Unicycle Planner

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1 Problem Definition

Let us consider a humanoid robot walking on a flat terrain. The main problem is to define the position and the orientation that the robot's feet should take in order to reach a specified position in the space. Timings should also be taken into account, since the robot will not be able to perform steps at arbitrary speed. In brief, for every foot and for every step taken the following parameters need to be defined:

- $x_{f,i}$
- $y_{f,i}$
- $\theta_{f,i}$
- $t_{f,i}$.

$x_{f,i}$ and $y_{f,i}$ are the x and y position of a frame fixed to the foot f when step i is completed, measured in a inertial reference frame $\mathbb{w}$, called *world*. The *world* frame is oriented such that the $z-$axis is aligned with the gravity and the origin belongs to the ground plane, supposed to be perpendicular to gravity too. The position of the origin of the feet fixed frame (measured in $\mathbb{w}$) has zero $z-$component when in touch with the ground, while the $z-$axis is parallel to gravity. In this configuration $\theta_{f,i}$ expresses the magnitude of the rotation to be performed around the *world* $z-$axis to align this frame with the corresponding foot fixed frame. Finally $t_{f,i}$ expresses the impact time, the instant at which foot f impacts the ground after completing the i^{th} step.

Suppose now that you want the robot to follow a generic point on the ground. It may be difficult to define directly all the quantities defined above, while using an optimization algorithm usually require the usage of integers variables which make the resulting problem hard to be solved. Another point that has to be taken into account is side stepping. Usually stepping aside is costly also for humans [1] while the robot could have difficulties in perform this motion correctly due to kinematic limitations. Referring again to human walking, the shape of the feet and the placement of the strong muscles makes the forward direction preferred. In view of all these considerations, the use of a unicycle model to plan footsteps provides a solution.

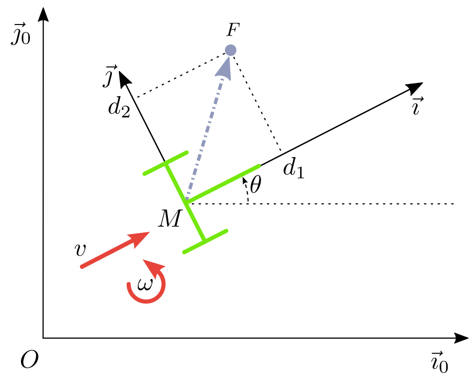

Figure 1: Notation.

2 The Unicycle Model

The unicycle model is a planar model of a robot having two wheels placed at a distance $d \in \mathbb{R}$ with a coinciding rotation axis (Figure 1). As a consequence, this mobile robot cannot move sideways, along the wheel axis, but it can turn by moving the wheel at different velocity.

In this section we are going to use the following notation:

- The i_{th} component of a vector x is denoted as x_i and, for the sake of brevity, $x_1\vec{u}+x_2\vec{w}$ is written as $(\vec{u}, \vec{w})x$.
- $\mathcal{I} = \{O; \vec{\imath}_0, \vec{\jmath}_0\}$ is a fixed inertial frame with respect to (w.r.t.) which the robot's absolute pose is measured.
- The point M is the middle point of the wheel's axis, and $\mathcal{B} = \{M; \vec{\imath}, \vec{\jmath}\}$ is a frame attached to the robot. The vector $\vec{\imath}$ is perpendicular to the wheel's axis.
- F denotes a point attached to the robot. Then, $\vec{OF} = (\vec{\imath}_0, \vec{\jmath}_0)x_f$ and $\vec{MF} = (\vec{\imath}, \vec{\jmath})d$, with d a constant vector.
- $e_1 := (1, 0)^T$ and $e_2 := (0, 1)^T$ denote the canonical basis vectors of $\mathbb{R}^2$.
- The rotation matrix of an angle θ is denoted as $R(\theta)$; $S = R(\pi/2)$ is the unitary skew-symmetric matrix, i.e.

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \quad S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The model equations are the following [2]:

$$\dot{x} = vR(\theta)e_1, \tag{1a}$$
$$\dot{\theta} = \omega, \tag{1b}$$

with v and ω the robot's rolling and rotational velocity, respectively. The variables v and ω are considered as kinematic control inputs.

2.1 Unicycle Controller

The control objective is to asymptotically stabilize the point F about a desired point F^*. For this purpose, define the coordinates of the geometric position error $F\vec{F}^*$ as follows

$$F\vec{F}^* = (\vec{\imath}_0, \vec{\jmath}_0)\tilde{x},$$

so that

$$\tilde{x} := x_f - x_f^*. \tag{2}$$

Then, the control objective is equivalent to the asymptotic stabilization of $\tilde{x}$ to zero. Since

$$x_f = x + R(\theta)d,$$

then differentiating Eq. (2) yields

$$\begin{aligned} \dot{\tilde{x}} &= \dot{x} + \dot{R}(\theta)d && \text{(3a)} \\ &= \dot{x} + \omega R(\theta)Sd. && \text{(3b)} \end{aligned}$$

Notice that

$$\begin{aligned} R(\theta) &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \\ \dot{R}(\theta) &= \begin{pmatrix} \sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}\omega \\ &= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\omega \\ &= R(\theta)S = SR(\theta). \end{aligned}$$

with

$$M := \left[\begin{array}{cc} 1 & -d_2 \\ 0 & d_1 \end{array}\right], \quad u := \left[\begin{array}{c} v \\ \omega \end{array}\right], \quad :=^T .$$

Classical control laws that asymptotically stabilize $\tilde{p} = 0$ when the point F is not along the wheels' axis, i.e. $d_1 \neq 0$. Assume that $d_1 \neq 0$ so that $\det(M) \neq 0$. Apply the control input

$$u \quad = \quad M^{-1}\left[-K\tilde{p}\right], \tag{4}$$

to System (**??**) with

$$K = \left[\begin{array}{cc} k_1 & 0 \\ 0 & k_2 \end{array}\right], \quad K > 0.$$

Then,

$$\dot{\tilde{p}} = -\omega S\tilde{p} - K\tilde{p}, \tag{5}$$

and $\tilde{p} = 0$ is a globally asymptotically stable equilibrium point for the closed-loop system.

This result ensures that when the control point F is not located on the wheels' axle, its stabilization to an arbitrary reference position P can be achieved by the use of simple feedback laws. This control strategy works very well when F is located ahead of the wheels' axle, which corresponds to the situation where the robot follows the user. Several limitations of this approach, however, must be mentioned. For instance, the control law is not defined when F is located on the wheel's axle and this gives rise to ill-conditioning problems when F is close to this axle (the matrix M is close to singular). Still, this situation is of practical interest for many applications where the user wants to remain close to the platform and keep it in his/her field of view. Another drawback of the control (4) is that when the robot is located behind the user, it tends to turn back if the user starts moving backward – this is the so-called *jack-knife effect*. We present below a new feedback control law to address these problems when F is located on the wheels' axle.

References

[1] M. L. Handford and M. Srinivasan, "Sideways walking: preferred is slow, slow is optimal, and optimal is expensive," *Biology letters*, vol. 10, no. 1, p. 20131006, 2014.

[2] D. Pucci, L. Marchetti, and P. Morin, "Nonlinear control of unicycle-like robots for person following," in *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on.* IEEE, 2013, pp. 3406–3411.