Bayesian Networks

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Introduction

A *Bayesian network* is a graphical model used for the analysis of probabilistic relationship between events, represented by variables.

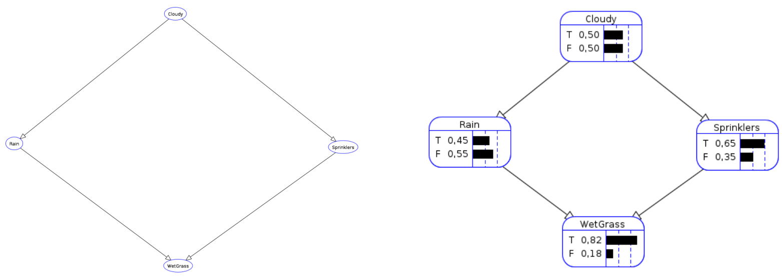
Every variable is sketched by a node and the nodes are connected by arrows, representing the relationship between the variables.

In this report, in particular, there will be analysed three problems with the help of *Belief and decision network Tool*:

- weather problem;
- coins problem;
- fire alarm problem.

1 Weather problem

The Bayesian network corresponding to the problem is shown in fig.1.



(a) Network. (b) Probability.

Figure 1: Bayesian network relative to weather problem.

In fig.1(b) there are the probabilities of every event in absence of observations.

By the chain rule and exploiting the conditional independence we can obtain the joint probability:

$$P(W,R,S,C) = P(W|R,S,C)P(R|S,C)P(S|C)P(C)$$
$$\Rightarrow P(W,R,S,C) = P(W|R,S)P(R|C)P(S|C)P(C)$$

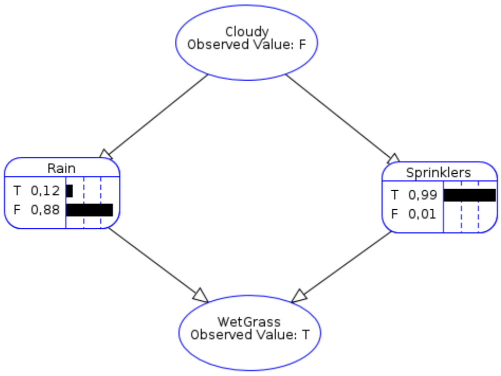


Figure 2: Probability with observations.

We introduce two observations: *WetGrass=True* and *Cloudy=False*. As it can be seen in fig.2, the probability it rained is $P(R) = 0.12$ and the probability the sprinklers were on is $P(S) = 0.99$

2 Coin problem

There are three biased coins. Every coin has equal probability to come out (1/3 each) but different probability to come up head:

- Coin 1: 20%
- Coin 2: 60%
- Coin 3: 80%

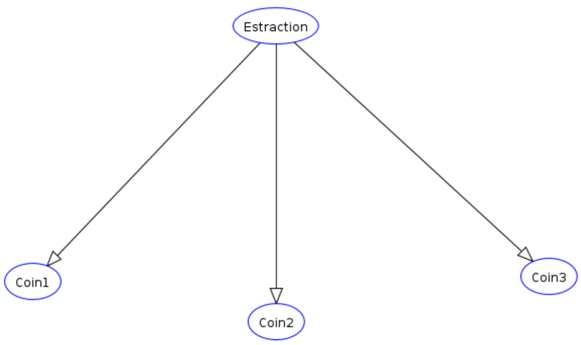


Figure 3: Bayesian network of coins problem.

We can set the associated probability with the tool, as in fig.4.

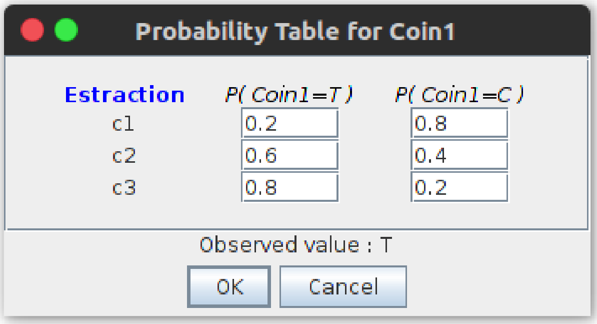


Figure 4: Probability of extraction.

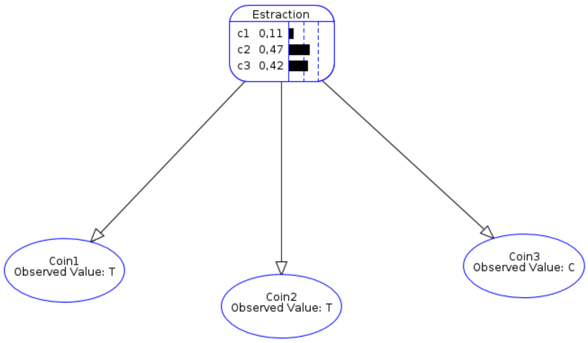


Figure 5: The solved Bayesian network.

After the observations (head twice and tails once) comes out that the coin that, most likely, gives this result is the second (fig.5).

3 Fire alarm problem

The joint probability of the *fire alarm problem* is calculated as in the *weather problem*:

$$P(R, L, A, S, T, F) = P(R|L)P(L|A)P(A|T, F)P(S|F)P(T)P(F)$$

The associated graph, with the extension *CallMom* if the alarm goes off is in fig.6.

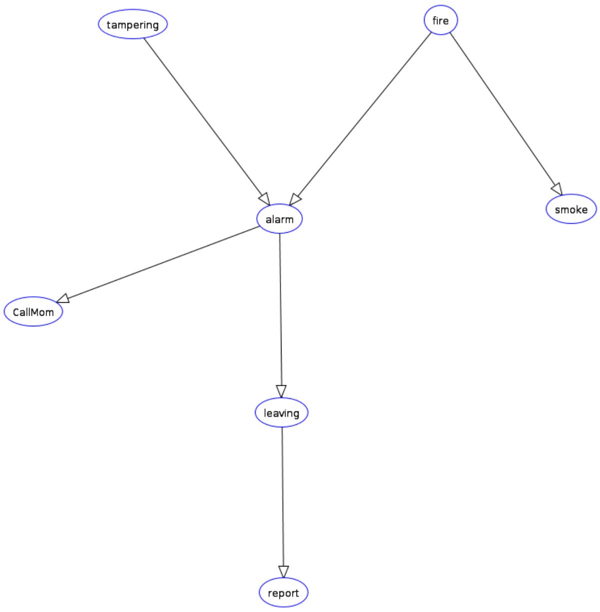


Figure 6: