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A Feedback Linearization Approach to Fault Tolerance in Quadrotor Vehicles

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Abstract: In this paper the control problem of a quadrotor vehicle experiencing a rotor failure is investigated. First we derive a nonlinear mathematical model for the quadrotor including both translational and rotational drag terms. Then we use a feedback linearization approach to design a controller whose task is to make the vehicle enter a constant angular velocity spin around its vertical axis, while retaining zero angular velocities around the other axis. These conditions can be exploited to design a second control loop, which is used to perform trajectory following. The proposed double control loop architecture allows the vehicle to perform both trajectory and roll/pitch control when a rotor failure is present.

Keywords: Fault tolerance, Helicopters, Mobile Robots, Aerospace Control

1. INTRODUCTION

Quadrotors are small aerial vehicles propelled by four rotors arranged to the extremities of a X-shaped frame, where all the arms make an angle of 90 degrees with one another. They are commonly designed to be used as Unmanned Vehicles (UVs) due to their high maneuverability, low maintenance cost and simple design.

In order to fly autonomously quadrotors must rely on sensors that provide information about the external environment or the internal system states and a controller that drives the actuators according to the measurements and the task that need to be accomplished. Both sensors and actuators, however, may be subject to faults or failures and it is important that the vehicle can be also controlled in a faulty scenario. The capability of dealing with faults is of vital importance for UVs, however in the case of the unmanned quadrotor vehicle only few researches have been devoted to the problem of Fault Detection and Isolation (FDI) Rafaralahy et al. (2008); Berbra et al. (2008a); Freddi et al. (2009) as well as to the problem of Fault Tolerant Control (FTC) Nejad et al. (2009); Berbra et al. (2008b). Moreover, at the best of the authors knowledge, a fault tolerant controller has never been proposed in case of actuator faults for this kind of vehicle.

The main contribution of this paper is to successfully develop a control law to apply in case of loss of one of the actuators in order to stabilize the attitude of the quadrotor and make it reach a desired position in space (usually a specified point on the ground), supposing that the vehicle is already equipped with a FDI module capable to detect, isolate faults and switch the controller from the fault-free to the faulty state.

Feedback linearization methods underpin the control law developed in this paper. Although robustness to model uncertainties (Lanzon and Papageorgiou (2009); Griggs et al. (2009); Lanzon (2009); Dehghani et al. (2009)) is not explicitly considered in the current work, the authors believe that this work lays down the nominal conceptual foundations for subsequent robust fault tolerant control designs in the presence of actuator failure. The nominal fault tolerant design is complex in itself because it is impossible to maintain full control of all the attitude states and all the translational states when a primary actuator has failed and the system becomes underactuated. This paper proposes a solution to this loss of control action by spinning the vehicle in the yaw direction, thereby sharing actuators.

The fault tolerant controller is developed following a double control loop architecture in which an inner and faster controller has the task to regulate the attitude angles and the altitude of the vehicle, while an outer and slower controller has the aim of modifying the desired values of the attitude angles in order to perform trajectory following.

The inner control law is developed exploiting the conservation of the angular momentum around the vertical axis of the quadrotor. When one of the rotor fails, the velocity of the rotor laying on the same axis of the faulty rotor is modulated until the value of the angle controlled by the faulty couple of rotors is zero. In this configuration the quadrotor is parallel to the ground, spinning around the vertical axis with a steady state rotational velocity depending on the rotational drag. Varying simultaneously the rotational velocity of the two rotors of the healthy couple it is also possible to set a desired altitude for the vehicle.

The outer control law has the task to supply a proper input to the healthy couple of rotors in order to make the quadrotor reach a desired position in space. The control law needs only to modify the desired direction of the total lift thrust, since the motion in space depends on that direction. Because the vehicle rotates around its vertical axis, in order to keep the total lift thrust at a desired direction it is necessary to vary each motor thrust with a proper frequency, which must be proportional to the angular velocity at which the vehicle is rotating.

The paper is organized as follows. In Section II the nonlinear model of the quadrotor is presented. Section III contains the mathematical formulation of the control laws. Section IV is devoted to the presentation of the simulation results obtained for various fault scenarios when the proposed solution is applied to the quadrotor. Conclusions and future works are presented at the end of the paper.

