ALAN geometrica (11 ott)

sabato 12 ottobre 2024 09:57

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & \dots & \dots & a_{nn} \end{bmatrix}$$ matrice quadrata

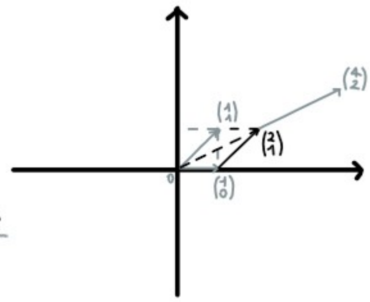
$$V = \begin{bmatrix} V_1 \\ \vdots \\ V_n \end{bmatrix}$$ matrice con n righe e 1 colonna $\longrightarrow$ vettore

operazioni tra vettori

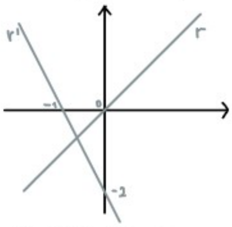
esempi:

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ somma tra vettori

$5 \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ moltiplicazione per uno scalare



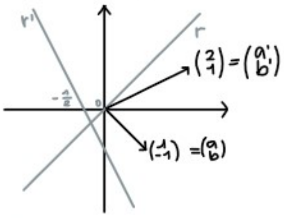
$$\begin{matrix} r) \\ r') \end{matrix} \begin{cases} y = mx + q \\ y = m'x + q' \end{cases}$$ intersezione di due rette



$m = 1, q = 0$
$\rightarrow y = x$
$m' = -2, q = -1$

non può rappresentare rette del tipo $x = c$ (costante)

$$\begin{cases} ax + by = c \\ a'x + b'y = c' \end{cases} \longrightarrow \begin{cases} a = 1, b = -1, c = 0 \\ a = 2, b = 1, c = -1 \end{cases} \rightarrow \begin{cases} x - y = 0 \\ 2x + y = -1 \end{cases}$$ $(1,-1) = (a,b)$ $(2,1) = (a',b')$



retta ortogonale al suo vettore

r) $x - y = 0$

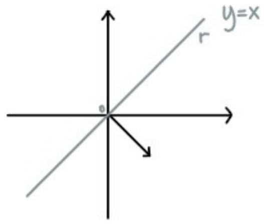
$(1,-1)$ $y = x$

$1 \cdot x + (-1) \cdot y = 0$

$1 \cdot t + (-1)t = 0$ $t = 1$ $v\binom{1}{1}$

$v = \binom{t}{t}$ t $\mathbb{R} \longrightarrow$ $t = 2$ $v\binom{2}{2}$

$t = -3$ $v\binom{-3}{-3}$



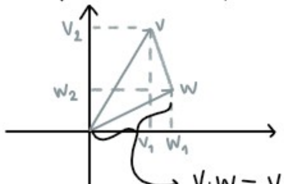
prodotto scalare

$\left\langle \binom{a}{b}, \binom{x}{y} \right\rangle = ax + by$ $\mathbb{R}^2 \cdot \mathbb{R}^2 \longmapsto \mathbb{R}$

esempio:

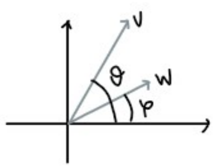
$\binom{1}{-1} \cdot \binom{3}{3} = 1 \cdot 3 + (-1)3 = 0$

$V = \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix}$ $W = \begin{pmatrix} W_1 \\ \vdots \\ W_n \end{pmatrix}$ $V \cdot W = \sum_{i=1}^{N} V_i W_i = V_1 W_1 + V_2 W_2 + \dots + V_N W_N \xrightarrow{\text{in 2 dimensioni}} \overset{a\ x}{V_1 W_1} + \overset{b\ y}{V_2 W_2}$



$V \cdot W = V_1 W_1 + V_2 W_2 \in \mathbb{R}$

$V_1 = \|V\| \cos \vartheta$
$V_2 = \|V\| \sin \vartheta$
$\|V\| = \sqrt{V_1^2 \ V_2^2}$
$W_1 = \|W\| \cos \varphi$
$W_2 = \|W\| \sin \varphi$



$V \cdot W = V_1 W_1 + V_2 W_2 = \|V\| \cos \vartheta \, \|W\| \cos \varphi + \|V\| \sin \vartheta \, \|W\| \sin \varphi = \|V\| \|W\| (\cos \vartheta \cos \varphi - \sin \vartheta \sin \varphi) = \|V\| \|W\| \cdot \cos(\vartheta - \varphi)$

$\rightarrow \vartheta - \varphi = \frac{\pi}{2}$