In[841]:=
```
Clear["Global`*"]
```

In[•]:=
```
KK2x2 = {{k1[-1], 0}, {0, k1[0]}};
BB2x2 = {{0, k3[-1]}, {k2[0], 0}};
M2x2 = Inverse[KK2x2].BB2x2;
KK3x3 = {{k1arm[-1], 0, 0}, {0, 2 * k1arm[0], 0}, {0, 0, k1arm[1]}};
BB3x3 = {{0, k3arm[-1], 0}, {k2arm[0], 0, k3arm[0]}, {0, k2arm[1], 0}};
M3x3 = Inverse[KK3x3].BB3x3;
```

In[•]:=
$$k1[n\_] := \frac{1}{4\,k^3\,\lambda x^3\,q[n]}\,i\,\Big(4\,k\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big]\,q[n]^3 + k\,q[n]\,\Big(4\,k^2\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] + 4\,k^2\,\lambda x^5\,\mu\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - \lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,n\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,n^2\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 8\,k^2\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] - 4\,k^2\,\lambda x^5\,\mu\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + \lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + 4\,n\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + 4\,n^2\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + 4\,k^3\,\gamma\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,g\,k\,\lambda x^3\,\rho\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big] + G[n]\,\Big(4\,k\,\big(k^2\,\gamma - g\,\lambda x^3\,\rho\big)\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] + \big(-4\,k^3\,\gamma + 4\,g\,k\,\lambda x^3\,\rho\big)\,\text{Cosh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + \lambda x\,\Big(4\,k^2\,\big(1+\lambda x^4\big)\,\mu - (1+2\,n)^2\,\lambda x^2\,\rho\,\omega^2\Big)\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big]\Big)\Big) + 4\,k^3\,\big(-k^2\,\gamma + g\,\lambda x^3\,\rho\big)\,\text{Sinh}\Big[\frac{H\,q[n]}{\lambda x}\Big] - \lambda x\,\Big(4\,k^2\,\big(2+\lambda x^4\big)\,\mu - (1+2\,n)^2\,\lambda x^2\,\rho\,\omega^2\Big)\,G[n]\,q[n]^2\,\text{Sinh}\Big[\frac{H\,q[n]}{\lambda x}\Big] + 4\,\lambda x\,\mu\,G[n]\,q[n]^4\,\text{Sinh}\Big[\frac{H\,q[n]}{\lambda x}\Big]\Big)$$

$$k1c[n\_] := -\frac{1}{4\,k^3\,\lambda x^3\,qc[n]}\,i\,\Big(4\,k\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big]\,qc[n]^3 + k\,qc[n]\,\Big(4\,k^2\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] + 4\,k^2\,\lambda x^5\,\mu\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - \lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,n\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,n^2\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - 8\,k^2\,\lambda x\,\mu\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] - 4\,k^2\,\lambda x^5\,\mu\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + \lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + 4\,n\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + 4\,n^2\,\lambda x^3\,\rho\,\omega^2\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + 4\,k^3\,\gamma\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big] - 4\,g\,k\,\lambda x^3\,\rho\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big] + Gc[n]\,\Big(4\,k\,\big(k^2\,\gamma - g\,\lambda x^3\,\rho\big)\,\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] + \big(-4\,k^3\,\gamma + 4\,g\,k\,\lambda x^3\,\rho\big)\,\text{Cosh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + \lambda x\,\Big(4\,k^2\,\big(1+\lambda x^4\big)\,\mu - (1+2\,n)^2\,\lambda x^2\,\rho\,\omega^2\Big)\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big]\Big)\Big) + 4\,k^3\,\big(-k^2\,\gamma + g\,\lambda x^3\,\rho\big)\,\text{Sinh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] - \lambda x\,\Big(4\,k^2\,\big(2+\lambda x^4\big)\,\mu - (1+2\,n)^2\,\lambda x^2\,\rho\,\omega^2\Big)\,Gc[n]\,qc[n]^2\,\text{Sinh}\Big[\frac{H\,qc[n]}{\lambda x}\Big] + 4\,\lambda x\,\mu\,Gc[n]\,qc[n]^4\,\text{Sinh}\Big[\frac{H\,qc[n]}{\lambda x}\Big]\Big)$$

$$k3[n\_] := \frac{1}{k\,q[1+n]}\,i\,\rho\,\Big(\Big(\text{Cosh}\Big[\frac{H\,k}{\lambda x}\Big] - \text{Cosh}\Big[\frac{H\,q[1+n]}{\lambda x}\Big]\Big)\,G[1+n] \times q[1+n] + q[1+n]\,\text{Sinh}\Big[\frac{H\,k}{\lambda x}\Big] - k\,\text{Sinh}\Big[\frac{H\,q[1+n]}{\lambda x}\Big]\Big)$$

