Concepts of Integrated Luminosity Optimization Strategy

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September 2021

Abstract

The Luminosity is the key figure of merit of a collider and its integrated value is closely related with the data collected by the physics detectors. In order to maximise the probability of physics discovery, the goal of a collider run is to collect the largest possible integrated luminosity of a circular collider over the fixed time allocated for physics. The aim of this work is to consider the objective to devising a strategy to maximise the integrated luminosity of a circular collider under the assumption of the knowledge of the distribution turn-around times, i.e. the times between the end of a physics fill, and the probability of failure of a physics fill.

Contents

1 Introduction

The process of luminosity production is sketched in Fig. 1 where the magnetic cycle is shown and the main quantities are introduced.

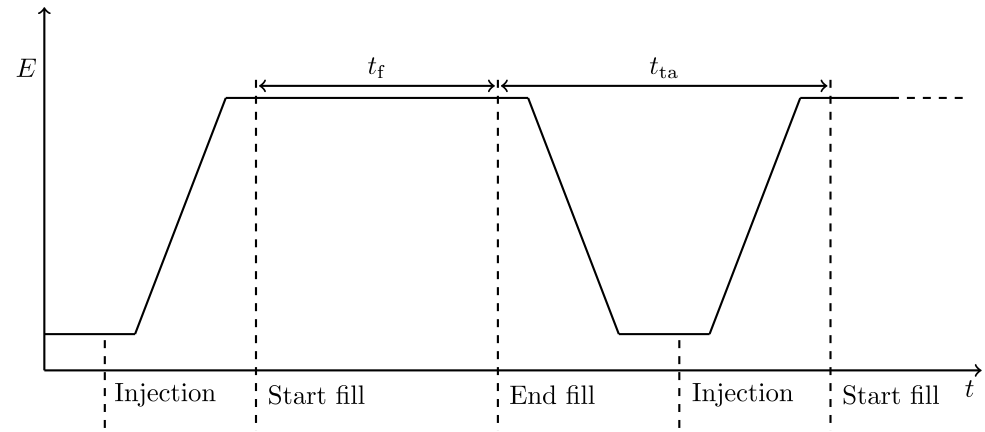


Figure 1: Sketch of the luminosity production process in a circular collider, where E is the beam energy.

In this study, $L(t)$ represents the evolution of the luminosity during a fill. The goal is to maximise the integrated luminosity, and, in a purely deterministic case, this corresponds to maximising

$$\begin{aligned} L_{\text{tot}}(t_{\text{f}}) &= N_{\text{fill}} \int_0^{t_{\text{f}}} \mathrm{d}t\, L(t) \\ &= \frac{T}{t_{\text{ta}} + t_{\text{f}}} \int_0^{t_{\text{f}}} \mathrm{d}t\, L(t)\,, \end{aligned} \tag{1}$$

where T is the total time for physics, t_{ta} is the so-called turnaround, i.e. the time elapse from the end a fill for physics and the start of the next one, t_{f} is the length of the physics fill. In this framework, the optimisation of is performed by noting that $L_{\text{tot}} = L_{\text{tot}}(t_{\text{f}})$ and the maximum can be found by solving the equation

$$\frac{\mathrm{d}L_{\text{tot}}}{\mathrm{d}t_{\text{f}}} = 0\,. \tag{2}$$

2 The L(t) model

As described in [1], the luminosity can be derived form the only time-dependent beam parameter: the intensity N[1]. Considering the burn off as the only relevant mechanism for a time-variation, in case of round beams ($\epsilon_x^* = \epsilon_y^* = \epsilon^*$) and round optics ($\beta_x^* = \beta_y^* = \beta^*$), it is possible to find

$$L(t) = \frac{\Xi N_{\text{i}}^2}{(1 + \sigma_{\text{int}} n_{\text{c}} \Xi N_{\text{i}} t)^2}\,, \tag{3}$$

[1] $N_i = k_{\text{b}} n_i$, where n_i represents the bunch population.

where $\sigma_{\rm int}$ is the cross section for interaction of charged particles, $n_{\rm c}$ stands for the number of collision points and the two colliding beams have been assumed to be of equal intensity (which is the best scenario as far as the luminosity is concerned). The term Ξ stands for

$$\Xi = \frac{\gamma_{\rm r} f_{\rm rev}}{4\pi\epsilon^* \beta^* k_{\rm b}} F(\theta_{\rm c}, \sigma_{\rm z}, \sigma^*),$$

where $\gamma_{\rm r}$ is the relativistic γ-factor, $f_{\rm rev}$ is the revolution frequency, $k_{\rm b}$ the number of colliding bunches, ϵ^* is the RMS normalized transverse emittance, β^* is the value of the beta-function at the collision point and the factor F accounts for the reduction in volume overlap between the colliding bunches due to the presence of a crossing angle

$$F(\theta_{\rm c}, \sigma_{\rm z}, \sigma^*) = 1/\sqrt{1 + (\frac{\theta_{\rm c}}{2}\frac{\sigma_{\rm z}}{\sigma_*})}, \qquad \text{with } \sigma^* = \sqrt{\beta^* \epsilon^*/(\beta_{\rm r}\gamma_{\rm r})},$$

where $\sigma_{\rm z}$ is the longitudinal RMS dimension, $\beta_{\rm r}$ is the relativistic β and $\theta_{\rm c}/2$ is the half crossing angle. At this point, assuming the simple case of equal intensities for both beams, it is possible to obtain for the burn off part

$$L_{\rm int}(t) = \int_0^t dt L(t) = \frac{N_{\rm i}^2 t_{\rm f} \Xi}{(N_{\rm i} t_{\rm f} \Xi n_{\rm c} \sigma_{\rm int} + 1)} = \frac{N_{\rm i}\Xi}{\varepsilon f_{\rm rev}} \frac{\varepsilon N_{\rm i} f_{\rm rev} t}{1 + \varepsilon N_{\rm i} f_{\rm rev} t}, \tag{4}$$

where $\varepsilon = \sigma_{\rm int} n_{\rm c} \Xi / f_{\rm rev}$. The next table (1) shows the values of the above-mentioned parameters for Run 2.

