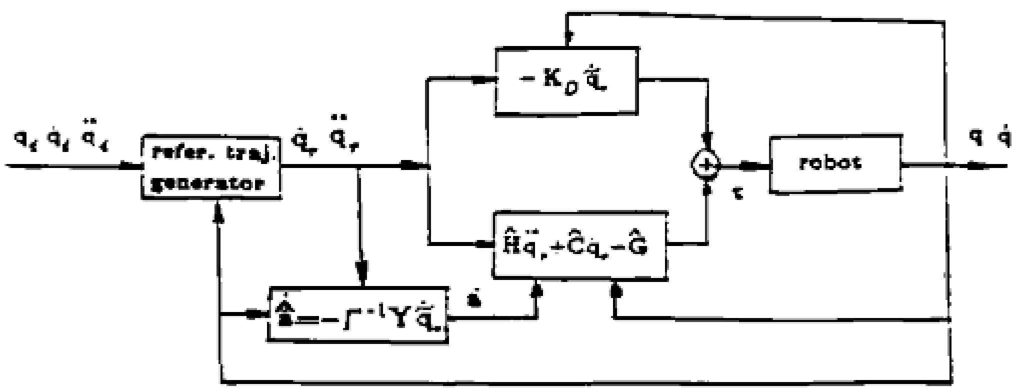
Objective

Design an Adaptive Controller for a two link manipulator system with all the parameters unknown to follow a desired trajectory.

Calculations

Applying the procedure described in the research paper on adaptive control for robot manipulators by Slotine, following model was implemented.



$$\begin{bmatrix} M_{aa} & M_{au} \\ M_{ua} & M_{uu} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} h\dot{q}_2 & h(\dot{q}_1+\dot{q}_2) \\ -h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_a \\ G_u \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$(M_{aa})\ddot{q}_1 + (M_{au})\ddot{q}_2 + 2h\dot{q}_1\dot{q}_2 + h\dot{q}_2^2 + G_a = u_1 \quad -(1)$$

$$(M_{ua})\ddot{q}_1 + (M_{uu})\ddot{q}_2 + (-h\dot{q}_1^2) + G_u = u_2 \quad -(2)$$

$$M_{aa} = m_1 l_{c1}^2 + m_2 l_1^2 + m_2 l_{c2}^2 + m_2(2 l_1 l_{c2} \cos q_2) + I_1 + I_2 \quad -(a)$$

$$M_{ua} = M_{au} = m_2 l_{c2}^2 + m_2 l_1 l_{c2} \cos q_2 + I_2 \quad -(b)$$

$$M_{uu} = m_2 l_{c2}^2 + I_2 \quad -(c)$$

$$h = -m_2 l_1 l_{c2} \sin q_2 \quad -(d)$$

$$G_a = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + (m_2 l_{c2} g)\cos(q_1+q_2) \quad -(e)$$

$$G_u = m_2 l_{c2} \cos(q_1+q_2) \quad -(f)$$

The expressions, let us assume following parameters:-

$a_1 = m_2 l_{c2}^2 + I_2$; $a_2 = m_2 l_1 l_{c2}$; $a_3 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1$

$a_4 = m_1 l_{c1} + m_2 l_1$; $a_5 = m_2 l_{c2}$.

$\therefore$ The expressions (a) to (f) become:-

$$\left.\begin{aligned} m_{aa} &= a_1 + a_3 + 2a_2 \cos q_2; \\ m_{ua} = m_{au} &= a_1 + a_2 \cos q_2; \\ m_{uu} &= a_1 \\ h &= -a_2 \sin q_2 \\ G_a &= a_4 g \cos q_1 + a_5 g \cos(q_1+q_2) \end{aligned}\right\} \text{ in terms of parameters}$$

$$G_u = a_5 \cos(q_1+q_2)$$

put these in eqn (1) & (2)

(1) $\rightarrow$

$$(a_1 + a_3 + 2a_2 \cos q_2)\ddot{q}_1 + (a_1 + a_2 \cos q_2)\ddot{q}_2 - 2a_2 \dot{q}_1 \dot{q}_2 \sin q_2 - a_2 \dot{q}_2^2 \sin q_2 + a_4 g \cos q_1 + a_5 g \cos(q_1+q_2) = u_1 \quad \text{— (3)}$$

$$\therefore a_1[\ddot{q}_1 + \ddot{q}_2] + a_2[2\ddot{q}_1 \cos q_2 + \ddot{q}_2 \cos q_2 - 2\dot{q}_1\dot{q}_2 \sin q_2 - \dot{q}_2^2 \sin q_2] + a_3[\ddot{q}_1] + a_4[g\cos q_1] + a_5[g\cos(q_1+q_2)] = u_1 \quad \text{— (A)}$$

