



# Exercises on Concurrency Control (part 2)

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## Exercise 1

Let  $C$  be the class of all the schedules  $S$  that, when given as input to a timestamp-based scheduler, is such that the scheduler accepts the schedule  $S$  and, when processing  $S$ , uses the Thomas rule at least once. Prove or disprove that every schedule in the class  $C$  is view-serializable.



## Solution to exercise 1

The schedule

$r_1(X) \ r_2(Y) \ w_2(X) \ c_2 \ w_1(X) \ c_1$

is accepted by the timestamp-based method and is processed by using the Thomas rule. However, it is not view-serializable.



## Esercise 2

Prove or disprove that every rigorous schedule is view-serializable



## Solution to exercise 2

We recall the definition of rigorous schedule: a (complete) schedule is rigorous if for every pair of conflicting actions  $\langle a_i(x), b_j(x) \rangle$ , where  $a_i(X)$  appears before  $b_j(X)$ , the commit operation of  $T_i$  appears between them. Let  $S$  be a rigorous schedule on transactions  $T_1, \dots, T_n$ , and let  $R$  be the serial schedule on  $T_1, \dots, T_n$  reflecting the order of the commit operations in  $S$ . We show that  $S$  is conflict-equivalent to  $R$ . It is sufficient to show that, if  $a_i(X), b_j(X)$  is a pair of conflicting actions in  $S$ , with  $a_i(X)$  appearing before  $b_j(X)$  in  $S$ , then they appear in the same order in  $R$ . The proof of this claim is easy: since  $S$  is rigorous, the commit action of transaction  $T_i$  appears between  $a_i(X)$  and  $b_j(X)$ , which means that  $T_i$  commits before  $T_j$  in  $S$ . Now, since  $R$  reflects the order of the commit actions,  $T_i$  comes before  $T_j$  in  $R$ , and therefore the claim is proven. Since we have shown that  $S$  is conflict-equivalent to a serial schedule on the same set of transactions, we have proved that  $S$  is conflict-serializable, and this concludes the proof of the theorem.



## Exercise 3

Let  $A$  and  $B$  be two elements of the database such that  $rts(A)=rts(B)=wts(A)=wts(B)=wts-c(A)=wts-c(B)=0$  and  $cb(A)=cb(B)=true$ . Suppose that the system clock is 0, and that the system uses the clock values as the timestamps to be assigned to the various transactions (at their first action). Illustrate the actions of the timestamp-based scheduler when receiving the following complete schedule

$r1(A) \ r2(B) \ r3(A) \ r2(A) \ w1(A) \ w3(A)$

and tell whether the resulting sequence of actions (obtained by ignoring aborted transactions) is a strong strict 2PL schedule (with shared and exclusive locks).



## Solution to exercise 3 (1)

➤ The initial situation is:

$wts(A) = wts-c(A) = wts(B) = wts-c(B) = rts(A) = rts(B) = 0$   
and  $cb(A)=cb(B)=true$

➤ The system responds as follows:

- $r1(A) \rightarrow ok \rightarrow ts(T1)=1, rts(A)=1$  -- because  $ts(T1) \geq wts(A)$  and  $cb(A)=true$
- $r2(B) \rightarrow ok \rightarrow ts(T2)=2, rts(B)=2$  -- because  $ts(T2) \geq wts(B)$  and  $cb(B)=true$
- $r3(A) \rightarrow ok \rightarrow ts(T3)=3, rts(A)=3$  -- because  $ts(T3) \geq wts(A)$  and  $cb(A)=true$
- $r2(A) \rightarrow ok \rightarrow rts(A)=3$  -- because  $ts(T2) \geq wts(A)$ ,  $cb(A)=true$ , and  $\max(ts(T2), rts(A))=3$
- $w1(A) \rightarrow no$ : T1 rollbacks -- because  $ts(T1) < rts(A)=3$  (write too late)
- $w3(A) \rightarrow ok \rightarrow wts(A)=3, cb(A)=false$  -- because  $ts(T3)=rts(A)$  and  $ts(T3) \geq wts(A)$



## Solution to exercise 3 (2)

Since T1 rollbacks, the accepted complete schedule (the one obtained from the original one by ignoring the actions of the aborted transactions) is the following:

$r_2(B) \ r_3(A) \ r_2(A) \ w_3(A)$

that is a strong strict 2PL schedule (with shared and exclusive locks).