Table 1: LHC Run 2 Parameters for the Luminosity Model [2] [3].

Parameters		**2016**	**2017**	**2018**
$L_{\rm Peak}$	[10^{34} cm^{-2} s^{-1}]	1.4	2.1	2.1
$E_{\rm beam}$	[TeV]	6.5	6.5	6.5
$f_{\rm rev}$	[kHz]	11.2	11.2	11.2
N	[10^{11}]	$1.0 - 1.25$	$1.0 - 1.25$	$1.0 - 1.25$
β^*	[cm]	0.4	$0.4 - 0.3$	$0.3 - 0.25$
ϵ^*	[µm]	~ 2.2	~ 2.2	~ 1.9
$n_{\rm c}$		2	2	2
$\sigma_{\rm z}$	[m]	0.102	0.102	0.102
$\sigma_{\rm int}$	[10^{-30} m^2]	7.95	7.95	7.95
$k_{\rm b}$		2220	2556/1868	2556
$\theta_{\rm c}/2$	[µrad]	$185 - 140$	$150 - 120$	$160 - 130$
$\gamma_{\rm r}$		6929.64	6929.64	6929.64
$\beta_{\rm r}$		~ 1	~ 1	~ 1
No. days of physics operations		146	140	145

3 Optimization with a fixed t_{ta}

A very first attempt of integrated luminosity optimization can then be carried out by solving the equation (2) or going through the Lagrange Multipliers method, which is based on the *Lagrange Multipliers Theorem.*

Lagrange Multipliers Theorem. *If $f, g \in \mathcal{C}^1(\mathbb{R}^2)$ and $(\hat{x}, \hat{y})$ is a constrained critical point of f, with $g(\hat{x}, \hat{y}) = c$ and $\nabla g(\hat{x}, \hat{y}) \neq (0,0)$, then it exists $\lambda \in \mathbb{R}$ (called Lagrange Multiplier) such that [4]:*

$$\nabla f(\hat{x}, \hat{y}) = \lambda \nabla g(\hat{x}, \hat{y})$$

In first approximation, it is possible to consider t_{ta} as given, and so the function to be optimized is

$$f(t_{\mathrm{f}}, N_{\mathrm{fill}}) = N_{\mathrm{fill}} \int_0^{t_{\mathrm{f}}} dt L(t), \tag{5}$$

and the constraint function is

$$g(t_{\mathrm{f}}, N_{\mathrm{fill}}) = N_{\mathrm{fill}} t_{\mathrm{f}} + N_{\mathrm{fill}} t_{\mathrm{ta}}. \tag{6}$$

Thus, the system to be solved will be:

$$\begin{cases} \left(\frac{\partial f(t_{\mathrm{f}}, N_{\mathrm{fill}})}{\partial t_{\mathrm{f}}}\right) = \lambda \left(\frac{\partial g(t_{\mathrm{f}}, N_{\mathrm{fill}})}{\partial t_{\mathrm{f}}}\right) \\ \\ \left(\frac{\partial f(t_{\mathrm{f}}, N_{\mathrm{fill}})}{\partial N_{\mathrm{fill}}}\right) = \lambda \left(\frac{\partial g(t_{\mathrm{f}}, N_{\mathrm{fill}})}{\partial N_{\mathrm{fill}}}\right) \\ \\ N_{\mathrm{fill}} t_{\mathrm{f}} + N_{\mathrm{fill}} t_{\mathrm{ta}} = T \end{cases} \Rightarrow \begin{cases} N_{\mathrm{fill}} L(t_{\mathrm{f}}) = \lambda N_{\mathrm{fill}} \\ \\ \mathbb{L}(t_{\mathrm{f}}) - \mathbb{L}(0) = \lambda (t_{\mathrm{f}} + t_{\mathrm{ta}}) \\ \\ N_{\mathrm{fill}} t_{\mathrm{f}} + N_{\mathrm{fill}} t_{\mathrm{ta}} = T \end{cases} \tag{7}$$

where $L(t_{\mathrm{f}})$ is the integrable function of (5) evaluated in t_{f}, and $\mathbb{L}$ is the primitive functions of $L(t)$. At this point, solving the system, it is possible to find that:

$$\begin{cases} \lambda = L(t_{\mathrm{f}}) \qquad \text{(considering that } N_{\mathrm{fill}} \neq 0) \\ \\ \mathbb{L}(t_{\mathrm{f}}) - \mathbb{L}(0) = L(t_{\mathrm{f}})(t_{\mathrm{f}} + t_{\mathrm{ta}}) \\ \\ N_{\mathrm{fill}} = \frac{T}{t_{\mathrm{f}} + t_{\mathrm{ta}}} \end{cases} \tag{8}$$

Substituting equations (3) and (4) in the system (8):

$$\begin{cases} \lambda = \frac{\Xi N_{\mathrm{i}}^2}{(1 + \sigma_{\mathrm{int}} n_{\mathrm{c}} \Xi N_{\mathrm{i}} t_f)^2} \\ \\ \frac{N_{\mathrm{i}}^2 t_{\mathrm{f}} \Xi}{(N_{\mathrm{i}} t_{\mathrm{f}} \Xi n_{\mathrm{c}} \sigma_{\mathrm{int}} + 1)} = \frac{\Xi N_{\mathrm{i}}^2}{(1 + \sigma_{\mathrm{int}} n_{\mathrm{c}} \Xi N_{\mathrm{i}} t_f)^2} (t_{\mathrm{f}} + t_{\mathrm{ta}}) \\ \\ N_{\mathrm{fill}} = \frac{T}{t_{\mathrm{f}} + t_{\mathrm{ta}}} \end{cases} \tag{9}$$

Solving the previous system is equivalent to solve the following equation:

$$\frac{N_{\mathrm{i}}^2 \Xi t_{\mathrm{f}}}{N_{\mathrm{i}} n_{\mathrm{c}} \Xi \sigma_{\mathrm{int}} t_{\mathrm{f}} + 1} = \frac{N_{\mathrm{i}}^2 \Xi (t_{\mathrm{f}} + t_{\mathrm{ta}})}{(N_{\mathrm{i}} n_{\mathrm{c}} \Xi \sigma_{\mathrm{int}} t_{\mathrm{f}} + 1)^2}, \tag{10}$$

whose real solutions for the optimal fill time t_{f} are:

$$t_{\mathrm{opt}} = \pm \frac{\sqrt{t_{\mathrm{ta}}}}{\sqrt{N_{\mathrm{i}} n_{\mathrm{c}} \Xi \sigma_{\mathrm{int}}}}. \tag{11}$$

It is therefore possible to conclude that for relatively consistent variations in the turnaround time, there will be rather small variations in the optimal fill time.

