MUSIC AND
ACOUSTIC ENGINEERING

# SOUND SYNTHESIS AND SPATIAL PROCESSING

## Homework - Report

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# 1 Problem presentation

In this report, we want to address the implementation of a Wave Digital Filter simulating the behaviour of MEMS speakers using the mechanical equivalent circuit.

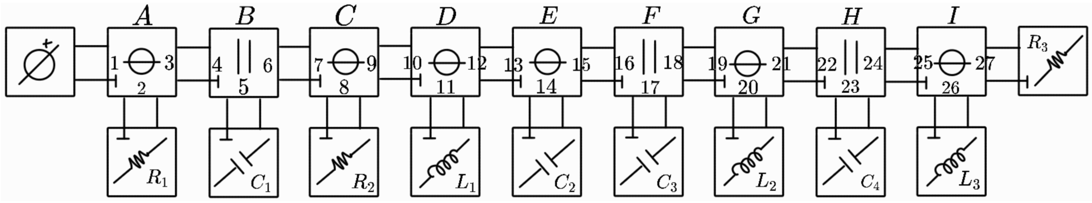


Figure 1: Wave domain model of the MEMS speaker - series/parallel adaptors (A-I) and ports (1-27)

The first step is to describe all the 1-port elements - resistors, capacitors, inductors and ideal voltage generator - and the adaptors - series and parallel. In this model, we define the reference resistances $Z_k$ of the passive elements such as:

$$Z_R = R, \quad Z_C = \frac{T_s}{2C}, \quad Z_L = \frac{2L}{T_s} \qquad T_s : \text{sampling period [s]}$$

The ports between junctions share the same reference resistances $Z_k$. The remaining condition to set, for each junction, is to adapt the left port based on the other two such as:

$$Z_{\text{series}} = Z_{\text{bottom}} + Z_{\text{right}}, \quad Z_{\text{parallel}} = Z_{\text{bottom}} // Z_{\text{right}}$$

The next step is to define the matrixes $[Q]$ and $[B]$ in order to evaluate the scattering matrixes $[S_K]$ for all 9 adaptors. We considered only "simple" junctions, where all currents and voltages for the single adaptors follow the same convention. Hence, we have:

$$[B] = [Q] = [1 \ 1 \ 1]$$

For each adaptor, we define the following diagonal matrix:

$$[Z] = \text{diag}\{Z_{\text{left}}, \ Z_{\text{bottom}}, \ Z_{\text{right}}\}$$

The scattering matrixes for series and parallel adaptors are computed as:

$$[S_{\text{series}}] = [I] - 2\,[Z]\,[B]^T \left([B]\,[Z]\,[B]^T\right)^{-1} [B]\,, \qquad [S_{\text{parallel}}] = 2 \ [Q]^T \left([Q]\,[Z]^{-1}\,[Q]^T\right)^{-1} [Q] \ [Z]^{-1} - [I]$$

# 2 WDF implementation

Regarding the coefficient computation, we introduce a loop for all samples $n$, in which we implement forward scan, local root scattering and backward scan. The instructions in the loop are the following:

1. Update the incident coefficients $a_k$ from the previous state accordingly:

$$\begin{cases} a_k(n) = 0, & \text{resistors} \\ a_k(n) = b_k(n-1), & \text{capacitors} \\ a_k(n) = -b_k(n-1), & \text{inductors} \end{cases}$$

We do not update the remaining incident waves, which are not related to actual circuital components but to the junctions, as they will be overwritten;

2. **Forward scan:** starting from the load $R_3$, we evaluate the reflected coefficient $b_k$ of the left port of the adaptors using the following relation:

$$\underline{b}_K(n) = [S_K]\underline{a}_K(n), \qquad K : \text{adaptor index} \in \{A, B, \cdots, I\}$$

The last coefficient to be evaluated will be $b_1$ for port 1;

3. **Local root scattering:** the ideal voltage generator has the following expressions for incident and reflected waves:

$$a_g(n) = b_1(n), \qquad b_g(n) = 2F_{\text{IN}}(n) - a_g(n)$$

This calculation influences the rest of the model starting from $a_1(n) = b_g(n)$;

4. **Backward scan:** starting from the adaptor (A), we evaluate the reflected coefficients $b_k$ of the bottom port and the right port of each junction - which are the remaining ports to be considered - using the usual relation:

$$\underline{b}_K(n) = [S_K]\underline{a}_K(n)$$

5. Store the reflected coefficients $b_k$ for the next state. In our implementation, this step is not present, as it is already included in the coefficients update at point (1).

# 3 Comments and results

The results of our WDF implementation look satisfactory and coherent with respect to the reference given by the circuit simulation.

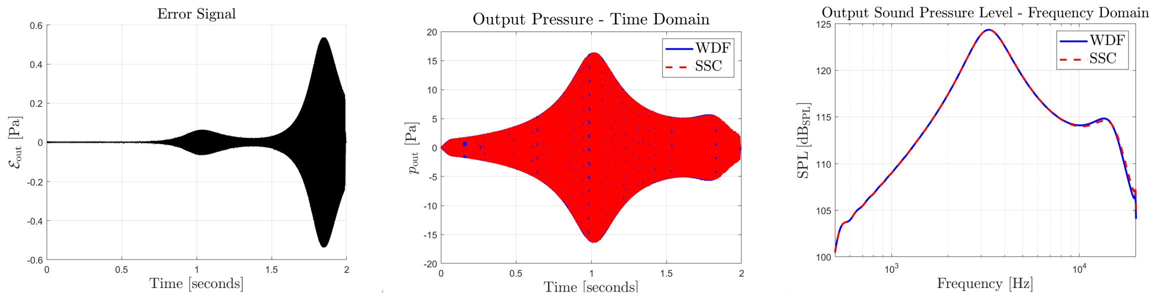


Figure 2: Error signal (left), output pressure (center) and output SPL (right)

The **MSE** value we get is **0.0119**. We can notice how the implementation is initially very good at modelling the circuit: for low frequencies (initial time instants, considering a sweep input sine), the computational error is almost zero and the SPL values are the same of the reference.

On the other hand, the model loses a small amount of accuracy at very high frequencies: the error starts having non-negligible pressure values, which are actually not that relevant in the SPL graph. The error computation is a numerical difference - for each time instant - between output pressure values of the WDF implementation and the circuit simulation: that means is very sensitive to any phase mismatch.

In terms of SPL values, an output pressure like that is around 88 dB, which is particularly relevant in terms of sound intensity. Nevertheless, the output SPL graph does not show significant differences between implementation and ground truth: this profile is related to the overall pressure envelope more than the actual pressure values in time. Thus, any small phase mismatch can result in non-negligible error values, but do not influence the perceived sound: this is coherent with the fact that the human auditory system is not sensitive to phase shifts (excluding any interference phenomena).