

A) Pasa-bajos, Butter, orden 2: $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$

Desnormalizamos con $\omega_c = 2\pi 1\text{KHz}$

↓

$$H(s) = \frac{1}{\left(\frac{s}{\omega_c}\right)^2 + \frac{s}{\omega_c} \sqrt{2} + 1} = \frac{\omega_c^2}{s^2 + s \cdot \sqrt{2} \omega_c + \omega_c^2}$$

luego, aplicamos la transformada Bilineal.

$$s = 2f_s \left(\frac{z-1}{z+1} \right) \rightarrow f_s = 100\text{KHz}$$

↓

$$H(z) = \frac{\omega_c^2}{\left(2f_s \left(\frac{z-1}{z+1}\right)\right)^2 + \sqrt{2} \cdot \omega_c \left[2f_s \left(\frac{z-1}{z+1}\right)\right] + \omega_c^2}$$

Finalmente:

$$H(z) = \frac{z^2(39478417) + z(78956835) + (39478417)}{z^2(41816631592) - z(79921043164) + (38262325242)}$$

B) Realizamos el mismo desarrollo, solo variando $f_s = 10\text{KHz}$.

$$H(z) = \frac{z^2(39478417) + (78956835) \cdot z + 39478417}{z^2(617193735) - (721043164) \cdot z + 38262325242}$$

C) Pasa-altos, Butter, orden 2: $H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$

Desnormalizamos con $\omega_c = 2\pi 6\text{KHz}$

↓

$$H(s) = \frac{(s/\omega_c)^2}{\left(\frac{s}{\omega_c}\right)^2 + \frac{s}{\omega_c} \sqrt{2} + 1} \rightarrow H(s) = \frac{s^2}{s^2 + \sqrt{2} \cdot \omega_c \cdot s + \omega_c^2}$$

luego, aplicamos la transformada Bilineal:

$$s = 2f_s \left(\frac{z-1}{z+1} \right) \rightarrow f_s = 100 \text{ kHz}$$

$$\downarrow$$

$$H(z) = \frac{\left[2f_s \left(\frac{z-1}{z+1} \right) \right]^2}{\left[2f_s \left(\frac{z-1}{z+1} \right) \right]^2 + \sqrt{2} \omega_c \left[2f_s \left(\frac{z-1}{z+1} \right) \right] + \omega_c^2}$$

Finalmente:

$$H(z) = \frac{z^2 (4 \times 10^{10}) - z (8 \times 10^{10}) + 4 \times 10^{10}}{z^2 (5 \times 10^{10}) - z (8 \times 10^{10}) + 3 \times 10^{10}}$$

d) Utilizamos pre-warping en el pasa-altos.