4 Optimization with a distribution of t_{ta}

At this point, it is possible to consider n values t_i, distributing according to a certain probability density function (p.d.f.), representing n realisations of the turnaround time. In this case the function to be maximised is:

$$L_{\mathrm{tot}}(\hat{t}) = n \int_0^{\hat{t}} \mathrm{d}t \, L(t) \; = n \frac{N_{\mathrm{i}}^2 \hat{t} \Xi}{(N_{\mathrm{i}} \hat{t} \Xi n_{\mathrm{c}} \sigma_{\mathrm{int}} + 1)}, \tag{12}$$

which assumes that the fills should be of equal length although the turnaround times are not, with the constraint

$$\sum_{i=1}^{n} t_i + n \, \hat{t} = T \, . \tag{13}$$

One can assume to replace the sum of the numbers by a term $n \, \tau$ so that

$$n = \frac{T}{\tau + \hat{t}} \tag{14}$$

and the optimisation of equation (12) becomes of the same type as the problem (1).
In this case, indeed, the equation to be optimized is

$$f(n, \hat{t}) = n \int_0^{\hat{t}} dt L(t), \tag{15}$$

and the constraint function is

$$g(n, \hat{t}) = n\tau + n\hat{t}. \tag{16}$$

Thus, the system to be solved will be:

$$\begin{cases} \left(\frac{\partial f(n,\hat{t})}{\partial \hat{t}}\right) = \lambda \left(\frac{\partial g(n,\hat{t})}{\partial \hat{t}}\right) \\ \\ \left(\frac{\partial f(n,\hat{t})}{\partial n}\right) = \lambda \left(\frac{\partial g(n,\hat{t})}{\partial n}\right) \\ \\ n\tau + n\hat{t} = T \end{cases} \Rightarrow \begin{cases} \lambda = L(\hat{t}) \qquad \text{(considering that } n \neq 0) \\ \\ \mathbb{L}(\hat{t}) - \mathbb{L}(0) = \lambda(\hat{t} + \tau) \\ \\ n = T/(\tau + \hat{t}) \end{cases} \tag{17}$$

which leads to a solution similar to the previous case (see equation (11)):

$$t_{\rm opt} = \pm \frac{\sqrt{\tau}}{\sqrt{N_{\rm i} n_{\rm c} \Xi \sigma_{\rm int}}}, \tag{18}$$

.

It can be checked *a posteriori* that the assumption of optimising the integrated luminosity by using equal fills lengths is the correct one. In fact, in this case

$$\begin{aligned} L_{\rm tot}(\hat{t}_1, \hat{t}_2) &= (n-1) \int_0^{\hat{t}_1} {\rm d}t\, L(t) + \int_0^{\hat{t}_2} {\rm d}t\, L(t) \\ &= \frac{T - \hat{t}_2 - \tau}{\tau + \hat{t}_1} \int_0^{\hat{t}_1} {\rm d}t\, L(t) + \int_0^{\hat{t}_2} {\rm d}t\, L(t)\,, \end{aligned} \tag{19}$$

where the constraint (13) has been adapted to this new case. The maximisation of $L_{\rm tot}(\hat{t}_1, \hat{t}_2)$ is obtained by considering as the function to be optimized

$$f(n, \hat{t}_1, \hat{t}_2) = (n-1) \int_0^{\hat{t}_1} dtL(t) + \int_0^{\hat{t}_2} dtL(t), \tag{20}$$

and as the constraint function

$$g(n, \hat{t}_1, \hat{t}_2) = n\tau + n\hat{t}_1 - \hat{t}_1 + \hat{t}_2. \tag{21}$$

Thus, the system to be solved will be:

$$\begin{cases} \left(\frac{\partial f(n,\hat{t}_1,\hat{t}_2)}{\partial n}\right) = \lambda\left(\frac{\partial g(n,\hat{t}_1,\hat{t}_2)}{\partial n}\right) \\ \left(\frac{\partial f(n,\hat{t}_1,\hat{t}_2)}{\partial \hat{t}_1}\right) = \lambda\left(\frac{\partial g(n,\hat{t}_1,\hat{t}_2)}{\partial \hat{t}_1}\right) \\ \left(\frac{\partial f(n,\hat{t}_1,\hat{t}_2)}{\partial \hat{t}_2}\right) = \lambda\left(\frac{\partial g(n,\hat{t}_1,\hat{t}_2)}{\partial \hat{t}_2}\right) \\ n\tau + (n-1)\hat{t}_1 = T \end{cases} \Rightarrow \begin{cases} \mathbb{L}(\hat{t}_1) - \mathbb{L}(0) = \lambda(\tau - \hat{t}_1) \\ \left.\begin{array}{l} nL(\hat{t}_1) - L(\hat{t}_1) = \lambda(n-1) \\ L(\hat{t}_2) = \lambda \end{array}\right\} \hat{t}_1 = \hat{t}_2 \equiv \hat{t} \\ n = T/(\tau + \hat{t}) \end{cases} \tag{22}$$

where is possible to verify that $\hat{t}_1 = \hat{t}_2$.

