minimi quadrati

20/06/2024

$g(x) = \alpha x^2 + \beta x + y$

x	-1	$-1/2$	0	$1/2$	1
y	7	4	-14	-4	7

a)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1/4 & -1/2 & 1 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 7 \\ 4 \\ -14 \\ -4 \\ 7 \end{bmatrix}$$

b)

A^2 = non calcolabile perchè la matrice A non è quadrata

$$AA^t = \begin{bmatrix} 1 & -1 & 1 \\ 1/4 & -1/2 & 1 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/4 & 0 & 1/4 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 7/4 & 1 & 3/4 & 1 \\ 7/4 & 21/16 & 1 & 13/16 & 3/4 \\ 1 & 1 & 1 & 1 & 1 \\ 3/4 & 13/16 & 1 & 21/16 & 7/4 \\ 1 & 3/4 & 1 & 7/4 & 3 \end{bmatrix}$$

$$A^tA = \begin{bmatrix} 1 & 1/4 & 0 & 1/4 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1/4 & -1/2 & 1 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 17/8 & 0 & 5/2 \\ 0 & 5/2 & 0 \\ 5/2 & 0 & 5 \end{bmatrix}$$

$$A^t y = \begin{bmatrix} 1 & 1/4 & 0 & 1/4 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 4 \\ -14 \\ -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 14 \\ -4 \\ 0 \end{bmatrix}$$

$$y^t A = \begin{bmatrix} 7 & 4 & -14 & -4 & 7 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1/4 & -1/2 & 1 \\ 0 & 0 & 1 \\ 1/4 & 1/2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -4 & 0 \end{bmatrix}$$

c)

tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1 a_1 + c_2 a_2 + c_3 a_3 y = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} 1 & -1 & 7 \\ 1/4 & -1/2 & 4 \\ 0 & 0 & -14 \\ 1/4 & 1/2 & -4 \\ 1 & 1 & 7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad c_1 \begin{bmatrix} 1 \\ 1/4 \\ 0 \\ 1/4 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ -1/2 \\ 0 \\ 1/2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 7 \\ 4 \\ -14 \\ -4 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} c_1 - c_2 + 7c_3 = 0 \\ 1/4\, c_1 - 1/2\, c_2 + 4c_3 = 0 \\ -14c_3 = 0 \\ 1/4\, c_1 + 1/2\, c_3 - 4c_3 = 0 \\ c_1 + c_2 + 7c_3 = 0 \end{cases} \Rightarrow \begin{cases} 2c_2 - c_2 = 0 \Rightarrow c_2 = 0 \\ 1/4\, c_1 = 1/2\, c_2 \Rightarrow c_1 = 2c_2 \\ c_3 = 0 \\ // \\ // \end{cases} \Rightarrow \begin{cases} c_2 = 0 \\ c_1 = 0 \\ c_3 = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d)

$$\begin{matrix} a_1^t \\ a_2^t \end{matrix} \begin{pmatrix} 1 & 1/4 & 0 & 1/4 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \\ \bar{y}_4 \\ \bar{y}_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Siccome $\bar{y}_3$ è una variabile libera, posso scegliere il vettore $\hat{y} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

e)

$$A^tA \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^t y \qquad \begin{pmatrix} 17/8 & 0 & 5/2 \\ 0 & 5/2 & 0 \\ 5/2 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 14 \\ -4 \\ 0 \end{pmatrix}$$

$$\begin{cases} 17/8\, x + 5/2\, z = 14 \\ 5/2\, y = -4 \\ 5/2\, x + 5z = 0 \end{cases} \Rightarrow \begin{cases} \cancel{17/8}^{\,1}(-\cancel{2}z) + 5/2\, z = 14 \Rightarrow -17/4\, z + 5/2\, z = 14 \Rightarrow -9/4\, z = 14 \Rightarrow z = \cancel{14}^{\,2}(-4/\cancel{7}) \Rightarrow z = -8 \\ y = -4 \cdot 2/5 \Rightarrow y = -8/5 \\ 5/2\, x = -5z \cdot 2/5 \Rightarrow x = -2z \Rightarrow x = -2(-8) \Rightarrow x = 16 \end{cases}$$

f)

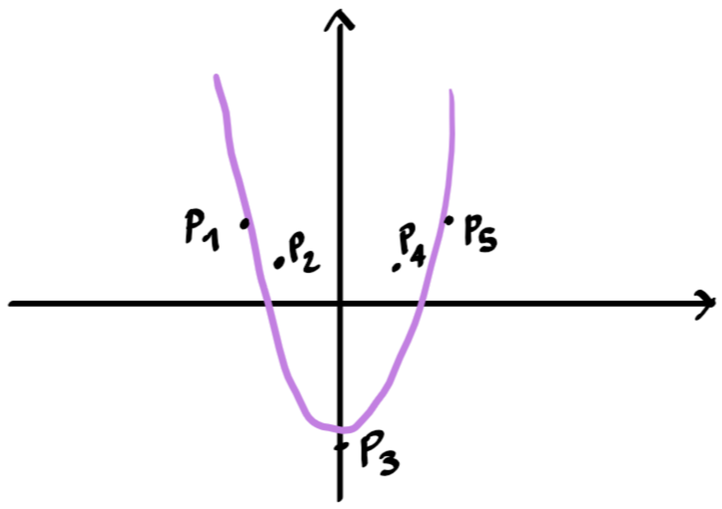
$x = 16$

$y = -8/5$

$z = -8$

$g(x) = \alpha x^2 + \beta x + y$

$= 16x^2 - 8/5\,x - 8$



19/07/2024

$g(x) = a\frac{6}{x} + bx + c$

x	-6	1	2	3
y	5	5	1	-1

a) $A = \begin{bmatrix} -1 & -6 & 1 \\ 6 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ $\quad y = \begin{bmatrix} 5 \\ 5 \\ 1 \\ -1 \end{bmatrix}$

b)

