ORC Assignment 2

Saviane Arianna mat. 240602

Titton Giulia mat. 240311

November 2023

Professor: Del Prete Andrea

Assistant Professor: Alboni Elisa

Contents

1 Question 1 **1**
1.1 Sampling strategy . . . 1
1.2 Considerations about the dataset . . . 1

2 Question 2 **2**
2.1 Viability kernel shape . . . 2
2.2 Considerations about control bounds . . . 2
2.3 Considerations about position and velocity limits . . . 3

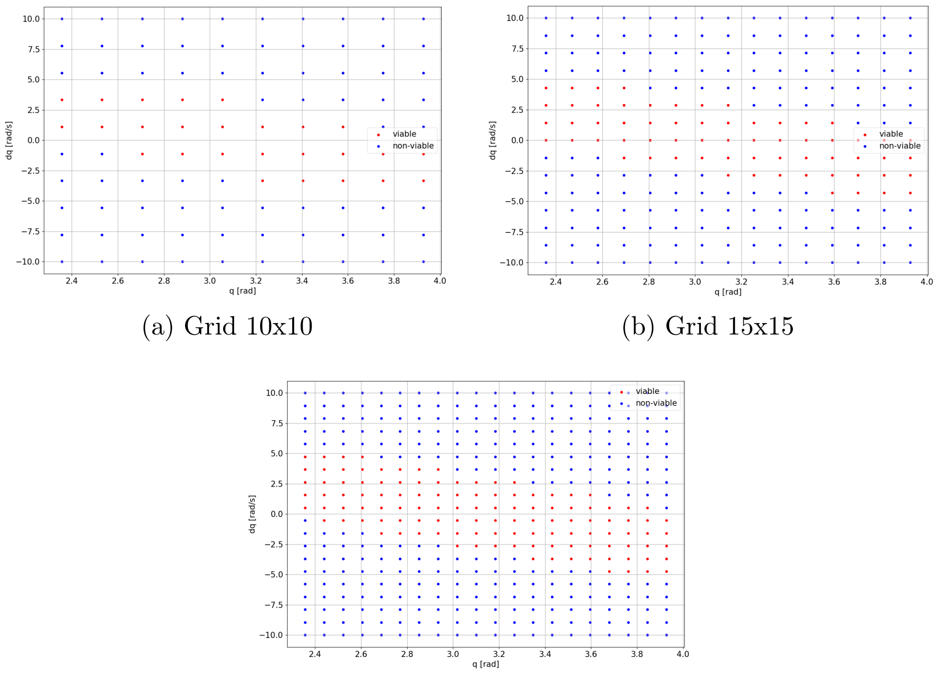
3 Question 3 **5**
3.1 Recursive feasibility in MPC . . . 5

1 Question 1

1.1 Sampling strategy

The first request is aimed at implementing a sampling strategy to select x_{sample} and generate a plot of the viability kernel based on the result of OCPs.

The strategy applied consists in taking a squared grid and computing the viability kernel for each point on the grid. Three experiments have been made in order to obtain the most accurate results possible. Firstly a 10x10 grid has been considered, then a 15x15 grid and lastly a 20x20 grid. Obviously the last one has reported to be the most accurate, since the steps of the grid are closer one to the other. Figure 1 shows the three results with the three different grids.



(a) Grid 10x10

(b) Grid 15x15

(c) Grid 20x20

Figure 1: Sampling strategy

1.2 Considerations about the dataset

As shown in Figure 1, the viable states lay inside a diagonal band which seems to be entirely viable despite the fact that the sampled states do not cover all the space inside the band. This hypothesis could be confirmed, for example, considering a sampling strategy which implements a finer grid.

Observing the viable kernel, it seems possible that some other viable states could be found outside the provided dataset by enlarging the joint bounds. The dataset considers joint positions ranging from $\frac{3}{4}\pi$ rad to $\frac{5}{4}\pi$ rad and joint velocities from -10 rad/s to 10 rad/s. Since the viable set clearly reaches the position bounds, they could be enlarged in order to check if other viable states could be present. Depending on the results, also velocity bounds might be increased.

