TUTORATO 03

Scritto 02 Ex 1
- Proprietà VA
-

Scritto 0 bis Ex 2
- P

PROBABILITÀ CONGIUNTA

ESEMPIO

A \ B	1	2	3	
1	0.25	0.3	0.05	0.6
2	0.15	0.05	0.2	0.4
	0.4	0.35	0.25	$\boxed{1}$

MARGINALE X

$0.6 + 0.4$: $\Sigma = 1$

MARGINALE Y

$\underbrace{0.4 \quad 0.35 \quad 0.25}_{\Sigma = 1}$

KP CONGIUNTA $\rightarrow$ MARGINALE

$\cancel{\leftarrow}$

Dalle congiunte posso ricavare le marginali ma NON viceversa

CONTROESEMPIO

X, Y 1/2

X⊥Y

	0	1	
0	1/4	1/4	1/2
1	1/4	1/4	1/2
	1/2	1/2	

X⊥Y

Y \ X	0	1	
0	0	1/2	1/2
1	1/2	0	1/2
	1/2	1/2	

X = 1 - Y

Y = 1 $\rightarrow$ X = 0 wp 1

Y = 0 $\rightarrow$ X = 1 wp 1

Stesse marginali,
diverse congiunte

MARGINALE

$$P_A(a) = \sum_{b \in B} P(A=a, B=b) =$$

$$= \sum_{b \in B} P_{AB}(a,b)$$

SOMMA PER RIGA

$$\text{Eg } P(A=1) = P_{AB}(1,1) + P_{AB}(1,2) + P_{AB}(1,3)$$

CONDIZIONATA

$$P_{A|B}(a|b) = \frac{P_{AB}(a,b)}{P_B(b)}$$

CORRELAZIONE

- Indicatore di dipendenza LINEARE

- $E[XY] - E[X] \cdot E[Y]$ ALTRA FORMULA

- VAR IND $\longrightarrow$ CORRELAZIONE ZERO

  $\not\longleftarrow$

Non vale il viceversa perché potrebbe esserci qualche altro tipo di dipendenza, ad esempio

SCRITTO 0 Ex 2

PROPRIETÀ VALORE ATTESO

Ind $\quad A, B$ ind $\quad E[A \cdot B] = E[A] \cdot E[B]$

linearity $\quad E[aX + bY] = E[aX] + E[bY] = aE[X] + bE[Y]$

$$\varepsilon_i \begin{cases} -1 & \text{pw } 1/2 \\ 1 & \text{pw } 1/2 \end{cases}$$

i) $X(\omega) = \varepsilon_1(\omega) \cdot \varepsilon_2(\omega)$

- $E[X(\omega)] = E[\varepsilon_1(\omega) \cdot \varepsilon_2(\omega)]$
$= \underbrace{E[\varepsilon_1(\omega)]}_{=0} \cdot \underbrace{E[\varepsilon_1(\omega)]}_{=0} = 0$

$E[\varepsilon_i(\omega)] = -1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 0$

$Var[X(\omega)] = E[X^2(\omega)] - [E[X(\omega)]]^2$
$= E[X^2(\omega)] = E[[\varepsilon_1(\omega) \cdot \varepsilon_2(\omega)]^2]$
$= E[\underbrace{\varepsilon_1^2(\omega)}_{=1} \cdot \underbrace{\varepsilon_2^2(\omega)}_{=1}] = 1$

$$\varepsilon_i^2 = \begin{cases} (-1)^2 = 1 & \text{wp } 1/2 \\ (1)^2 = 1 & \text{wp } 1/2 \end{cases} \Longrightarrow \varepsilon_i^2 = 1 \text{ wp } 1$$