Obviously, in the realistic case the n values of the turnaround are not known *a priori*, which modifies the scheme previously described. Let us assume that t_j, $1 \leq j \leq i$ and $\hat{t}_j$, $1 \leq j \leq i-1$ are the turnaround times and the optimal fill times, respectively for all the fills from 1 to i. The problem is to determine $\hat{t}_i$ so to maximise

$$L_{\rm tot}(\hat{t}_i, \hat{t}) = \sum_{j=1}^{i-1} L_{\rm tot}(\hat{t}_j) + \int_0^{\hat{t}_i} {\rm d}t\, L(t) + \frac{T - \sum_{j=1}^{i} \left(t_j + \hat{t}_j\right)}{\frac{1}{i}\sum_{j=1}^{i} t_j + \hat{t}} \int_0^{\hat{t}} {\rm d}t\, L(t)\,, \tag{23}$$

where and additional optimisation parameter has been introduced, namely $\hat{t}$, which represents the optimum time of future fills. The third term in Eq. (23) introduces a relationship between $\hat{t}_i$ and $\hat{t}$. Moreover the term

$$\frac{1}{i} \sum_{j=1}^{i} t_j \tag{24}$$

is intended to provide the average turnaround time. The optimization is performed as follows:

$$\begin{cases} \dfrac{\partial L_{\text{tot}}}{\partial \hat{t}_i} & = L(\hat{t}_i) - \dfrac{1}{\frac{1}{i}\sum_{j=1}^{i} t_j + \hat{t}} \displaystyle\int_0^{\hat{t}} \mathrm{d}t\, L(t) = 0 \\ \\ \dfrac{\partial L_{\text{tot}}}{\partial \hat{t}} & = L(\hat{t}) - \dfrac{1}{\frac{1}{i}\sum_{j=1}^{i} t_j + \hat{t}} \displaystyle\int_0^{\hat{t}} \mathrm{d}t\, L(t) = 0\,, \end{cases} \tag{25}$$

which gives

$$\begin{cases} \hat{t}_i & = \hat{t} \\ \\ L(\hat{t}) & = \dfrac{1}{\frac{1}{i}\sum_{j=1}^{i} t_j + \hat{t}} \displaystyle\int_0^{\hat{t}} \mathrm{d}t\, L(t)\,. \end{cases} \tag{26}$$

Substituting the equations (3) and (4) in the previous system and considering $\frac{1}{i}\sum_{j=1}^{i} t_j = \tau$, we obtain a solution of the type (18):

$$t_{\text{opt}} = \pm \frac{\sqrt{\tau}}{\sqrt{N_{\text{i}} n_{\text{c}} \Xi \sigma_{\text{int}}}}. \tag{27}$$

A Statistics of the Turnaround Times

In this chapter, we analyzed the turnaround times concerning the years 2016, 2017 and 2018, excluding those that represent exceptional conditions, and therefore must be ignored for statistical studies. An overview on Run 2 data is presented in [3].

A.1 Kolmogorov-Smirnov Test

The first step of data analysis was the execution of a statistical test of Kolmogorov-Smirnov on pairs of samples [5]. These test statistics are used to determine whether two distributions (e.g. F(x), G(x)) differ or whether an underlying probability distribution differs from a hypothesized distribution.

Considering two independent samples at time, there will be two possible hypothesis:

$$\begin{cases} H_0 : \; F(x) = G(x) \\ H_1 : \; F(x) \neq G(x) \end{cases} \tag{28}$$

In this case, there are three datasets to compare: the 2016 sample, the 2017 sample and the 2018 sample. The results of the test are shown in Table **2**.

For this test, a significance level, $1-\alpha$, of 95% was imposed, which entails $\alpha = 0.05$.

To interpret the results first we observe the p-values: if the p-value is bigger than α it is not possible to reject H_0, while if it is smaller the rejection is possible.

Looking at the obtained p-values, we have that:

1. p-value(2016-2017)$>\alpha$: The distribution of the 2016 sample is **the same** of the 2017 one;

Table 2: Results of the Kolmogorov-Smirnov test.

Samples	**2016 - 2017**	**2016 - 2018**	**2017 - 2018**
KS Statistics $(D_{\#,n})$	0.112	0.161	0.064
p-value	0.208	0.016	0.7523

2. p-value(2016-2018)$<\alpha$: The distribution of the 2016 sample is **not the same** of the 2018 one;
3. p-value(2017-2018)$>\alpha$: The distribution of the 2017 sample is **the same** of the 2018 one.

Secondly, we look at the statistics. A 95% confidence level critical value is defined as follows:

$$D_{\mathrm{crit},0.05} = 1.36\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \tag{29}$$

In this case, if the critical value D_{crit} is bigger than the obtained statistics D_n it is possible to accept H_0, while if it is smaller H_0 has to be rejected. Considering that $n_1 = 156$, $n_2 = 194$ and $n_3 = 218$, we have that:

$$\begin{cases} D_{1,\mathrm{crit},0.05} = 0.1463 \\ D_{2,\mathrm{crit},0.05} = 0.1428 \\ D_{3,\mathrm{crit},0.05} = 0.1344 \end{cases} \tag{30}$$

This means that:

1. $D_{1,\mathrm{crit},0.05} > D_{1,n}$: The distribution of the 2016 sample is **the same** of the 2017 one;
2. $D_{2,\mathrm{crit},0.05} < D_{2,n}$: The distribution of the 2016 sample is **not the same** of the 2018 one;
3. $D_{3,\mathrm{crit},0.05} > D_{3,n}$: The distribution of the 2017 sample is **the same** of the 2018 one.

In conclusion, it is possible to say that data coming from 2016 and 2017 seems to be distributed according to the same distribution, as data coming from 2017 and 2018. However, for the 2016 and 2018 samples, it is not possible to accept the hypothesis of equal distributions. This last conclusion can be linked to the consistent difference in the number of objects in the two samples. Furthermore, taking into account the results as a whole, it is possible to state, albeit with a certain limit of uncertainty, that the turn-around times of different years are distributed following the same distribution.