2 Question 2

2.1 Viability kernel shape

The aim of the control is to stabilize a pendulum around its equilibrium position. For this reason, as shown in Figure 2, for each position in the dataset there exists a set of possible velocities able to make the state viable.

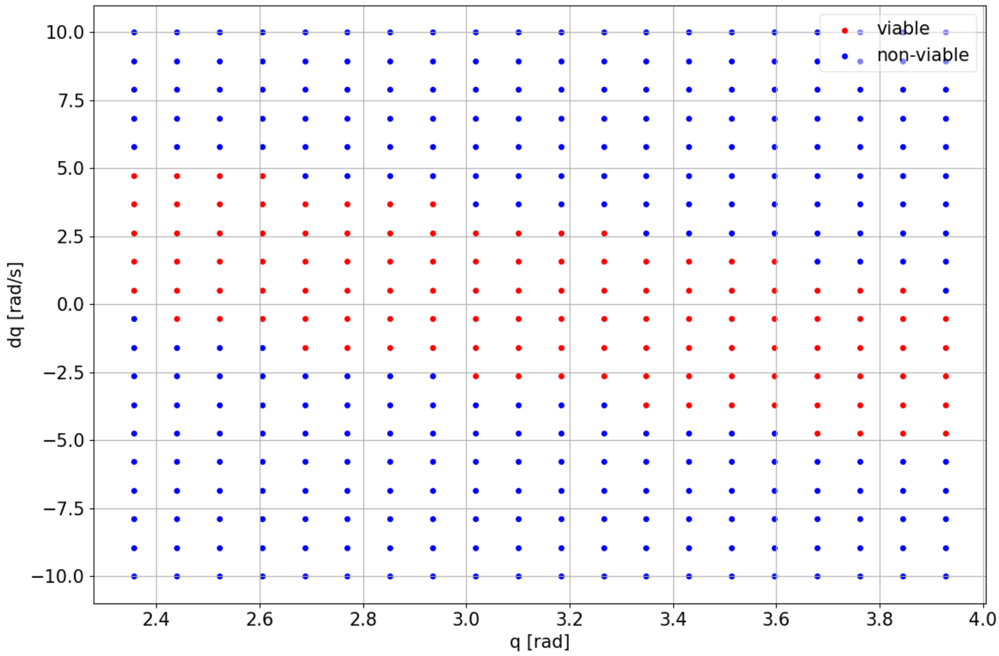


Figure 2: Viability kernel.

The viable kernel has the shape of a diagonal band because as the position increases (moving from the right to the left of the equilibrium position) the velocity decreases in order to allow the pendulum to reach the equilibrium position.
For example, for a position of $\frac{3}{4}\pi$ rad the possible velocities allowing the state to be viable range from 0.0 to 5.0 rad/s, while for the opposite position ($\frac{5}{4}\pi$) velocities have the opposite sign because of the symmetry of the problem with respect to the equilibrium point.

2.2 Considerations about control bounds

Considering the case in which the absolute value of the control bounds are increased, taking $5 * g$ as constant value, the shape of the viability kernel is slightly modified, becoming wider, as shown in Figure 3.

