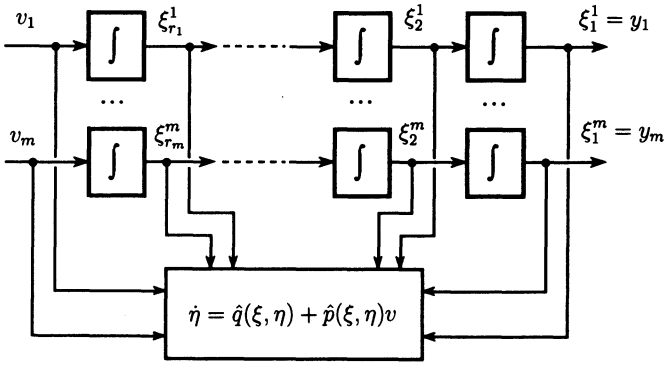

Fig. 5.1.

relative degree at this point, the noninteracting control problem can be solved, by means of a static state feedback which is defined for all x in a neighborhood of the point $x = 0$. It will be shown now that the existence of a (vector) relative degree it is also a *necessary* condition for the existence of solutions of the problem in question.

Proposition 5.3.1. *Consider a multivariable nonlinear system with m inputs and m outputs*

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x)u_i \\ y_1 &= h_1(x) \\ &\cdots \\ y_m &= h_m(x) \, . \end{aligned}$$

The Noninteracting Control Problem is solvable if and only if the matrix $A(0)$ is nonsingular, i.e. if the system has a some vector relative degree $\{r_1, \ldots, r_m\}$ at $x = 0$.

Proof. Suppose, for some integer r_i,

$$L_{g_j} L_f^k h_i(x) = 0$$

for all $1 \leq j \leq m$, for all $k < r_i - 1$, and for all x in some neighborhood of $x = 0$, and

$$\left(L_{g_1} L_f^{r_i - 1} h_i(x) \quad \cdots \quad L_{g_m} L_f^{r_i - 1} h_i(x) \right)$$

is not identically zero in some neighborhood of $x = 0$. Then (see Lemma 5.2.1) also

$$L_{(g\beta)_j} L_{f+g\alpha}^k h_i(x) = 0$$