A.2 Evaluation of the t_{ta} distribution

Knowledge of the statistical distribution of turn-around times is essential for the building of a strategy for integrated luminosity optimization. To do this, we analyzed the data of the LHC Run 2 by producing histograms, that describe the distribution of the turnaround times, and by looking for a model that would better fit them (see figure **2**). The first model chosen for the fit of the experimental data is the exponential one with an offset, which is able to represent rather well the tail of the data but not the peak:

$$f(x) = \lambda \exp\big(-\lambda(x - Off)\big), \tag{31}$$

whose expectation value is $E[x] = e^{\lambda b}/\lambda$. For the bad representation of the peak, we have also considered a power law, which acts contrary to the exponential, representing quite well the peak of the data and badly the tail:

$$f(x) = \frac{A}{(x - Off)^n}. \tag{32}$$

Finally, combining the two previous models we have come to the truncated power law model:

$$f(x) = \frac{A}{(x - Off)^n} \exp(-\lambda x), \tag{33}$$

whose first momentum is $E[t] = \int_{t_{\min}}^{t_{\max}} dt' f(t')t'$. In figure **2**, it is possible to view all the fits performed, with the respective reduced chi-square values, which are now also shown in the next table for greater clarity.

Table 3: Results of the $\tilde{\chi}^2$ test.

Samples	**2016**	**2017**	**2018**	**Total**
Power Law $\tilde{\chi}^2$	0.00014	0.0005	0.00069	0.00026
Exponential Law $\tilde{\chi}^2$	0.0007	0.00092	0.00059	0.00042
Truncated Power Law $\tilde{\chi}^2$	0.0002	0.0005	0.0006	0.00025

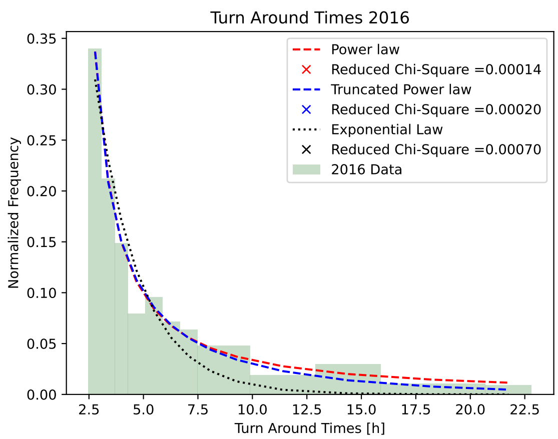
Observing the table **3**, it is possible to state that the best fit is the one obtained with the truncated power-law model. A further analysis, to verify this conclusion, is presented in figure **2**, with the residual plots. The latter guides on the goodness of fit in the case of linear regression. A *residual* ϵ can be defined as the vertical distance between the regression line (the predicted value) and the observed value:

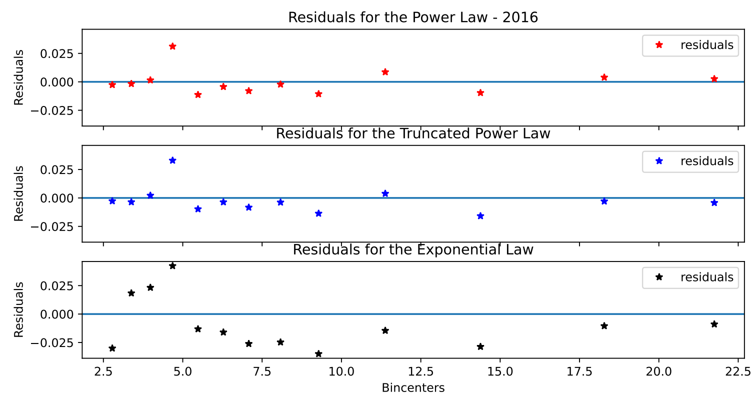
$$\epsilon = y_{obs} - \hat{y}_{fit} \tag{34}$$

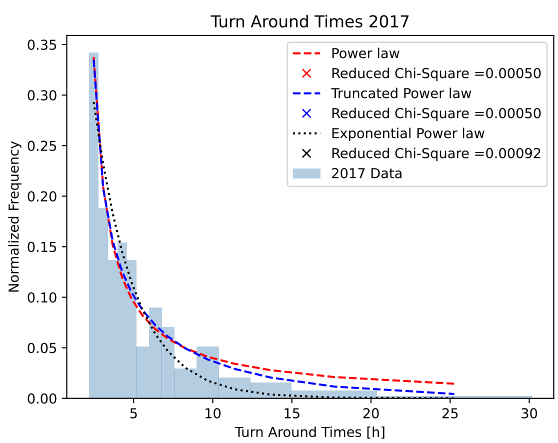
Taking into account that the model that we want to capture will be intrinsically composed of a deterministic part and a stochastic one, this part of the study aims to try to capture all the predictive information coming from our data. This implies that if the chosen model is correct, the residuals should be completely random and unpredictable. Thus, a residual plot that describes a good fit pattern must be symmetrical around the origin and with a higher density of points near the origin than elsewhere. What has just been described is majorly present in the residual plots of figure **2** which represent the fit with the truncated power law, giving further confirmation of how this model is the one better capable to describe the behaviour of our data. In figure **3**, we have reported the model chosen with the fit parameters (presented also in table **4**) obtained for the various years analyzed.

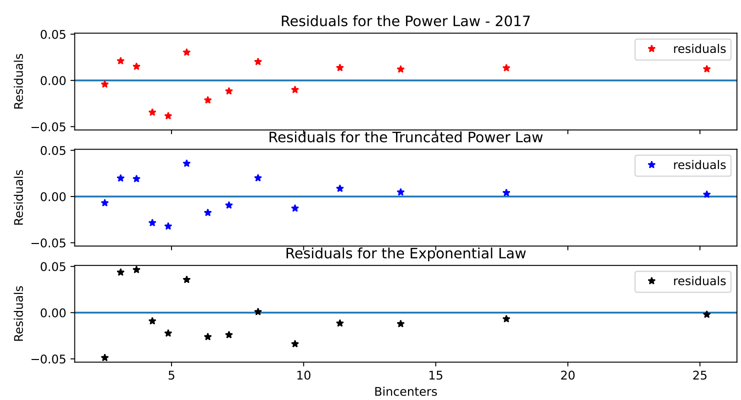
Table 4: Parameters from the fits of data.

