

## Non parametric tests

### Are the Panini cards packaged at random?

We have a suspicion that the Panini cards are not packaged completely at random because we tend to get a lot of duplicates, especially of some card types. We'll now use a simulation test to test our hypothesis.

The null hypothesis is that the cards are packaged at random with replacement. The alternative is the following: the cards are packaged at random with replacement, but  $k = 100$  of the card types are 5 times more common than the others.

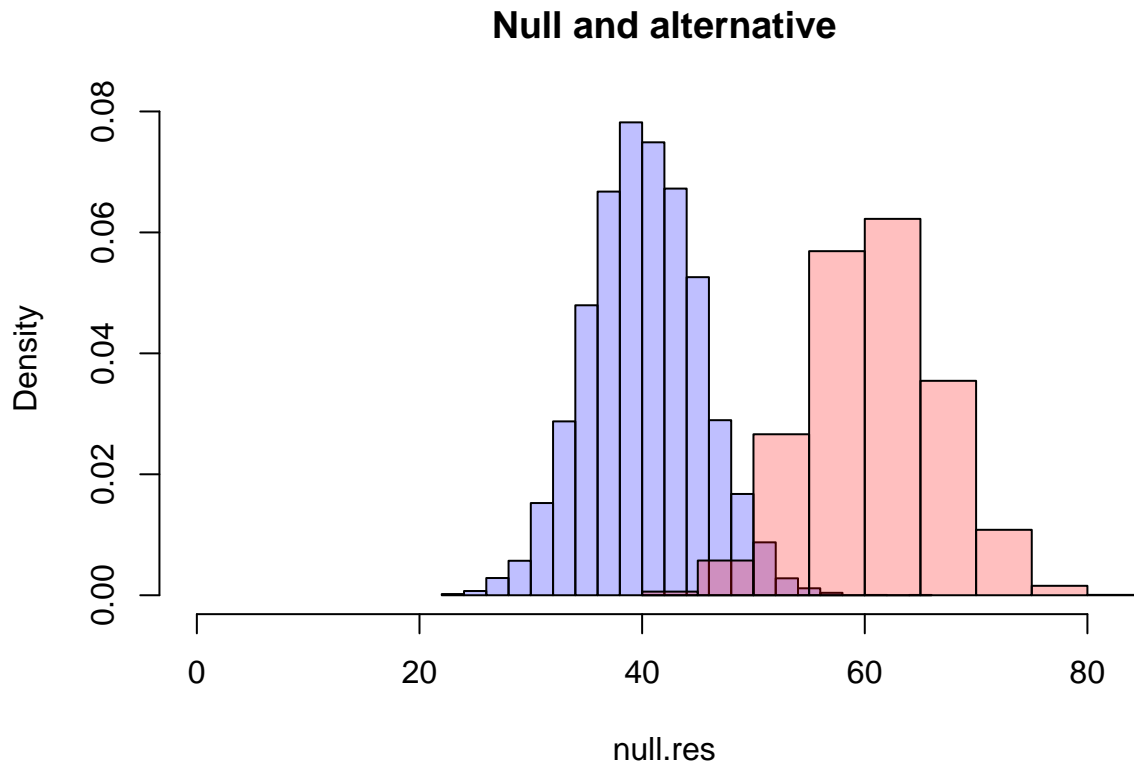
```
npacks <- 50
ncards <- 682
nsim <- 10000

sim.from.null <- function(npacks=50){
  sampled <- sample(1:ncards, size=5*npacks, replace=T)
  # count the number of duplicates
  dupl <- 5*npacks - length(unique(sampled))
  return(dupl)
}
alt.cards <- c(101:682, rep(1:100, 5))
sim.from.alt <- function(npacks=50){
  sampled <- sample(alt.cards, size=5*npacks, replace=T)
  dupl <- 5*npacks - length(unique(sampled))
  return(dupl)
}
```

Let's simulate from both hypothesis and look at what we get.

```
null.res <- replicate(nsim, sim.from.null())
alt.res <- replicate(nsim, sim.from.alt())

p1 <- hist(null.res, plot = F)
p2 <- hist(alt.res, plot=F)
plot(p1, col=rgb(0,0,1,1/4), xlim=c(0,max(null.res,alt.res)), main="Null and alternative", freq = F)
plot(p2, col=rgb(1,0,0,1/4), add=T, freq=F)
```



