Householder

19/07/2024

Si consideri il vettore $x=\begin{pmatrix}-3\\4\end{pmatrix}\rightarrow\begin{pmatrix}\eta\\0\end{pmatrix}$

a)

$\eta=\|x\|_2=\sqrt{(-3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

$u^tu=(-8\ \ 4)\begin{pmatrix}-8\\4\end{pmatrix}=(-8)^2+4^2=64+16=80$

$u=x+\eta e_1=\begin{pmatrix}-3\\4\end{pmatrix}-5\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-3\\4\end{pmatrix}-\begin{pmatrix}5\\0\end{pmatrix}=\begin{pmatrix}-8\\4\end{pmatrix}$

$uu^t=\begin{pmatrix}-8\\4\end{pmatrix}(-8\ \ 4)=\begin{pmatrix}64&-32\\-32&16\end{pmatrix}$

$P=I-2\cdot\frac{uu^t}{u^tu}=I-2\cdot\frac{1}{80}\cdot\begin{pmatrix}-8\\4\end{pmatrix}(-8\ \ 4)=I-\cancel{2}\cdot\frac{1}{\cancel{80}\,40}\cdot\begin{pmatrix}64&-32\\-32&16\end{pmatrix}=I-\begin{pmatrix}64/40&-32/40\\-32/40&16/40\end{pmatrix}=$

$=I-\begin{pmatrix}32/20&-16/20\\-16/20&8/20\end{pmatrix}=I-\begin{pmatrix}16/10&-8/10\\-8/10&4/10\end{pmatrix}=I-\begin{pmatrix}8/5&-4/5\\-4/5&2/5\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}-\begin{pmatrix}8/5&-4/5\\-4/5&2/5\end{pmatrix}=\begin{pmatrix}-3/5&4/5\\4/5&3/5\end{pmatrix}$

b) La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}-3\\4\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\eta\\0\end{pmatrix}$ con $\eta=5$

c) $P\cdot x=\eta e_1$?

$\begin{pmatrix}-3/5&4/5\\4/5&3/5\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}=\begin{pmatrix}(-3/5)(-3)+(4/5)(4)\\(4/5)(-3)+(3/5)(4)\end{pmatrix}=\begin{pmatrix}9/5+16/5\\-12/5+12/5\end{pmatrix}=\begin{pmatrix}5\\0\end{pmatrix}$

04/06/2019

Si consideri il vettore $x=\begin{pmatrix}4\\0\\-3\end{pmatrix}\rightarrow\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$

a)

$\alpha=\|x\|_2=\sqrt{4^2+(-3)^2}=\sqrt{16+9}=\sqrt{25}=5$

$u=x-\alpha e_1=\begin{pmatrix}4\\0\\-3\end{pmatrix}-5\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}-1\\0\\-3\end{pmatrix}$

$u^tu=(-1\ \ 0\ \ -3)\begin{pmatrix}-1\\0\\-3\end{pmatrix}=1+9=10$

$uu^t=\begin{pmatrix}-1\\0\\-3\end{pmatrix}(-1\ \ 0\ \ -3)=\begin{bmatrix}1&0&3\\0&0&0\\3&0&9\end{bmatrix}$

$P=I-\cancel{2}\cdot\frac{1}{\cancel{10}\,5}\cdot\begin{bmatrix}1&0&3\\0&0&0\\3&0&9\end{bmatrix}=I-\begin{bmatrix}1/5&0&3/5\\0&0&0\\3/5&0&9/5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}1/5&0&3/5\\0&0&0\\3/5&0&9/5\end{bmatrix}=\begin{bmatrix}4/5&0&-3/5\\0&1&0\\-3/5&0&-4/5\end{bmatrix}$

b) La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}4\\0\\-3\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$ con $\alpha=5$

c) $P\cdot x=\alpha e_1$?

$\begin{bmatrix}4/5&0&-3/5\\0&1&0\\-3/5&0&-4/5\end{bmatrix}\begin{bmatrix}4\\0\\-3\end{bmatrix}=\begin{bmatrix}(4/5)(4)+(-3/5)(-3)\\0\\(-3/5)(4)+(-4/5)(-3)\end{bmatrix}=\begin{bmatrix}16/5+9/5\\0\\-12/5+12/5\end{bmatrix}=\begin{bmatrix}25/5\\0\\0\end{bmatrix}=\begin{bmatrix}5\\0\\0\end{bmatrix}$