Parameters	**2016**	**2017**	**2018**	**Total**
Amplitude	0.371 ± 0.144	0.297 ± 0.084	0.332 ± 0.167	0.394 ± 0.137
Offset [h]	2.000 ± 0.005	2.000 ± 1.898	1.50 ± 0.12	2.000 ± 1.236
Exponent	0.735 ± 0.693	0.507 ± 0.620	0.272 ± 0.858	0.542 ± 0.566
λ [h^{-1}]	0.100 ± 0.035	0.104 ± 0.139	0.164 ± 0.172	0.111 ± 0.095









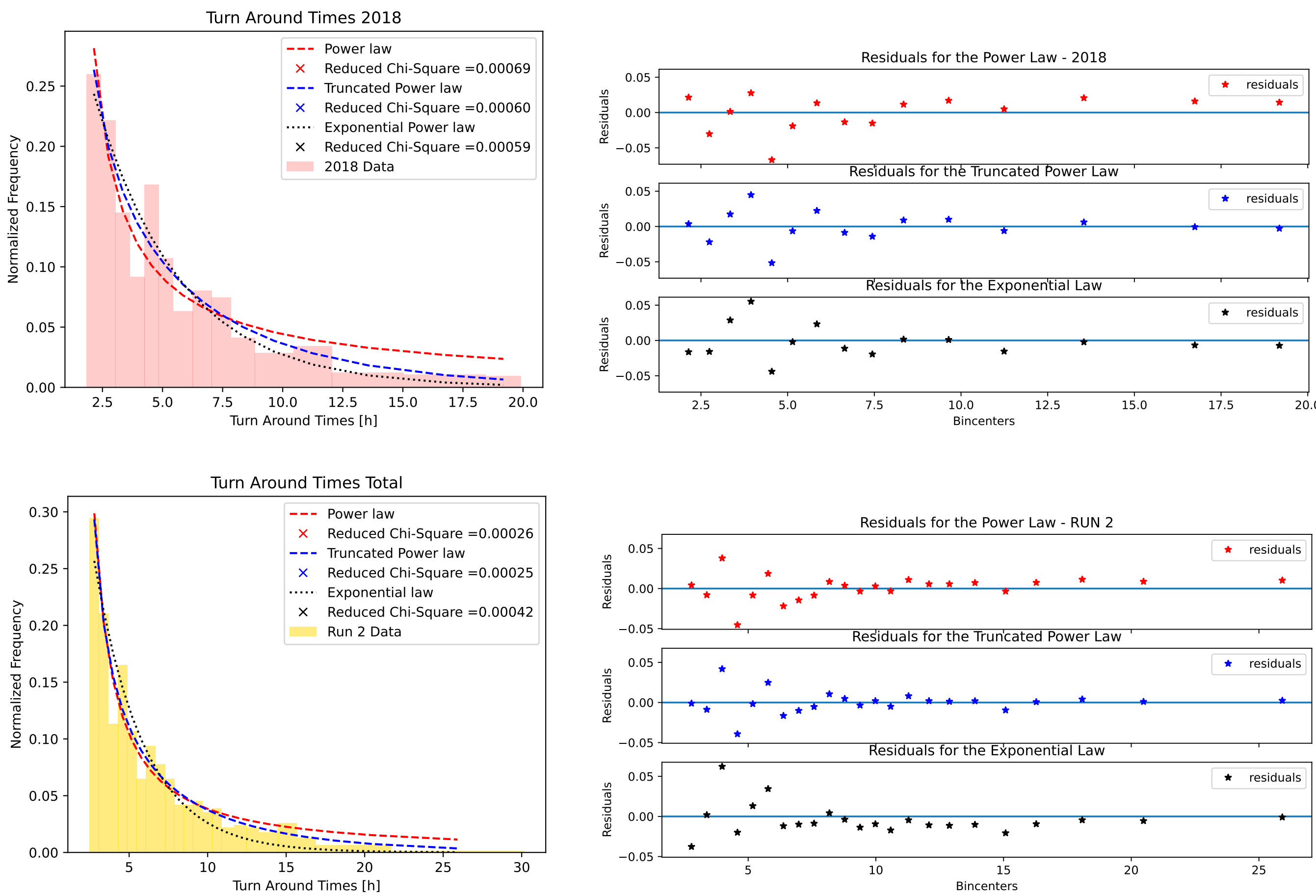


Figure 2: *All the plots present in the figure have been obtained with the* `lmfit` *python function. On the left: we have the fits for the 2016, 2017, 2018 and the total sample with the previously described functions; On the right: the residual plots for all the fits.*

Once the truncated power-law as distribution for turnaround times was determined, the distribution averages for each year were also calculated and they are shown in the table **5**.

Table 5: Averages turnaround times for the years 2016, 2017, 2018.