## Exercise 4

Let A and B be two elements of the database such that  $rts(A)=rts(B)=wts(A)=wts(B)=wts-c(A)=wts-c(B)=0$  and  $cb(A)=cb(B)=true$ . Suppose that the system clock is 0, and that the system uses the clock values as the timestamps to be assigned to the various transactions (at their first action). Illustrate the actions of the timestamp-based scheduler when receiving the following complete schedule

$r1(B) \ w1(A) \ w2(B) \ w1(B) \ r2(A)$

and tell whether such a schedule is a strong strict 2PL schedule (with shared and exclusive locks)



## Solution to exercise 4 (1)

➤ The initial situation is:

$wts(A) = wts-c(A) = wts(B) = wts-c(B) = rts(A) = rts(B) = 0$   
and  $cb(A)=cb(B)=true$

➤ The system responds as follows:

- $r1(B) \rightarrow ok \rightarrow ts(T1)=1, rts(B)=1$  -- because  $ts(T1) \geq wts(B)$ ,  $cb(B)=true$  and  $rts(B)=\max(ts(T1), rts(B))$
- $w1(A) \rightarrow ok \rightarrow wts(A)=1$  and  $cb(A)=false$  -- because  $ts(T1) \geq wts(A)$ ,  $cb(A)=true$  and  $ts(T1) \geq rts(A)$
- $w2(B) \rightarrow ok \rightarrow ts(T2)=3, wts(B)=3$  and  $cb(B)=false$  -- because  $ts(T2) \geq wts(B)$ ,  $cb(B)=true$  and  $ts(T2) \geq rts(B)$
- $w1(B) \rightarrow T1$  waiting for the commit or rollback of  $T2$  -- because  $cb(B)=false$ ,  $ts(T1) = rts(B)$  and  $ts(T1) < wts(B)$
- $r2(A) \rightarrow T2$  waiting for the commit or rollback of  $T1$  -- because  $cb(A)=false$ , and  $ts(T2) > wts(A) \implies$

**DEADLOCK!**



## Solution to exercise 4 (2)

It is easy to see that the schedule is not in the class of 2PL schedules (with shared and exclusive locks).

Indeed, in order to release the lock on B, T2 should acquire the shared lock on A, but this is impossible, because T1 has such lock, and it cannot release the exclusive lock it has on A without breaking the 2PL rule, because it needs it for reading A.



## Exercise 5

Let A and B be two elements of the database such that  $rts(A)=rts(B)=wts(A)=wts(B)=wts-c(A)=wts-c(B)=0$  and  $cb(A)=cb(B)=true$ . Suppose that the system clock is 0, and that the system uses the clock values as the timestamps to be assigned to the various transactions (at their first action). Illustrate the actions of the timestamp-based scheduler when receiving the following complete schedule (“c” stands for commit and “a” for rollback)

$r1(A) \ w2(A) \ c2 \ r3(B) \ w3(A) \ w1(A) \ a3 \ r1(B)$



## Solution to exercise 5

➤ The initial situation is:

$$\text{wts}(A) = \text{wts-c}(A) = \text{wts}(B) = \text{wts-c}(B) = \text{rts}(A) = \text{rts}(B) = 0 \text{ and } \text{cb}(A) = \text{cb}(B) = \text{true}$$

➤ The system responds as follows:

- $r1(A) \rightarrow \text{ok} \rightarrow \text{ts}(T1)=1, \text{rts}(A)=1$ , because  $\text{ts}(T1) \geq \text{wts}(A)$ ,  $\text{cb}(A)=\text{true}$  and  $\text{rts}(A)=\max(\text{ts}(T1), \text{rts}(A))$
- $w2(A) \rightarrow \text{ok} \rightarrow \text{ts}(T2)=2, \text{wts}(A)=2$  and  $\text{cb}(A)=\text{false}$ , because  $\text{ts}(T2) \geq \text{wts}(A)$ ,  $\text{ts}(T2) \geq \text{rts}(A)$  and  $\text{cb}(A)=\text{true}$
- $c2 \rightarrow \text{ok} \rightarrow \text{wts-c}(A)=\text{wts}(A)=2, \text{cb}(A)=\text{true}$
- $r3(B) \rightarrow \text{ok} \rightarrow \text{ts}(T3)=4, \text{rts}(B)=4$ , because  $\text{ts}(T3) \geq \text{wts}(B)$ ,  $\text{cb}(B)=\text{true}$  and  $\text{rts}(B)=\max(\text{ts}(T3), \text{rts}(B))$
- $w3(A) \rightarrow \text{ok} \rightarrow \text{wts}(A)=4$  and  $\text{cb}(A)=\text{false}$ , because  $\text{ts}(T3) \geq \text{wts}(A)$ ,  $\text{cb}(B)=\text{true}$  and  $\text{ts}(T3) \geq \text{rts}(A)$
- $w1(A) \rightarrow T1$  waiting for the commit or rollback of  $T3$ , because  $\text{cb}(A)=\text{false}$ ,  $\text{ts}(T1) \geq \text{rts}(A)$  and  $\text{ts}(T1) < \text{wts}(A)$
- $a3 \rightarrow \text{cb}(A)=\text{true}, \text{wts}(A)=\text{wts-c}(A)=2$
- $w1(A) \rightarrow$  ignored by Thomas rule
- $r1(B) \rightarrow \text{ok} \rightarrow \text{rts}(B)=4$ , because  $\text{ts}(T1) \geq \text{wts}(B)$ , and  $\text{rts}(B)=\max(\text{ts}(T1), \text{rts}(B))$



## Exercise 6

In the DBMS called Misty, a total order  $D$  is defined on the set of elements in the database, and the concurrency control strategy adopted by Misty is the following:

- a) if a transaction  $T_1$  reads an element  $X$  written by transaction  $T_2$  (i.e.,  $T_2$  was the last transaction that wrote the element when  $T_1$  reads it), and the first element used (i.e., read or written) by  $T_2$  does not precede the first element used by  $T_1$  according to  $D$ , then  $T_1$  is aborted, otherwise  $T_1$  continues;
- b) analogously, if a transaction  $T_1$  writes on an element  $X$  written by transaction  $T_2$ , or read by transaction  $T_2$  (i.e.,  $T_1$  was the first transaction that wrote  $X$  after such reading), and the first element used by  $T_2$  does not precede the first element used by  $T_1$  according to  $D$ , then  $T_1$  is aborted, otherwise  $T_1$  continues.

Prove or disprove the following claim: every schedule accepted by Misty is serializable.



## Solution to exercise 6 (1)

Consider a schedule  $S$  accepted by Misty, and let  $G$  be the precedence graph associated to  $S$ . We will prove that if the precedence graph  $G$  associated to  $S$  is cyclic, then we have a contradiction. This is obviously equivalent to say that if a schedule  $S$  is accepted by Misty, then the precedence graph  $G$  associated to  $S$  is acyclic, and therefore  $S$  is conflict-serializable, and hence serializable.

We proceed by proving three claims. In all of them, we assume that  $S$  is a schedule accepted by Misty, and  $G$  is the associated precedence graph.



## Solution to exercise 6 (2)

**First claim:** if there is an edge from  $T_i$  to  $T_j$  in  $G$ , then the first element used by  $T_i$  in  $S$  precedes the first element used by  $T_j$  in  $S$  according to  $D$ .

Proof: the edge from  $T_i$  to  $T_j$  in  $G$  comes from  $\alpha(x)$  in  $T_i$  and  $\beta(x)$  in  $T_j$ , where at least one of  $\alpha$  and  $\beta$  is “write”. Therefore,  $S$  has the form:

...,  $\alpha(x)$ , ...,  $\gamma_1(x)$ , ...,  $\gamma_k(x)$ , ...,  $\beta(x)$ , ...

Where  $\alpha(x), \gamma_1(x), \dots, \gamma_k(x), \beta(x)$  are all the actions on  $x$  in  $S$ . We proceed by induction on  $k$ .

If  $k = 0$ , then there are two cases:

- $\alpha$  is “read” (therefore  $\beta$  is “write”):  $T_j$  writes on an element read by  $T_i$ , and therefore the first element used by  $T_i$  precedes the first element used by  $T_j$  according to  $D$ .
- $\alpha$  is “write” (therefore  $\beta$  is “read”, or “write”):  $T_j$  reads an element or writes on an element written by  $T_i$  and therefore the first element used by  $T_i$  precedes the first element used by  $T_j$  according to  $D$ .





## Solution to exercise 6 (3)

If  $k > 0$ , then there are two cases:

- $\gamma_1(x), \dots, \gamma_k(x)$  are all “read”.
  - If beta is “read”, then alpha is “write”, which means that  $T_j$  reads an element written by  $T_i$ ; so, the first element used by  $T_i$  precedes the first element used by  $T_j$  according to D.
  - If beta is “write” and alpha is “read”, then  $T_j$  writes on an element read by  $T_i$ ; so, the first element used by  $T_i$  precedes the first element used by  $T_j$  according to D.
  - If beta is “write” and alpha is “write”, then  $T_j$  writes on an element written by  $T_i$ ; so, the first element used by  $T_i$  precedes the first element used by  $T_j$  according to D.

- at least one (say  $\gamma_h$ ) of  $\gamma_1(x), \dots, \gamma_k(x)$  is “write” (and therefore  $h$  is different from  $i$  and  $j$ ). Thus,  $S$  is constituted by

$$S1 = \alpha(x), \dots, \gamma_h(x) \quad \text{and} \quad S2 = \gamma_h(x), \dots, \beta(x)$$

By induction hypothesis on  $S1$  and  $S2$ , the first element used by  $T_j$  precedes the first element used by  $T_h$  according to D, and the first element used by  $T_h$  precedes the first element used by  $T_i$  according to D. By transitivity, we can conclude that the first element used by  $T_i$  precedes the first element used by  $T_j$  according to D.



## Solution to exercise 6 (4)

**Second claim:** if there is a path from  $T_i$  to  $T_j$  in  $G$ , then the first element used by  $T_i$  precedes the first element used by  $T_j$  according to  $D$ .

Proof: easy, by induction on the length of the path, where the base step of the induction is provided by the first claim.

**Third claim:** a cycle is a path of length greater than 1 from  $T_i$  to  $T_j$  (for some  $T_i$ ) in  $G$ . Therefore, by the second claim, if there is a cycle in  $G$ , then the first element used by  $T_i$  precedes the first element used by  $T_j$  according to  $D$ . Since this is impossible, we conclude that  $G$  is acyclic, and therefore  $S$  is conflict-serializable and hence serializable.