2. DYNAMICAL MODEL OF THE QUADROTOR

As shown in Fig. 1, the front and rear motors ($\mathcal{M}_1$ and $\mathcal{M}_3$) spin in the clockwise direction with angular velocities $\boldsymbol{\omega_1}$ and $\boldsymbol{\omega_3}$, while the other two rotors ($\mathcal{M}_2$ and $\mathcal{M}_4$) spin in the counter-clockwise direction with angular velocities $\boldsymbol{\omega_2}$ and $\boldsymbol{\omega_4}$. The mathematical model developed here is
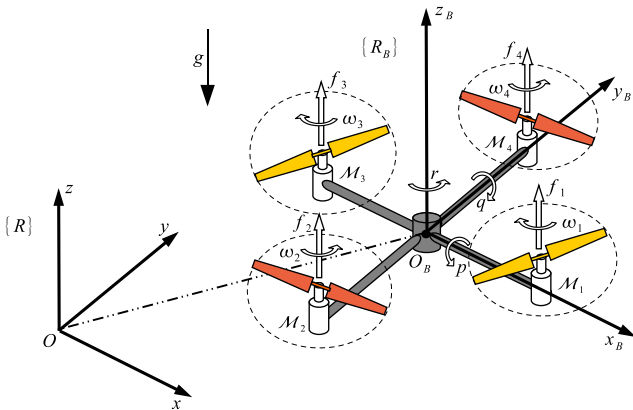

Fig. 1. The quadrotor scheme used for the development of the mathematical model.

based on some basic assumptions as given below:

- Design is symmetrical.
- Quadrotor body is rigid.
- Propellers are rigid.
- Free stream air velocity is zero.
- The motors dynamics is relatively fast and can be neglected.
- The flexibility of the blade is relatively small and can be neglected.
- Drag is supposed to be linear, thus obeying Stoke's law.

Two frames are used to study the system motion: a frame integral with the earth $\{R\}\,(O, x, y, z)$, which is supposed to be inertial, and a body-fixed frame $\{R_B\}\,\{O_B, x_B, y_B, z_B\}$, where O_B is fixed to the center of mass of the quadrotor. $\{R_B\}$ is related to $\{R\}$ by a position vector $\boldsymbol{\xi} = [x\ y\ z]^T$, describing the position of the center

of gravity in $\{R_B\}$ relative to $\{R\}$ and by a vector of three independent angles $\boldsymbol{\eta} = [\phi\ \theta\ \psi]^T$, which represent the orientation of the body-fixed frame $\{R_B\}\,\{O_B, x_B, y_B, z_B\}$ with respect to the earth frame $\{R\}\,(O, x, y, z)$, using the so-called yaw, pitch and roll notation. In this way $\boldsymbol{\xi} = [x\ y\ z]^T$ and $\boldsymbol{\eta} = [\phi\ \theta\ \psi]^T$ fully describe, respectively, the translational and the rotational movement of the rotorcraft with respect to the earth frame.

Given a force $\boldsymbol{F}_B$, expressed using the coordinates of the body frame, the force $\boldsymbol{F}$ expressed in the coordinates of the earth frame is:

$$\boldsymbol{F} = \boldsymbol{R}_{B \to E}\boldsymbol{F}_B \tag{1}$$

where $\boldsymbol{R}_{B \to E}$ is the rotation transformation from vectors read in the body reference frame to vectors read in the earth reference frame given by

$$\boldsymbol{R}_{B \to E} \doteq \begin{bmatrix} C_\theta C_\psi & C_\psi S_\theta S_\phi - C_\phi S_\psi & C_\phi C_\psi S_\theta + S_\phi S_\psi \\ C_\theta S_\psi & S_\theta S_\phi S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix} \tag{2}$$

where $S_{(.)}$ and $C_{(.)}$ represent $\sin{(.)}$ and $\cos{(.)}$ respectively

In a similar way, given the angular velocity vector $\boldsymbol{\omega} = [p\ q\ r]^T$, where p, q and r represent the instantaneous angular velocities around the x_B-axis, y_B-axis and z_B-axis respectively, it is related to the rate of change of the yaw, pitch and roll angles by Fossen (2002):

$$\boldsymbol{\omega} = \mathbb{W}_\eta\dot{\boldsymbol{\eta}} \tag{3}$$

where

$$\mathbb{W}_\eta \doteq \begin{bmatrix} 1 & 0 & -S_\theta \\ 0 & C_\phi & S_\phi C_\theta \\ 0 & -S_\phi & C_\phi C_\theta \end{bmatrix}. \tag{4}$$