	2016	**2017**	**2018**
Average Turnaround Times [h]	8.86	8.884	7.22

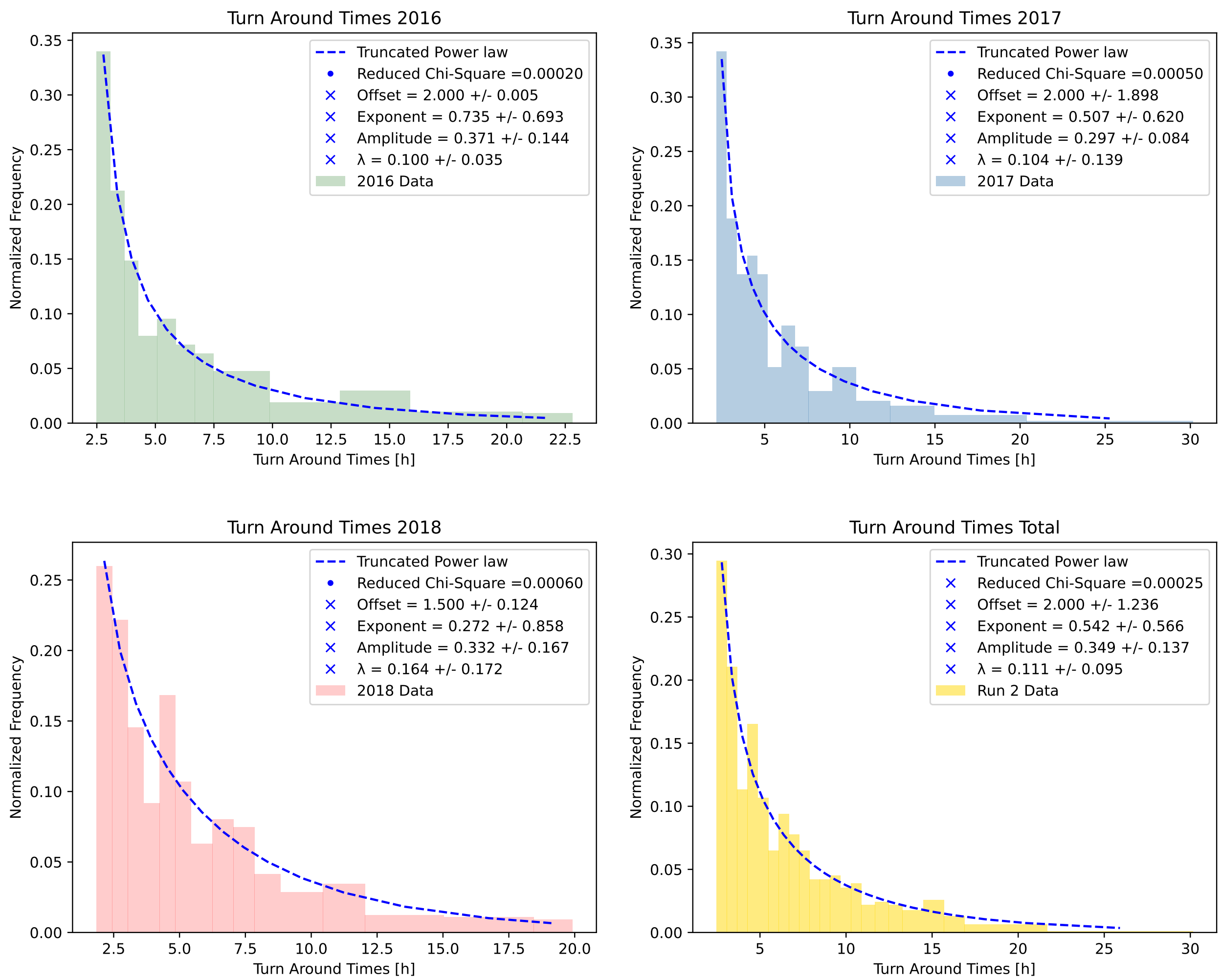


Figure 3: *Plots of the data with the Truncated Power law model for the turnaround times distributions. All the plots present in the figure have been obtained with the* `lmfit` *python function.*

A.2.1 Evaluation of the L_{tot} and t_{opt} distribution

A function of a random variable is itself a random variable. Supposing that x follows a pdf, $f(x)$, considering then a function of x, $a(x)$, its pdf $g(a)$ will be [6]:

$$g(a) = (f(x(a)))\frac{dx}{da}. \tag{35}$$

In the studied case, there are two function of the random variable t_a (i.e. the turn-around times) which is distributed according to a truncated power law (33), the total luminosity L_{tot} (1) and the optimal fill time t_{opt} (11) . Considering the equation (35), the total luminosity p.d.f. is

$$g_{L_{\text{tot}}} = -\frac{AT\int_0^{t_{\text{f}}} dtL(t)}{L_{\text{tot}}^2\Big[\Big(\frac{T}{L_{\text{tot}}}\int_0^{t_{\text{f}}} dtL(t) - t_{\text{f}}\Big) - b\Big]^n} \exp\Big\{-\lambda\Big[\Big(\frac{T}{L_{\text{tot}}}\int_0^{t_{\text{f}}} dtL(t) - t_{\text{f}}\Big) - b\Big]\Big\}, \tag{36}$$

while the optimal time one is:

$$g_{t_{\text{opt}}} = \frac{2AN_{\text{i}}n_{\text{c}}\Xi\sigma_{\text{int}}t_{\text{opt}}}{(N_{\text{i}}n_{\text{c}}\Xi\sigma_{\text{int}}t_{\text{opt}}^2 - b)^n} \exp\{-\lambda N_{\text{i}}n_{\text{c}}\Xi\sigma_{\text{int}}t_{\text{opt}}^2\}. \tag{37}$$

With regard to the latter distribution it was also possible to verify its correctness with a direct comparison with the data, as shown in figure **4**.

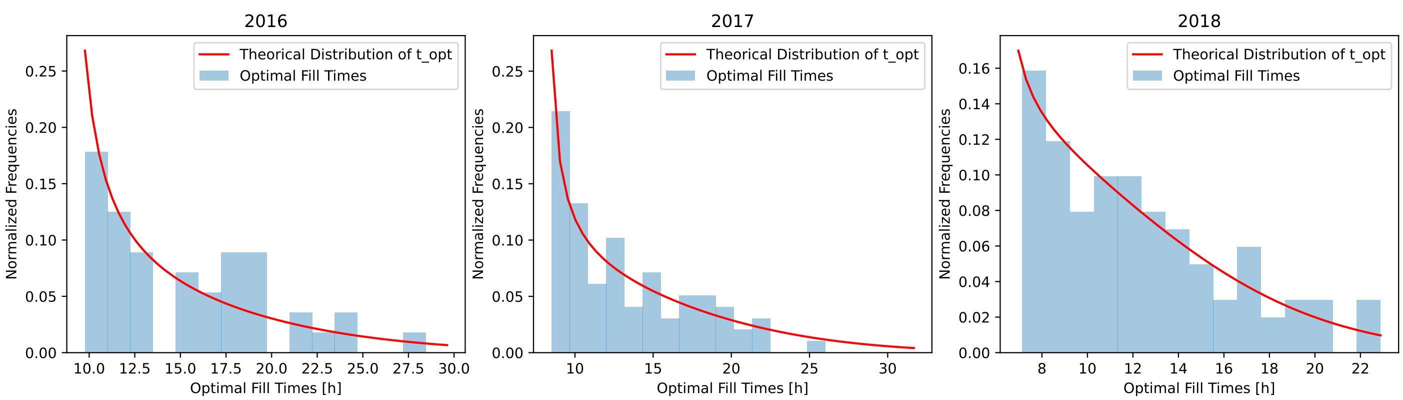
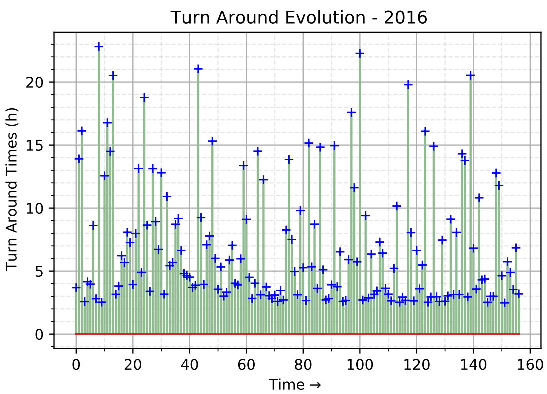