$A^tA = \begin{bmatrix} -1 & 6 & 3 & 2 \\ -6 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & -6 & 1 \\ 6 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 50 & 24 & 10 \\ 24 & 50 & 0 \\ 10 & 0 & 4 \end{bmatrix}$

$y A^t = \underset{4\cdot\underline{1}}{\begin{bmatrix} 5 \\ 5 \\ 1 \\ -1 \end{bmatrix}} \underset{\underline{3}\cdot 4}{\begin{bmatrix} -1 & 6 & 3 & 2 \\ -6 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$ = non possibile

$AA^t = \begin{bmatrix} -1 & -6 & 1 \\ 6 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 6 & 3 & 2 \\ -6 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 38 & -11 & -14 & -19 \\ -11 & 38 & 21 & 16 \\ -14 & 21 & 14 & 13 \\ -19 & 16 & 13 & 14 \end{bmatrix}$

$A^ty = \underset{3\cdot\underline{4}}{\begin{bmatrix} -1 & 6 & 3 & 2 \\ -6 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}} \underset{\underline{4}\cdot 1}{\begin{bmatrix} 5 \\ 5 \\ 1 \\ -1 \end{bmatrix}} = \begin{bmatrix} 26 \\ -26 \\ 10 \end{bmatrix}$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1 + c_2a_2 + c_3a_3\,y = 0$ è $c_1 = c_2 = c_3 = 0$

$M = \begin{bmatrix} -1 & -6 & 5 \\ 6 & 1 & 5 \\ 3 & 2 & 1 \\ 2 & 3 & -1 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{cases} -c_1 - 6c_2 + 5c_3 = 0 \\ 6c_1 + c_2 + 5c_3 = 0 \\ 3c_1 + 2c_2 + c_3 = 0 \\ 2c_1 + 3c_2 - c_3 = 0 \end{cases}$ $\begin{cases} -c_1 - 6(-6c_1 - 5c_3) + 5c_3 = 0 \\ c_2 = -6c_1 - 5c_3 \\ // \\ // \end{cases}$ $\begin{cases} -c_1 + 36c_1 + 30c_3 + 5c_3 = 0 \\ c_2 = -6c_1 - 5c_3 \\ // \\ // \end{cases}$ $\begin{cases} 35c_1 + 35c_3 = 0 \\ c_2 = -6c_1 - 5c_3 \\ // \\ // \end{cases}$

$\begin{cases} c_1 = -c_3 \\ c_2 = -6(-c_3) - 5c_3 \\ // \\ // \end{cases}$ $\begin{cases} c_1 = -c_3 \\ c_2 = 6c_3 - 5c_3 \\ // \\ // \end{cases}$ $\begin{cases} c_1 = -c_3 \\ c_2 = c_3 \\ 3(-c_3) + 2(c_3) + c_3 = 0 \\ // \end{cases}$ $\begin{cases} c_1 = -c_3 \\ c_2 = c_3 \\ -3c_3 + 2c_3 + c_3 = 0 \\ // \end{cases}$ $\begin{cases} c_1 = -c_3 \\ c_2 = c_3 \\ c_3 = 0 \\ 0 = 0 \end{cases}$

$\begin{cases} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \\ 0 = 0 \end{cases}$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d)

$\begin{bmatrix} -1 \\ 6 \\ 3 \\ 2 \end{bmatrix} + \begin{bmatrix} -6 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 7 \\ 5 \\ 5 \end{bmatrix} \rightarrow$ vettore $\hat{y}$ è linearmente dipendente da a_1 e a_2 e diverso da loro

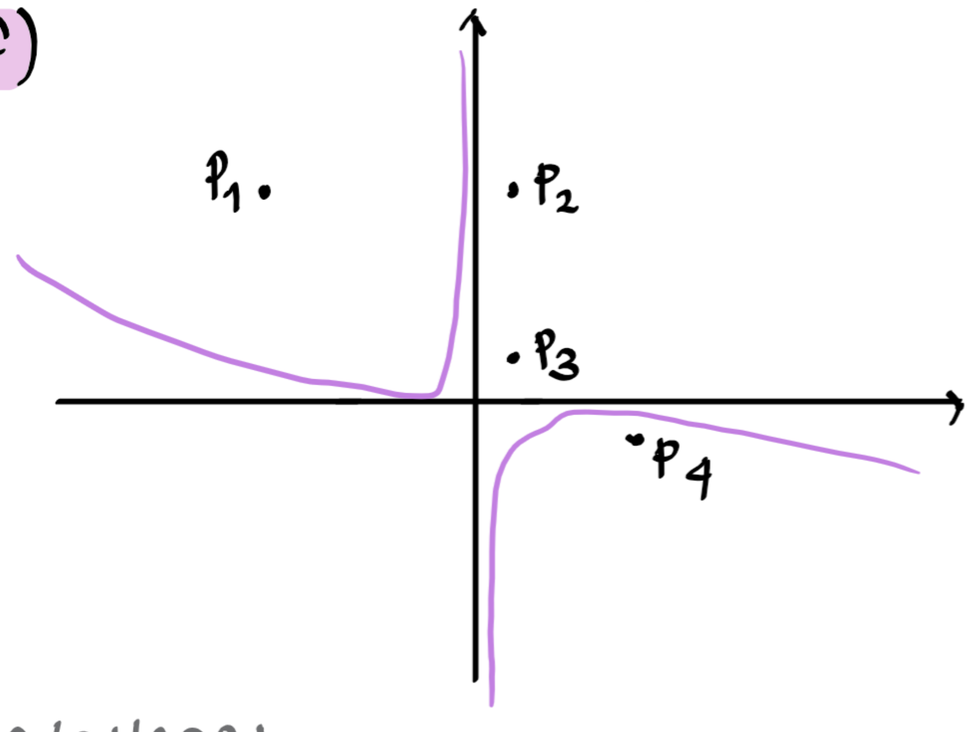
e) $A^tA\begin{pmatrix}a\\b\\c\end{pmatrix}=A^ty \qquad \begin{pmatrix}50&24&10\\24&50&0\\10&0&4\end{pmatrix}=\begin{pmatrix}26\\-26\\10\end{pmatrix}$