↳ diventa una costante

$$E[Y] = E[\varepsilon_1 \cdot (\varepsilon_2 - \varepsilon_3)] \overset{\text{IND}}{=}$$

$$= \underbrace{E[\varepsilon_1]}_{=0} \cdot \underbrace{E[\varepsilon_2 - \varepsilon_3]}_{\text{NON SERVE CALCOLARLO}} = 0$$

$$Var[Y] = E[Y^2] - (\underbrace{E[Y]}_{=0})^2 = E[Y^2]$$

$$= E[[\varepsilon_1^2 \cdot (\varepsilon_2 - \varepsilon_3)]^2] =$$

$$E[\underbrace{\varepsilon_1^2}_{=1} \cdot (\varepsilon_2 - \varepsilon_3)^2] = E[(\varepsilon_2 - \varepsilon_3)^2]$$

$$= E[\varepsilon_2^2 + \varepsilon_3^2 - 2(\varepsilon_2 \varepsilon_3)]$$

$$= E[\varepsilon_2^2] + E[\varepsilon_3^2] - 2E[$$

ε $\quad E[a] = a$ Il valore atteso di una costante è

$\varepsilon_1^2 = 1$ $\quad 1$

COMMENTO

A, B, C ind

$X = B + C$

A e X sono ind ?

$$E[A\,X] = E[A(B+C)] = E(A\cdot B) + E(A\,C)$$
$$= E[A]\cdot E[B] + E[A]\cdot E[C]$$

$$= E[A]\cdot[E(B) + E(C)]$$
$$\|$$
$$= E[A]\cdot[E(\underbrace{B+C}_{X})] = E[A]\cdot E[X]$$

RP La somma **NON** altera l'indipendenza

ε_1	ε_2	$X = \varepsilon_1 \cdot \varepsilon_2$
1	1	1
1	-1	-1
-1	1	-1
-1	-1	1

ε_1	ε_2	ε_3	$\varepsilon_2-\varepsilon_3$	$\times\varepsilon_1$
1	1	1	0	0
1	1	-1	2	2
1	-1	1	-2	-2
1	-1	-1	0	0
-1	1	1	0	0
-1	1	-1	2	-2
-1	-1	1	-2	2
-1	-1	-1	0	0

$X \backslash Y$	-2	0	2
-1	1/4	1/4	0
1	0	1/4	1/4

- Riempiamo tabella
- 6 casi

$1-\frac{3}{4}=\frac{1}{4}$

$P(X=-1, Y=-2) = P(\varepsilon_1=1, \varepsilon_2=-1, \varepsilon_3=1)$
$+P(\varepsilon_1=-1, \varepsilon_2=1, \varepsilon_3=-1)$
$= P(\ )P(\ )\cdot P(\ ) + P(\ )P(\ )P(\ )$
$= 1/2 \cdot 1/2 \cdot 1/2 + 1/2 \cdot 1/2 \cdot 1/2 = 1/8 + 1/8$
$= 1/4$

$P(X=-1, Y=0) = P(\varepsilon_1=1, \varepsilon_2=-1, \varepsilon_3=1)$
$+P(-1, 1, 1) = 1/4$

$$\mathrm{Cov}[X,Y]$$
$$= E[X\cdot Y] - E[X]\cdot E[Y]$$

$$E[XY] = -1\cdot(-2)\,P(-1,-2)$$
$$+ \underbrace{-1\cdot 0}_{=0}\,P(-1,0) + \underbrace{-1\cdot 2\cdot P(-1,2)}_{=0}$$

$$+ 1\cdot\underbrace{-2\cdot P(1,-2)}_{=0} + 1\cdot 0\,\underbrace{P}_{=0}(1,0)$$

$$+ 1\cdot 2\,P(1,2) = 2\cdot\frac{1}{4} + 2\cdot\frac{1}{4} =$$
$$= 1/2 + 1/2 = 1$$

SCRITTO 6 BIS EX 2

- $X_1, X_2 \dots X_n$
  $X_i \sim exp(\lambda)$ $f_{X_i}(x) = \lambda e^{-\lambda x}$