(2) $\rightarrow$

$$(a_1 + (a_2)\cos q_2)\ddot{q}_1 + a_1 \ddot{q}_2 + a_2 \dot{q}_1^2 \sin q_2 + a_5 \cos(q_1+q_2) = u_2 \quad \text{— (4)}$$

$$a_1[\ddot{q}_1 + \ddot{q}_2] + a_2[\ddot{q}_1 \cos q_2 + \dot{q}_1^2 \sin q_2] + a_3(0) + a_4(0) + a_5[\cos(q_1+q_2)] = u_2 \quad \text{— (B)}$$

$$\therefore H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u = Y(q,\dot{q},\ddot{q})\, a$$

$$\therefore Y(q,\dot{q},\ddot{q}) = \begin{bmatrix} \ddot{q}_1 + \ddot{q}_2 & \cos q_2(2\ddot{q}_1 + \ddot{q}_2) - \sin q_2(\dot{q}_1^2 + 2\dot{q}_1\dot{q}_2) & \ddot{q}_1 & g\cos q_1 & g\cos(q_1+q_2) \\ \ddot{q}_1 + \ddot{q}_2 & \ddot{q}_1 \cos q_2 + \dot{q}_1^2 \sin q_2 & 0 & 0 & \cos(q_1+q_2) \end{bmatrix}_{2\times 5}$$

where $a = [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T$

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Now,

$$\hat{H}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) = \hat{Y}(q,\dot{q}_r,\dot{q},\ddot{q}_r)\,\hat{a}$$

Let $\ddot{q}_1 = \ddot{q}_{r_1}$; $\ddot{q}_2 = \ddot{q}_{r_2}$; $\dot{q}_1^2 = \dot{q}_{r_1}\cdot\dot{q}_1$; $\dot{q}_2^2 = \dot{q}_{r_2}\cdot\dot{q}_2$

$$\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 = \dot{q}_2(\dot{q}_2+\dot{q}_1) + \dot{q}_1\dot{q}_2 = \dot{q}_{r_2}(\dot{q}_1+\dot{q}_2) + \dot{q}_{r_1}\dot{q}_2$$

∴ Therefore put above values in $Y(q,\dot{q},\ddot{q})$ we get,

$\hat{Y}(q,\dot{q}_r,\ddot{q}_r,\dot{q}) =$

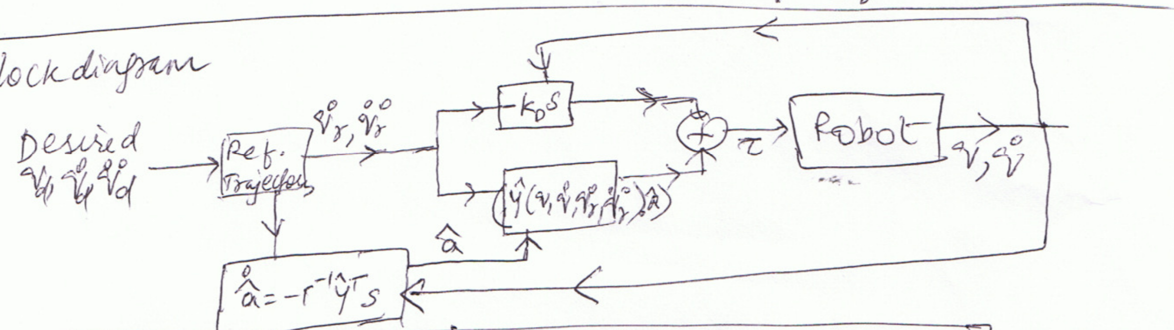
$$\begin{bmatrix} \ddot{q}_{r_1}+\ddot{q}_{r_2} & \cos q_2(2\ddot{q}_{r_1}+\ddot{q}_{r_2}) - \sin q_2(\dot{q}_{r_1}\dot{q}_2) - \sin q_2(\dot{q}_2\dot{q}_{r_2}) - \sin q_2(\dot{q}_{r_2}\dot{q}_1) & \ddot{q}_{r_1} & g\cos q_1 & g\cos(q_1+q_2) \\ \ddot{q}_{r_1}+\ddot{q}_{r_2} & \ddot{q}_{r_1}\cos q_2 + \dot{q}_{r_1}\dot{q}_1\sin q_2 & 0 & 0 & \cos(q_1+q_2) \end{bmatrix}_{2\times 5}$$