$\begin{cases}50a+24b+10c=26\\24a+50b=-26\\10a+4c=10\end{cases}\begin{cases}//\\//\\10a=10-4c\end{cases}\begin{cases}50(3/5\,c)+24b+10c=26\\//\\a=3/5\,c\end{cases}\begin{cases}30c+24b+10c=26\\//\\a=3/5c\end{cases}$

$\begin{cases}40c+24b=26\\//\\a=3/5c\end{cases}\begin{cases}40c=26-24b\\//\\a=3/5c\end{cases}\begin{cases}c=1/20\,b\\//\\a=3/5c\end{cases}\begin{cases}c=1/20\,b\\//\\a=3/5(1/20\,b)\end{cases}\begin{cases}c=1/20\,b\\24(3/100b)+50b=-26\\a=3/100\,b\end{cases}$

$\begin{cases}c=1/20\,b\\18/25\,b+50b=-26\\a=3/100\,b\end{cases}\begin{cases}c=1/20\,b\\1268/25b=-26\\a=3/100\,b\end{cases}\begin{cases}c=1/20\,b\\b=-325/634\\a=3/100\,b\end{cases}\begin{cases}c=-65/2536\\b=-325/634\\a=-39/2536\end{cases}\begin{cases}c=-0{,}03\\b=-0{,}5\\a=-0{,}02\end{cases}$

f)



$g(x)=(-0{,}02)\frac{6}{x}+(-0{,}5)x+(-0{,}03)$

22/01/2024

$g(x)=\alpha x+\beta$

x	-4	-1	0	2	3
y	3	5	-2	5	2

a)

$A=\begin{bmatrix}-4&1\\-1&1\\0&1\\2&1\\3&1\end{bmatrix} \qquad y=\begin{bmatrix}3\\5\\-2\\5\\2\end{bmatrix}$

b)

$AA^t=\begin{bmatrix}-4&1\\-1&1\\0&1\\2&1\\3&1\end{bmatrix}\begin{bmatrix}-4&-1&0&2&3\\1&1&1&1&1\end{bmatrix}=\begin{bmatrix}17&5&1&-7&-11\\5&2&1&-1&-2\\1&1&1&1&1\\-7&-1&1&5&7\\-11&-2&1&7&10\end{bmatrix}$

$A^tA=\begin{bmatrix}-4&-1&0&2&3\\1&1&1&1&1\end{bmatrix}\begin{bmatrix}-4&1\\-1&1\\0&1\\2&1\\3&1\end{bmatrix}=\begin{bmatrix}30&0\\0&5\end{bmatrix}$

$yA^t=\underset{5\cdot 1}{\begin{bmatrix}3\\5\\-2\\5\\2\end{bmatrix}}\underset{5\cdot 2}{\begin{bmatrix}-4&-1&0&2&3\\1&1&1&1&1\end{bmatrix}}=$ non possibile $\qquad A^ty=\underset{2\cdot 5}{\begin{bmatrix}-4&-1&0&2&3\\1&1&1&1&1\end{bmatrix}}\underset{5\cdot 1}{\begin{bmatrix}3\\5\\-2\\5\\2\end{bmatrix}}=\begin{bmatrix}-1\\13\end{bmatrix}$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1+c_2a_2+c_3a_3\,y=0$ è $c_1=c_2=c_3=0$

$M=\begin{bmatrix}-4&1&3\\-1&1&5\\0&1&-2\\2&1&5\\3&1&2\end{bmatrix}\quad x\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$

$$\begin{cases} -4c_1 + c_2 + 3c_3 = 0 \\ -c_1 + c_2 + 5c_3 = 0 \\ c_2 - 2c_3 = 0 \\ 2c_1 + c_2 + 5c_3 = 0 \\ 3c_1 + c_2 + 2c_3 = 0 \end{cases} \quad \begin{cases} -4c_1 + 2c_3 + 3c_3 = 0 \\ // \\ c_2 = 2c_3 \\ // \\ // \end{cases} \quad \begin{cases} -4c_1 + 5c_3 = 0 \\ // \\ c_2 = 2c_3 \\ // \\ // \end{cases} \quad \begin{cases} c_1 = -5c_3 + 4 \\ // \\ c_2 = 2c_3 \\ // \\ // \end{cases} \quad \begin{cases} c_1 = c_3 \\ -c_3 + 2c_3 + 5c_3 = 0 \\ c_2 = 2c_3 \\ // \\ // \end{cases}$$

$$\begin{cases} c_1 = c_3 \\ 6c_3 = 0 \\ c_2 = 2c_3 \\ // \\ // \end{cases} \quad \begin{cases} c_1 = c_3 \\ c_3 = 0 \\ c_2 = 2c_3 \\ // \\ // \end{cases} \quad \begin{cases} c_1 = 0 \\ c_3 = 0 \\ c_2 = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d)

$$\begin{bmatrix} -4 \\ -1 \\ 0 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

Un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1 a_1 + k_2 a_2$

Scegliamo quindi $k_1 = k_2 = 1$

d)

$$\begin{pmatrix} -4 & -1 & 0 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \bar{y}_3 \\ \bar{y}_4 \\ \bar{y}_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -4y_1 - y_2 + 2y_4 + 3y_5 = 0 \\ y_1 + y_2 + y_3 + y_4 + y_5 = 0 \end{cases} \quad \ldots$$

