Probability Basics

Question 1: A die is rolled. What is the probability of getting:

(a) An even number (b) A number greater than 4

(a) Probability of an even number:
Even numbers on a die: {2, 4, 6}→ 3 outcomes

$$P(\text{Even}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(b) Probability of a number greater than 4:
Numbers greater than 4: {5, 6}→ 2 outcomes

$$P(> 4) = \frac{2}{6} = \frac{1}{3}$$

Question 2: In a class of 50 students: 20 like Mathematics (M) 15 like Science (S) 5 like both subjects What is the probability that a student chosen at random likes Mathematics or Science?

The formula is:

$$P(M \cup S) = \frac{n(M) + n(S) - n(M \cap S)}{N}$$

Substitute the numbers:

$$P(M \cup S) = \frac{20 + 15 - 5}{50} = \frac{30}{50} = 0.6$$

Question 3: A bag has 3 red and 2 blue balls. If one ball is drawn randomly and is red, what is the probability that the next ball is also red (without replacement)?

Bag: 3 Red, 2 Blue → total 5 balls
Event: Draw 2 balls **without replacement**
Want: Probability both are Red

Step 1: Probability the **first ball is Red**

$$P(R_1) = \frac{3}{5}$$

Step 2: Probability the **second ball is Red** given the first was Red

$$P(R_2 \mid R_1) = \frac{2}{4} = \frac{1}{2}$$

Step 3: Joint probability (both Red)

$$P(R_1 \cap R_2) = P(R_1) \times P(R_2 \mid R_1) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

Question 4: The population of a school is divided into 60% boys and 40% girls. If you want equal representation of both genders in the sample, which method should you use: Simple Random Sampling or Stratified Sampling? Why.

Given:

- Population: 60% boys, 40% girls
- Goal: Get a sample with **equal representation of boys and girls**

Step 1: Simple Random Sampling (SRS)

- In SRS, every individual has an equal chance of being selected.
- If you randomly pick, the sample will **likely reflect the population proportions** (about 60% boys, 40% girls).
- So, you **won't get equal numbers of boys and girls** consistently.

Step 2: Stratified Sampling

- In stratified sampling, the population is divided into **strata (groups) based on a characteristic**, here **gender**.
- You can then **choose an equal number from each stratum**.
- This ensures your sample has **exactly 50% boys and 50% girls**, meeting your goal.

Question 5: The average height of 1000 students = 160 cm. A sample of 100 students shows an average height = 158 cm. Find the sampling error.

Given:

- Population mean ($\bar{X}_{\text{population}}$) = 160 cm

- Sample mean ($\bar{X}_{\text{sample}}$) = 158 cm

Step 1: Recall the formula for Sampling Error

$$\text{Sampling Error} = \text{Sample Mean} - \text{Population Mean}$$

Step 2: Substitute the values

$$\text{Sampling Error} = 158 - 160 = -2 \text{ cm}$$

Question 6: The population mean salary is ₹50,000 with σ = ₹5,000. If we take a sample of 100 employees, what is the standard error of the mean (SEM)?

Given:

- Population mean salary: $\mu = ₹50{,}000$(not directly needed for SEM)
- Population standard deviation: $\sigma = ₹5{,}000$
- Sample size: $n = 100$

Step 1: Recall the formula for Standard Error of the Mean (SEM)

$$\text{SEM} = \frac{\sigma}{\sqrt{n}}$$

Step 2: Substitute the values

$$\text{SEM} = \frac{5000}{\sqrt{100}} = \frac{5000}{10} = 500$$

Question 7: In a group of 100 students: 40 like Cricket (C) 30 like Football (F) 10 like both Cricket and Football Find the probability that a student likes at least one sport.

Given:

- Total students: $N = 100$
- Likes Cricket: $n(C) = 40$
- Likes Football: $n(F) = 30$
- Likes both: $n(C \cap F) = 10$

Step 1: Recall the formula for probability of union

$$P(C \cup F) = \frac{n(C) + n(F) - n(C \cap F)}{N}$$

Step 2: Substitute the numbers

$$P(C \cup F) = \frac{40 + 30 - 10}{100} = \frac{60}{100} = 0.6$$

Question 8: From a deck of 52 cards, two cards are drawn without replacement. What is the probability that both are Aces.

Given:

- Standard deck: 52 cards
- Number of Aces in the deck: 4
- Cards drawn **without replacement**
- Want: Probability both cards are Aces

Step 1: Probability the first card is an Ace

$$P(\text{First Ace}) = \frac{4}{52} = \frac{1}{13}$$

Step 2: Probability the second card is an Ace (given first was Ace)

- After drawing 1 Ace, remaining Aces = 3
- Remaining total cards = 51

$$P(\text{Second Ace} \mid \text{First Ace}) = \frac{3}{51} = \frac{1}{17}$$

Step 3: Joint probability (both Aces)

$$P(\text{Both Aces}) = P(\text{First Ace}) \times P(\text{Second Ace} \mid \text{First Ace}) = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

Question 9: A factory produces bulbs with 2% defective rate. If 5 bulbs are chosen at random, what is the probability that all are non-defective?

Given:

- Defective rate = 2% → probability a bulb is **non-defective** = 100% − 2% = 98% = 0.98
- Number of bulbs chosen = 5
- Want: probability **all 5 are non-defective**

Step 1: Probability all bulbs are non-defective

Since bulbs are chosen **independently**, multiply the probabilities:

$$P(\text{all non} - \text{defective}) = 0.98^5$$

Step 2: Calculate

$$0.98^5 = 0.98 \times 0.98 \times 0.98 \times 0.98 \times 0.98$$

Let's compute step by step:

1. $0.98 \times 0.98 = 0.9604$
2. $0.9604 \times 0.98 = 0.941192$
3. $0.941192 \times 0.98 = 0.92236816$
4. $0.92236816 \times 0.98 \approx 0.9039$

Question 10: Differentiate between discrete and continuous random variables with examples

Here is a clear differentiation between **Discrete** and **Continuous** random variables, with examples:

Basis	**Discrete Random Variable**	**Continuous Random Variable**
Definition	Takes **countable** values (finite or countably infinite)	Takes **uncountable** values within a range
Values	Whole numbers (0, 1, 2, 3, …)	Any real number (decimals included)
Measurement	Obtained by **counting**	Obtained by **measuring**
Probability	Probability is assigned to each exact value	Probability is defined over an **interval**, not at a single point
Probability at a point	($P(X = x) > 0$)	($P(X = x) = 0$)
Distribution	Probability Mass Function (PMF)	Probability Density Function (PDF)

Examples

Discrete Random Variable:

- Number of students in a class
- Number of heads when tossing 3 coins
- Number of defective bulbs in a box

Continuous Random Variable:

- Height of students
- Weight of a person
- Time taken to complete an exam