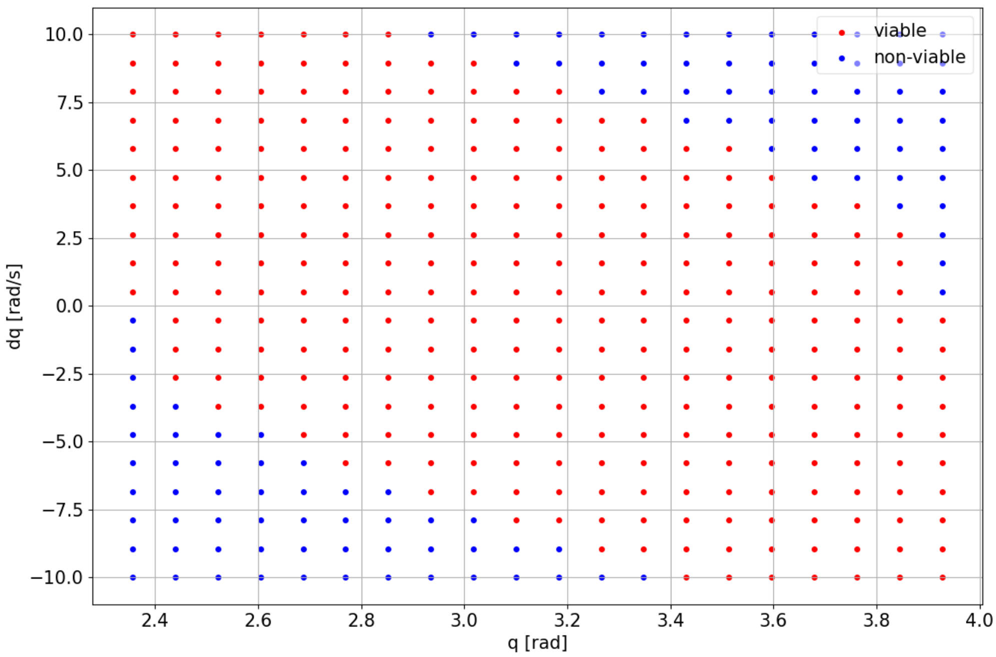
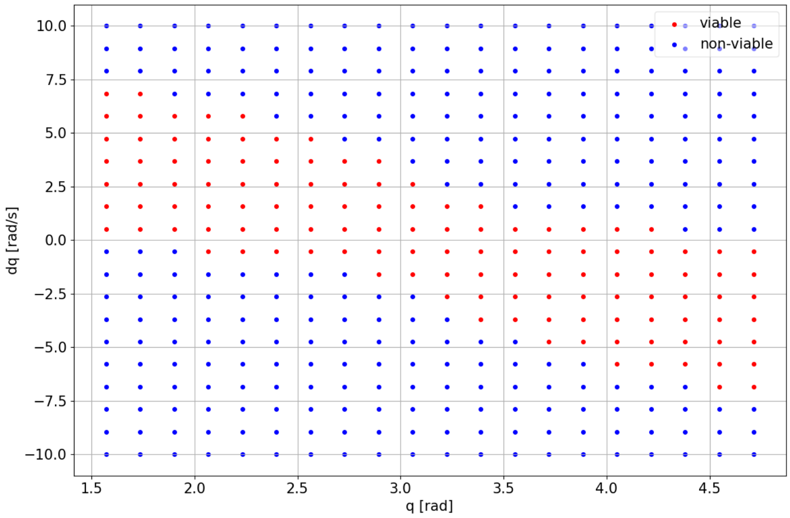


Figure 3: Viability kernel with absolute value of control bounds increased.

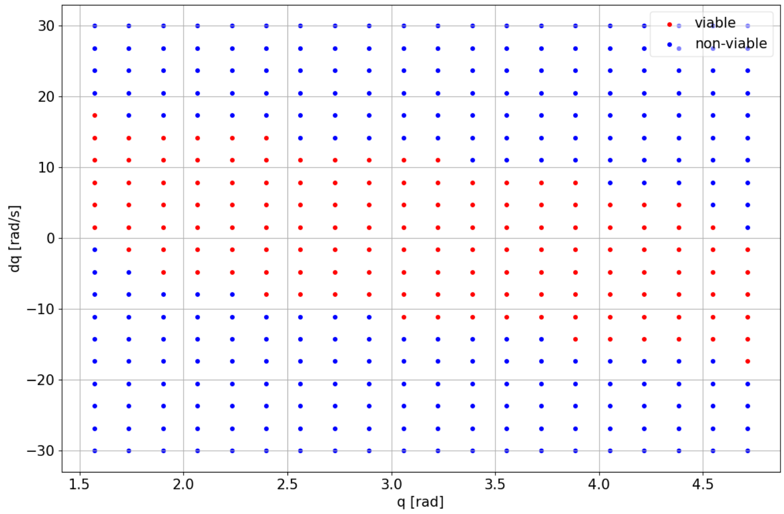
The results confirm the expectations because increasing the possible control values the set of possible states from which the pendulum can still reach the equilibrium position increases too (because of the extended control capability of the system). In this case, comparing Figure 3 and 2, for each starting position the possible initial velocities for which the state is still viable are larger.