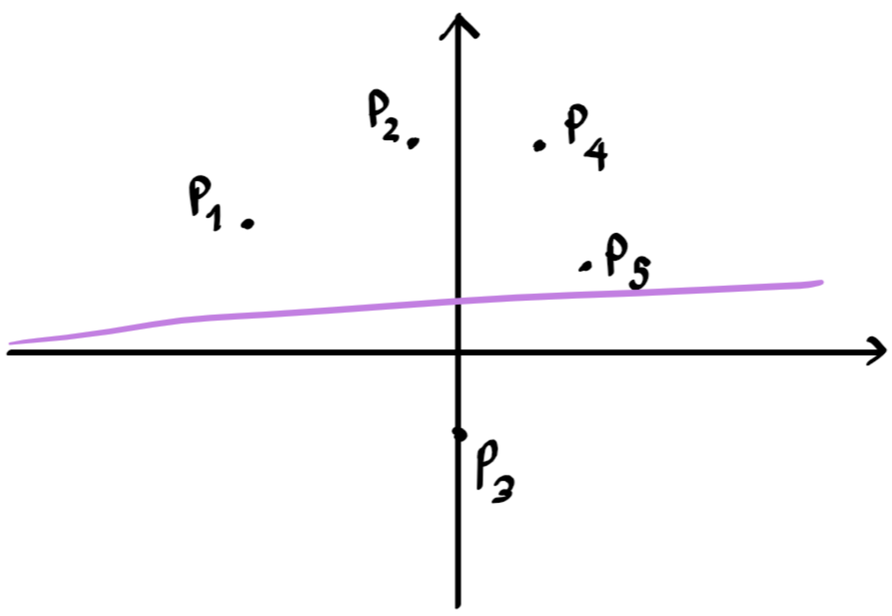
e)

$$A^t A \begin{pmatrix} a \\ b \end{pmatrix} = A^t y \qquad \begin{pmatrix} 30 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 13 \end{pmatrix}$$

$$\begin{cases} 30a = -1 \\ 5b = 13 \end{cases} \quad \begin{cases} a = -1 \cdot 1/30 \\ b = 13 \cdot 1/5 \end{cases} \quad \begin{cases} a = -1/30 \\ b = 13/5 \end{cases} \quad \begin{cases} a = -0{,}03 \\ b = 2{,}6 \end{cases}$$

f)

$$g(x) = -0{,}03x + 2{,}6$$



12/02/2024

$$g(x) = \alpha x + \beta$$

x	-2	-1	0	1	2	4
y	0	0	-10	2	2	4

a)

$$A = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 4 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 2 \\ 2 \\ 4 \end{bmatrix}$$

b)

$$AA^t = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 & 0 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 1 & -1 & -3 & -7 \\ 3 & 2 & 1 & 0 & -1 & -3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 2 & 3 & 5 \\ -3 & -1 & 1 & 3 & 5 & 9 \\ -7 & -3 & 1 & 5 & 9 & 17 \end{bmatrix}$$

$$A^tA = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 26 & 4 \\ 4 & 6 \end{bmatrix}$$

$$yA^t = \begin{bmatrix} 0 \\ 0 \\ -10 \\ 2 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} -2 & -1 & 0 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \text{non possibile}$$

$5 \cdot \underline{1}$ $\quad$ $\underline{2} \cdot 5$

$$A^ty = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -10 \\ 2 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 22 \\ -2 \end{bmatrix}$$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1 + c_2a_2 + c_3a_3 y = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} -2 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & -10 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2c_1 + c_2 = 0 \\ -c_1 + c_2 = 0 \\ c_2 - 10c_3 = 0 \\ c_1 + c_2 + 2c_3 = 0 \\ 2c_1 + c_2 + 2c_3 = 0 \\ 4c_1 + c_2 + 4c_3 = 0 \end{cases} \begin{cases} // \\ c_2 = c_1 \\ // \\ c_1 + c_1 + 2c_3 = 0 \\ // \\ // \end{cases} \begin{cases} // \\ c_2 = c_1 \\ // \\ 2c_3 = -2c_1 \\ // \\ // \end{cases} \begin{cases} // \\ c_2 = c_1 \\ // \\ c_3 = -c_1 \\ 2c_1 + c_1 - 2c_1 = 0 \\ // \end{cases} \begin{cases} // \\ c_2 = c_1 \\ // \\ c_3 = -c_1 \\ c_1 = 0 \\ // \end{cases} \begin{cases} 0 = 0 \\ c_2 = 0 \\ 0 = 0 \\ c_3 = 0 \\ c_1 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

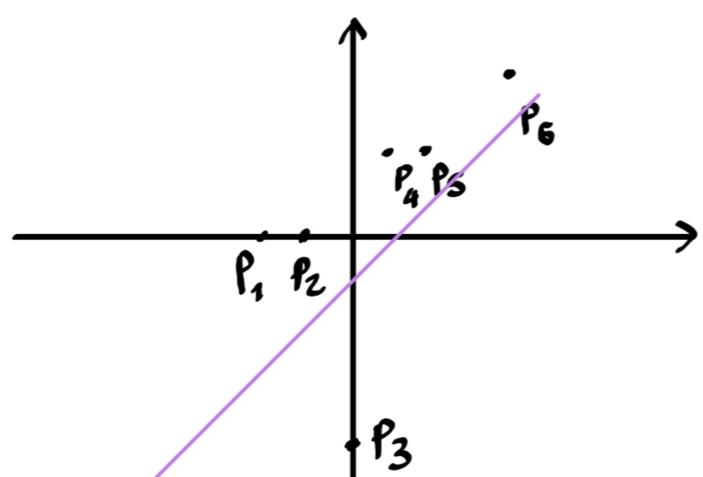
d) un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1a_1 + k_2a_2$. Scegliamo quindi $k_1 = k_2 = 1$.

