\frac{y}{\sqrt{a^2-y^2}}-(e^u+ye^u\frac{-y}{a\sqrt{a^2-y^2}})$.

$$\frac{dy}{dx}=-\frac{F_x}{F_y}=\frac{\frac{y}{a}e^u}{\frac{y^2e^u}{a\sqrt{a^2-y^2}}-e^u-\frac{y}{\sqrt{a^2-y^2}}}$$ 将 $e^u=\frac{a+\sqrt{a^2-y^2}}{y}$ 代入.

$$=\frac{y(a+\sqrt{a^2-y^2})}{\sqrt{a^2-y^2}(a+\sqrt{a^2-y^2})}=\frac{y}{\sqrt{a^2-y^2}}$$

$$\frac{d^2y}{dx^2}=\frac{\sqrt{a^2-y^2}\frac{dy}{dx}+\frac{y^2}{\sqrt{a^2-y^2}}\frac{dy}{dx}}{(a^2-y^2)}=\frac{a^2y}{(a^2-y^2)^2}$$

(6) 解：令 $F(x,y,z) = z - f(x+y+z,\ xyz)$

$F_x(x,y,z) = -f_1' - yz f_2'$ $\quad F_y = -f_1' - xz f_2'$

$F_z = 1 - f_1' - xy f_2'$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{f_1' + yz f_2'}{1 - f_1' - xy f_2'} \qquad \frac{\partial z}{\partial y} = -\frac{f_1' + xz f_2'}{f_1' + yz f_2'}$$

$$\frac{\partial y}{\partial z} = \frac{1 - f_1' - xy f_2'}{f_1' + xz f_2'}$$

4. 解：令 $F(x,y) = x^2 - xy + y^2 - 1$ $\quad F_x(x,y) = 2x - y$ $\quad F_y(x,y) = -x + 2y$.

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = \frac{2x-y}{x-2y}$$

$\because z = x^2 + y^2$ $\quad dz = 2x\,dx + 2y\,dy$ $\therefore$

$$\therefore \frac{dz}{dx} = 2x + 2y\frac{dy}{dx} = \frac{2x^2 - 2y^2}{x-2y}$$

$$\frac{d^2 z}{dx^2} = \frac{(4x - 4yy')(x-2y) - (2x^2 - 2y^2)(1-2y')}{(x-2y)^2} =$$

$$= \frac{(x-2y)(4x-2y) - 6xy}{(x-2y)^2} + \frac{6x(x^2 - y^2)}{(x-2y)^3}$$

$$= \frac{4x-2y}{(x-2y)} + \frac{6x(x^2 + y^2 - xy)}{(x-2y)^3} \qquad x^2 + y^2 - xy = 1$$

$$= \frac{4x-2y}{x-2y} + \frac{6x}{(x-2y)^3}$$

7、(2)解：~~$G=F$~~ $G(x,y,z)=F(x,x+y,x+y+z)$

$$G_x=F_1'+F_2'+F_3' \qquad G_y=F_2'+F_3' \qquad G_z=F_3'$$

$$\frac{\partial z}{\partial x}=\frac{F_1'+F_2'+F_3'}{F_3'} \qquad \frac{\partial z}{\partial y}=\frac{F_2'+F_3'}{F_3'}$$

$$\frac{\partial^2 z}{\partial x^2}=\frac{F_3'\left(\frac{\partial F_1'}{\partial x}+\frac{\partial F_2'}{\partial x}+\frac{\partial F_3'}{\partial x}\right)-(F_1'+F_2'+F_3')\frac{\partial F_3'}{\partial x}}{(F_3')^2}$$

$$=\frac{1}{(F_3')^2}\left\{F_3'\left[F_{11}''+F_{12}''+F_{13}''\left(1+\frac{\partial z}{\partial x}\right)+F_{21}''+F_{22}''+F_{23}''\left(1+\frac{\partial z}{\partial x}\right)+F_{31}''+F_{32}''+F_{33}''\left(1+\frac{\partial z}{\partial x}\right)\right]-(F_1'+F_2'+F_3')\left[F_{31}''+F_{32}''+F_{33}''\left(1+\frac{\partial z}{\partial x}\right)\right]\right\}$$

