grafi (30/9, 3/10)

mercoledì 2 ottobre 2024 09:50

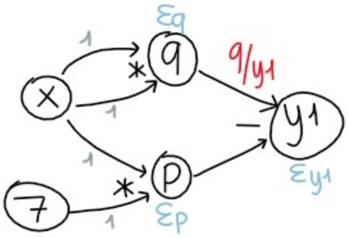
algoritmi $\longleftrightarrow$ **grafo**

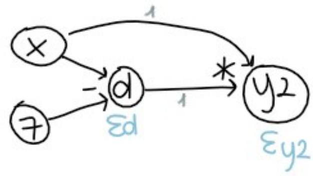
- nodi $\to$ numeri (dati costanti, risultati intermedi)
- archi $\to$ operazioni

+ ETICHETTE
  - nodi: errori locali (solo per risultati intermedi)
  - archi: coefficienti di amplificazione $\to c_f = \frac{x f'(x)}{f(x)}$

esempio 1:

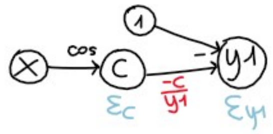
$$f(x) = x^2 - 7x \to \text{alg } 1$$
$$= x(x-7) \to \text{alg } 2$$

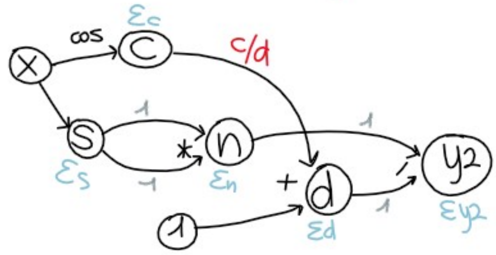




esempio 2:

$$f(x) = 1 - \cos x \to \text{alg}1$$
$$\parallel \quad \frac{\sin^2 x}{1+\cos x} \to \text{alg}2$$





REGOLA: $\varepsilon_{alg} = \sum (\text{err locali})(\text{coeff})$

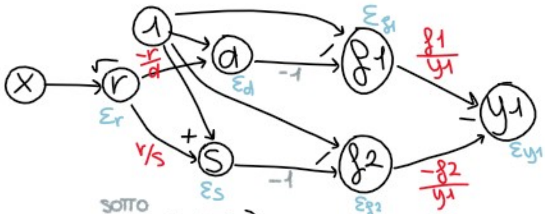
COEFF = π (etichetta archi uscenti) [1° CAMMINO] + (etichetta archi uscenti) [2° CAMMINO] + ...

$\to$ SOLO PER ARCHI USCENTI DA NODI ETICHETTATI

$$\varepsilon_n = 1 + \varepsilon_d\{-1\} + \varepsilon_s\{1+1+1\cdot 1\} + \varepsilon_c\{c/d \cdot (-1)\} + \varepsilon_{y2}$$

esempio 3:

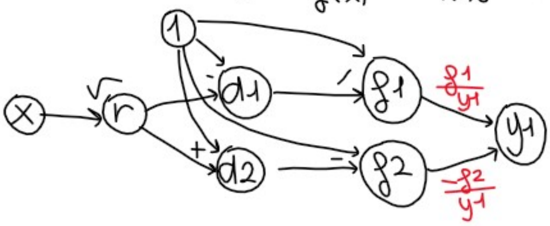
$$f(x) = \frac{1}{1-\sqrt{x}} - \frac{1}{1+\sqrt{x}}$$



$$\varepsilon_{alg1} = \varepsilon_r\left\{\underbrace{\left(\frac{-r}{d}\right)\cdot(-1)\cdot\frac{f1}{y1}}_{\text{SOPRA}} + \underbrace{\frac{r}{s}\cdot(-1)\cdot\left(\frac{-f2}{y1}\right)}_{\text{SOTTO}}\right\} + \varepsilon_s\left\{(-1)\cdot\left(\frac{-f2}{y1}\right)\right\} + \varepsilon_{f1}\left\{\frac{f1}{y1}\right\} + \varepsilon_{f2}\left\{\frac{-f2}{y1}\right\} + \varepsilon_{y1}$$

$$f(x) = \frac{1}{1-\sqrt{x}} - \frac{1}{1+\sqrt{x}}, \quad x \approx 0^+ \to f(x) = \frac{1+\sqrt{x}-1+\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})} = \frac{2\sqrt{x}}{1-x}$$

$$\to c_f = \frac{x f'(x)}{f(x)} \to \lim_{x\to 0^+} c_f$$

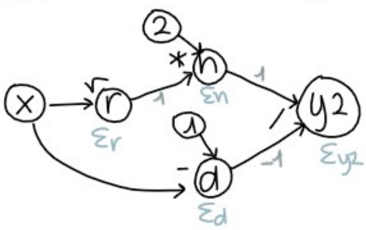


$$\lim_{x\to 0^+} \frac{f1}{y1} = \frac{\frac{1}{1-\sqrt{x}} \to 1}{\underbrace{\frac{1}{1+\sqrt{x}} - \frac{1}{1+\sqrt{x}}}_{1 \; - \; 1} \to 0} = +\infty$$

$$f'(x) = -\frac{1}{(1-\sqrt{x})^2}\left(-\frac{1}{2\sqrt{x}}\right) + \frac{1}{(1+\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}}\overbrace{\left[\frac{1}{(1-\sqrt{x})^2} + \frac{1}{(1+\sqrt{x})^2}\right]}^{2}$$

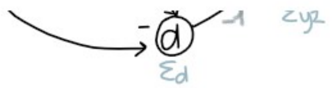
$$c_f = \frac{x f'(x)}{f(x)} = \frac{x\frac{1}{2\sqrt{x}}[\overset{2}{\uparrow}]}{\frac{2\sqrt{x}}{1-x}} = \frac{x[\overset{2}{\uparrow}]}{2\sqrt{x}}\cdot\frac{1-x}{2\sqrt{x}} = \frac{\cancel{x}[\overset{2}{\uparrow}](1-x)}{4\cancel{x}} = \frac{[\overset{2}{\uparrow}](1-x)}{4}$$

$$\Rightarrow \lim_{x\to 0^+} c_f = \frac{1}{2} \Rightarrow \varepsilon_{in} = \frac{1}{2}\varepsilon_x$$



$$\varepsilon_{alg2} = \varepsilon_r\{1\cdot 1\} + \varepsilon_n\{1\} + \varepsilon_d\{-1\} + \varepsilon_{y2} =$$
$$= \varepsilon_r + \varepsilon_n - \varepsilon_d + \varepsilon_{y2}$$

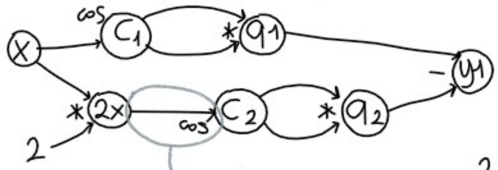
esempio ...



esempio 4: $f(x) = \overbrace{\cos^2 x}^{\simeq 1} - \overbrace{\cos^2(2x)}^{\simeq 1}, \; x \simeq 0$

$= (4\underbrace{\cos^2 x}_{\simeq 1} - 1)\sin^2 x$

$C_f = \dfrac{x f'(x)}{f(x)}$



etichetta $c_{\cos} \longrightarrow -\dfrac{2x\sin(2x)}{\cos(2x)}$

$g(x) = \cos x$

$C_g = \dfrac{x g'(x)}{g(x)} = \dfrac{x \sin x}{\cos x}$