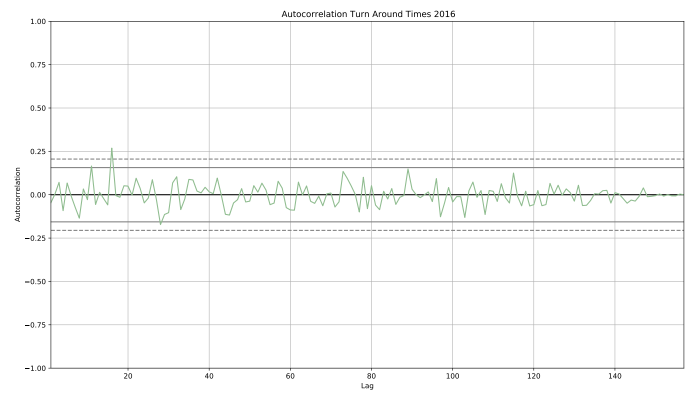
Figure 4: *Comparison between the theoretical distribution of* t_{opt} (37) *and the distribution of the* t_{opt} *evaluated from data.*

A.3 Autocorrelation

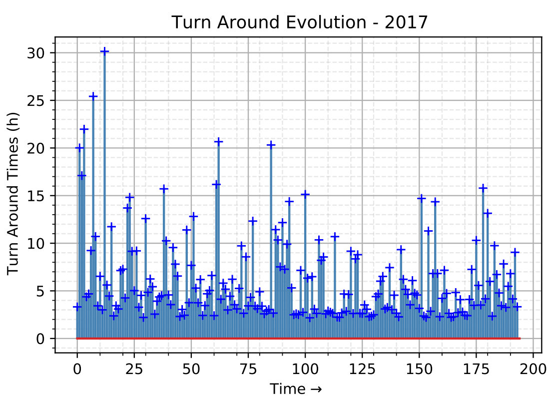
Auto-correlation is a mathematical representation of the degree of similarity between a given time series and a lagged version of itself over successive time intervals. We studied the auto-correlation of the data samples for the years 2016, 2017 and 2018, as shown in figure **5**. This figure displays, on the left, the data samples in chronological order, while, on the right, the auto-correlation plots. The latter show that all three studied samples do not exhibit auto-correlation: in fact, variations on auto-correlation are very close to zero for each chosen lag.



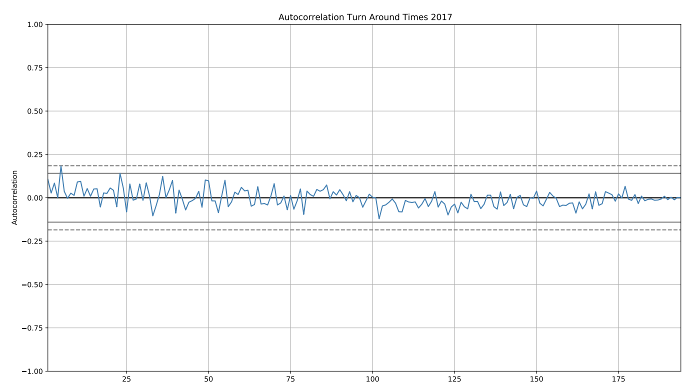
(a)



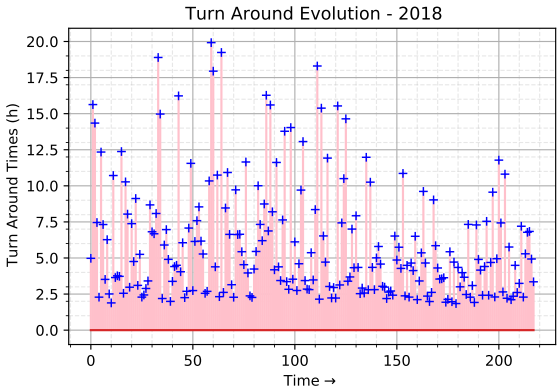
(b)



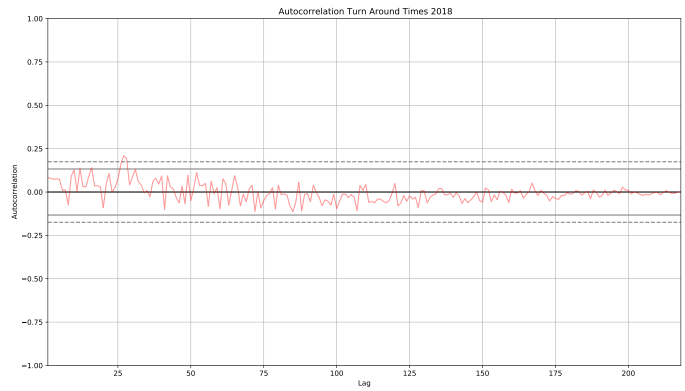
(c)



(d)



(e)



(f)

Figure 5: ***(a)*** *Turnaround Times of 2016 plotted in chronological order in a python* `matplotlib` *stem plot;* ***(b)*** *Autocorrelation Plot of 2016 Turnaround Times obtained with the* `plotting.autocorrelation_plot` *function of python* `pandas` *library;* ***(c)*** *Turnaround Times of 2017 plotted in chronological order in a stem plot;* ***(d)*** *Autocorrelation Plot of 2017 Turnaround Times;* ***(e)*** *Turnaround Times of 2018 plotted in chronological order in a stem plot;* ***(f)*** *Autocorrelation Plot of 2018 Turnaround Times.*

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