Algorithm Design - Homework 2

Academic year 2018/2019

Instructors: Prof. Stefano Leonardi, Prof. Chris Schwiegelshohn

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Due date: January 15th 2019, 11.59pm

Make sure that the solutions are typewritten or clear to read. A complete answer consists of a clear description of an algorithm (an English description is fine), followed by an analysis of its running time and a proof that it works correctly.

For this exercise, we have included a page limit per exercise, using reasonable margins and 11pt character size. These page limits are strict: if the solution of the exercise exceeds this length, the solution will only be graded up to the page limit.

Hand in your solutions and keep a copy for yourself. Solutions will be posted or presented after due date. In the final exam, you will be asked to explain your solutions and/or to go over your mistakes.

Collaboration policy. Groups are formed by at most two students. You must write up the assignment alone, in isolation. Also, you must understand well your solutions and be able to discuss your choices and their motivations in detail with the instructor. Finally, you should cite any sources you use in working on a homework problem.

Late policy: Every homework must be returned by its due date. Homeworks that are late will lose 10% of the grade if they are up to 1 day (24h) late, 20% if they are 2 days late, 30% if they are 3 days late, and they will receive no credit if they are late by more than 3 days.

Length: For this homework, we would like your solution to comply with the following lengths. Please use 11pt, and reasonable margins.

- Exercise 1: Less than 1 page
- Exercise 2: Less than 2 pages
- Exercise 3: Less than 2 pages
- Exercise 4: Less than 1 page
- Exercise 5: Less than 1 page
- Exercise 6: Less than 1 page

Please refer to course's Web page for detailed information about above aspects.

Exercise 1. (Approximation Algorithms for Michele's Party Problem) Leonardo was not invited to Michele's birthday party because of Michele's unreasonable demands. Leonardo observed that if Michele had more female friends, then there would have been an option to invite him (Leonardo himself seems to be more popular with the ladies than Michele). Leonardo therefore introduced a number of his own female friends and asked Michele to reconsider the invitations.

Formally, assume that after Leonardo's intervention, the set of male friends M has the exact same amount as the set of female friends F. Moreover, for each two persons x, y, we denote with $w(x, y)$ the measure of friendship between x and y. The task is to find a set of friends I such that

- $\frac{1}{|I|}\sum_{x,y\in I} w(x, y)$ is maximized and
- $|I \cap M| = |I \cap F|$.

Goal: Show that with the assumption $|M| = |F|$, there exists a 2-approximation algorithm.

Exercise 2. (Randomized Rounding for Set Cover with Redundancies) Valerio has to supervise a project. For the project, he needs a number of skills to be presents (e.g. programming, public relations, coffee making, etc). Normally, it would be enough to ensure that he includes sufficiently many people in the project such that all skills are covered, according to the set cover problem. However, since this is Valerio's first project, he is overly cautious and insists on having a large number redundancies, i.e. every skill has to be present in at least 3 different people.

Goal: Design an approximation algorithm for this modified version of the set cover problem with redundancies (every set must appear 3 times). Specifically, modify the randomized rounding algorithm for set cover and discuss the analysis of the approximation factor of the algorithm.

Exercise 3. We are given an undirected graph G = (V,E), costs $c_e \geq 0$ for all edges $e \in E$, and k distinguished vertices $s_1, \dots, s_k$. The goal is to remove a minimum-cost set of edges F such that no pair of distinguished vertices s_i and s_j are in the same connected component of $(V, E - F)$. We know that when $k = 2$ the problem is just the min-cut problem, and it can be solved in polynomial time via max-flow. Consider the following algorithm for the problem with k vertices. For $i = 1, \dots, k$, let F_i be the mincut that separates vertex s_i and vertices $s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_k$. Solution $i = 1, \dots, k$, is obviously a feasible solution, i.e. it separates all $s_1, \dots, s_k$. Show that it is also a 2-approximation of the optimal solution.

Exercise 4. Cristina is playing with a DNA sequence D of length n over the alphabet $\{A, C, G, T\}$. Her task will be to replicate this DNA-sequence using a set of genes $G := \{G_1, \dots, G_m\}$ (each gene is a string over the alphabet), i.e. she wants to piece together D by concatenating genes $G' \subset G$. The lab tells Cristina to be careful with the selection of G', as special equipment is needed to synthesize each one. Once the equipment for gene G_i is bought (at price w_i), Cristina can generate as many copies of G_i as she likes.

Goal: Formulate this problem as a linear program. Having done that, determine the dual program.

Exercise 5. It is Christmas afternoon and there is as usual discussion between the two lazy reindeers Comet and Dasher on who should do the heavy job tonight. The only way to convince the reindeers is to let them to play a game with hay bales that they love more anything else. Santa gives two magic biased coins to the two reindeers and proposes the following game:

- If there are two heads, Comet gets 4 hay bales from Dasher.
- If there are two tails, Comet gets 2 hay bales from Dasher.

- If there is head for Comet and tail for Dasher, Dasher gets 1 hay bale from Comet.
- If there is tail for Comet and head for Dasher, Dasher gets 2 hay bales from Comet.

Goal: Tell Santa how to bias the two magic coins in order to convince Comet and Dasher to play the game.

Who will win more hay bales will go with Santa. Happy Christmas!

Exercise 6. Bonus Exercise Giorgio was partying a bit too much and is now slightly tipsy. For some reason he found himself one step away from the highway. While no longer sober, he is aware enough of his surroundings that with probability p he takes a step towards the highway, and with probability $1-p$ he moves away from the highway. If Giorgio touches the highway, he will wake up in a hospital, otherwise he will make it home safely.

Goal: Assuming that Giorgio makes an infinite amount of steps on a straight line, for what value of p is the probability that Giorgio lands in the hospital at most $1/2$? For which value of p is Leonardo always going to wake up in the hospital? Recall that Giorgio starts at position 0, and if he ever reaches position -1, he steps onto the highway.