IoT Homework #3

RFID

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1. Efficiency Computation for Different Initial Frame Sizes

Assuming that after the first frame, the frame size is correctly set to the current backlog size, and using the given durations:

$$L_2 = 4, \quad L_3 = \frac{51}{8}$$

Below are the subsections for the initial frame sizes $r_1 = 1, 2, 3, 4, 5, 6$. Each one includes the probabilities of having $s = 0$ to 4 successful transmissions in the first frame, the arbitration duration formula $L_4^\star$, and the efficiency $\eta = \frac{N}{L_4^\star}$.

r1 = 1

Efficiency: 0.4071

- $P(s = 0) = 10/64$
- $P(s = 1) = 3/16$
- $P(s = 2) = 9/16$
- $P(s = 3) = 0$
- $P(s = 4) = 6/64$

$$L_4^\star = 1 + L_4$$

$$L_4 = \left(\frac{10}{64} \cdot L_4\right) + \left(\frac{3}{16} \cdot L_3\right) + \left(\frac{9}{16} \cdot L_2\right)$$

$$\eta = \frac{4}{L_4^\star}$$

r1 = 2

Efficiency: 0.4166

- $P(s = 0) = 1/2$
- $P(s = 1) = 1/2$
- $P(s = 2) = 0$
- $P(s = 3) = 0$
- $P(s = 4) = 0$

$$L_4^\star = 1 + \left(\frac{1}{2} \cdot L_4\right) + \left(\frac{1}{2} \cdot L_3\right)$$

$$\eta = \frac{4}{L_4^\star}$$

r1 = 3

Efficiency: 0.4467

- $P(s = 0) = 7/27$
- $P(s = 1) = 8/27$
- $P(s = 2) = 12/27$
- $P(s = 3) = 0$
- $P(s = 4) = 0$

$$L_4^\star = 1 + \left(\frac{7}{27} \cdot L_4\right) + \left(\frac{8}{27} \cdot L_3\right) + \left(\frac{12}{27} \cdot L_2\right)$$

$$\eta = \frac{4}{L_4^\star}$$

r1 = 4

Efficiency: 0.4533

- $P(s = 0) = 10/64$
- $P(s = 1) = 3/16$
- $P(s = 2) = 9/16$
- $P(s = 3) = 0$
- $P(s = 4) = 6/64$

$$L_4^\star = 1 + \left(\frac{10}{64} \cdot L_4\right) + \left(\frac{3}{16} \cdot L_3\right) + \left(\frac{9}{16} \cdot L_2\right)$$

$$\eta = \frac{4}{L_4^\star}$$

r1 = 5

Efficiency: 0.4426

- $P(s = 0) = 65/625$
- $P(s = 1) = 16/125$
- $P(s = 2) = 360/625$
- $P(s = 3) = 0$
- $P(s = 4) = 120/625$

$$L_4^\star = 1 + \left(\frac{65}{625} \cdot L_4\right) + \left(\frac{16}{125} \cdot L_3\right) + \left(\frac{360}{625} \cdot L_2\right)$$

$$\eta = \frac{4}{L_4^\star}$$

r1 = 6

Efficiency: 0.4225

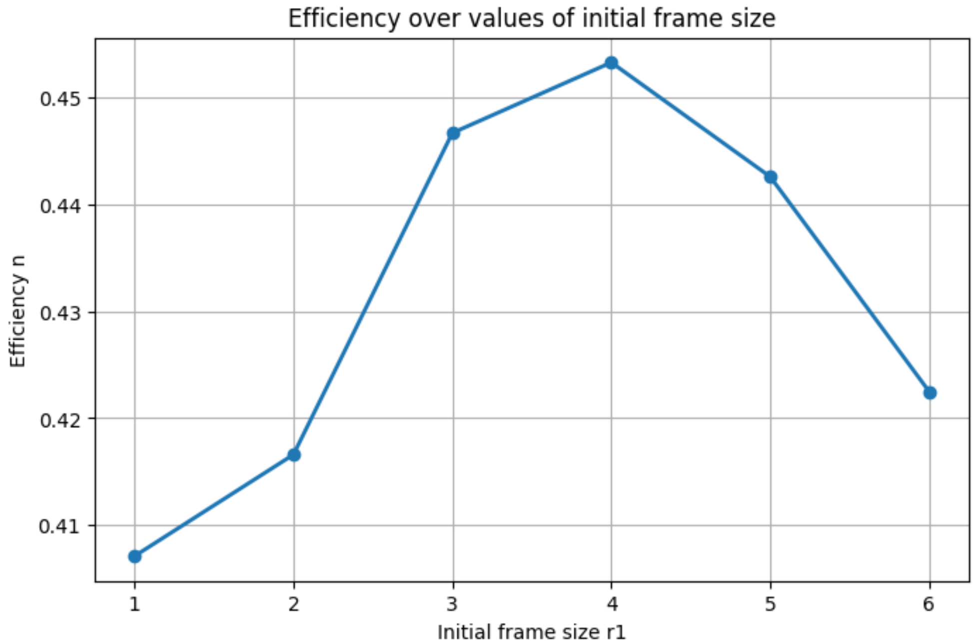
- $P(s = 0) = 96/1296$
- $P(s = 1) = 120/1296$
- $P(s = 2) = 720/1296$
- $P(s = 3) = 0$
- $P(s = 4) = 360/1296$

$$L_4^\star = 1 + \left(\frac{96}{1296} \cdot L_4\right) + \left(\frac{120}{1296} \cdot L_3\right) + \left(\frac{720}{1296} \cdot L_2\right)$$

$$\eta = \frac{4}{L_4^\star}$$

2. Efficiency Plot

The following plot shows the computed efficiency η over different initial frame sizes r_1.



3. Analysis of Maximum Efficiency

The goal of this analysis is to determine which initial frame size r_1 yields the highest overall efficiency η, defined as the ratio between the number of tags successfully identified and the total duration of the arbitration process.

Based on the computed efficiencies for each initial frame size:

- For small values of r_1 (e.g., $r_1 = 1, 2$), the frame is too short, leading to a high collision probability and hence an increase in the number of required retransmissions. This results in longer arbitration periods and lower efficiency.
- For large values of r_1 (e.g., $r_1 = 5, 6$), the frame tends to be underutilized, with many empty slots. Although collisions are reduced, the overhead from unused slots causes a drop in efficiency.
- At intermediate values (notably $r_1 = 4$), the frame is well balanced: the number of collisions is limited, and the number of empty slots is still low. As a result, the system achieves its best trade-off, leading to the highest efficiency.

From the plot and from the values of η, we observe that the maximum efficiency occurs when:

$$r_1 = 4$$

This suggests that choosing an initial frame size close to the actual backlog size allows Dynamic Frame ALOHA to operate most effectively, minimizing the expected arbitration time and maximizing throughput.