2.3 Considerations about position and velocity limits

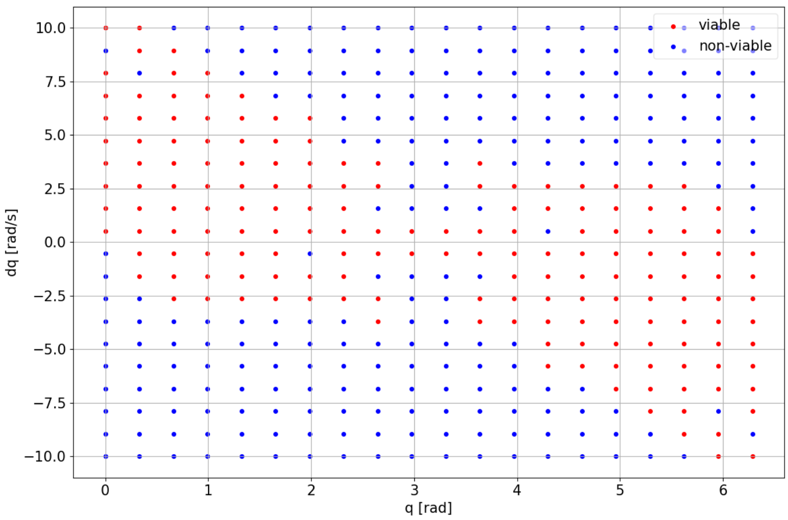
Changing the values of the position and velocity limits, it may seem logical that the viability kernel would expand accordingly. Instead, the tests reported below show a non linear trend depending on the applied position and velocity bounds. For example, Figure 4 represents three different tests in which the viability kernel shape changes depending on the applied limits, which are specified in Table 1.



(a) Test 1.



(b) Test 2.



(c) Test 3.

Figure 4: Viability kernel shape depending on position and velocity limits.

Test limits			
Test number	Position limits $[rad]$	Velocity limits $[rad/s]$	Control bounds
1	[$-\pi/2$, $3\pi/2$]	[-10, 10]	[-9.81, 9.81]
2	[$-\pi/2$, $3\pi/2$]	[-30, 30]	[-9.81*5, 9.81*5]
3	[0, 2π]	[-10, 10]	[-9.81, 9.81]

Table 1: Position and velocity limits applied to each test.

As shown in Figure 4, the shape of the viability kernel strongly depends not only on control bounds (as explained in Section 2.2), but also on position and velocity limits.
Observing the results, it may be noticed that some states that are viable in the original configuration become non-viable once position and/or velocity limits are enlarged; moreover, the shape of the viability kernel becomes more and more irregular with respect to the initial one (Figure 2). This is due to the fact that enlarging the limits (for example in position) some new possible trajectories are generated for each considered state and if the pendulum follows these new trajectories it may violate other constraints (for example velocity limits or final velocity constraint), transforming a previously viable state into a non-viable one.

3 Question 3

3.1 Recursive feasibility in MPC

From the theory we know that if the set of feasible states (X_f) is control-invariant, then the MPC is recursively feasible, which means that the feasibility of the first OCP implies the feasibility of all successive OCPs.

As a second consideration, from the theory we know also that a set is control-invariant if for each state there exists a control so that if the computation starts in the set, the result will still be in the set.

Since the viability kernel is the largest set of feasible states starting from which the system can always remain inside the set without violating the constraints, it could be used to address the problem of recursive feasibility. In particular, if the states of the MPC problem are taken directly from the viability kernel, recursive feasibility is guaranteed.

To select the states belonging to the viability kernel, the kernel itself may be included in the problem as a constraint for the states. Alternatively, a higher cost may be associated to states that don't belong to the viability kernel in order to include it as a less rigid constraint.