$\frac{\partial z}{\partial x}$代入 $=-\frac{1}{(F_3')^3}\left[(F_3')^2(F_{11}''+2F_{12}''+F_{22}'')-2F_3'(F_1'+F_2')(F_{13}''+F_{23}'')+F_{33}''(F_1'+F_2')^2\right]$

8、证明：
$$\begin{vmatrix} F_{xx} & F_{xy} & F_x \\ F_{xy} & F_{yy} & F_y \\ F_x & F_x & 0 \end{vmatrix}=2F_xF_yF_{xy}-F_y^2F_{xx}-F_x^2F_{yy}$$

$$y'=-\frac{F_x}{F_y}$$

$$y''=-\frac{F_y\frac{\partial F_x}{\partial x}-F_x\frac{\partial F_y}{\partial x}}{F_y^2}=-\frac{1}{F_y^3}\left[F_y^2F_{xx}-2F_xF_yF_{xy}+F_x^2F_{yy}\right]$$

$$\therefore F_y^3y''=\begin{vmatrix} F_{xx} & F_{xy} & F_x \\ F_{xy} & F_{yy} & F_y \\ F_x & F_y & 0 \end{vmatrix}$$

补充题：解：$G(x,y,z)=F(xz,yz)$ $G_x=zF_1$ $G_y=zF_2$ $G_z=xF_1+yF_2$

$$\frac{\partial z}{\partial x}=-\frac{zF_1}{xF_1+yF_2} \quad \frac{\partial z}{\partial y}=-\frac{zF_2}{xF_1+yF_2}$$

$$\frac{\partial^2 z}{\partial x\partial y}=-\frac{\frac{\partial zF_1}{\partial y}(xF_1+yF_2)-\frac{\partial(xF_1+yF_2)}{\partial y}(zF_1)}{(xF_1+yF_2)^2}$$

$$=-\frac{(\frac{\partial z}{\partial y}F_1+\frac{\partial F_1}{\partial y}z)(xF_1+yF_2)-(x\frac{\partial F_1}{\partial y}+F_2+y\frac{\partial F_2}{\partial y})(zF_1)}{(xF_1+yF_2)^2}$$

$$=\frac{2zF_1F_2-yz^2F_2F_{12}-yz^2F_1F_{22}}{(xF_1+yF_2)^2}$$

P144.

1、解 令 $\begin{cases} F(x,y,z)=x^2+y^2-\frac{1}{2}z^2 & G(x,y,z)=x+y+z-2. \\ F(1,-1,2)=0 & ,G(1,-1,2)=0. \end{cases}$

$F_x=2x \quad F_y=2y \quad F_z=-z \quad G_x=1 \quad G_y=1 \quad G_z=1$

各原函数与各一阶偏导数显然在 R^3 上连续.

$$J=\frac{\partial(F,G)}{\partial(x,y)}=\begin{vmatrix} F_x & F_y \\ G_x & G_y \end{vmatrix}=\begin{vmatrix} 2x & 2y \\ 1 & 1 \end{vmatrix}=2(x-y)$$

在 $(1,-1,2)$ 处 $J=4\neq 0$

因此方程组在 $(1,-1,2)$ 附近能确定形如 $x=f(z)$, $y=g(z)$ 的隐函数组

2.(3) 方程组两边分别对 x 求偏导,整理

$$\begin{cases} (1-xf_1')u_x-f_2'v_x=uf_1' \\ g_1'u_x+(2yvg_2'-1)v_x=g_1' \end{cases}$$

$$\frac{\partial u}{\partial x}=\frac{\begin{vmatrix} uf_1' & -f_2' \\ g_1' & 2yvg_2'-1 \end{vmatrix}}{\begin{vmatrix} 1-xf_1' & -f_2' \\ g_1' & 2yvg_2'-1 \end{vmatrix}}=\frac{uf_1'(2yvg_2'-1)+f_2'g_1'}{(1-xf_1')(2yvg_2'-1)+f_2'g_1'}$$

$$\frac{\partial v}{\partial x}=\frac{\begin{vmatrix} 1-xf_1' & uf_1' \\ g_1' & g_1' \end{vmatrix}}{\begin{vmatrix} 1-xf_1' & -f_2' \\ g_1' & 2yvg_2'-1 \end{vmatrix}}=\frac{g_1'(1-xf_1')-uf_1'g_1'}{(1-xf_1')(2yvg_2'-1)+f_2'g_1'}$$