23/06/2021

Si consideri il vettore $x=\begin{pmatrix}-2\\-1\\2\end{pmatrix} \to \begin{pmatrix}\alpha\\0\\0\end{pmatrix}$

a)

$$\alpha = \|x\|_2 = \sqrt{(-2)^2+(-1)^2+2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$u = x - \alpha e_1 = \begin{pmatrix}-2\\-1\\2\end{pmatrix} - 3\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-5\\-1\\2\end{pmatrix}$$

$$u^t u = (-5\ -1\ 2)\begin{pmatrix}-5\\-1\\2\end{pmatrix} = (-5)\cdot(-5)+(-1)\cdot(-1)+2\cdot 2 = 25+1+4 = 30$$

$$u u^t = \begin{pmatrix}-5\\-1\\2\end{pmatrix}(-5\ -1\ 2) = \begin{bmatrix}25 & 5 & -10\\ 5 & 1 & -2\\ -10 & -2 & 4\end{bmatrix}$$

$$P = I - 2\cdot\frac{uu^t}{u^tu} = I - \cancel{2}\cdot\frac{1}{\cancel{30}_{15}}\cdot\begin{bmatrix}25 & 5 & -10\\ 5 & 1 & -2\\ -10 & -2 & 4\end{bmatrix} = I - \begin{bmatrix}25/15 & 5/15 & -10/15\\ 5/15 & 1/15 & -2/15\\ -10/15 & -2/15 & 4/15\end{bmatrix} = I - \begin{bmatrix}5/3 & 1/3 & -2/3\\ 1/3 & 1/15 & -2/15\\ -2/3 & -2/15 & 4/15\end{bmatrix} =$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}5/3 & 1/3 & -2/3\\ 1/3 & 1/15 & -2/15\\ -2/3 & -2/15 & 4/15\end{bmatrix} = \begin{bmatrix}-2/3 & -1/3 & 2/3\\ -1/3 & 14/15 & 2/15\\ 2/3 & 2/15 & 11/15\end{bmatrix}$$

b) La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}-2\\-1\\2\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$ con $\alpha = 3$

c) $P\cdot x = \alpha e_1$?

$$\begin{bmatrix}-2/3 & -1/3 & 2/3\\ -1/3 & 14/15 & 2/15\\ 2/3 & 2/15 & 11/15\end{bmatrix}\begin{bmatrix}-2\\-1\\2\end{bmatrix} = \begin{bmatrix}(-2/3)(-2)+(-1/3)(-1)+(2/3)(2)\\ (-1/3)(-2)+(14/15)(-1)+(2/15)(2)\\ (2/3)(-2)+(2/15)(-1)+(11/15)(2)\end{bmatrix} = \begin{bmatrix}4/3+1/3+4/3\\ 2/3-14/15+4/15\\ -4/3-2/15+22/15\end{bmatrix} = \begin{bmatrix}3\\0\\0\end{bmatrix}$$

12/02/2020

Si consideri il vettore $x=\begin{pmatrix}-3\\0\\-4\end{pmatrix} \to \begin{pmatrix}\alpha\\0\\0\end{pmatrix}$

a)

$$\alpha = \|x\|_2 = \sqrt{(-3)^2+0^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$u = x - \alpha e_1 = \begin{pmatrix}-3\\0\\-4\end{pmatrix} - 5\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-8\\0\\-4\end{pmatrix}$$

$$u^t u = (-8\ 0\ -4)\begin{pmatrix}-8\\0\\-4\end{pmatrix} = 80$$

$$u u^t = \begin{pmatrix}-8\\0\\-4\end{pmatrix}(-8\ 0\ -4) = \begin{bmatrix}64 & 0 & 32\\ 0 & 0 & 0\\ 32 & 0 & 16\end{bmatrix}$$

$$P = I - \cancel{2}\cdot\frac{1}{\cancel{80}_{40}}\cdot\begin{bmatrix}64 & 0 & 32\\ 0 & 0 & 0\\ 32 & 0 & 16\end{bmatrix} = I - \begin{bmatrix}64/40 & 0 & 32/40\\ 0 & 0 & 0\\ 32/40 & 0 & 16/40\end{bmatrix} = I - \begin{bmatrix}16/10 & 0 & 8/10\\ 0 & 0 & 0\\ 8/10 & 0 & 4/10\end{bmatrix} = I - \begin{bmatrix}8/5 & 0 & 4/5\\ 0 & 0 & 0\\ 4/5 & 0 & 2/5\end{bmatrix} =$$

$$= \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}8/5 & 0 & 4/5\\ 0 & 0 & 0\\ 4/5 & 0 & 2/5\end{bmatrix} = \begin{bmatrix}-3/5 & 0 & -4/5\\ 0 & 1 & 0\\ -4/5 & 0 & 3/5\end{bmatrix}$$

b) La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}-3\\0\\-4\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$ con $\alpha=5$

c) $P\cdot x = \alpha e_1$?

