$A \to B = \neg A \vee B$

Represent $A \wedge B$ with '$\neg$' and '$\to$'

$$A \wedge B = \neg(\neg A \vee \neg B) = \neg(A \to \neg B)$$

Represent $A \leftrightarrow B$ with '$\to$' and '$\neg$'

$$A \leftrightarrow B = \underbrace{(A \to B)}_{X} \wedge \underbrace{(B \to A)}_{Y}$$

$$X \wedge Y$$

$$= \neg(X \to \neg Y)$$

$$= \neg((A \to B) \to \neg(B \to A))$$

DNF : Contains $\wedge$, $\vee$, $\neg$ only $(\cdots \wedge \cdots) \vee (\cdots \wedge \cdots) \vee (\cdots \wedge \cdots)$

$A \cdot B + C \cdot D + E \cdot F$

"sums of product".

A	A is true	A is False
T	T	F
F	F	T

$\therefore$ "A is true" $\models A$, "A is false" $\models \neg A$

$$(\neg P \to \neg Q) \wedge Q = (P \vee \neg Q) \wedge Q = P \wedge Q \vee \neg Q \wedge Q$$
$$= P \wedge Q \vee \text{False} = P \wedge Q$$

CNF : Contains $\wedge$, $\vee$, $\neg$ only $(\cdots \vee \cdots) \wedge (\cdots \vee \cdots) \wedge (\cdot \vee \cdot)$

$(A + B) \cdot (C + D) \cdot (E + F)$

"product of sums"

Set : a collection

let A be a set : $A = \{a_1 \cdots a_n\}$

Define $B = \{b_1, \ldots, b_m\}$, and $\forall b_i \in B$, $b_i \in A$

Then: B is subset of A. $B \subseteq A$.

$A : \{0, 1, 2\}$

Size 0, $\emptyset = \{\}$

Size 1, $\{0\}, \{1\}, \{2\}$

Size 2, $\{0,1\}, \{1,2\}, \{0,2\}$

Size 3, $\{0, 1, 2\}$

Powerset A : the sets of all subsets of A.

$$\{\{\}, \{0\}, \{1\}, \{2\}, \{0,1\}, \{1,2\}, \{0,2\}, \{0,1,2\}\}$$

Boolean Algebra : $(B, 0, 1, +, -)$

① $0 \in B$, $1 \in B$

② $\forall b, b_2 \in B$, $b_1 + b_2 \in B$

③ $\forall b \in B$, $-b \in B$

Example. $(Pow(X), 0, 1, \cup, X\setminus)$

Duality : if $A = B$ is true

then $A^* = B^*$ is true

$A^* :=$ swap + and $\cdot$, swap 0 and 1

Any FOL is $\Sigma = \{F, P\}$

F is a set of functions $\{f_1, f_2 \ldots\}$

If $x_1, x_2, \ldots, x_k$ are terms, then $f_i(x_1, \ldots x_k)$ is a term

P is a set of predicates $\{P_1, P_2 \ldots\}$

$\forall y \ \exists x, \ x = S(y)$, Any natural number has a successor T

$\exists x \ \forall y, \ x = S(y)$ There exists a number which is a successor of all nature number F.