3.(1) 解：将方程组分别对 x, y 求偏导，

$$\begin{cases}(e^u+\sin v)u_x+(u\cos v)v_x=1, \\ (e^u-\cos v)u_x+(u\sin v)v_x=0,\end{cases}$$ 解得 $$\begin{cases}u_x=\dfrac{\sin v}{1+e^u(\sin v-\cos v)} \\ v_x=\dfrac{\cos v-e^u}{u+ue^u(\sin v-\cos v)}\end{cases}$$

$$\begin{cases}(e^u+\sin v)u_y+(u\cos v)v_y=0 \\ (e^u-\cos v)u_y+(u\sin v)v_y=1,\end{cases}$$ 解得 $$\begin{cases}u_y=\dfrac{-\cos v}{1+e^u(\sin v-\cos v)} \\ v_y=\dfrac{e^u+\sin v}{u+ue^u(\sin v-\cos v)}\end{cases}$$

5.(2) $du=y\,dx+x\,dy \quad dv=\frac{1}{y}dx-\frac{x}{y^2}dy$

$$dz=\frac{\partial z}{\partial u}du+\frac{\partial z}{\partial v}dv=\frac{\partial z}{\partial u}(y\,dx+x\,dy)+\frac{\partial z}{\partial v}\left(\frac{1}{y}dx-\frac{x}{y^2}dy\right)$$

$$=\left(y\frac{\partial z}{\partial u}+\frac{1}{y}\frac{\partial z}{\partial v}\right)dx+\left(x\frac{\partial z}{\partial u}-\frac{x}{y^2}\frac{\partial z}{\partial v}\right)dy.$$

$$\therefore \frac{\partial z}{\partial x}=y\frac{\partial z}{\partial u}+\frac{1}{y}\frac{\partial z}{\partial v} \quad \frac{\partial z}{\partial y}=x\frac{\partial z}{\partial u}-\frac{x}{y^2}\frac{\partial z}{\partial v}$$

$$\frac{\partial^2 z}{\partial x^2}=y\left(yz_{uu}+\frac{1}{y}z_{uv}\right)+\frac{1}{y}\left(yz_{vu}+\frac{1}{y}z_{vv}\right)=y^2z_{uu}+2z_{uv}+\frac{1}{y^2}z_{vv}$$

$$\frac{\partial^2 z}{\partial y^2}=x\left(xz_{uu}-\frac{x}{y^2}z_{uv}\right)+\frac{2x}{y^3}z_v-\frac{x}{y^2}\left(xz_{vu}-\frac{x}{y^2}z_{vv}\right)$$

$$=x^2z_{uu}-\frac{2x^2}{y^2}z_{uv}+\frac{x^2}{y^4}z_{vv}+\frac{2x}{y^3}z_v$$

$$x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}=4x^2\left(z_{uv}-\frac{1}{2xy}z_v\right)=0$$

$$\therefore \frac{\partial^2 z}{\partial u\partial v}-\frac{1}{2u}\frac{\partial z}{\partial v}=0$$

7. 证明. $\frac{\partial u}{\partial s} = u_x x_s + u_y y_s + u_z z_s$ $\quad \frac{\partial u}{\partial t} = u_x x_t + u_y y_t + u_z z_t$

$\frac{\partial v}{\partial s} = v_x x_t + v_y y_s + v_z z_s$ $\quad \frac{\partial v}{\partial t} = v_x x_t + v_y y_t + v_z z_t$

$$\begin{vmatrix} u_s & u_t \\ v_s & v_t \end{vmatrix} = (u_x x_s + u_y y_s + u_z z_s)(v_x x_t + v_y y_t + v_z z_t) - (u_x x_t + u_y y_t + u_z z_t)(v_x x_s + v_y y_s + v_z z_s)$$

左边 $= (u_x v_y - u_y v_x)(x_s y_t - x_t y_s) + (u_y v_z - v_y u_z)(y_s z_t - z_s y_t) + (u_z v_x - u_x v_z)(z_s x_t - z_t x_s)$