As it is possible to see from the plot above the test has a high power, meaning that the distributions are easily distinguishable. But let's put some numbers into that claim.

```
# power of a test = probability of the rejection region of the null , given that the alternative is true
# To get the power we need 2 things:
# the rejection region for the level alpha=0.05
# the density of the alternative - which we have computed by simulation above

# note that our alternative claims to have more duplicates than the null, hence the rejection region is
rej.reg <- quantile(null.res, probs = 0.95) + 1 #we subtract one because of the discretization of the d
rej.reg

## 95%
## 50

# let's now count how many of the results under the alternative fall under this threshold
power <- sum(alt.res >= rej.reg)/nsim
power

## [1] 0.9804
```

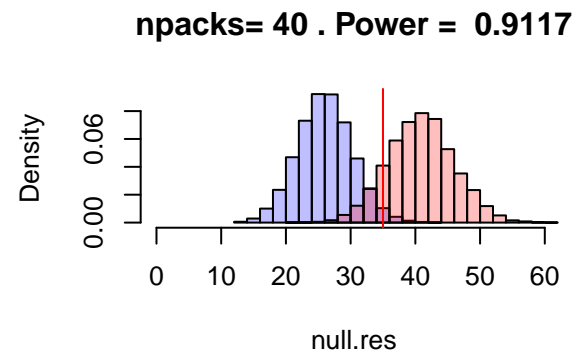
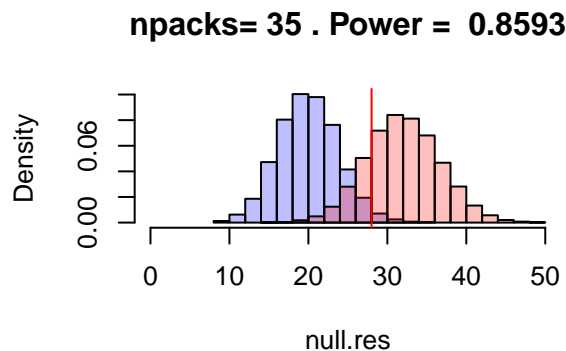
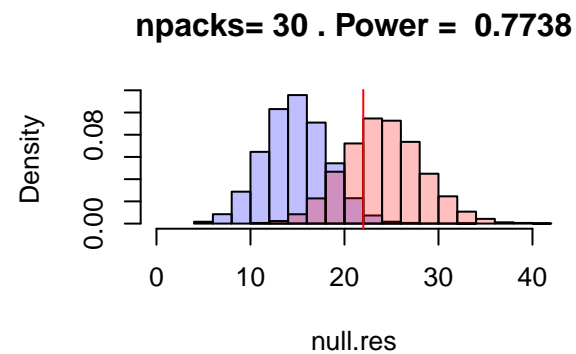
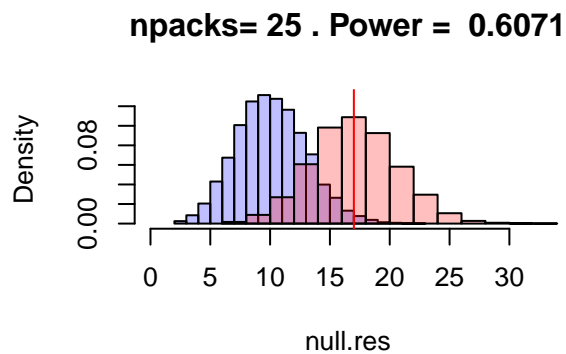
Our calculations confirm our intuition: the test has high power. Let's try to change the number of packs opened and see how this affects the power.

```
npacks <- c(25,30,35,40)
powers <- rep(0,4)
par(mfrow=c(2,2))
for(i in 1:4){
  # simulating
```

```

np <- npacks[i]
null.res <- replicate(nsim, sim.from.null(np))
alt.res <- replicate(nsim, sim.from.alt(np))
# power
rej.reg <- quantile(null.res, probs = 0.95) + 1
power <- sum(alt.res >= rej.reg)/nsim
powers[i] <- power
# plotting
p1 <- hist(null.res, plot = F)
p2 <- hist(alt.res, plot=F)
plot(p1, col=rgb(0,0,1,1/4), xlim=c(0,max(null.res,alt.res)), main=paste("npacks=",np,". Power = ",power),
plot(p2, col=rgb(1,0,0,1/4), add=T, freq = F)
abline(v=rej.reg, col="red")
}

```



We can conclude that to obtain power  $\geq 85\%$  from our test we need to open at least 35 packs.