$$\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

e) $A^tA \begin{pmatrix} a \\ b \end{pmatrix} = A^ty \qquad \begin{pmatrix} 26 & 4 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 22 \\ -2 \end{pmatrix}$

$$\begin{cases} 26a + 4b = 22 \\ 4a + 6b = -2 \end{cases} \begin{cases} 26a + 4b = 22 \\ 6b = -2 - 4a \end{cases} \begin{cases} 26a + 4b = 22 \\ b = -\frac{6}{6}a \end{cases} \begin{cases} 26a - 4a = 22 \\ b = -a \end{cases} \begin{cases} 22a = 22 \\ b = -a \end{cases} \begin{cases} a = 1 \\ b = -1 \end{cases}$$

f) $g(x) = x - 1$



13/09/2024

$g(x) = \alpha\sqrt{x} + \beta$

x	0	1	4	9
y	-1	0	1	2

a)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

b)

$$A^tA = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} \qquad AA^t = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{bmatrix}$$

$$yA^t = \underset{4\cdot 1}{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}}\underset{2\cdot 4}{\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}} = \text{non possibile} \qquad A^ty = \underset{2\cdot 4}{\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}}\underset{4\cdot 1}{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

c)

tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1 + c_2a_2 + c_3a_3\, y = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} c_2 - c_3 = 0 \\ c_1 + c_2 = 0 \\ 2c_1 + c_2 + c_3 = 0 \\ 3c_1 + c_2 + 2c_3 = 0 \end{cases} \begin{cases} c_2 = c_3 \\ c_1 = -c_2 \\ -2c_3 + c_3 + c_3 = 0 \\ -3c_3 + c_3 + 2c_3 = 0 \end{cases} \begin{cases} c_2 = c_3 \\ c_1 = -c_2 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d)

Un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1a_1 + k_2a_2$. Scegliamo quindi $k_1 = k_2 = 1$.

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

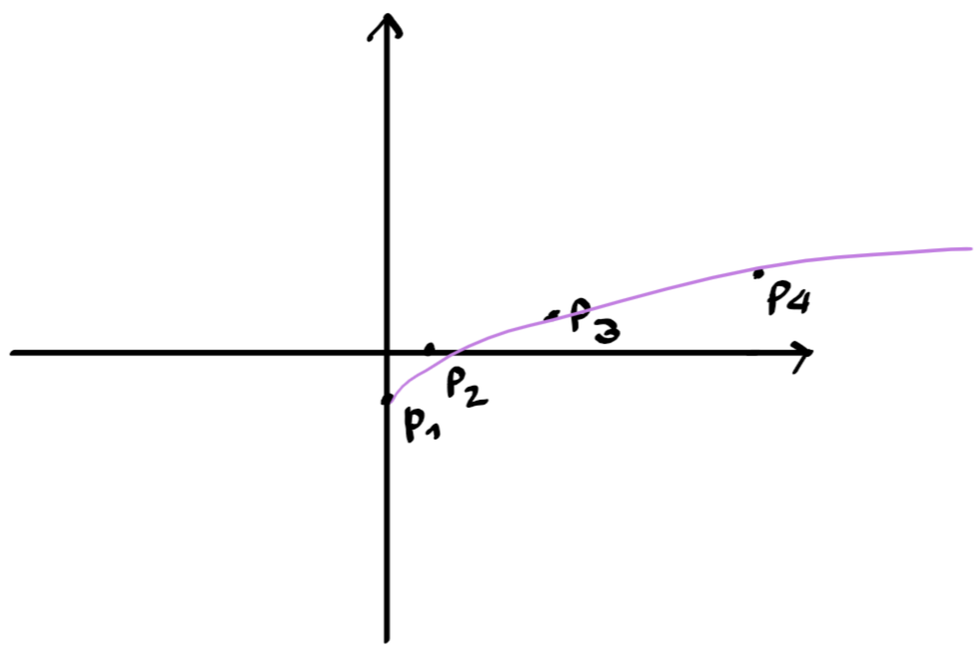
e)

$$A^tA\begin{pmatrix} a \\ b \end{pmatrix} = A^ty \qquad \begin{pmatrix} 14 & 6 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$

$$\begin{cases} 14a + 6b = 8 \\ 6a + 4b = 2 \end{cases} \begin{cases} 14a + 6b = 8 \\ 6a = 2 - 4b \end{cases} \begin{cases} -14b + 6b = 8 \\ a = -b \end{cases} \begin{cases} -8b = 8 \\ a = -b \end{cases} \begin{cases} b = -1 \\ a = 1 \end{cases}$$

f)

$g(x) = \sqrt{x} - 1$



23/01/2023

$g(x) = \alpha|x-1| + \beta$

x	-1	0	1	2	3
y	1	1	1	0	1

a)

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

b)

$$A^tA = \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 6 & 5 \end{bmatrix} \qquad AA^t = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 1 & 3 & 5 \\ 3 & 2 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 2 & 3 \\ 5 & 3 & 1 & 3 & 5 \end{bmatrix}$$

$$yA^t = \underset{5\cdot 1}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}} \underset{2\cdot 5}{\begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}} = \text{non possibile} \qquad A^ty = \begin{bmatrix} 2 & 1 & 0 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1 + c_2a_2 + c_3a_3 y = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 2c_1 + c_2 + c_3 = 0 \\ c_1 + c_2 + c_3 = 0 \\ c_2 + c_3 = 0 \\ c_1 + c_2 = 0 \\ 2c_1 + c_2 + c_3 = 0 \end{cases} \quad \begin{cases} // \\ c_3 - c_3 + c_3 = 0 \\ c_2 = -c_3 \\ c_1 = -c_2 \\ // \end{cases} \quad \begin{cases} 0 = 0 \\ c_3 = 0 \\ c_2 = 0 \\ c_1 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d) un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1a_1 + k_2a_2$. Scegliamo quindi $k_1 = k_2 = 1$.

$$\begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

e) $A^tA \begin{pmatrix} a \\ b \end{pmatrix} = A^ty \qquad \begin{pmatrix} 10 & 6 \\ 6 & 5 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

