```
In[849]:= Clear["Global`*"]
ln[850] = KK3x3 = \{\{k1arm[-1], 0, 0\}, \{0, 2 * k1arm[0], 0\}, \{0, 0, k1arm[1]\}\};
                                KK5x5 = \{\{k1arm[-2], 0, 0, 0, 0\}, \{0, k1arm[-1], 0, 0, 0\},
                                                    \{0, 0, 2 * klarm[0], 0, 0\}, \{0, 0, 0, klarm[1], 0\}, \{0, 0, 0, 0, klarm[2]\}\};
                                BB3x3 = \{\{0, k3arm[-1], 0\}, \{k2arm[0], 0, k3arm[0]\}, \{0, k2arm[1], 0\}\};
                                BB5x5 = \{\{0, k3arm[-2], 0, 0, 0\},
                                                   {k2arm[-1], 0, k3arm[-1], 0, 0}, {0, k2arm[0], 0, k3arm[0], 0},
                                                   {0, 0, k2arm[1], 0, k3arm[0]}, {0, 0, 0, k2arm[2], 0}};
                                M3x3 = Inverse[KK3x3].BB3x3;
                               M5x5 = Inverse[KK5x5].BB5x5;
In[856]:= k1arm[n_] :=
                                     -3 \pm \mu \, \text{Cosh}[\text{H qarm}[n]] \, \text{If}[n == 0, 0, 1] + \frac{\pm n^2 \, \rho \, \omega^2 \, \text{Cosh}[\text{H qarm}[n]] \, \text{If}[n == 0, 0, 1]}{k^2} - \frac{1}{2} \, \frac{1}{2}
                                             ik\gamma Cosh[Hqarm[n]] Garm[n] \times If[n = 0, 0, 1] +
                                              ig \rho Cosh[Hqarm[n]] Garm[n] \times If[n == 0, 0, 1]
                                              \frac{i n^2 \rho \omega^2 \operatorname{Cosh}[H \, k] \operatorname{Garm}[n] \times \operatorname{If}[n == 0, 1, 0]}{\iota^3}
                                              \frac{3 \pm \mu \operatorname{Cosh}[\operatorname{H} \operatorname{qarm}[n]] \operatorname{Garm}[n] \times \operatorname{If}[n = 0, 1, 0]}{\nu} +
                                              \frac{i}{m} n^2 \rho \omega^2  Cosh[H qarm[n]] Garm[n] × If[n == 0, 1, 0] +
                                            2 \pm \mu \, \text{Cosh}[\text{H k}] \, \text{If}[\text{n} == 0, \, \text{H}, \, 1] - \frac{\pm \, \text{n}^2 \, \rho \, \omega^2 \, \text{Cosh}[\text{H k}] \, \text{If}[\text{n} == 0, \, \text{H}, \, 1]}{k^2} +
                                            ikγCosh[Hk] Garm[n] \times If[n == 0, H, 1] -\frac{igρCosh[Hk] Garm[n] \times If[n == 0, H, 1] + k
                                              \frac{i \mu \operatorname{Cosh}[H \operatorname{qarm}[n]] \operatorname{If}[n = 0, 0, 1] \operatorname{qarm}[n]^{2}}{\nu^{2}} +
                                              \underline{\dot{\mathbf{n}}} \; \mu \; \mathsf{Cosh}[\mathsf{H} \; \mathsf{qarm}[\mathsf{n}]] \; \mathsf{Garm}[\mathsf{n}] \times \mathsf{If}[\mathsf{n} = 0, 1, 0] \; \mathsf{qarm}[\mathsf{n}]^2
                                             \frac{ \mbox{$\dot{\text{i}}$ n}^2 \, \rho \, \omega^2 \, \text{If}[n=0\,,\,1,\,0] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$k^3$}} + \mbox{$\dot{\text{i}}$ k } \gamma \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$k$}} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$k$}} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$k$}} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]} + 2 \, \mbox{$\dot{\text{i}}$ $\mu$ Garm}[n] \times \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] - \\ \frac{\mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]}{ \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}]} + 2 \, \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] + 2 \, \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] + 2 \, \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{Sinh}[\text{H}\,\text{k}] + 2 \, \mbox{$\dot{\text{i}}$ g} \, \rho \, \, \text{If}[n=0\,,\,\text{H},\,1] \, \, \text{If}[n=
                                              \frac{i n^2 \rho \omega^2 \operatorname{Garm}[n] \times \operatorname{If}[n = 0, H, 1] \operatorname{Sinh}[H k]}{I}
                                              \frac{i k^2 \gamma \text{If}[n = 0, 0, 1] \text{Sinh}[\text{H} \text{qarm}[n]]}{\text{garm}[n]} + \frac{i g \rho \text{If}[n = 0, 0, 1] \text{Sinh}[\text{H} \text{qarm}[n]]}{\text{garm}[n]} -
                                                                                                                                                                                                                                                                                                                                                     qarm[n]
                                                                                                                               qarm[n]
                                              \frac{i k \gamma Garm[n] \times If[n = 0, 1, 0] Sinh[H qarm[n]]}{+}
                                                                                                                                                       qarm[n]
                                              \frac{i g \rho Garm[n] \times If[n = 0, 1, 0] Sinh[H qarm[n]]}{.}
                                              \frac{3 \pm \mu \operatorname{Garm}[n] \times \operatorname{If}[n = 0, 0, 1] \times \operatorname{qarm}[n] \operatorname{Sinh}[\operatorname{H} \operatorname{qarm}[n]]}{\mathsf{k}} +
                                            \frac{1}{\iota^3} \pm n^2 \rho \omega^2 \operatorname{Garm}[n] \times \operatorname{If}[n = 0, 0, 1] \times \operatorname{qarm}[n] \operatorname{Sinh}[H \operatorname{qarm}[n]] +
```

```
i\mu Garm[n] × If[n = 0, 0, 1] qarm[n]<sup>3</sup> Sinh[H qarm[n]]
                k3arm[n_{-}] := -\frac{i\rho Cosh[Hqarm[1+n]] Garm[1+n] \times If[1+n=0,0,1]}{.} + \frac{i\rho Cosh[Hqarm[1+n]] Garm[1+n]}{.} + \frac{i\rho Cosh[Hqarm[1+n]]}{.} + \frac{i\rho Cosh[Hqarm[1+n]]}
                       \frac{i \rho Cosh[H k] Garm[1+n] \times If[1+n == 0, H, 1]}{} +
                      \frac{i \rho \text{ If}[1+n=0, H, 1] \text{ Sinh}[H k]}{-i \rho \text{ If}[1+n=0, 0, 1] \text{ Sinh}[H \text{ qarm}[1+n]]}
                                                                                                                                                            qarm[1+n]
                       \underline{i} \rho Garm[1+n] \times If[1+n == 0, 1, 0] Sinh[Hqarm[1+n]]
               k2arm[n_{-}] := -\frac{1}{k} i \rho Cosh[Hqarm[-1+n]] Garm[-1+n] \times If[-1+n == 0, 0, 1] +
                       \frac{i \rho Cosh[H k] Garm[-1+n] \times If[-1+n == 0, H, 1]}{i}+