$= u_x x_s v_y y_t + u_y y_s v_x x_t + u_y y_s v_z z_t + u_z z_s v_y y_t + u_z z_s v_x x_t + u_x x_s v_z z_t$

$-(u_x x_t v_y y_s + u_y y_t v_x x_s + u_y y_t v_z z_s + u_z z_t v_y y_s + u_z z_t v_x x_s + u_x x_t v_z z_s)$

$+ u_x x_s v_x x_t + u_y y_s v_y y_t + u_z z_s v_z z_t$

$- u_x x_t v_x x_s - u_y y_t v_y y_s + u_z z_t v_z z_s$

$= (u_x x_s + u_y y_s + u_z z_s)(v_x x_t + v_y y_t + v_z z_t)$
$- (u_x x_t + u_y y_t + u_z z_t)(v_x x_s + v_y y_s + v_z z_s)$ = 左边

得证

10、解 $u^2+v^2+w^2=\frac{1}{r^4}(x^2+y^2+z^2)=\frac{1}{r^2}$

$x=ur^2=\frac{u}{u^2+v^2+w^2}$; $y=\frac{v}{u^2+v^2+w^2}$, $z=\frac{w}{u^2+v^2+w^2}$

(2) $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{vmatrix}$

$$=\begin{vmatrix}r^{-2}-2x^2r^{-4} & -2xyr^{-4} & -2xzr^{-4}\\ -2xyr^{-4} & r^{-2}-2y^2r^{-4} & -2yzr^{-4}\\ -2xzr^{-4} & -2yzr^{-4} & r^{-2}-2z^2r^{-4}\end{vmatrix}$$

$=-\frac{1}{r^6}$

1、解: $F(x,y)=x^{\frac{2}{3}}+y^{\frac{2}{3}}-a^{\frac{2}{3}}$ $\therefore F_x=\frac{2}{3}x^{-\frac{1}{3}}$ $F_y=\frac{2}{3}y^{-\frac{1}{3}}$

$\therefore$ 曲线上任意一点,(x_0,y_0)处的切线方程。

$$\frac{2}{3}x^{-\frac{1}{3}}(x-x_0)+\frac{2}{3}y^{-\frac{1}{3}}(y-y_0)=0$$

即 $xx_0^{-\frac{1}{3}}+yy_0^{-\frac{1}{3}}=a^{\frac{2}{3}}$

分别令$x=0$, $y=0$,可求在y轴、x轴上的截距为 $a^{\frac{2}{3}}y_0^{\frac{1}{3}}$, $a^{\frac{2}{3}}x_0^{\frac{1}{3}}$

$$l=\sqrt{(a^{\frac{2}{3}}y_0^{\frac{1}{3}})^2+(a^{\frac{2}{3}}x_0^{\frac{1}{3}})^2}=a$$

$\therefore$ 切线被坐标轴所截取线段数.

2.(2)解: $F(x,y,z)=2x^2+3y^2+z^2-9$ $G(x,y,z)=z^2-3x^2-y^2$.

在$(1,-1,2)$处 $F_x=4x=4$ $F_y=6y=-6$ $F_z=2z=4$

$G_x=-6x=-6$ $G_y=-2y=2$ $G_z=2z=4$

$$\therefore \frac{\partial(F,G)}{\partial(y,z)}=-32 \quad \frac{\partial(F,G)}{\partial(z,x)}=-40 \quad \frac{\partial(F,G)}{\partial(x,y)}=-28$$

$\therefore$ 切向量为$(8,10,7)$,切线方程为

$$\frac{x-1}{8}=\frac{y+1}{10}=\frac{z-2}{7}.$$

法平面方程 $8(x-1)+10(y+1)+7(z-2)=0$.

3.(1) $F(x,y,z)=y-e^{2x-z}$ ~~F_x~~ $=$

在$(1,1,2)$处 $F_x=-2$ $F_y=1$ $F_z=1$.

法向量. $n=(-2,1,1)$.

切平面 $-2(x-1)+(y-1)+(z-2)=0$

法线方程 $\frac{x-1}{-2}=\frac{y-1}{1}=\frac{z-2}{1}$

5.解: 平面 $x+4y+6z=0$ 法向量为$(1,4,6)$.