The dynamics of the quadrotor can be described analyzing the forces acting on it, which are the weight force, the thrust forces and the drag terms. The weight force is applied to the center of gravity and directed along the negative z-axis in the earth frame. The thrust force $\boldsymbol{f}_j$, where $j = 1, 2, 3, 4$, is applied to the center of the j-th motor, distant l from the center of mass, and directed along the positive z_B-axis: $f_j \geq 0$ for $j = 1, \dots, 4$. The drag terms obey Stoke's law: the translational drag is proportional to the linear velocity and the rotational drag term is proportional to the angular velocitiy.

An Euler-Lagrange approach is adopted in order to write the equations which describe the translational motion of the quadrotor. The kinetic energy of the rotorcraft can be divided into translational and rotational components Raffo et al. (2010). The translational component is

$$T_{trans} \triangleq \frac{1}{2}m\dot{\boldsymbol{\xi}}^T\dot{\boldsymbol{\xi}} \tag{5}$$

where m denotes the whole mass of the rotorcraft. The rotational component is

$$T_{rot} \triangleq \frac{1}{2}\dot{\boldsymbol{\eta}}^T\mathbb{J}\dot{\boldsymbol{\eta}} \tag{6}$$

in which $\mathbb{J}$ represents the inertia matrix in terms of the generalized coordinates $\boldsymbol{\eta}$. Defining the inertia matrix in the body frame as

$$\mathbb{I} \doteq \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{7}$$

which is diagonal due to the symmetry of the quadrotor, the inertia matrix in terms of the generalized coordinates can be expressed as

$$\mathbb{J} = \mathbb{W}_\eta^T \mathbb{I} \mathbb{W}_\eta \tag{8}$$

The only potential energy which needs to be considered is due to the gravitational field. Therefore, potential energy is expressed as

$$U = -mgz \tag{9}$$

Let $\mathbf{q} = \begin{bmatrix} \boldsymbol{\xi}^T & \boldsymbol{\eta}^T \end{bmatrix}^T = [x, y, z, \phi, \theta, \psi]^T \in \mathbb{R}^6$ be the generalized coordinates vector for the aerial vehicle, the Lagrangian is given by

$$\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = T_{trans} + T_{rot} + U = \frac{1}{2} m \dot{\boldsymbol{\xi}}^T \dot{\boldsymbol{\xi}} + \frac{1}{2} \dot{\boldsymbol{\eta}}^T \mathbb{J} \dot{\boldsymbol{\eta}} - mgz \tag{10}$$

Since the Lagrangian contains no cross-terms in the kinetic energy combining $\dot{\boldsymbol{\xi}}$ and $\dot{\boldsymbol{\eta}}$, the Euler-Lagrange equation relative to the translational motion can be written as:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{\xi}}} - \frac{\partial \mathcal{L}}{\partial \boldsymbol{\xi}} = \boldsymbol{F}_\xi \tag{11}$$

where $\boldsymbol{F}_\xi$ defines the translational generalized force acting on the aerial vehicle and relative to the frame $\{R\}$.

Labeling as f_1, f_2, f_3 and f_4 the upward lifting forces generated by the propellers, the force in the z_B direction due to the control inputs, expressed into the body frame, is:

$$u_f = f_1 + f_2 + f_3 + f_4 \tag{12}$$

Substituting (10) in (11) and expanding $\boldsymbol{F}_\xi$ leads to

$$m\ddot{\boldsymbol{\xi}} + \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = \boldsymbol{R}_{B\to E} \begin{bmatrix} 0 \\ 0 \\ u_f \end{bmatrix} - k_t \dot{\boldsymbol{\xi}} \tag{13}$$

where k_t is the translational drag coefficient, assumed to be equal in all directions for simplicity.

The vector equation that describes the rotational motion of the quadrotor, referred to the body frame coordinates, is:

$$\boldsymbol{\tau}_B = \mathbb{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbb{I}\boldsymbol{\omega}) + k_r \boldsymbol{\omega} \tag{14}$$

where k_r is the rotational drag coefficient, assumed to be constant in all directions for simplicity and "×" denotes the vector cross product.

The torques around the body frame axis are described by:

$$\boldsymbol{\tau}_B = \begin{bmatrix} \tau_p \\ \tau_q \\ \tau_r \end{bmatrix} = \begin{bmatrix} l(f_4 - f_2) \\ l(f_3 - f_1) \\ d(f_1 - f_2 + f_3 - f_4) \end{bmatrix} \tag{15}$$

where l is the arm length and d is the ratio between the drag and the thrust coefficients of the blade.

3. THE FAULT TOLERANT CONTROLLER

In literature, several techniques have been proposed to control a fault-free quadrotor vehicle: dynamic inversion (Lewis (2009)), nested saturations Castillo et al. (2005), nonlinear H_∞ control, Model Predictive Control (MPC) Raffo et al. (2010), feedback linearization Voos (2009) and backstepping Bouabdallah and Siegwart (2007) are among the most used approaches. These controller structures are based upon the already mentioned double loop architecture.

The Fault Tolerant Controller (FTC) is based on the assumption that one of the four actuators is experiencing a failure (no longer able to provide an upward lift force), while the other three actuators are fully working. When one of the rotor fails two problems which are not present in the fault-free case arise. First the quadrotor looses the ability to control independently the three torques necessary to fully control the attitude of the vehicle. Furthermore this rotor failure implies the loss of controllability of one variable from roll, pitch, yaw and altitude. It is our claim that, from a physical point of view, the most important variables to control are roll, pitch and altitude. The roll and pitch angles are of vital importance because a small change in their values may affect the stability of the vehicle. On the other hand altitude must always be kept above a positive threshold in order to avoid collision with the ground. The impossibility to control yaw displacement when a rotor failure occurs, instead, only implies loosing the heading of the vehicle. For these reasons the FTC proposed in this paper is developed sacrificing the controllability of the ψ state.

In case of fault the control structure can be realized using the double loop architecture shown in fig. 2. The inner control loop controls roll, pitch and altitude, while the outer control loop sets the desired values of the ϕ and θ angles in order to control position in the xy-plane. Since roll, pitch and altitude have the highest priority during flight, the inner controller works much faster than the outer controller. Finally the quadrotor states must be accessible in order to develop the controller: the inner controller needs the states $[\phi, \theta, \psi, p, q, r, z]$, while $[x, y]$ are also needed by the outer control loop. In this section the
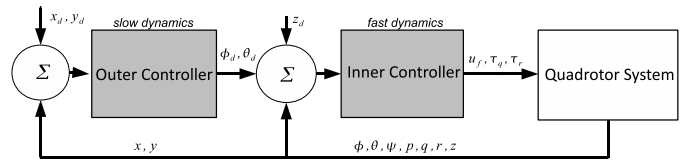

Fig. 2. The control system scheme: the inner control loop controls the quadrotor altitude and attitude, while the slower outer controller has the task to control position.

FTC in case of failure on the actuator 2 is described. Due to the symmetry of the system a similar controller can be developed in case of failure on actuators 1, 3 or 4.

In case of failure on $\mathcal{M}_2$ the control inputs are chosen as u_f, τ_q and τ_r, where the relation

$$\begin{bmatrix} u_f \\ \tau_q \\ \tau_r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -l & l & 0 \\ d & d & -d \end{bmatrix} \begin{bmatrix} f_1 \\ f_3 \\ f_4 \end{bmatrix} \tag{16}$$

is bijective. Choosing the state vector as

$$\boldsymbol{x} = [\phi \ \theta \ \psi \ p \ q \ r \ x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T \tag{17}$$

and the input vector as

$$\boldsymbol{u} = [u_f \ \tau_q \ \tau_r]^T \tag{18}$$

then equations (13)-(14) can be written in state-space form as

$$\dot{x}_1 = x_4 + x_5 S_{x_1} T_{x_2} + x_6 C_{x_1} T_{x_2} \tag{19}$$

$$\dot{x}_2 = x_5 C_{x_1} - x_6 S_{x_1} \tag{20}$$

$$\dot{x}_3 = \frac{1}{C_{x_2}} [x_5 S_{x_1} + x_6 C_{x_1}] \tag{21}$$

$$\dot{x}_4 = \frac{1}{I_{xx}} [-k_r x_4 - x_5 x_6 (I_{zz} - I_{xx}) + \frac{l}{2}(u_f - \frac{\tau_r}{d})] \tag{22}$$

$$\dot{x}_5 = \frac{1}{I_{xx}} [-k_r x_5 - x_4 x_6 (I_{xx} - I_{zz}) + \tau_q] \tag{23}$$

$$\dot{x}_6 = \frac{1}{I_{zz}} (-k_r x_6 + \tau_r) \tag{24}$$

$$\dot{x}_7 = x_{10} \tag{25}$$

$$\dot{x}_8 = x_{11} \tag{26}$$

$$\dot{x}_9 = x_{12} \tag{27}$$

$$\dot{x}_{10} = \frac{C_{x_1} S_{x_2} C_{x_3} + S_{x_1} S_{x_3}}{m} u_f - \frac{k_t}{m} x_{10} \tag{28}$$

$$\dot{x}_{11} = \frac{C_{x_1} S_{x_2} S_{x_3} - S_{x_1} C_{x_3}}{m} u_f - \frac{k_t}{m} x_{11} \tag{29}$$

$$\dot{x}_{12} = \frac{1}{m} [u_f C_{x_1} C_{x_2} - k_t x_{12} - mg] \tag{30}$$

in which I_{xx} has been chosen equal to I_{yy} due to the symmetry of the quadrotor.

3.1 Inner Control Loop

From the equations (19)-(30) it can be seen that the dynamics of the state variables x_1, x_2, x_4, x_5, x_6, x_9, x_{12}, which we will call $\bar{x}$ for notation simplicity, can be written as

$$\dot{\bar{x}} = \boldsymbol{f}(\bar{\boldsymbol{x}}) + \boldsymbol{h}(\bar{\boldsymbol{x}})\boldsymbol{u} \tag{31}$$

The dynamics of the states x_1, x_2 and x_9 can be similarly written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_9 \end{bmatrix} = \begin{bmatrix} x_4 + x_5 S_{x_1} T_{x_2} + x_6 C_{x_1} T_{x_2} \\ x_5 C_{x_1} - x_6 S_{x_1} \\ x_{12} \end{bmatrix} \doteq \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}}) \tag{32}$$

which, in this case, is independent of the input vector $\boldsymbol{u}$. This property becomes useful when calculating the second derivative of $[x_1 \ x_2 \ x_9]^T$:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_9 \end{bmatrix} = \frac{d\hat{\boldsymbol{f}}(\bar{\boldsymbol{x}})}{dt} = \frac{\partial \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}})}{\partial \bar{\boldsymbol{x}}} \dot{\bar{\boldsymbol{x}}} = \frac{\partial \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}})}{\partial \bar{\boldsymbol{x}}} \boldsymbol{f}(\bar{\boldsymbol{x}}) + \frac{\partial \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}})}{\partial \bar{\boldsymbol{x}}} \boldsymbol{h}(\bar{\boldsymbol{x}})\boldsymbol{u} \tag{33}$$

Denoting the Jacobian matrix with

$$\boldsymbol{J}(\bar{\boldsymbol{x}}) \doteq \frac{\partial \hat{\boldsymbol{f}}(\bar{\boldsymbol{x}})}{\partial \bar{\boldsymbol{x}}} \tag{34}$$

the previous dynamics can be written as

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_9 \end{bmatrix} = \boldsymbol{J}(\bar{\boldsymbol{x}})\boldsymbol{f}(\bar{\boldsymbol{x}}) + \boldsymbol{J}(\bar{\boldsymbol{x}})\boldsymbol{h}(\bar{\boldsymbol{x}})\boldsymbol{u} \tag{35}$$

It can be proved that matrix $\boldsymbol{J}(\bar{\boldsymbol{x}})\boldsymbol{h}(\bar{\boldsymbol{x}})$ is invertible if and only if

$$x_2 \neq arctan(\frac{I_{zz}l}{2I_{xx}d}C_{x_1}) \tag{36}$$

In most practical scenarios this condition is satisfied. In our case, with the parameters adopted for the simulation, the invertibility condition is guaranteed as long as the pitch and roll angles are limited to 80°, which is a value never reached by the quadrotor during non aerobatic flight.

Let x_{id}, $\dot{x}_{id}$, $\ddot{x}_{id}$ be the desired values for x_i, $\dot{x}_i$, $\ddot{x}_i$ and define the i-th error as

$$e_i \doteq x_i - x_{id} \tag{37}$$

If the control inputs are chosen as

$$\begin{bmatrix} u_f^* \\ \tau_q^* \\ \tau_r^* \end{bmatrix} \doteq -(\boldsymbol{J}(\bar{\boldsymbol{x}})\boldsymbol{h}(\bar{\boldsymbol{x}}))^{-1}\boldsymbol{J}((\bar{\boldsymbol{x}})\boldsymbol{f}(\bar{\boldsymbol{x}}))$$
$$-(\boldsymbol{J}(\bar{\boldsymbol{x}})\boldsymbol{h}(\bar{\boldsymbol{x}}))^{-1} \left(2\xi_{in}c_{in}\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_9 \end{bmatrix} + c_{in}^2 \begin{bmatrix} e_1 \\ e_2 \\ e_9 \end{bmatrix} - \begin{bmatrix} \ddot{x}_{1d} \\ \ddot{x}_{2d} \\ \ddot{x}_{9d} \end{bmatrix} \right),$$

where ξ_{in} and c_{in} are positive constants, then the error dynamics can be written as

$$\begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \\ \ddot{e}_9 \end{bmatrix} + 2\xi_{in}c_{in}\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_9 \end{bmatrix} + c_{in}^2 \begin{bmatrix} e_1 \\ e_2 \\ e_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

which yields stable second-order dynamics providing exponential decay of e_i for $i = 1, 2, 9$ (respectively the roll, pitch and altitude errors).

Furthermore if x_{1d} and x_{2d} are chosen to converge to zero and x_{9d} to converge to a constant, then x_4 and x_5 converge to zero while x_6 retains boundedness, i.e.:

$$x_6 \to \frac{dmgI_{zz}}{k_r} \tag{39}$$

From a physical point of view it means that when one of the rotor fails, the proposed control law can bring the faulty quadrotor back to hover condition (roll and pitch stabilization at zero degress, altitude stabilization at any desired value above the ground), while spinning in the yaw direction with constant angular velocity.

3.2 Outer Control Loop

The horizontal motion of the quadrotor depends on the direction that the horizontal component of the thrust vector, that is to say the vector sum of each rotor thrust, assumes in the space. Since this direction depends on the roll and pitch angles, it is possible to develop an outer control loop whose task is to generate the desired values for the roll and pitch angles in order to reach a desired position in the earth fixed frame and with a desired profile for both the x and y velocities and accelerations (see fig. 2).

In order to make the two controllers work together without compromising stability, it is necessary that the outer control loop acts much slower than the inner control loop: this can be achieved choosing properly the values of c_{in} and c_o. The constant c_{in} represents the natural frequency of the dynamics of the subsystem described by the state variables x_1, x_2 and x_9. If we similarly define c_{out} the natural frequency of the dynamics of the subsystem described by the state variables x_7, x_8, x_{10} and x_{11}, it is necessary to choose $c_{out} \ll c_{in}$ in order to ensure the overall stability to the closed-loop system.

Suppose x_1 and x_2 are small angles: this can be achieved simply activating the inner controller with x_{1d} and x_{2d} set close to zero. If x_{1d} and x_{2d} are chosen as

$$\begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix} = -\frac{m}{u_f} \begin{bmatrix} S_{x_3} & -C_{x_3} \\ C_{x_3} & S_{x_3} \end{bmatrix} \begin{bmatrix} -\frac{k_t}{m}x_{10} + 2\xi_o c_o e_{10} + c_o^2 e_7 - \dot{x}_{10d} \\ -\frac{k_t}{m}x_{11} + 2\xi_o c_o e_{11} + c_o^2 e_8 - \dot{x}_{11d} \end{bmatrix}, \tag{40}$$

where ξ_o and c_o are positive constants, then error dynamics for the horizontal displacements in closed loop is

$$\begin{bmatrix} \ddot{e}_x + 2\xi_o c_o \dot{e}_x + c_o^2 e_x \\ \ddot{e}_y + 2\xi_o c_o \dot{e}_y + c_o^2 e_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (41)$$

which is asymptotically stable.

From a physical point of view it means that when one of the rotor fails, the inner control law stabilizes roll, pitch and altitude, while the outer control law exploits the near hover condition to slowly change the pitch and roll angles in order to move the vehicle to a desired position in space.

4. SIMULATION RESULTS

Several simulations have been run using the Matlab and Simulink® softwares in order to validate the theoretical results.

The simulation system consists of 3 modules:

- the nonlinear system module, which includes all the differential equations described in section 2;
- the nonlinear controller module, which includes the equations described in section 3;
- the initialization module, which includes all the parameters that are necessary to run the simulation.

In the present paper the results of two different simulations are reported. Both the simulations have been performed with the controller activated at $t = 0$ and with the initial conditions chosen far from the hover equilibrium point.

In the first simulation only the inner controller is working while the outer control is deactivated. The desired values for the state variables are all set to 0 exception made for the altitude, which is chosen to be $10m$. The simulation has been run for $20s$ which is a sufficient time to reach hover.

As it can be seen in fig. 3 the attitude angles ϕ and θ go to zero in less than $15s$ and with a smooth profile. The ψ angle does not reach a steady state value, but increases with a constant rate . The x and y variables reach instead a steady state value that depends on the initial conditions, while the altitude z is quickly regulated to the desired value of $10m$. The angular and linear velocities are all stabilized as it can be seen in fig. 4, with the exception of r: this behaviour is actually not a surprise since the controllability of the ψ variable has been sacrificed in the design phase of the fault tolerant controller. Thus r converges to a constant and the yaw angle ψ increases according to a linear law.

In the second simulation both the inner controller and the outer controller are working. The desired values for the state variables are all set to 0, exception made for the altitude, which is set to $10m$ as long as the quadrotor has not reached the desired lateral and longitudinal position, then it is set to a ramp with negative slope until it becomes zero (landing procedure). The time needed to reach the desired set points is greater than that needed to grant attitude stabilization in the previous simulation: this is caused by the fact that the outer controller tends to force the attitude angles to differ from zero as long as the desired position is not reached, while the inner control tends to force those angles to zero. The simulation has
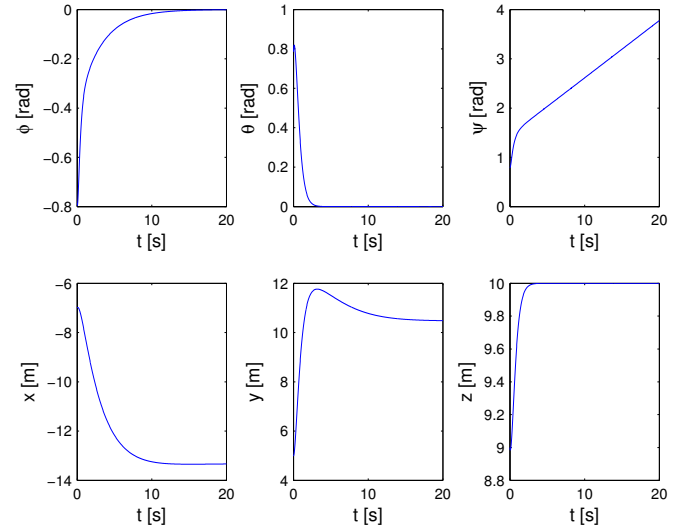

Fig. 3. Linear and angular positions, in function of time, when the stabilizing controller is running while the outer controller is disabled. The ϕ and θ angles go to zero and the altitude quickly reaches the desired set point.
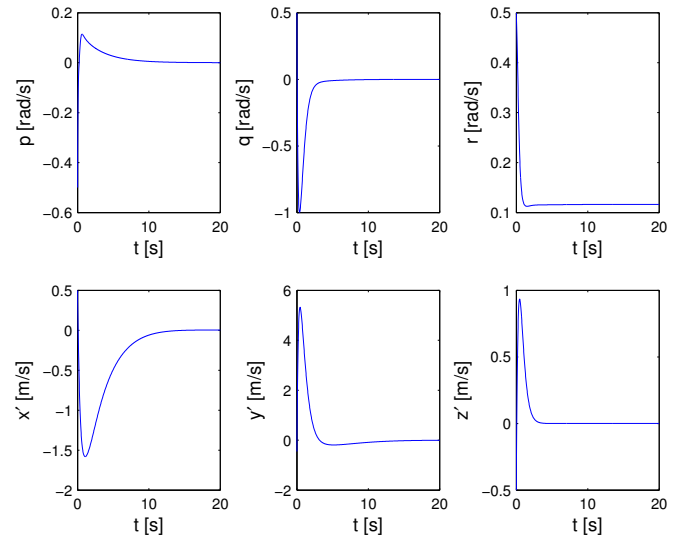

Fig. 4. Linear and angular velocities, in function of time, when the stabilizing controller is running while the outer controller is disabled. All velocities go to zero while the quadcopter is stabilized, exception made for r whose controllability has been sacrificed during the design phase of the fault tolerant controller.

been therefore run for $400s$, which is a sufficient time to reach the origin with stable pitch and yaw angles.