This is used to estimate the parameters using following equation:

$$\hat{a} = \int\dot{\hat{a}} = \int -\Gamma^{-1}\hat{Y}^T_{(q,\dot{q},\dot{q}_r,\ddot{q}_r)}\, s$$

; where Γ = pos. def. matrix
$\Gamma = \alpha$ Ident. matrix
& $s = \dot{q} - \dot{q}_r$

Block diagram



$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$

$\ddot{q}_r = \ddot{q}_d - \Lambda\dot{\tilde{q}}$

$s = \dot{q} - \dot{q}_r$

$\tilde{q} = q(t) - q_d(t)$

$\dot{\tilde{q}} = \dot{q} - \dot{q}_d$

Control & Adaptation laws:—

$\tau = \hat{H}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G}(q) - k_D s$

$\dot{\hat{a}} = -\Gamma^{-1} Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\, s$

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Following values were obtained for estimating the parameters.

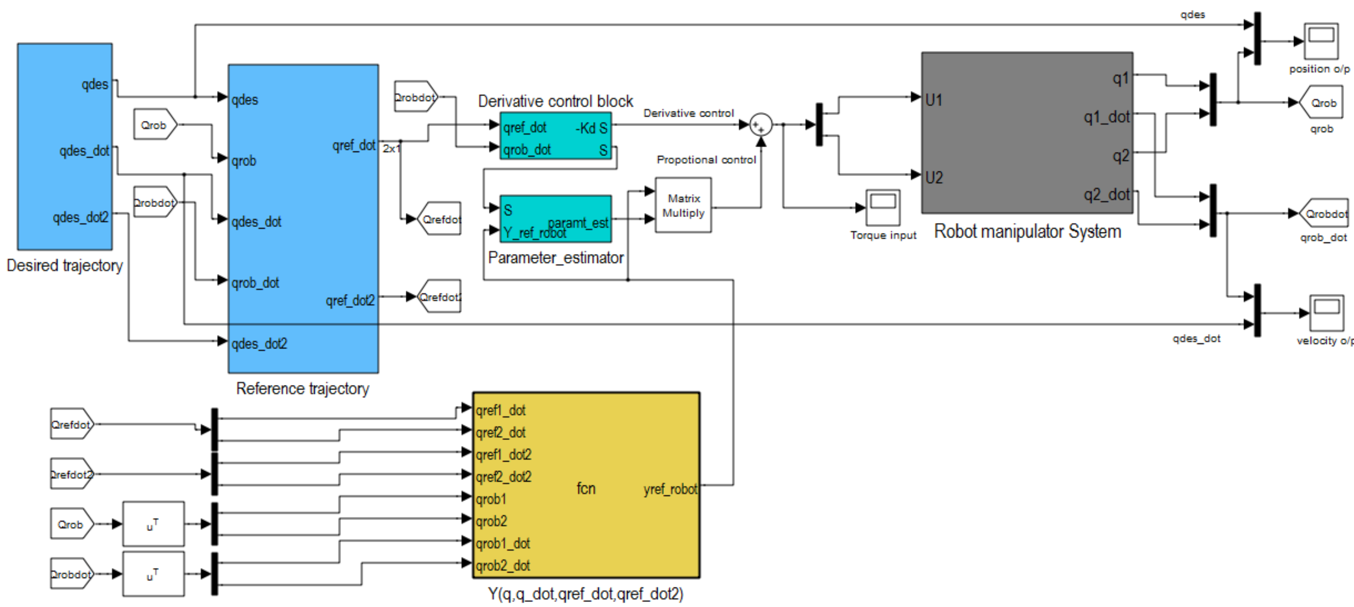
$\Gamma = 7I$; I is a 5x5 identity matrix (positive definite)

$\Lambda = 2.5\ I$; I is a 2x2 identity matrix (positive definite)

$Kd = 11I$; I is a 2x2 identity matrix (positive definite)