$$\begin{bmatrix}-3/5 & 0 & -4/5\\ 0 & 1 & 0\\ -4/5 & 0 & 3/5\end{bmatrix}\begin{bmatrix}-3\\0\\-4\end{bmatrix} = \begin{bmatrix}(-3/5)(-3)+(0)(0)+(-4/5)(-4)\\ (0)(-3)+(1)(0)+(0)(-4)\\ (-4/5)(-3)+(0)(0)+(3/5)(-4)\end{bmatrix} = \begin{bmatrix}9/5+16/5\\ 0\\ 12/5-12/5\end{bmatrix} = \begin{bmatrix}25/5\\0\\0\end{bmatrix} = \begin{bmatrix}5\\0\\0\end{bmatrix}$$

— 04/06/2019

Si consideri il vettore $x=\begin{pmatrix}4\\0\\-3\end{pmatrix}\rightarrow\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$

a)

$$\alpha=\|x\|_2=\sqrt{4^2+(-3)^2}=\sqrt{16+9}=\sqrt{25}=5$$

$$u=x-\alpha e_1=\begin{pmatrix}4\\0\\-3\end{pmatrix}-5\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}-1\\0\\-3\end{pmatrix}$$

$$u^tu=(-1\ \ 0\ \ -3)\begin{pmatrix}-1\\0\\-3\end{pmatrix}=1+9=10$$

$$uu^t=\begin{pmatrix}-1\\0\\-3\end{pmatrix}(-1\ \ 0\ \ -3)=\begin{bmatrix}1&0&3\\0&0&0\\3&0&9\end{bmatrix}$$

$$P=I-\cancel{2}\cdot\frac{1}{\cancel{10}\,5}\cdot\begin{bmatrix}1&0&3\\0&0&0\\3&0&9\end{bmatrix}=I-\begin{bmatrix}1/5&0&3/5\\0&0&0\\3/5&0&9/5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}1/5&0&3/5\\0&0&0\\3/5&0&9/5\end{bmatrix}=\begin{bmatrix}4/5&0&-3/5\\0&1&0\\-3/5&0&-4/5\end{bmatrix}$$

b)

La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}4\\0\\-3\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$ con $\alpha=5$

c)

$P\cdot x=\alpha e_1$?

$$\begin{bmatrix}4/5&0&-3/5\\0&1&0\\-3/5&0&-4/5\end{bmatrix}\begin{bmatrix}4\\0\\-3\end{bmatrix}=\begin{bmatrix}(4/5)(4)+(-3/5)(-3)\\0\\(-3/5)(4)+(-4/5)(-3)\end{bmatrix}=\begin{bmatrix}16/5+9/5\\0\\-12/5+12/5\end{bmatrix}=\begin{bmatrix}25/5\\0\\0\end{bmatrix}=\begin{bmatrix}5\\0\\0\end{bmatrix}$$

— 19/09/2017

Si consideri il vettore $x=\begin{pmatrix}1\\2\\-2\end{pmatrix}\rightarrow\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$

a)

$$\alpha=\|x\|_2=\sqrt{1^2+2^2+(-2)^2}=\sqrt{1+4+4}=\sqrt{9}=3$$

$$u=x-\alpha e_1=\begin{pmatrix}1\\2\\-2\end{pmatrix}-3\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}-2\\2\\-2\end{pmatrix}$$

$$u^tu=(-2\ \ 2\ \ -2)\begin{pmatrix}-2\\2\\-2\end{pmatrix}=(-2)^2+(2)^2+(-2)^2=4+4+4=12$$

$$uu^t=\begin{pmatrix}-2\\2\\-2\end{pmatrix}(-2\ \ 2\ \ -2)=\begin{bmatrix}4&-4&4\\-4&4&-4\\4&-4&4\end{bmatrix}$$

$$P=I-\cancel{2}\cdot\frac{1}{\cancel{12}\,6}\cdot\begin{bmatrix}4&-4&4\\-4&4&-4\\4&-4&4\end{bmatrix}=I-\begin{bmatrix}4/6&-4/6&4/6\\-4/6&4/6&-4/6\\4/6&-4/6&4/6\end{bmatrix}=I-\begin{bmatrix}2/3&-2/3&2/3\\-2/3&2/3&-2/3\\2/3&-2/3&2/3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}2/3&-2/3&2/3\\-2/3&2/3&-2/3\\2/3&-2/3&2/3\end{bmatrix}=\begin{bmatrix}1/3&2/3&-2/3\\2/3&1/3&2/3\\-2/3&2/3&1/3\end{bmatrix}$$

b)

La riflessione di Householder P riflette il vettore $x=\begin{pmatrix}1\\2\\-2\end{pmatrix}$ rispetto al piano ortogonale al vettore $w=\frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix}\alpha\\0\\0\end{pmatrix}$ con $\alpha=3$

c)

$P\cdot x=\alpha e_1$?

