EBERHARD KARLS UNIVERSITÄT TÜBINGEN
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MASTER THESIS

Master thesis

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Chapter 1

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1.1 Derivation of the Lindblad equation from microscopic dynamics

The most common derivation of the Lindblad master equation is based on open quantum theory. The Lindblad equation is then an effective motion equation for a subsystem that belongs to a more complicated system. This derivation can be found in several textbooks such as Breuer and Petruccione [1] A total system belonging to a Hilbert space $\mathcal{H}_T$ is divided into our system of interest, belonging to a Hilbert space $\mathcal{H}_S$, and the environment living in $\mathcal{H}_E$.

The evolution of the total system is given by the von Neumann equation,

$$\dot{\rho}_T(t) = -i[H_T, \rho_T(t)]. \tag{1.1}$$

As we are interested in the dynamics of the system without the environment, we trace over the environment degrees of freedom to obtain the reduced density matrix of the system $\rho(t) = \mathrm{Tr}_E[\rho_T]$. The total Hamiltonian can be separated as

Step 1: Interaction picture

$$H_T = H_S \otimes \mathbb{1}_E + \mathbb{1}_S \otimes H_E + \alpha H_I, \tag{1.2}$$

where $H_S \in \mathcal{H}_S$, $H_E \in \mathcal{H}_E$, and $H_I \in \mathcal{H}_T$ represents the interaction between the system and the environment with coupling strength α. The interaction term is typically decomposed as

$$H_I = \sum_i S_i \otimes E_i, \tag{1.3}$$

where $S_i \in \mathcal{B}(\mathcal{H}_S)$ and $E_i \in \mathcal{B}(\mathcal{H}_E)$.

To describe the system dynamics, we move to the interaction picture where the operators evolve with respect to $H_S + H_E$,

$$\hat{O}(t) = e^{i(H_S+H_E)t} O e^{-i(H_S+H_E)t}. \tag{1.4}$$

The time evolution in the interaction picture is given by

$$\dot{\hat{\rho}}_T(t) = -i\alpha[\hat{H}_I(t), \hat{\rho}_T(t)], \tag{1.5}$$

Step 2: Expand which can be formally integrated as

$$\hat{\rho}_T(t) = \hat{\rho}_T(0) - i\alpha \int_0^t ds [\hat{H}_I(s), \hat{\rho}_T(s)]. \tag{1.6}$$

$$\dot{\hat{\rho}}_T(t) = -i\alpha \left[\hat{H}_I(t), \hat{\rho}_T(0)\right] - \alpha^2 \int_0^t \left[\hat{H}_I(t), \left[\hat{H}_I(t'), \hat{\rho}_T(t')\right]\right] dt'. \tag{1.7}$$

$$\int_t^{t'} ds \dot{\hat{\rho}}_T(s) = -i \int_t^{t'} \left[\hat{H}_I(s), \hat{\rho}_T(s)\right] ds, \tag{1.8}$$

$$\hat{\rho}_T(t') - \hat{\rho}_T(t) = -i\alpha \int_t^{t'} \left[\hat{H}_I(s), \hat{\rho}_T(s)\right] ds. \tag{1.9}$$

$$\dot{\hat{\rho}}_T(t) = -i\alpha \left[\hat{H}_I(t), \hat{\rho}_T(0)\right] - \alpha^2 \int_0^t \left[\hat{H}_I(t), \left[\hat{H}_I(t'), \hat{\rho}_T(t)\right]\right] + \mathcal{O}(\alpha^3). \tag{1.10}$$

Step 3: Partial trace

$$\dot{\rho}_S(t) = -i\alpha[\hat{H}_I(t), \hat{\rho}_T(0)] - \alpha^2 \int_0^t ds \mathrm{Tr}_E[\hat{H}_I(t), [\hat{H}_I(s), \rho_S(t) \otimes \rho_E]]. \tag{1.11}$$

Approximation

$$\hat{\rho}_T(0) = \hat{\rho}_S(0) \otimes \hat{\rho}_E(0) \tag{1.12}$$

using eq.

$$\sum_i \mathrm{Tr}_E[S_i \otimes E_i, \hat{\rho}_S(0) \otimes \hat{\rho}_E(0)] = \sum_i (S_i \hat{\rho}_S(0) - \hat{\rho}_S(0) S_i) \cdot \mathrm{Tr}_E[E_i \hat{\rho}_E(0)] \tag{1.13}$$

Argue that

$$< E_i >_E = \mathrm{Tr}_E[E_i \hat{\rho}_E(0)] \tag{1.14}$$

with new Hamiltonian ...

$$\hat{H}'_S = \hat{H}_S + \sum_i S_i \otimes (E_i - < E_i >_E) \tag{1.15}$$

TODO FINISH this derivation

Appendix A

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Bibliography

[1] Heinz-Peter Breuer and Francesco Petruccione. *The theory of open quantum systems*. 1. publ. in paperback, [Nachdr.] Oxford: Clarendon Press, 2009. 613 pp. ISBN: 978-0-19-852063-4 978-0-19-921390-0.