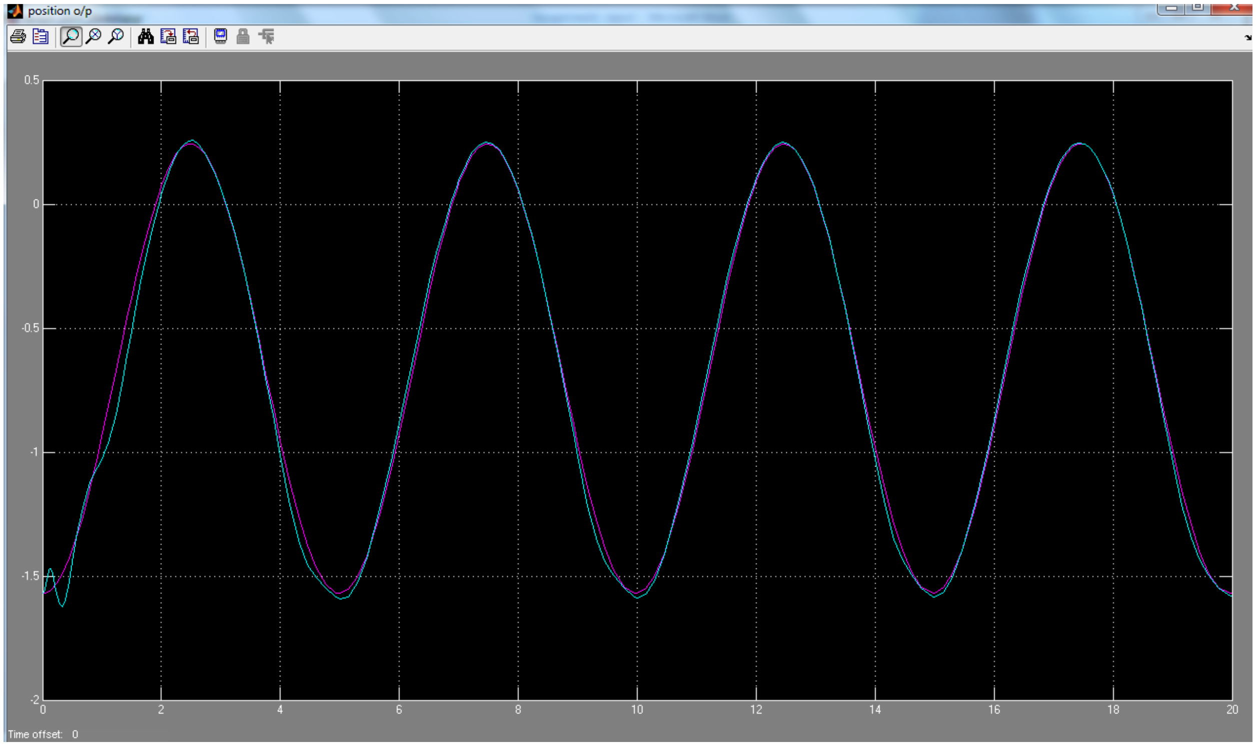
Simulink Model

Adaptive control for Robot Manipulator subsystem

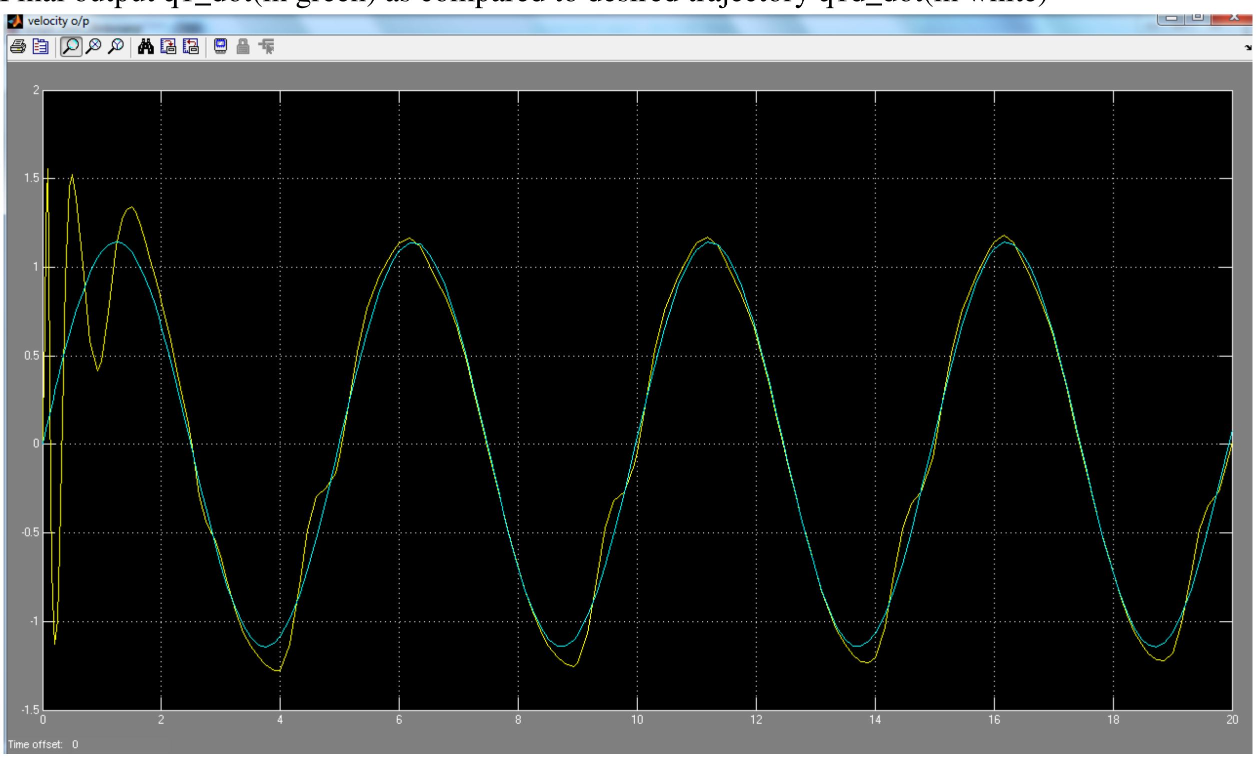


Simulation Results

