15.6 EXAMPLE: COMPARISON BETWEEN COMPLIANT AND COLLAPSIBLE TUBES

To better understand the difference between compliant and collapsible tubes, we use the expression of deformable tubes (15.44) and (15.47) to study a hydraulic system made of three different fluid compartments. The compartments have different hydraulic resistances, as described in Fig. 15.5. A pressure gradient $\Delta p = p_{in} - p_{out}$ is maintained between the first and the third fluid compartment by the analogy of independent voltage sources.

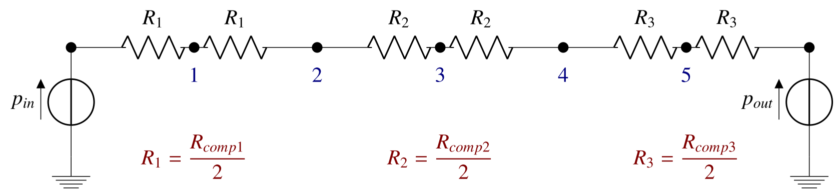

FIGURE 15.5

Electric analogy of a fluid system made by three distinct compartments, $comp1$, $comp2$, and $comp3$, having different characteristics. The compartments $comp1$ and $comp3$ are deformable compliant tubes having the same characteristics. The compartment $comp2$ is a deformable tube and assumes two different behaviors during the simulations. In the first test case, $comp2$ is considered a compliant resistor, whereas in the second test case, $comp2$ is considered a collapsible resistor. The black bullets, numbered 1 to 5, are the nodes of the circuit.

The three compartments, indicated by $comp1$, $comp2$, and $comp3$, have the following characteristics:

- $comp1$ and $comp3$ are assumed to be deformable compliant tubes;
- $comp2$ is a deformable tube and assumes two different behaviors during the simulations. In the first test case it is modeled as a compliant resistor, whereas in the second test case it is modeled as a collapsible resistor.

The hydraulic resistance for compartments $comp1$ and $comp3$ is calculated with Eq. (15.44). To model the hydraulic resistance of compartment $comp2$, Eq. (15.44) is used when the tube is modeled as compliant, whereas Eq. (15.47) is used when the tube is modeled as collapsible.

Let us now consider the electric circuit of Fig. 15.5. We note that the resistances in the three compartments are split in two equal portions, in such a way that

$$R_1 = \frac{R_{comp1}}{2}, \quad R_2 = \frac{R_{comp2}}{2}, \quad R_3 = \frac{R_{comp3}}{2}. \tag{15.61}$$

This is done to better describe the pressure drop within each compartment. The electric circuit comprises six unknowns: the total flow Q_v and the five nodal pressures p_i, $i = 1, \ldots, 5$. Applying the KVL (14.14) on the whole loop in Fig. 15.5 we can express the total flow Q_v as a function of the total resistance of the circuit, namely,

$$p_{in} - p_{out} = R_{tot}\, Q_v, \quad \text{where} \quad R_{tot} = R_{comp1} + R_{comp2} + R_{comp3}. \tag{15.62}$$

The remaining five unknowns can be calculated by applying the KCL (14.13) at each node of the electric circuit in Fig. 15.5. The nonlinear algebraic system to be solved is then

$$\underline{\underline{\mathbf{Y}}}\,\underline{\mathbf{P}} = \underline{\mathbf{I}}, \tag{15.63}$$

where $\underline{\mathbf{P}} = \left[p_1, p_2, p_3, p_4, p_5\right]^T \in \mathbb{R}^5$ is a column vector of the unknown nodal fluid pressures, $\underline{\underline{\mathbf{Y}}} \in \mathbb{R}^{5\times 5}$ is the admittance matrix

$$\underline{\underline{\mathbf{Y}}} = \begin{bmatrix} -G_1 & 0 & 0 & 0 & 0 \\ G_1 & -G_1 & 0 & 0 & 0 \\ 0 & G_2 & -G_2 & 0 & 0 \\ 0 & 0 & G_2 & -G_2 & 0 \\ 0 & 0 & 0 & G_3 & G_3 \end{bmatrix}, \tag{15.64}$$

having set

$$G_1 := \frac{1}{R_{comp1}}, \; G_2 := \frac{1}{R_{comp2}}, \; G_3 := \frac{1}{R_{comp3}},$$

whereas the right-hand side $\underline{\mathbf{I}}$ is the column vector defined as

$$\underline{\mathbf{I}} = \left[\frac{\Delta p}{2\,R_{tot}} - \frac{p_{in}}{R_{comp1}}, \frac{\Delta p}{2\,R_{tot}}, \frac{\Delta p}{2\,R_{tot}}, \frac{\Delta p}{2\,R_{tot}}, \frac{\Delta p}{2\,R_{tot}}\right]^T \in \mathbb{R}^5. \tag{15.65}$$