$$\begin{bmatrix}1/3&2/3&-2/3\\2/3&1/3&2/3\\-2/3&2/3&1/3\end{bmatrix}\begin{bmatrix}1\\2\\-2\end{bmatrix}=\begin{bmatrix}(1/3)(1)+(2/3)(2)+(-2/3)(-2)\\(2/3)(1)+(1/3)(2)+(2/3)(-2)\\(-2/3)(1)+(2/3)(2)+(1/3)(-2)\end{bmatrix}=\begin{bmatrix}1/3+4/3+4/3\\2/3+2/3-4/3\\-2/3+4/3-2/3\end{bmatrix}=\begin{bmatrix}9/3\\4/3-4/3\\-4/3+4/3\end{bmatrix}=\begin{bmatrix}3\\0\\0\end{bmatrix}$$

04/07/2016

Si consideri il vettore $x = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \to \begin{pmatrix} \alpha \\ 0 \end{pmatrix}$

a)

$$\alpha = \|x\|_2 = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$u = x - \alpha_{e_1} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} - 5\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$$

$$u^t u = (-9 \quad 3)\begin{pmatrix} -9 \\ 3 \end{pmatrix} = 81 + 9 = 90$$

$$u u^t = \begin{pmatrix} -9 \\ 3 \end{pmatrix}(-9 \quad 3) = \begin{pmatrix} 81 & -27 \\ -27 & 9 \end{pmatrix}$$

$$P = I - 2 \cdot \frac{uu^t}{u^t u} = I - \cancel{2} \cdot \frac{1}{\cancel{90}_{45}} \cdot \begin{pmatrix} 81 & -27 \\ -27 & 9 \end{pmatrix} = I - \begin{pmatrix} 81/45 & -27/45 \\ -27/45 & 9/45 \end{pmatrix} = I - \begin{pmatrix} 9/5 & -3/5 \\ -3/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 9/5 & -3/5 \\ -3/5 & 1/5 \end{pmatrix} =$$

$$= \begin{pmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{pmatrix}$$

b) La riflessione di Householder P riflette il vettore $x = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ rispetto al piano ortogonale al vettore $w = \frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix} \alpha \\ 0 \end{pmatrix}$ con $\alpha = 5$

c) $P \cdot x = \alpha_{e_1}$?

$$\begin{bmatrix} -4/5 & 3/5 \\ 3/5 & 4/5 \end{bmatrix}\begin{bmatrix} -4 \\ 3 \end{bmatrix} = \begin{bmatrix} (-4/5)(-4) + (3/5)(3) \\ (3/5)(-4) + (4/5)(3) \end{bmatrix} = \begin{bmatrix} 16/5 + 9/5 \\ -12/5 + 12/5 \end{bmatrix} = \begin{bmatrix} 25/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

07/07/2014

Si consideri il vettore $x = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \to \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}$

a)

$$\alpha = \|x\|_2 = \sqrt{(2)^2 + (-1)^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$u = x - \alpha_{e_1} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$$

$$u^t u = (-1 \quad -1 \quad -2)\begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix} = 1 + 1 + 4 = 6$$

$$u u^t = \begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}(-1 \quad -1 \quad -2) = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix}$$

$$P = I - \cancel{2} \cdot \frac{1}{\cancel{6}_3} \cdot \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{pmatrix} = I - \begin{pmatrix} 1/3 & 1/3 & 2/3 \\ 1/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 4/3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1/3 & 1/3 & 2/3 \\ 1/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 4/3 \end{pmatrix} = \begin{pmatrix} 2/3 & -1/3 & -2/3 \\ -1/3 & 2/3 & -2/3 \\ -2/3 & -2/3 & -1/3 \end{pmatrix}$$

b) La riflessione di Householder P riflette il vettore $x = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ rispetto al piano ortogonale al vettore $w = \frac{u}{\|u\|_2}$ trasformando nella forma $\begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix}$ con $\alpha = 3$

c) $P \cdot x = \alpha_{e_1}$?

$$\begin{bmatrix} 2/3 & -1/3 & -2/3 \\ -1/3 & 2/3 & -2/3 \\ -2/3 & -2/3 & -1/3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} (2/3)(2) + (-1/3)(-1) + (-2/3)(-2) \\ (-1/3)(2) + (2/3)(-1) + (-2/3)(-2) \\ (-2/3)(2) + (-2/3)(-1) + (-1/3)(-2) \end{bmatrix} = \begin{bmatrix} 4/3 + 1/3 + 4/3 \\ -2/3 - 2/3 + 4/3 \\ -4/3 + 2/3 + 2/3 \end{bmatrix} = \begin{bmatrix} 9/3 \\ -4/3 + 4/3 \\ -4/3 + 4/3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$