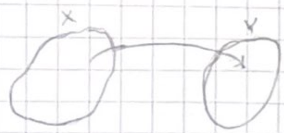
Functions:

$f: X \to Y$



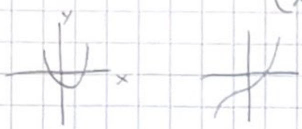
$\text{domain}(f) = \{x \mid f(x) \text{ is defined}\} = X$

$\text{image}(f) = f(X)$

- Injection / <u>Encodings</u>:

  $f: X \to Y$ is an <u>injection</u> if for any x_1 and x_2 from X:

  $$(x_1 \neq x_2) \to (f(x_1) \neq f(x_2))$$



Not injection — Injection

(function with unique x and $f(x)$ values)

- Surjection (a.k.a. <u>onto</u> mapping)

  $f: X \to Y$ is a surjection if $f(X) = Y$ (range = given set)

  $$\forall y \in Y, \exists x \in X, \ f(x) = y$$

- Bijection / 1-1 correspondence

  $f: X \to Y$ is a bijection if f is injective and surjective.

  Inverse bijection $f^{-1}(y) = x$ exists if $f(x) = y$

- Sequential composition of functions

  let: $f: X \to Y$ and $g: Y \to Z$

  $h = f ; g \longrightarrow h(x) = g(f(x))$

  1st apply f, then, apply g

- Composition of injections

  let: $f: X \to Y$ and $g: Y \to Z$ (both injective)

  $h = f ; g$, $h: X \to Z$

  injection from $X \to Z$

- Composition of surjections

  let: $f: X \to Y$ and $g: Y \to Z$ (both surjective)

  $h: X \to Z$

  surjection $X \to Z$

- Composition of bijection

  let: $f: X \to Y$ and $g: Y \to Z$ (both bijective)

  $h: X \to Z$

  bijection $X \to Z$, & exists inverse bijection $h^{-1}: Z \to X$

Cardinality:

- $|X| \leq |Y|$ if there is an injection function $f: X \to Y$
- $|X| = |Y|$ if there exists a bijection / one-one correspondence h between X and Y. $h: X \leftrightarrow Y$
- Discrete Math: $|\mathbb{Q}| = |\mathbb{Z}| = |\mathbb{N}| = \aleph_0$ (countable), since continuum is <u>not</u> countable.