$$k3c[n\_] := \frac{i\rho\left(\left(-\text{Cosh}\left[\frac{Hk}{\lambda x}\right]+\text{Cosh}\left[\frac{H\,qc[1+n]}{\lambda x}\right]\right)Gc[1+n]-\text{Sinh}\left[\frac{Hk}{\lambda x}\right]+\frac{k\,\text{Sinh}\left[\frac{H\,qc[1+n]}{\lambda x}\right]}{qc[1+n]}\right)}{k}$$

$$k2[n\_] := i\rho\left(\frac{\left(\text{Cosh}\left[\frac{Hk}{\lambda x}\right]-\text{Cosh}\left[\frac{H\,q[-1+n]}{\lambda x}\right]\right)G[-1+n]}{k}+\frac{\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{k}-\frac{\text{Sinh}\left[\frac{H\,q[-1+n]}{\lambda x}\right]}{q[-1+n]}\right)$$

$$k2c[n\_] :=$$

$$\frac{1}{k}i\rho\left(\left(-\text{Cosh}\left[\frac{Hk}{\lambda x}\right]+\text{Cosh}\left[\frac{H\,qc[-1+n]}{\lambda x}\right]\right)Gc[-1+n]-\text{Sinh}\left[\frac{Hk}{\lambda x}\right]+\frac{k\,\text{Sinh}\left[\frac{H\,qc[-1+n]}{\lambda x}\right]}{qc[-1+n]}\right)$$

In[•]:=

$$k1arm[n\_] := \frac{i\mu\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]}{\lambda x^2}+i\lambda x^2\mu\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]-\frac{in^2\rho\omega^2\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]}{k^2}-$$
$$\frac{2i\mu\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{\lambda x^2}-i\lambda x^2\mu\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]+\frac{in^2\rho\omega^2\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{k^2}+$$
$$\frac{ik\gamma\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]Garm[n]}{\lambda x^3}-\frac{ig\rho\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]Garm[n]}{k}-\frac{ik\gamma\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]Garm[n]}{\lambda x^3}+$$
$$\frac{ig\rho\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]Garm[n]}{k}+\frac{i\mu\,\text{Cosh}\left[\frac{H\,qarm[n]}{\lambda x}\right]qarm[n]^2}{k^2\lambda x^2}+\frac{ik\gamma\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{\lambda x^3}-$$
$$\frac{ig\rho\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{k}+\frac{i\mu\,Garm[n]\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{\lambda x^2}+i\lambda x^2\mu\,Garm[n]\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]-$$
$$\frac{in^2\rho\omega^2\,Garm[n]\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{k^2}-\frac{ik^2\gamma\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{\lambda x^3\,qarm[n]}+\frac{ig\rho\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{qarm[n]}-$$
$$\frac{2i\mu\,Garm[n]\times qarm[n]\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{k\lambda x^2}-\frac{i\lambda x^2\mu\,Garm[n]\times qarm[n]\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{k}+$$
$$\frac{in^2\rho\omega^2\,Garm[n]\times qarm[n]\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{k^3}+\frac{i\mu\,Garm[n]\,qarm[n]^3\,\text{Sinh}\left[\frac{H\,qarm[n]}{\lambda x}\right]}{k^3\lambda x^2}$$

$$k3arm[n\_] := \frac{i\rho\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]Garm[1+n]}{k}-\frac{i\rho\,\text{Cosh}\left[\frac{H\,qarm[1+n]}{\lambda x}\right]Garm[1+n]}{k}+$$
$$\frac{i\rho\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{k}-\frac{i\rho\,\text{Sinh}\left[\frac{H\,qarm[1+n]}{\lambda x}\right]}{qarm[1+n]}$$

$$k2arm[n\_] := \frac{i\rho\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]Garm[-1+n]}{k}-\frac{i\rho\,\text{Cosh}\left[\frac{H\,qarm[-1+n]}{\lambda x}\right]Garm[-1+n]}{k}+$$
$$\frac{i\rho\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]}{k}-\frac{i\rho\,\text{Sinh}\left[\frac{H\,qarm[-1+n]}{\lambda x}\right]}{qarm[-1+n]}$$

