Assignment 3

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Contents

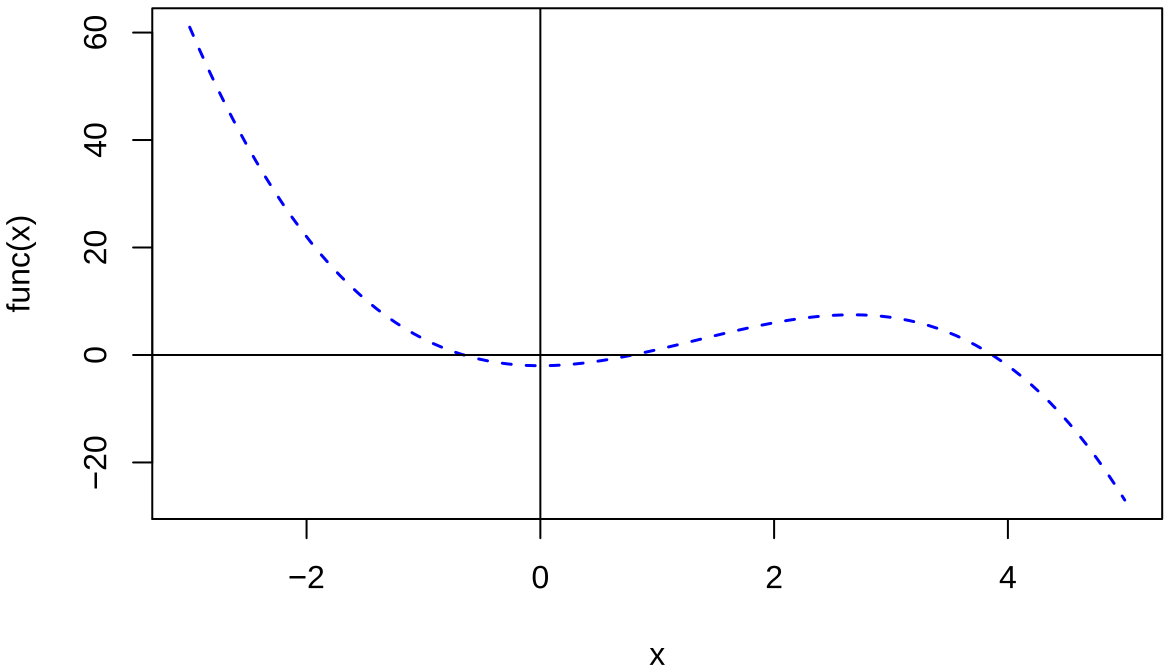

```r
library(NLRoot)
library(GenSA)
```

Problem 1.

Using the bisection method calculate at least one zero for $f(x) = -x^3 + 4x^2 - 2$ starting for a suitable initial guess. You may want to reuse the code provided in the exercise session.

```r
func <- function(x) {
  -x^3 + 4 * x^2 -2
}

curve(func, xlim = c(-3,5), col='blue', lwd=1.5, lty=2)
abline(h=0)
abline(v=0)
```



We will use the interval [-1,4]. We use the bisection method BFfzero().

```
BFfzero(func, -1, 4)
```

```
## [1] 1
## [1] -0.6554451
## [1] 1.775058e-05
## [1] 2
## [1] 0.7892418
## [1] -1.034969e-05
## [1] 3
## [1] 3.866199
## [1] -1.712537e-05
## [1] "finding root is successful"
```

The output of the function shows the following roots: x_1 = -0.6554451; x_2 = 0.7892418; x_3 = 3.866199.

Problem 2.

1. Apply an iteration of the gradient method by performing the line search in an exact way, starting from the point $A = (-1, 4)^T$. Report all the steps of the method, not just the result.

Step 0. We want to minimize the following problem:

$$f(x_1, x_2) = 2x_1^2 + x_1x_2 + 2(x_2 - 3)^2$$

Step 1. So we calculate

$$\nabla f(x) = [4x_1 + x_2, x_1 + 4(x_2 - 3)]$$

Step 2. Let's pick x_0 that corrispond to our A:

$$x_0 = A = [-1, 4]^T$$

so then:

$$\nabla f(x_0) = [4(-1) + 4, -1 + 4(4 - 3)] = [0, 3]$$

Step 3. Let's check to see if the stop criteria is statisfied, assuming that $\varepsilon = 0.1$.

$$||\nabla f(x_0)|| < \varepsilon$$

$$||\nabla f(x_0)|| = \sqrt{0 + 9} = 3 > \varepsilon$$

The stop criteria is not satisfied, then find next point.

Step 4:

$$d_0 = -\nabla f(x_0) = -[0, 3]$$

$$x_1 = [-1, 4] + \alpha_0 [0, -3] = [-1, 4 - 3\alpha_0]$$

$$f(-1, 4 - 3\alpha_0) = 18\alpha_0^2 - 9\alpha_0$$

$$f'(-1, 4 - 3\alpha_0) = 36\alpha_0 - 9$$

$$\alpha_0 = 1/4$$

$$x_1 = [-1, 13/4]$$

So then:

$$\nabla f(x_1) = [4(-1) + 13/4, -1 + 4(13/4 - 3)] = [-3/4, 0]$$

Step 5. Let's check to see if the stop criteria is now statisfied, always assuming that $\varepsilon = 0.1$.

$$||\nabla f(x_1)|| < \varepsilon$$

$$||\nabla f(x_1)|| = \sqrt{9/16 + 0} = 0.75 > \varepsilon$$

The stop criteria is not statisfied. This means that we should try with more iteration until the stop criteria is satisfied.