                      \frac{i \rho \text{ If}[-1 + n == 0, H, 1] \text{ Sinh}[H k]}{-i \rho \text{ If}[-1 + n == 0, 0, 1] \text{ Sinh}[H \text{ qarm}[-1 + n]]}
                                                                                                                                                                  qarm[-1+n]
                       \underline{i} \rho Garm[-1+n] \times If[-1+n == 0, 1, 0] Sinh[H qarm[-1+n]]
                                                                                   k qarm[-1+n]
In[689]:= Garm[n_] :=
                   (k (-2 Cosh[Hk] If[n = 0, 1, 0] \times qarm[n] - 2 k If[n = 0, H, 1] \times qarm[n] Sinh[Hk] +
                                   k^2 If [n == 0, 0, 1] Sinh[H qarm[n]] + If [n == 0, 0, 1] qarm[n]<sup>2</sup> Sinh[H qarm[n]])) /
                      (-k^2 Cosh[H qarm[n]] If[n = 0, 0, 1] \times qarm[n] + 2k^2 Cosh[H k]
                                If [n = 0, H, 1] \times qarm[n] - Cosh[Hqarm[n]] If [n = 0, 0, 1] qarm[n]^3 +
                             2 \text{ k If}[n = 0, 1, 0] \times \text{qarm}[n] \text{ Sinh}[H k] - k^2 \text{ If}[n = 0, 1, 0] \text{ Sinh}[H \text{ qarm}[n]] -
                             If [n = 0, 1, 0] qarm[n]^2 Sinh[H qarm[n]])
ln[690]:= qarm[0] := 1/H Sqrt[k^2 H^2];
               qarm[-1] := 1/H Sqrt[k^2H^2 - \alpha\omega^2];
               qarm[1] := 1/H Sqrt[k^2H^2 - \alpha\omega^2];
               qarm[-2] := 1/H Sqrt[k^2H^2 - 4\alpha\omega^2];
               qarm[2] := 1/H Sqrt[k^2 H^2 - 4 \alpha \omega^2];
ln[695] = \lambda x = 1;
               \omega = \operatorname{Sqrt}[\mu / \rho] (\alpha \omega / H);
               \gamma = \mu H \alpha \gamma;
               \mu = (\rho g H) / \alpha g;
ln[699]:= KK3x3a = Simplify[Simplify[KK3x3] /. k \rightarrow kk/H];
               KK5x5a = Simplify[Simplify[KK5x5] /. k → kk/H];
               BB3x3a = Simplify[Simplify[BB3x3] /. k → kk/H];
               BB5x5a = Simplify[Simplify[BB5x5] /. k → kk/H];
               M3x3a = Simplify[g * Inverse[KK3x3a].BB3x3a];
   Inf@]:= limite\alpha y3x3 = Limit[M3x3a, <math>\alpha y \rightarrow 0];
               limite\alpha \gamma 5 x 5 = \text{Limit}[M5 x 5 a, \alpha \gamma \rightarrow 0];
```

```
In[*]:= upval = 300;
     div = 100;
     \alpha \gamma val = 0;
     \alphagval = 1;
     \alpha\omegaval = 1.5;
     H = 1;
     For[i = 0, i < upval, i += 1;
        bb = (1 * i / div);
        state3x3 = False;
        For [r = 1, r < 3, r++,
          For [t = 1, t < 3, t++,
           If [(Chop[(limite\alpha\gamma3x3 /. \{\alpha\omega \rightarrow N[\alpha\omega val], kk \rightarrow N[bb], \alphag \rightarrow N[\alpha gval]\})])[[
                r, t]] === Indeterminate, state3x3 = True
         1
        ];
        state5x5 = False;
        For [r = 1, r < 3, r++,
          For [t = 1, t < 3, t++,
           If [(Chop[(limite\alpha\gamma5x5 /. \{\alpha\omega \rightarrow N[\alpha\omega val], kk \rightarrow N[bb], \alphag \rightarrow N[\alpha gval]\})])[[
                r, t]] === Indeterminate, state5x5 = True
         1
        ];
        If [state3x3, eigen3x3 = 10^{(16)}, eigen3x3 =