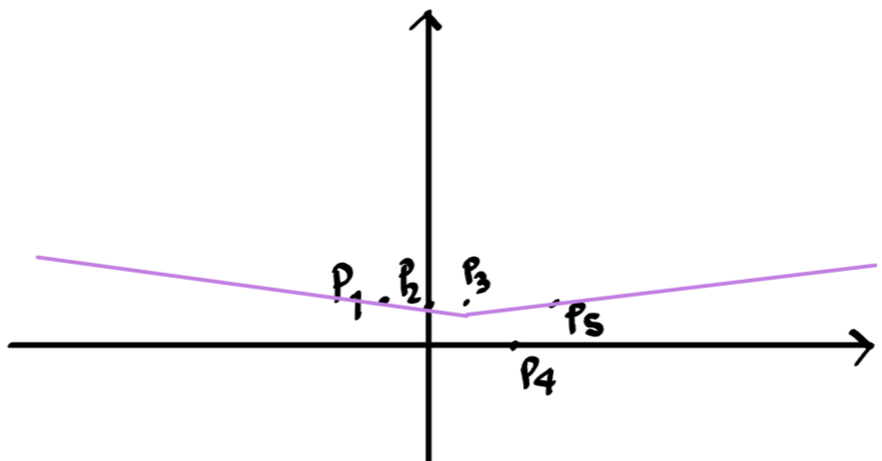
$$\begin{cases} 10a + 6b = 5 \\ 6a + 5b = 4 \end{cases} \begin{cases} a = \frac{5-6b}{10} \\ 6\cdot\left(\frac{5-6b}{10}\right) + 5b = 4 \end{cases} \begin{cases} // \\ \frac{30}{10} - \frac{36b}{10} + 5b = 4 \end{cases} \begin{cases} // \\ 3 + \frac{50b - 36b}{10} = 4 \end{cases} \begin{cases} // \\ 3 + \frac{14b}{10} = 4 \end{cases}$$

$$\begin{cases} // \\ \frac{14}{10}b = 4 - 3 \end{cases} \begin{cases} // \\ b = 10/14 = 5/7 \end{cases} \begin{cases} // \\ b = 5/7 \end{cases} \begin{cases} 10a + 6(5/7) = 5 \\ b = 5/7 \end{cases} \begin{cases} 10a = 5 - 30/7 \\ b = 5/7 \end{cases}$$

$$\begin{cases} a = 5/7 \cdot 1/10 \\ b = 5/7 \end{cases} \begin{cases} a = 5/70 \\ b = 5/7 \end{cases} \begin{cases} a = 1/14 \\ b = 5/7 \end{cases}$$

f)

$$g(x) = 1/14\,|x-1| + 5/7$$



13/02/2023

$g(x) = \alpha x + \beta$

x	-3	-2	-1	0	1	2	3
y	-9	-9	-3	-1	2	5	8

a)

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} -9 \\ -9 \\ -3 \\ -1 \\ 2 \\ 5 \\ 8 \end{bmatrix}$$

b)

$$A^tA = \begin{bmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 28 & 0 \\ 0 & 7 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 7 & 4 & 1 & -2 & -5 & -8 \\ 7 & 5 & 3 & 1 & -1 & -3 & -5 \\ 4 & 3 & 2 & 1 & 0 & -1 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 & 3 & 4 \\ -5 & -3 & -1 & 1 & 3 & 5 & 7 \\ -8 & -5 & -2 & 1 & 4 & 7 & 10 \end{bmatrix}$$

$$yA^t = \begin{bmatrix} -9 \\ -9 \\ -3 \\ -1 \\ 2 \\ 5 \\ 8 \end{bmatrix} \begin{bmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \text{non possibile} \qquad A^ty = \begin{bmatrix} -3 & -2 & -1 & 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -9 \\ -9 \\ -3 \\ -1 \\ 2 \\ 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 84 \\ -7 \end{bmatrix}$$

$2 \cdot 7$

$7 \cdot 1$

c)

tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1 a_1 + c_2 a_2 + c_3 a_3 y = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} -3 & 1 & -9 \\ -2 & 1 & -9 \\ -1 & 1 & -3 \\ 0 & 1 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 5 \\ 3 & 1 & 8 \end{bmatrix} \qquad x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -3c_1 + c_2 - 9c_3 = 0 \\ -2c_1 + c_2 - 9c_3 = 0 \\ -c_1 + c_2 - 3c_3 = 0 \\ c_2 - c_3 = 0 \\ c_1 + c_2 + 2c_3 = 0 \\ 2c_1 + c_2 + 5c_3 = 0 \\ 3c_1 + c_2 + 8c_3 = 0 \end{cases} \quad \begin{cases} // \\ // \\ // \\ c_2 = c_3 \\ c_1 + c_2 + 2c_2 = 0 \\ // \\ // \end{cases} \quad \begin{cases} // \\ // \\ // \\ c_2 = c_3 \\ c_1 = -3c_2 \\ // \\ // \end{cases} \quad \begin{cases} // \\ // \\ 3c_3 + c_3 - 3c_3 = 0 \\ c_2 = c_3 \\ c_1 = -3c_2 \\ // \\ // \end{cases} \quad \begin{cases} 0 = 0 \\ 0 = 0 \\ c_3 = 0 \\ c_2 = 0 \\ c_1 = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d)