令 $F(x,y,z)=x^2+2y^2+3z^2-21$,

在点(x_0,y_0,z_0)处 $F_x=2x_0$, $F_y=4y_0$ $F_z=6z_0$.

切平面法向量 为 $(x_0,2y_0,3z_0)$ 两法向量平行

$$\begin{cases} x_0^2+2y_0^2+3z_0^2=21 \\ \frac{x}{1}=\frac{2y}{4}=\frac{3z}{6} \end{cases}$$

解得$(\frac{1}{2},1,1)$ 或$(-\frac{1}{2},-1,-1)$,

所求切平面为 $(x-\frac{1}{2})+4(y-1)+6(z-1)=0$

或 $(x+\frac{1}{2})+4(y+1)+6(z+1)=0$.

7.解: 曲线在此处方向向量为 $(1,4,-8)$

方向余弦为 $\cos\alpha=\frac{1}{9}$ $\cos\beta=\frac{4}{9}$ $\cos\gamma=-\frac{8}{9}$ $s^0=(\frac{1}{9},\frac{4}{9},-\frac{8}{9})$.

$\mathrm{grad}\,u(M)=(\frac{8}{27},-\frac{2}{27},\frac{2}{27})$.

$$\frac{\partial u}{\partial s}\Big|_M=\mathrm{grad}\,u(M)\cdot s^0=\frac{1}{9}\times\frac{8}{27}+\frac{4}{9}\times(-\frac{2}{27})+(-\frac{8}{9})\times\frac{2}{27}=-\frac{16}{243}$$

1.解：令 $L(x,y,z,\lambda,\mu)=xyz+\lambda(x^2+y^2+z^2-1)+\mu(x+y+z)=0$

$$\begin{cases} L_x=yz+2\lambda x+\mu=0 & ① \\ L_y=xz+2\lambda y+\mu=0 & ② \\ L_z=xy+2\lambda z+\mu=0 & ③ \\ L_\lambda=x^2+y^2+z^2-1=0 & ④ \\ L_\mu=x+y+z=0 & ⑤ \end{cases}$$

①×x+②×y+③×z 解得 $\lambda=-\frac{3}{2}xyz$

①+②+③ $\mu=-\frac{1}{3}(yz+xz+xy)$

①.②③两相减得

$$\begin{cases} z(x-y)(1+3xy)=0 & ⑥ \\ x(y-z)(1+3zy)=0 & ⑦ \\ y(x-z)(1+3xz)=0 & ⑧ \end{cases}$$

易知：当且仅当 $x-y=0$, $y-z=0$, $x-z=0$ 有且仅有一个成立.

无论大于等于两个成立，还是都不成立，最终都会导致 $x=y=z=0$ 推出矛盾.

不妨假设 $x-y=0$. 则有 $1+3zy=0$, $1+3xz=0$

$$\begin{cases} xz=-\frac{1}{3} \\ 2x+z=0 \end{cases} \quad \therefore x=y=\pm\frac{\sqrt{6}}{6} \quad z=\mp\frac{\sqrt{6}}{3}$$

同理令 $y-z=0$, $x-z=0$, 可得同解.

共6解 $\left(\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{3}\right)$, $\left(-\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3}\right)$, $\left(\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6}\right)$ $\left(-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6}\right)$

$\left(-\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{6}\right)$, $\left(\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6}\right)$.

由方程组 $\begin{cases} x^2+y^2+z^2=1 \\ x+y+z=0 \end{cases}$ 确定隐函数 $y=y(x)$, $z=z(x)$.

由 $\begin{cases} 2x+2yy'+2zz'=0 \\ 1+y'+z'=0 \end{cases}$ $y'=\frac{z-x}{y-z}$ $z'=\frac{x-y}{y-z}$ $y''=\frac{(y-z)(z'-1)-(z-x)(y'-z')}{(y-z)^2}$

$$z''=\frac{(y-z)(1-y')-(x-y)(y'-z')}{(y-z)^2}.$$

$f(x,y,z)=xyz$ $f'=yz+xzy'+xyz'$ $f''=xy''z+xyz''+xy'z'+2y'z+2yz'$

在 $P_1(\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{3})$ $P_2(\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6})$ $P_3(-\frac{\sqrt{6}}{3},\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{6})$ 处 $f''>0$