As it can be seen in fig. 5 the attitude angles ϕ and θ are stabilized, but this time there are oscillations due to the presence of the outer controller. The ψ angle does not reach a steady state value, but increases according to a linear law as before. The linear positions reach instead the origin, even if the regulation of z is much faster than that of x and y due to the fact that altitude regulation is operated by the inner and faster controller. The angular and linear velocities are all stabilized as it can be seen in fig. 6, with the exception of r, for the reasons already stated in the
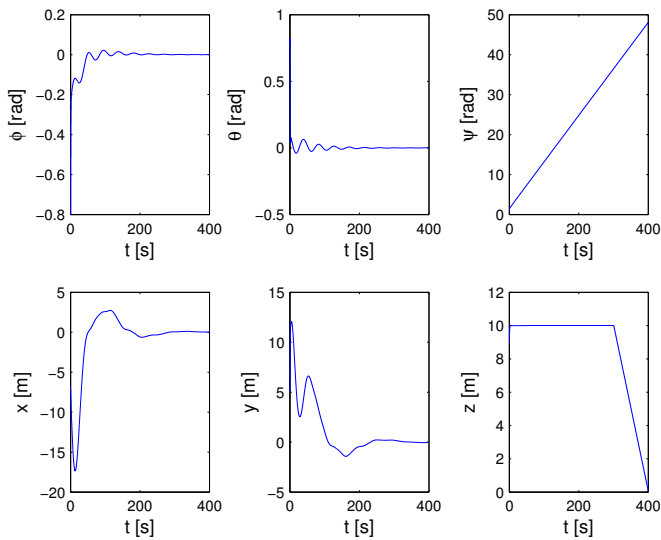
Fig. 5. Linear and angular positions, in function of time, when both the controllers are running. The ϕ and θ angles go to zero and all the positions reach the desired set points with different time constants.

paragraph in which we described the first simulation, and $\dot{z}$ which becomes negative during the landing procedure.
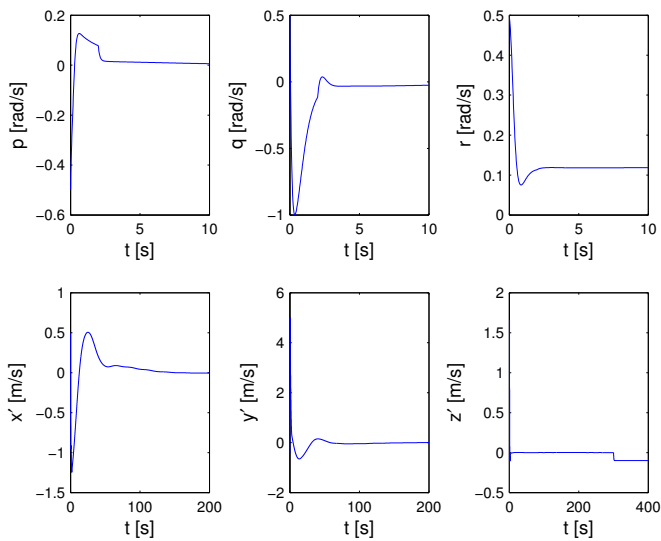

Fig. 6. Linear and angular velocities, in function of time, when both the controllers are running. All velocities go to zero, exception made for r.

5. CONCLUDING REMARKS

In the present paper the problem of controlling a quadrotor aerial vehicle when one of the actuators fails is proposed. First a mathematical model for the quadrotor aerial vehicle is presented, then this model is exploited to build a fault tolerant controller based on a double control loop architecture. The control law has finally been tested in two different simulated scenarios. The derived fault tolerant controller shows that despite a rotor failure, the quadrotor can remain flying with only three functional rotors. It can perform trajectory control in x, y and z perfectly and can also control roll and pitch angles.

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