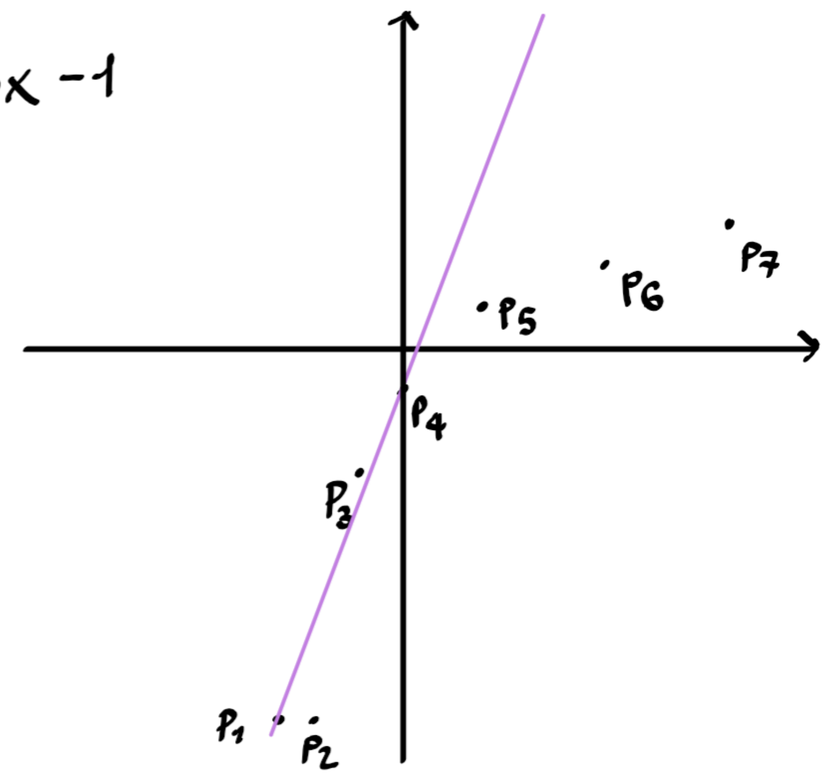
Un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1 a_1 + k_2 a_2$. Scelgo quindi $k_1 = k_2 = 1$.

$$\begin{bmatrix} -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

e) $A^tA\begin{pmatrix}a\\b\end{pmatrix}=A^ty \qquad \begin{pmatrix}28&0\\0&7\end{pmatrix}=\begin{bmatrix}84\\-7\end{bmatrix}$

$\begin{cases}28a=84\\7b=-7\end{cases} \begin{cases}a=3\\b=-1\end{cases}$

f) $g(x)=3x-1$



22/06/2023

$g(x)=\alpha x+\beta$

x	-2	-1	0	1	3	4
y	-2	-8	0	2	5	7

a)

$$A=\begin{bmatrix}-2&1\\-1&1\\0&1\\1&1\\3&1\\4&1\end{bmatrix} \qquad y=\begin{bmatrix}-2\\-8\\0\\2\\5\\7\end{bmatrix}$$

b)

$$A^tA=\begin{bmatrix}-2&-1&0&1&3&4\\1&1&1&1&1&1\end{bmatrix}\begin{bmatrix}-2&1\\-1&1\\0&1\\1&1\\3&1\\4&1\end{bmatrix}=\begin{bmatrix}31&5\\5&6\end{bmatrix}$$

$$AA^t=\begin{bmatrix}-2&1\\-1&1\\0&1\\1&1\\3&1\\4&1\end{bmatrix}\begin{bmatrix}-2&-1&0&1&3&4\\1&1&1&1&1&1\end{bmatrix}=\begin{bmatrix}5&3&1&-1&-5&-7\\3&2&1&0&-2&-3\\1&1&1&1&1&1\\-1&0&1&2&4&5\\-5&-2&1&4&10&13\\-7&-3&1&5&13&17\end{bmatrix}$$

$$yA^t=\begin{bmatrix}-2\\-8\\0\\2\\5\\7\end{bmatrix}\begin{bmatrix}-2&-1&0&1&3&4\\1&1&1&1&1&1\end{bmatrix}=\text{non possibile} \qquad A^ty=\begin{bmatrix}-2&-1&0&1&3&4\\1&1&1&1&1&1\end{bmatrix}\begin{bmatrix}-2\\-8\\0\\2\\5\\7\end{bmatrix}=\begin{bmatrix}57\\4\end{bmatrix}$$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1a_1+c_2a_2+c_3a_3y=0$ è $c_1=c_2=c_3=0$

$$M=\begin{bmatrix}-2&1&-2\\-1&1&-8\\0&1&0\\1&1&2\\3&1&5\\4&1&7\end{bmatrix} \qquad x=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

$$\begin{cases}-2c_1+c_2-2c_3=0\\-c_1+c_2-8c_3=0\\c_2=0\\c_1+c_2+2c_3=0\\3c_1+c_2+5c_3=0\\4c_1+c_2+7c_3=0\end{cases} \begin{cases}//\\-c_3-2c_3=0\\c_2=0\\c_1=1/2c_3\\//\\//\end{cases} \begin{cases}//\\c_3=0\\c_2=0\\c_1=1/2c_3\\//\\//\end{cases} \begin{cases}0=0\\c_3=0\\c_2=0\\c_1=0\\0=0\\0=0\end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1=c_2=c_3=0$ e quindi i tre vettori sono linearmente indipendenti

d) Un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1 a_1 + k_2 a_2$. Scegliamo quindi $k_1 = k_2 = 1$.