- $M_n(\omega) = \max_{i \in 1..,n} X_i(\omega)$

- F Funzione ripartizione X_i

- $G(x) = exp(-e^{-x})$ $x \in \mathbb{R}$
  $= e^{-e^{-x}}$ IVO DIST

o

FUNZIONE DI RIPARTIZIONE

OCCHIO! $e^{-e^{-x}} = e^{(-e^{-x})}$

NB $a^{x^4} \neq [a^x]^4 = a^{x \cdot 4}$ $\quad 2^{3^2} = 2^9 \quad [2^3]^2 = 2^6$

4 PROPRIETÀ

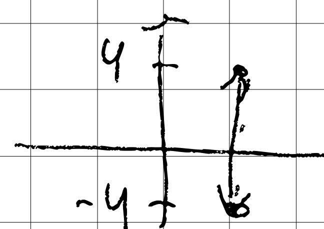
1) $\lim_{x \to \infty} F(x) = 1$
2) $\lim_{x \to -\infty} F(x) = 0$
3) Non decrescente
4) Continua a dx

REMARK FUNZIONI E SEGNI

Data $y = f(x)$

- $-f(x)$ inverte asse x

  $f(x) \to -f(x)$

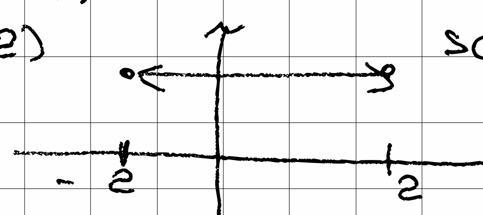


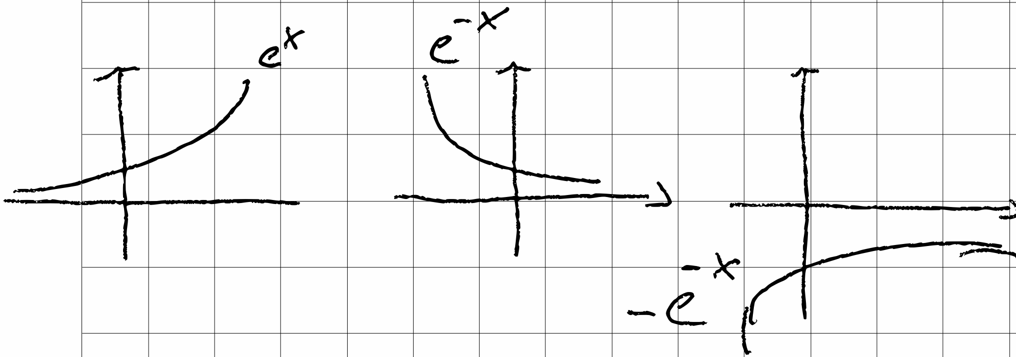
- $f(-x)$ inverte asse y

  $-a \to f(-(-a)) = f(a)$

  $a \to f(-a)$

SCAMBIO



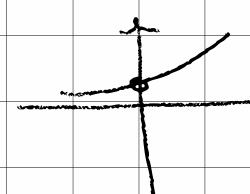


Delle figure deduciamo che

$$\lim_{x\to\infty} -e^{-x} = 0 \quad \& \quad \lim_{x\to -\infty} -e^{-x} = -\infty$$

Abbiamo finito? No

✓ $$\lim_{x\to\infty} e^{-e^{-x}} = e^{0} = 1$$



✓ $$\lim_{x\to -\infty} e^{-e^{-x}} = e^{-\infty} = 0$$

REMARK DERIVATE

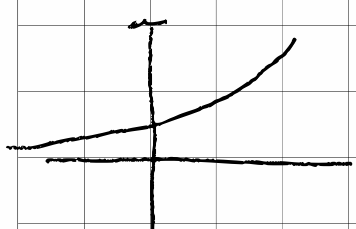
CHAIN RULE

- $[g(f(x))]' = g'(f(x)) \cdot f'(x)$
- $[e^x]' = e^x$

$$[e^{g(f(x))}]' = e^{g[f(x)]} \cdot [g(f(x))]' = e^{g[f(x)]} \cdot g'(f(x)) \cdot f'(x)$$