           N[Eigenvalues[(limite\alpha\gamma3x3/. \{\alpha\omega \rightarrow N[\alpha\omega val], kk \rightarrow N[bb], \alphag \rightarrow N[\alpha gval]\}),
              Method → "Direct"]]];
        If [state5x5, eigen5x5 = 10^{(16)}, eigen5x5 =
           N[Eigenvalues[(limite\alpha\gamma5x5 /. \{\alpha\omega \rightarrow N[\alpha\omega val], kk \rightarrow N[bb], \alphag \rightarrow N[\alpha gval]\}),
              Method → "Direct"]]];
        inveigen3x3[i] = 1 / Max[Re[eigen3x3]];
        inveigen5x5[i] = 1 / Max[Re[eigen5x5]];
       ];
     lista3x3 = Table[\{(1*i/div), inveigen3x3[i]\}, \{i, 1, upval\}];
     lista5x5 = Table [\{(1*i/div), inveigen5x5[i]\}, \{i, 1, upval\}];
In[*]:= vet3x3 = {};
     For[ll = 0, ll < Length[lista3x3], ll += 1;</pre>
        AppendTo[vet3x3, lista3x3[[ll]][[2]]]
       ];
     ord3x3 = Sort[vet3x3];
     ac3x3 = ord3x3[[2]];
     N[ac3x3]
     pos3x3 = (Position[lista3x3, ac3x3] - {{0, 1}}) // Flatten;
     k3x3 = N[Extract[lista3x3, pos3x3]]
```

```
In[*]:= vet5x5 = {};
     For[ll = 0, ll < Length[lista5x5], ll += 1;</pre>
        AppendTo[vet5x5, lista5x5[[ll]][[2]]]
     ord5x5 = Sort[vet5x5];
     ac5x5 = ord5x5[[3]];
     N[ac5x5]
     pos5x5 = (Position[lista5x5, ac5x5] - {{0, 1}}) // Flatten;
     k5x5 = N[Extract[lista5x5, pos5x5]]
     \beta = 0;
     For \alpha = 0, \alpha < 622 / 100, \alpha + = 2 / 10;
      Print["\alphag= ", N[\alpha]];
      \beta += 1;
      p = 0;
      nn = 0;
      For [jj = 0, jj < 155/100, jj += 1/100;
       Print["\alpha\omega= ", N[jj]];
        upval = 250;
        div = 100;
        \alpha \gamma val = 0;
        nn = N[jj];
        For[i = 0, i < upval, i += 1;
         bb = (1 * i / div);
         stateARM = False;
         For [r = 1, r < 3, r++,
           For [t = 1, t < 3, t++,
            If [(Chop[(ARMMadim /. \{\alpha\omega \rightarrow nn, kk \rightarrow N[bb], \alpha\gamma \rightarrow N[\alpha\gamma val], \alphag \rightarrow N[\alpha]\})])[[
                r, t]] === Indeterminate, stateARM = True
          ]
         ];
         If[stateARM, eigenARM = 10^(16), eigenARM =
            N[Eigenvalues[(ARMMadim /. \{\alpha\omega \rightarrow nn, kk \rightarrow N[bb], \alpha\gamma \rightarrow N[\alpha\gamma val], \alphag \rightarrow N[\alpha]\}),
               Method → "Direct"]]];
         inveigenARM[i] = 1 / Max[Re[eigenARM]];
        stato = False;
        listaARM[p] = Table[\{(1*i/div), inveigenARM[i]\}, \{i, 1, upval\}];
        vet = {};
        For[ll = 0, ll < Length[listaARM[p]], ll += 1;</pre>
         AppendTo[vet, listaARM[p][[ll]][[2]]]
        ];
        ord = Sort[vet];
        Print[N[ord[[1]]]];
        If [ord[[1]] == 10^(-16), stato = True];
        If[stato, acr[p] = ord[[2]], acr[p] = ord[[1]]];
```

```
pos = (Position[listaARM[p], acr[p]] - {{0, 1}}) // Flatten;
    kcr[p] = N[Extract[listaARM[p], pos]];
    Print["k= ", N[kcr[p]], " a= ", PowerExpand[acr[p]]];
    αωad[p] = jj;
];
listaac[β] = Table[{αωad[jj], acr[jj]}, {jj, 1, p}];
listakc[β] = Table[{αωad[jj], kcr[jj]}, {jj, 1, p}];
]
graficoa = Table[listaac[α], {α, 1, β}];
graficok = Table[listakc[α], {α, 1, β}];
```