NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
A.A. 2024/2025

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HOMEWORK 1

Consider the following wave propagation problem:

$$\begin{cases} \rho(x)\omega u(x) + \dfrac{\partial}{\partial x}(\mu(x)\dfrac{\partial u}{\partial x})(x) = f(x) \quad x \in (0, L), \\ u(0) = g_D, \\ u(L) - \dfrac{i}{\rho(x)\omega}\dfrac{\partial u}{\partial x}(L) = g_A, \end{cases} \tag{1}$$

where g_D and g_A are given constants, μ and ρ represent the (variable) stiffness and mass density of the medium, and i is the imaginary unit.

1. Write the weak formulation of problem (1) and the corresponding finite element discretization. Consider linear basis functions. Show that the Galerkin discretization leads to the following system

   $$M\boldsymbol{u} - A\boldsymbol{u} = \boldsymbol{F}.$$

   Define precisely the entries of the matrices M and A and of the right hand side $\boldsymbol{F}$.

2. Implement in Matlab a finite element solver for problem (1), and verify your implementation on a test problem for which you know the exact solution u. Report the behaviour of the norm $\|u - u_h\|_{L^2(0,L)}$ for different choiches of the discretization parameter h (mesh size). Consider both constant and variable (piece-wice constant) model parameters μ and ρ. Comment on the results.

3. Consider the data: $L = 5$, $f = g_A = 0$,

   $$g_D = \frac{2w^2}{\sqrt{\pi}f_0^2}\exp{(-\frac{w^2}{f_0^2})},$$

   wiht $f_0 = 3$ and the parameters $\mu(x)$ and $\rho(x)$ such that

   a) $c(x) = 1$ for $x \in [0, L]$
   b) $c(x) = 0.1$ for $x \in [0, L/2]$ and $c(x) = 2$ for $x \in [L/2, L]$.

   Solve problem (1) for $\omega \in (0, 20)$ and choose h to have at least ten points per wavelength $\lambda = \frac{\min c(x)}{3f_0}$. Report the results and comment on them.

4. Report a snapshot of the function $v(x,t) = \exp(i\omega t)u(x)$ in the space-time domain $(0, L) \times (0, T)$ for a large enough T. Comment on the plot.