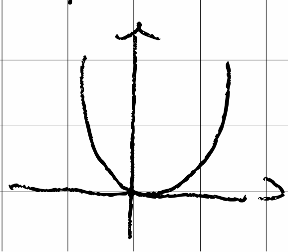
$$[e^{-e^{-x}}]' = e^{-e^{-x}} \cdot [-e^{-x}]' = e^{-e^{-x}} \cdot -e^{-x} \cdot \underbrace{(-x)'}_{=-1} = e^{-e^{-x}} \cdot e^{-x}$$

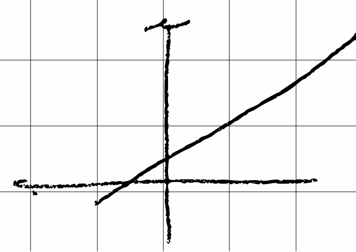
$$= e^{-e^{-x}-x} = e^{y} > 0$$



DERIVATA E' STRETTAMENTE POSITIVA

eg





- **PUNTO 2**

$$P(\lambda M_n - \log(n) \leq x) \overset{?}{=} F\left(\frac{x+\log n}{\lambda}\right)^n$$

$$P(\lambda M_n - \log n \leq x)$$

$$= P\left(M_n \leq \frac{x + \log n}{\lambda}\right)$$ MA M_n è max!

MAX $$= P\left(X_1 \leq \frac{x+\log n}{\lambda}, \dots, X_n \leq \frac{x+\log n}{\lambda}\right)$$

IND $$= P\left(X_1 \leq \frac{x+\log n}{\lambda}\right) \cdots P\left(X_n \leq \frac{x+\log n}{\lambda}\right)$$

$$= \prod \underbrace{P\left(X_i \leq \frac{x+\log n}{\lambda}\right)}_{F_X\left(\frac{x+\log n}{\lambda}\right)}$$ STESSA COSA PER n volte

$$= \prod F\left(\frac{x+\log n}{\lambda}\right)$$

NB1 λ e $\log n$ sono COSTANTI

NB2 Non ci serve conoscere quale sia la $F(x)$

PUNTO 3

$$F\left(\frac{x+\log(n)}{\lambda}\right)^n \overset{?}{=} \left(1-\frac{e^{-x}}{n}\right)^n \mathbb{1}_{[-\log(n),\infty)}x$$

RIP EXP

$$F(x) = (1-e^{-\lambda x})\,\mathbb{1}_{[0,\infty)}(x)\;,\; x \in \mathbb{R}$$

ok, adesso ci serve sapere

$$F(a) = 1-e^{-\lambda a}\,\mathbb{1}_{[0,+\infty)}(a)$$

allo stesso modo

SIMPLIFY λ

$$F\left(\frac{x+\log(n)}{\lambda}\right) = 1-e^{-\lambda\cdot\left(\frac{x+\log(n)}{\lambda}\right)}\,\mathbb{1}_{[0,+\infty)}\left(\frac{x+\log n}{\lambda}\right)$$

NB $$1-e^{-(x+\log n)} = 1-e^{-x}\cdot e^{-\log n} = 1-\frac{1}{n}e^{-x}$$

NB $$e^{\log_e a} = a \qquad \log a = x \;\; \text{lc} \;\; e^x = a$$

$$\lim_{n \to \infty} \left(1 - \frac{2}{n}\right)^n = e^{-2}$$

RIPASSO

DISTR MARGINALE

$$P_A(a) = \sum_{b \in B} P_{A,B}(A = a, B = b)$$

VERIFICARE SE E' F DI RIPARTIZIONE

4 condizioni da verificare

- $\lim_{x \to \infty} F(x) = 1$
- $\lim_{x \to -\infty} F(x) = 0$
-
-