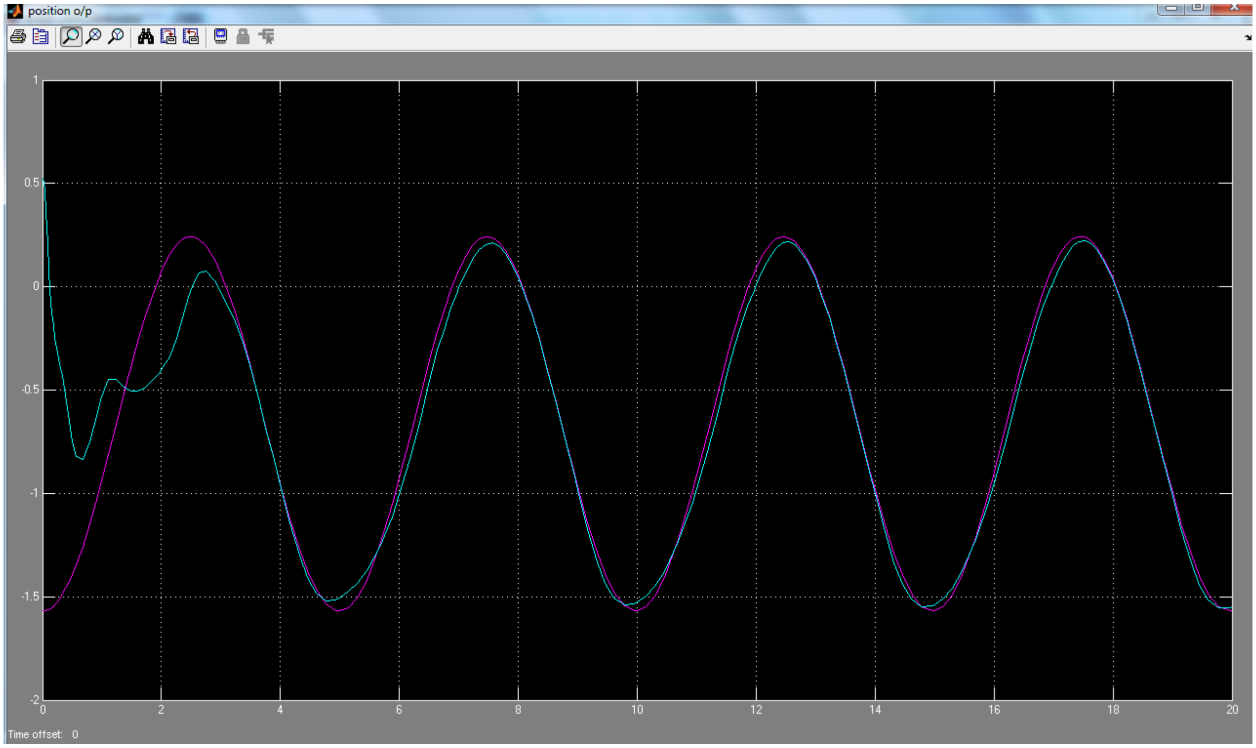
1. Final output q1(in white) as compared to desired trajectory q1d(in dark pink)
   It can be seen in the graph that both the curves start from -90 initial positions.