In[•]:=

$$G[n\_] := -\frac{k\left(-2k\,q[n]\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]+k^2\,\text{Sinh}\left[\frac{H\,q[n]}{\lambda x}\right]+q[n]^2\,\text{Sinh}\left[\frac{H\,q[n]}{\lambda x}\right]\right)}{k^2\left(-2\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]+\text{Cosh}\left[\frac{H\,q[n]}{\lambda x}\right]\right)q[n]+\text{Cosh}\left[\frac{H\,q[n]}{\lambda x}\right]q[n]^3}$$

$$Gc[n\_] := -\frac{k\left(-2k\,qc[n]\,\text{Sinh}\left[\frac{Hk}{\lambda x}\right]+k^2\,\text{Sinh}\left[\frac{H\,qc[n]}{\lambda x}\right]+qc[n]^2\,\text{Sinh}\left[\frac{H\,qc[n]}{\lambda x}\right]\right)}{k^2\left(-2\,\text{Cosh}\left[\frac{Hk}{\lambda x}\right]+\text{Cosh}\left[\frac{H\,qc[n]}{\lambda x}\right]\right)qc[n]+\text{Cosh}\left[\frac{H\,qc[n]}{\lambda x}\right]qc[n]^3}$$

```
In[•]:= Garm[n_] := -(k (-2 k qarm[n] Sinh[H k/λx] + k^2 Sinh[H qarm[n]/λx] + qarm[n]^2 Sinh[H qarm[n]/λx]))/(k^2 (-2 Cosh[H k/λx] + Cosh[H qarm[n]/λx]) qarm[n] + Cosh[H qarm[n]/λx] qarm[n]^3)

In[•]:= q[0] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - αω^2];
        qc[0] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - αω^2];
        q[-1] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - αω^2];
        qc[-1] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - αω^2];
        q[1] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - 9 αω^2];
        qc[1] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - 9 αω^2];
        q[-2] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - 9 αω^2];
        qc[-2] = λx / (2 H) * Sqrt[4 k^2 H^2 λx^2 - 9 αω^2];

In[•]:= qarm[0] := (λx / H) Sqrt[k^2 H^2 λx^2];
        qarm[-1] := (λx / H) Sqrt[λx^2 k^2 H^2 - αω^2];
        qarm[1] := (λx / H) Sqrt[λx^2 k^2 H^2 - αω^2];

In[•]:= ω = Sqrt[μ / ρ] (αω / H);
        γ = μ H αγ;
        μ = (ρ g H) / αg;

In[•]:= SUBKKadim = Simplify[Simplify[KK2x2] /. k → kk / H];
        SUBBBadim = Simplify[Simplify[BB2x2] /. k → kk / H];

In[•]:= ARMKKadim = Simplify[Simplify[KK3x3] /. k → kk / H];
        ARMBBadim = Simplify[Simplify[BB3x3] /. k → kk / H];

In[•]:= SUBMadim = Simplify[g * Inverse[SUBKKadim].SUBBBadim];
        ARMMadim = Simplify[g * Inverse[ARMKKadim].ARMBBadim];

        β = 0 ;
        For[α = 6 / 10, α < 14 / 10, α += 1 / 10;
         β += 1;
         αgval = 1 / 1000;
         αγval = 0;
         p = 0;
         nn = 0;
         For[jj = 0, jj < 23 / 10, jj += 1 / 1000;
          p += 1;
          Print["αω= ", N[jj]];
          upval = 200;
          div = 100;
          nn = N[jj];
          For[i = 0, i < upval, i += 1;
           bb = (1 * i / div);
           stateARM = False;
           For[r = 1, r < 3, r++,
            For[t = 1, t < 3, t++,
             If[(Chop[(ARMMadim /. {αω → nn, kk → N[bb], αγ → αγval, αg → N[αgval],
                      λx → N[α]})])[[r, t]] === Indeterminate, stateARM = True]
```

```
    ]
   ];
   If[stateARM, eigenARM = 10^(16),
    eigenARM = N[Eigenvalues[(ARMMadim /. {αω → nn, kk → N[bb],
          αγ → αγval, αg → N[αgval], λx → N[α]})], Method → "Direct"]]];
   inveigenARM[i] = 1/Max[Re[eigenARM]];
  ];
  stato = False;
  listaARM[p] = Table[{(1 * i/div), inveigenARM[i]}, {i, 1, upval}];
  vet = {};
  For[ll = 0, ll < Length[listaARM[p]], ll += 1;
   AppendTo[vet, listaARM[p][[ll]][[2]]]
  ];
  ord = Sort[vet];
  Print[N[ord[[1]]]];
  If[ord[[1]] == 10^(-16), stato = True];
  If[stato, acr[p] = ord[[2]], acr[p] = ord[[1]]];
  pos = (Position[listaARM[p], acr[p]] - {{0, 1}}) // Flatten;
  kcr[p] = N[Extract[listaARM[p], pos]];
  Print["k= ", N[kcr[p]], " a= ", PowerExpand[acr[p]]];
  αωad[p] = jj;
 ];
 listaac[β] = Table[{αωad[jj], acr[jj]}, {jj, 1, p}];
 listakc[β] = Table[{αωad[jj], kcr[jj]}, {jj, 1, p}];
]
graficoa = Table[listaac[α], {α, 1, β}];
graficok = Table[listakc[α], {α, 1, β}];
```