2. Apply an iteration of Newton's method from point A. Verify that the point found is the minimum of function f. Report all the steps of the method, not just the result.

Step 0. We want to minimize the same previous problem:

$$f(x_1, x_2) = 2x_1^2 + x_1 x_2 + 2(x_2 - 3)^2$$

Step 1. So we calculate

$$\nabla f(x) = [4x_1 + x_2, x_1 + 4(x_2 - 3)]$$

Step 2. The Hessian matrix is:

$$H(x_k) = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

and we need the inverse of the Hessian matrix:

$$H(x_k)^{-1} = \begin{bmatrix} 4/15 & -1/15 \\ -1/15 & 4/15 \end{bmatrix}$$

Step 3. Let's pick x_0 that corrispond to our A:

$$x_0 = A = [-1, 4]^T$$

then we calculate the gradient:

$$\nabla f(x_0) = [4(-1) + 4, -1 + 4(4-3)] = [0, 3]$$

Step 4. We calculate the new $\mathbf{x}$: $x_1 = x_0 - H(x_0)^{-1}\nabla f(x_0)$

$$x_1 = \begin{bmatrix} -1 \\ 4 \end{bmatrix} - \begin{bmatrix} 4/15 & -1/15 \\ -1/15 & 4/15 \end{bmatrix} * \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 3.2 \end{bmatrix}$$

Step 5. We calculate the gradient

$$\nabla f(x_1) = [4(-0.8) + 3.2, -0.8 + 4(3.2-3)] = [0, 0]$$

Since the gradient is zero, the method has converged.

3. How many iterations of Newton's method are required to optimize a quadratic function?

Newton's method uses the 2^{nd} derivative so to optimize a quadratic function it can find the optimum point in one iteration.

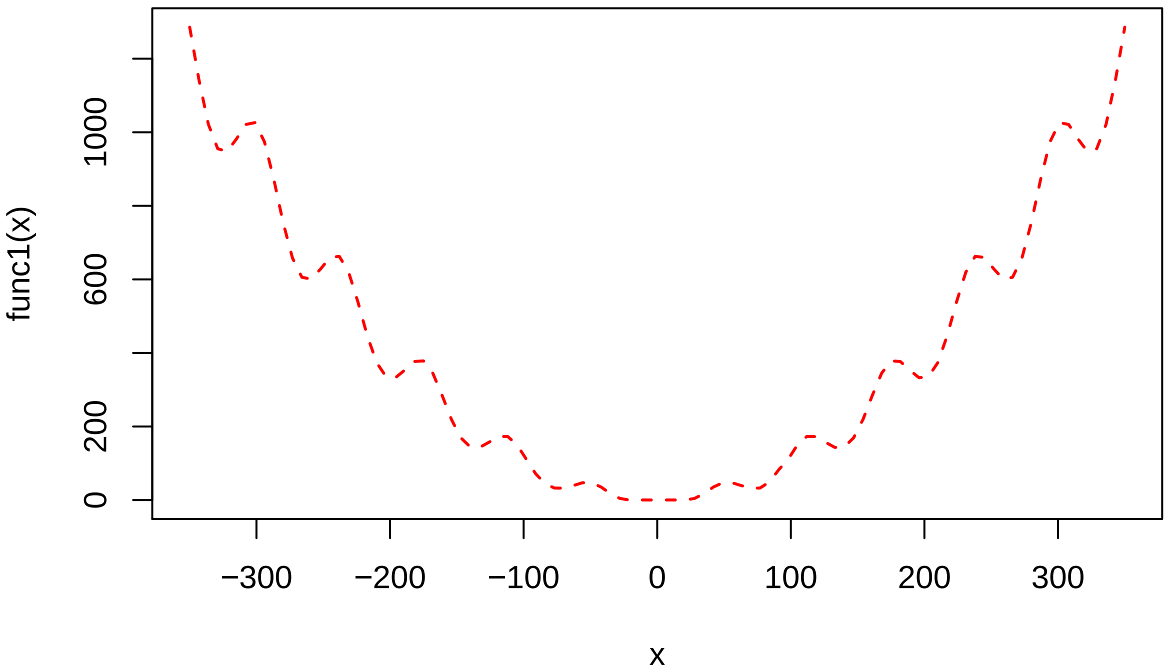
Problem 3.

Use the Simulated annealing algorithm to find the global minimum of the following function.

$$f(x) = 34e^{-\frac{1}{2}\left(\frac{x-88}{2}\right)^2} + \left(\frac{x}{10} - 2sin\left(\frac{x}{10}\right)\right)^2$$

Notiche that $f(x)$ may have several local optima, thus restarting and a careful selection of the algorithm parameters may be necessary.

```
func1 <- function(x) {
  34*(exp(1))^(-0.5*((x-88)/2)^2)+((x/10)-2*sin(x/10))^2
}
curve(func1, xlim=c(-350,350), col='red', lwd=1.5, lty=2)
```



It seems the function has the minimum very close to 0 so we will set the interval [-100,100], then we will use the GenSA package using that interval.

```
res <- GenSA(fn = func1, lower = c(-100), upper = c(100))
final_par <- res$par
final_res <- res$value
print(paste0("The global minimum of the function is equal to: ",final_par))
```

```
## [1] "The global minimum of the function is equal to: 4.06212988119108e-15"
```

```
print(paste0("The value of the function in that point is equal to: ",final_res))
```

```
## [1] "The value of the function in that point is equal to: 1.65008991716655e-31"
```