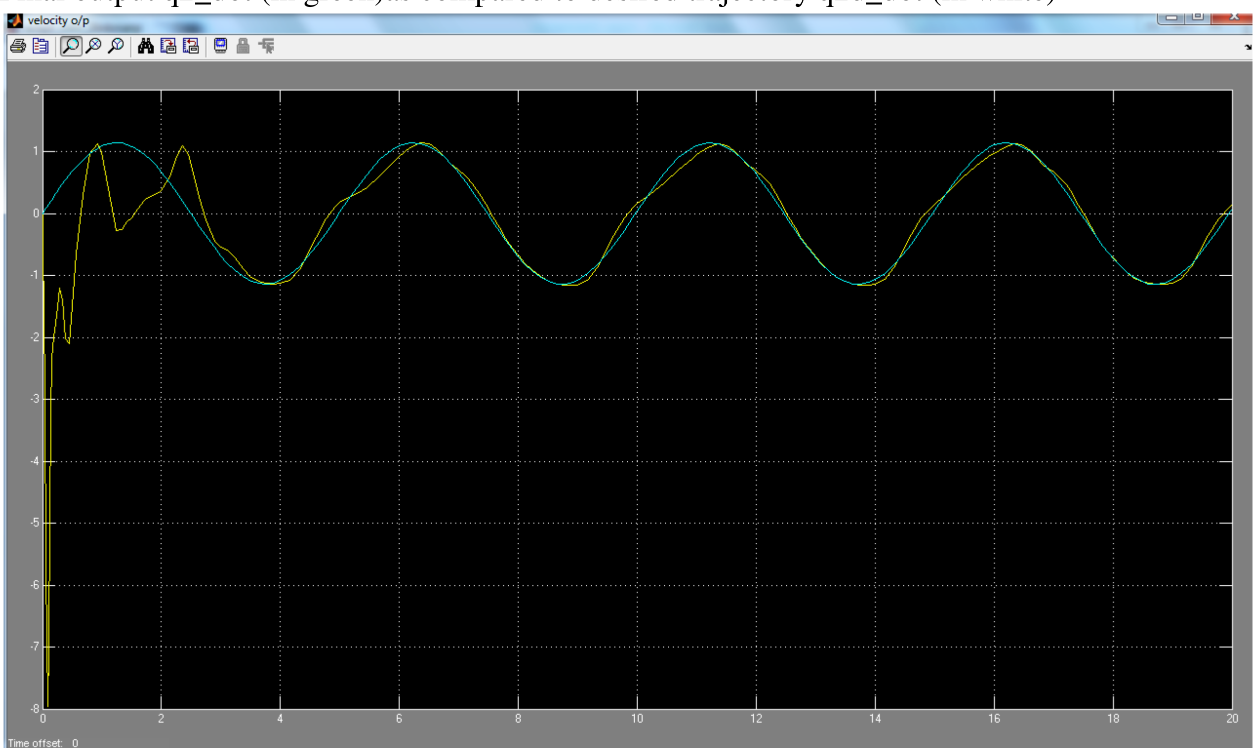
2. Final output q1_dot(in green) as compared to desired trajectory q1d_dot(in white)



3. Final output q2 (in white) as compared to desired trajectory q2d (in dark pink)
   It can be seen in the graph that curve for q2 starts from 30 and for q2d from -90



4. Final output q2_dot (in green)as compared to desired trajectory q2d_dot (in white)



5. The input control torques to the manipulator - τ_1 (in green) and τ_2 (in pink)