In the first simulation test we assume to deal with a compliant resistance in all of the three compartments. In this case the hydraulic resistances are calculated as

$$R_{comp1} = \frac{k_r \ell}{\mathcal{A}_{ref}^2}\left(\frac{p_1 - p_{ext}}{k_p k_L} + 1\right)^{-4}, \tag{15.66}$$

$$R_{comp2} = \frac{k_r \tilde{\ell}}{\tilde{\mathcal{A}}_{ref}^2}\left(\frac{p_2 - p_{ext}}{\tilde{k}_p \tilde{k}_L} + 1\right)^{-4}, \tag{15.67}$$

$$R_{comp3} = \frac{k_r \ell}{\mathcal{A}_{ref}^2}\left(\frac{p_3 - p_{ext}}{k_p k_L} + 1\right)^{-4}. \tag{15.68}$$

In the second simulation test we assume to deal with a collapsible resistance in the second compartment *comp*2. In this case the hydraulic resistances are calculated as

$$R_{comp1} = \frac{k_r \ell}{\mathcal{A}_{ref}^2}\left(\frac{p_1 - p_{ext}}{k_p k_L} + 1\right)^{-4}, \tag{15.69}$$

$$R_{comp2} = \begin{Bmatrix} \dfrac{k_r \tilde{\ell}}{\tilde{\mathcal{A}}_{ref}^2} \left(1 - \dfrac{p_2 - p_{ext}}{\tilde{k}_p}\right)^{\frac{4}{3}} & \text{for } p_2 \leq p_{ext}, \\ \dfrac{k_r \tilde{\ell}}{\tilde{\mathcal{A}}_{ref}^2} \left(\dfrac{p_2 - p_{ext}}{\tilde{k}_p \tilde{k}_L} + 1\right)^{-4} & \text{for } p_2 > p_{ext}, \end{Bmatrix} \tag{15.70}$$

$$R_{comp3} = \frac{k_r \ell}{\mathcal{A}_{ref}^2} \left(\frac{p_3 - p_{ext}}{k_p k_L} + 1\right)^{-4}. \tag{15.71}$$

The parameters in Eqs. (15.66) and (15.69) are reported in Table 15.1.

Table 15.1 Parameters used in the modeling of the hydraulic resistances in Eqs. (15.66) and (15.69).

Symbol	Description	Value
k_r	specific factor for Poiseuille's flow	$0.07113\ \mathrm{N\,m^2\,s}$
ℓ	length of *comp*1 and *comp*3	$175\ \mu\mathrm{m}$
$\mathcal{A}_{ref}$	cross-section surface of *comp*1 and *comp*3	$2.40528 \cdot 10^{-4}\ \mathrm{cm^2}$
k_p	constant given by Eq. (15.40) for the tubes *comp*1 and *comp*3	$1.11034 \cdot 10^2\ \mathrm{N\,m^2}$
k_L	adimensional constant given by Eq. (15.40) for *comp*1 and *comp*3	$5.3333 \cdot 10^2$
$\tilde{\ell}$	length of *comp*2	$238\ \mu\mathrm{m}$
$\tilde{\mathcal{A}}_{ref}$	cross-section surface of *comp*2	$4.448809 \cdot 10^{-4}\ \mathrm{cm^2}$
$\tilde{k}_p$	constant given by Eq. (15.40) for the tube *comp*2	$1.4444\ \mathrm{N\,m^2}$
$\tilde{k}_L$	adimensional constant given by Eq. (15.40) for *comp*2	$1.5306 \cdot 10^4$

The `Matlab` script illustrated in Section 28.2.1 is used to numerically solve the nonlinear system (15.63). The script calls the `Matlab` functions `el_fluid1`, `el_fluid2`, `cra`, `crv`, and `crv_col`, which implement the constitutive laws for the fluid compartments *comp*1, *comp*2, and *comp*3 and are reported in Section 29.2.1.1.

Figs. 15.6 and 15.7 show the simulation results of both cases: the blue curves (dark gray in print version) are obtained by assuming compliant resistances for all compartments; the red curves (light gray in print version) are obtained by modeling the compartment *comp*2 as a collapsible tube. The solutions of the system (15.63) are obtained considering a continuous variation of the external pressure p_{ext} in the interval $[0, 250]$ mmHg while $p_{in} = 40$ mmHg and $p_{out} = 15$ mmHg. Fig. 15.6 reports the fluid flow Q_v and the pressure drops $\Delta p_1 = p_{in} - p_2$, $\Delta p_2 = p_2 - p_4$, and $\Delta p_3 = p_4 - p_{out}$. Fig. 15.7 reports the resistances R_{comp1}, R_{comp2}, and R_{comp3} of the three fluid compartments and the total resistance R_{tot} of the fluid system.

Fig. 15.6 shows that when the external pressure p_{ext} reaches the value of 50 mmHg and the tube in *comp*2 is considered collapsible, the fluid flow starts to decrease and goes to zero when $p_{ext} =$ 200 mmHg. In the same time interval, the pressure drop in *comp*1 and in *comp*3 is going to zero while in *comp*2 is rising to $p_{in} - p_{out}$. Fig. 15.7 shows that when *comp*2 starts to collapse the hydraulic resistances of *comp*1 and *comp*3 continue to increase linearly with the increase of the external pressure p_{ext}. Conversely, the resistance in *comp*2, and hence the total resistance R_{tot} in