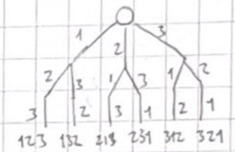
Finite Functions. Permutations S_n

given a bijection — permutation

$$\begin{array}{ccccc} 1 & 2 & 3 & \dots & n \\ \downarrow & \downarrow & \downarrow & & \downarrow \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{array} \rightarrow \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \dots & \sigma(n) \end{pmatrix} \rightarrow$$

Counting Permutations



thus, $|S_n| = n!$

- Order of a permutation σ is smallest positive integer k, such that

$$\sigma^k = \varepsilon, \quad \varepsilon(x) = x$$

- Sign of permutation

$$\mathrm{syn}(\sigma) \begin{cases} +1 & \text{if } \ell \text{ even} \\ -1 & \text{if } \ell \text{ odd} \end{cases}, \quad \ell = \text{n° of "disorders"}$$

Binary Relations:

- 2 variable relations

- Equivalence relations $E(x,y)$, Partitions.
  - "Reflexivity" $\forall x\, E(x,x)$ for any x, where $x == x$
  - "Symmetry" $\forall x,y\, (E(x,y) \rightarrow E(y,x))$
  - "Transitivity"

Ex.

$E_2(x,y) =$ "$x - y$ is even" $\rightarrow$ $[0]_2$ = even $= \{0, 2, -2 \dots 2n, -2n\}$ (representative)

$[k]_2 = \{y \mid E_2(k,y)\}$ $\qquad$ $[1]_2$ = odd $= \{1, 3, -1 \dots 2n+1\}$

bit operations

$0 + 0 = 0$
$1 + 0 = 1$
$1 + 1 = 0$

- Equivalence: $x = y \pmod m$

$E_m(x,y) =$ "$(x-y)$ is divisible by m" $\qquad G_m \{0, 1, 2, \dots, m-2, m-1\}$

$a \pmod m = [a]_m = \{y \mid (y-a) \text{ is divisible by } m\}$

- Congruence

if $a_1 = a_2 \pmod m$ & $b_1 = b_2 \pmod m$, then

$$\begin{cases} a_1 + b_1 = a_2 + b_2 \pmod m \\ a_1 \cdot b_1 = a_2 \cdot b_2 \pmod m \end{cases}, \text{ then } \rightarrow$$

+ (mod 3)	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

x (mod 3)	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$$\begin{cases} 2 + 1 = 3 \pmod 3 = 0 \pmod 3 \\ 2 \cdot 2 = 4 \pmod 3 = 1 \pmod 3 \end{cases}$$

Purpose: Introduce Groups "+", "x"