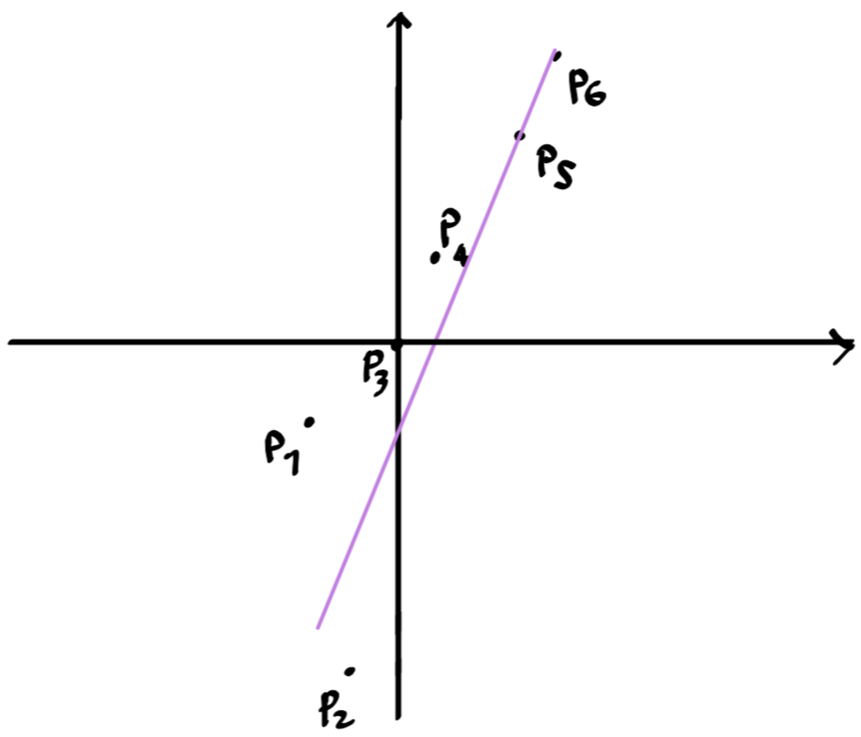
$$\begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \\ 4 \\ 5 \end{bmatrix}$$

e) $A^t A \begin{pmatrix} a \\ b \end{pmatrix} = A^t y \qquad \begin{pmatrix} 31 & 5 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 57 \\ 4 \end{pmatrix}$

$$\begin{cases} 31a + 5b = 57 \\ 5a + 6b = 4 \end{cases} \begin{cases} 31a = 57 - 5b \\ // \end{cases} \begin{cases} a = \frac{57}{31} - \frac{5}{31} b \\ 5\left(\frac{57}{31} - \frac{5}{31} b\right) + 6b = 4 \end{cases} \begin{cases} // \\ \frac{285}{31} - \frac{25}{31} b + 6b = 4 \end{cases} \begin{cases} // \\ \frac{285}{31} + \frac{161}{31} b = 4 \end{cases}$$

$$\begin{cases} // \\ 285 + 161b = 124 \end{cases} \begin{cases} // \\ 161b = 124 - 285 \end{cases} \begin{cases} // \\ 161b = -161 \end{cases} \begin{cases} a = \frac{57}{31} + \frac{5}{31} \\ b = -1 \end{cases} \begin{cases} a = 2 \\ b = -1 \end{cases}$$

f) $g(x) = 2x - 1$



21/07/2023

$g(x) = \alpha |x| + \beta$

x	-3	-2	-1	0	1	4
y	0	2	4	2	3	6

)

$$M = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \qquad y = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 2 \\ 3 \\ 6 \end{bmatrix}$$

b)

$$A^t A = \begin{bmatrix} 3 & 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 31 & 11 \\ 11 & 6 \end{bmatrix}$$

$$A A^t = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 7 & 4 & 1 & 4 & 13 \\ 7 & 5 & 3 & 1 & 3 & 9 \\ 4 & 3 & 2 & 1 & 2 & 5 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 & 2 & 5 \\ 13 & 9 & 5 & 1 & 5 & 17 \end{bmatrix}$$

$$y A^t = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 2 \\ 3 \\ 6 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \text{non possibile} \qquad A^t y = \begin{bmatrix} 3 & 2 & 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 4 \\ 2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 35 \\ 17 \end{bmatrix}$$

c) tre vettori sono linearmente indipendenti se l'unica soluzione dell'equazione $c_1 a_1 + c_2 a_2 + c_3 a_3 = 0$ è $c_1 = c_2 = c_3 = 0$

$$M = \begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 2 \\ 1 & 1 & 4 \\ 0 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 1 & 6 \end{bmatrix} \qquad X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} 3c_1 + c_2 + 6c_3 = 0 \\ 2c_1 + c_2 + 6c_3 = 0 \\ c_1 + c_2 + 4c_3 = 0 \\ c_2 + 2c_3 = 0 \\ c_1 + c_2 + 3c_3 = 0 \\ 4c_1 + c_2 + 6c_3 = 0 \end{cases} \begin{cases} // \\ // \\ c_1 - 2c_3 + 4c_3 = 0 \\ c_2 = -2c_3 \\ // \\ // \end{cases} \begin{cases} -6c_3 - 2c_3 + 6c_3 = 0 \\ // \\ c_1 = -2c_3 \\ c_2 = -2c_3 \\ // \\ // \end{cases} \begin{cases} 2c_3 = 0 \\ // \\ c_1 = -2c_3 \\ c_2 = -2c_3 \\ // \\ // \end{cases} \begin{cases} c_3 = 0 \\ 0 = 0 \\ c_1 = 0 \\ c_2 = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

Come vediamo dal sistema che abbiamo ottenuto $c_1 = c_2 = c_3 = 0$ e quindi i tre vettori sono linearmente indipendenti

d) un vettore $\bar{y}$ è linearmente dipendente se esistono scalari k_1 e k_2 tali che $\bar{y} = k_1 a_1 + k_2 a_2$. Scelgiamo quindi $k_1 = k_2 = 1$.

$$\begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 2 \\ 5 \end{bmatrix}$$

e) $A^t A \begin{pmatrix} a \\ b \end{pmatrix} = A^t y \qquad \begin{pmatrix} 31 & 11 \\ 11 & 6 \end{pmatrix} = \begin{pmatrix} 35 \\ 17 \end{pmatrix}$

$$\begin{cases} 31a + 11b = 35 \\ 11a + 6b = 17 \end{cases}$$

... non ne ho più voglia

frase riassunto per punto f)

La frazione $g(x) =$ *nuova frazione* approssima ai minimi quadrati i dati riportati, minimizzando la somma degli scarti che assume e i dati y