三者为极小值点 $f(P_1)=f(P_2)=f(P_3)=-\frac{\sqrt{6}}{18}$

在 $P_4(\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6})$ $P_5(-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3},-\frac{\sqrt{6}}{6})$ $P_6(-\frac{\sqrt{6}}{6},-\frac{\sqrt{6}}{6},\frac{\sqrt{6}}{3})$ $f''<0$

三者为极大值点 $f(P_4)=f(P_5)=f(P_6)=\frac{\sqrt{6}}{18}$

3解: $L(x,y,z;\lambda)=(x-x_0)^2+(y-y_0)^2+(z-z_0)^2+\lambda(Ax+By+Cz+D)$

$$\begin{cases} L_x=2(x-x_0)+\lambda A=0 & ① \\ L_y=2(y-y_0)+\lambda B=0 & ② \\ L_z=2(z-z_0)+\lambda C=0 & ③ \\ L_\lambda=Ax+By+Cz+D=0 & ④ \end{cases}$$

① $\times A$ + ② $\times B$ + ③ $\times C$. $2(Ax+By+Cz)-2(Ax_0+By_0+Cz_0)+\lambda(A^2+B^2+C^2)=0$

$$\lambda_0=\frac{2(Ax_0+By_0+Cz_0+D)}{A^2+B^2+C^2}$$

$x_1=x_0-\frac{A}{2}\lambda_0$ $y_1=y_0-\frac{B}{2}\lambda_0$ $z_1=z_0-\frac{C}{2}\lambda_0$.

点 (x_0,y_0,z_0) 到平面最短距离

$$d=\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

6.(1) $F(x,y,u)=x^2+u^2-f(x,u)-g(x,y,u)$.

$$\frac{\partial F}{\partial x}=2x-f_1'-g_1' \quad \frac{\partial F}{\partial y}=-g_2' \quad \frac{\partial F}{\partial u}=2u-f_2'-g_3'.$$

$$\frac{\partial u}{\partial x}=\frac{2x-f_1'-g_1'}{f_2'+g_3'-2u} \qquad \frac{\partial u}{\partial y}=\frac{-g_2'}{f_2'+g_3'-2u}.$$

10. 解. 由 $x=\varphi(u,v)$, $y=\psi(u,v)$.

$$dx=\varphi_u du+\varphi_v dv \qquad dy=\psi_u du+\psi_v dv$$

$$1=\varphi_u\frac{du}{dx}+\varphi_v\frac{dv}{du}\frac{du}{dx} \quad \therefore \frac{du}{dx}=\frac{1}{\varphi_u+\varphi_v v_u}$$

$$\frac{dy}{dx}=\frac{\psi_u+\psi_v\frac{dv}{du}}{\varphi_u+\varphi_v\frac{dv}{du}}$$

$$\frac{d^2y}{dx^2}=(\varphi_u+\varphi_v v_u)^{-1}[\psi_{uu}u_x+\psi_{uv}v_u u_x+\psi_{vu}u_x v_u+\psi_{vv}(v_u)^2u_x+\psi_v v_{uu}u_x]$$

$$-(\psi_u+\psi_v v_u)(\varphi_u+\varphi_v v_u)^{-2}[\varphi_{uu}u_x+\varphi_{uv}v_u u_x+\varphi_{vu}v_u u_x+\varphi_{vv}(v_u)^2u_x+\varphi_v v_{uu}u_x]$$

$$=\left(\varphi_u+\varphi_v\frac{dv}{du}\right)^{-3}\left\{\left(\varphi_u+\varphi_v\frac{dv}{du}\right)\left[\psi_{uu}+2\psi_{uv}\frac{dv}{du}+\psi_{vv}\left(\frac{dv}{du}\right)^2+\psi_v\frac{d^2v}{du^2}\right]-\left(\psi_u+\psi_v\frac{dv}{du}\right)\left[\varphi_{uu}+2\varphi_{uv}\frac{dv}{du}+\varphi_{vv}\left(\frac{dv}{du}\right)^2+\varphi_v\frac{d^2v}{du^2}\right]\right\}.$$