SGN – Assignment #1

Student Name, 123456

1 Periodic orbit

Exercise 1

Consider the 3D Earth–Moon Circular Restricted Three-Body Problem with $\mu = 0.012150$.

1) Find the x-coordinate of the Lagrange point L_1 in the rotating, adimensional reference frame with at least 10-digit accuracy.

Solutions to the 3D CRTBP satisfy the symmetry

$$\mathcal{S} : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t).$$

Thus, a trajectory that crosses perpendicularly the $y = 0$ plane twice is a periodic orbit.

2) Given the initial guess $\mathbf{x}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})$, with

$$\begin{aligned} x_0 &= 0.829380407710981 \\ y_0 &= 0 \\ z_0 &= 0.0901786827424052 \\ v_{x0} &= 0 \\ v_{y0} &= 0.181868545812666 \\ v_{z0} &= 0 \end{aligned}$$

Find the periodic halo orbit that passes through x_0; that is, develop the theoretical framework and implement a differential correction scheme that uses the STM either approximated through finite differences or achieved by integrating the variational equation.

The periodic orbits in the CRTBP exist in families. These can be computed by continuing the orbits along one coordinate, e.g., x_0. This is an iterative process in which one component of the state is varied, while the other components are taken from the solution of the previous iteration.

3) By gradually increasing x_0 and using numerical continuation, compute the families of halo orbits until $x_0 = 0.873$.

(8 points)

Write your answer here

- Develop the exercise in the file `lastname123456_Assign1_Ex1.m`
- Organize the script in sections, one for each point; use local functions if needed.
- Download the PDF from the Main menu.
- Create a single .zip file containing both the report in PDF and the MATLAB files. The name shall be `lastname123456_Assign1.zip`.
- Red text indicates where answers are needed; be sure there is no red stuff in your report.
- In your answers, be concise: to the point.
- **Deadline for the submission: Nov 13 2023, 23:30.**
- **Load the compressed file to the Assignments folder on Webeep.**

2 Impulsive guidance

Exercise 2

The Aphophis close encounter with Earth will occur on April 2029. You shall design a planetary protection guidance solution aimed at reducing the risk of impact with the Earth.

The mission shall be performed with an impactor spacecraft, capable of imparting a $\Delta \mathbf{v} = 0.005\,\mathbf{v}(t_{\text{imp}})$, where $\mathbf{v}$ is the spacecraft velocity and t_{imp} is the impact time. The spacecraft is equipped with a chemical propulsion system that can perform impulsive manoeuvres up to a total Δv of 5 km/s.

The objective of the mission is to maximize the distance from the Earth at the time of the closest approach. The launch shall be performed between 2024-10-01 (LWO, Launch Window Open) and 2025-02-01 (LWC, Launch Window Close), while the impact with Apophis shall occur between 2028-08-01 and 2029-02-28. An additional Deep-Space Maneuver (DSM) can be performed between LWO+6 and LWC+18 months.

1) Analyse the close encounter conditions reading the SPK kernel and plotting in the time window [2029-01-01; 2029-07-31] the following quantities:
   a) The distance between Apophis and the Sun, the Moon and the Earth respectively.
   b) The evolution of the angle Earth-Apophis-Sun
   c) The ground-track of Apophis for a time-window of 12 hours centered around the time of closest approach (TCA).
2) Formalize an unambiguous statement of the problem specifying the considered optimization variables, objective function, the linear and non-linear equality and inequality constraints, starting from the description provided above. Consider a multiple-shooting problem with $N = 3$ points (or equivalently 2 segments) from t_0 to t_{imp}.
3) Solve the problem with multiple shooting. Propagate the dynamics of the spacecraft considering only the gravitational attraction of the Sun; propagate the post-impact orbit of Apophis using a full n-body integrator. Use an event function to stop the integration at TCA to compute the objective function; read the position of the Earth at t_0 and that of Apophis at t_{imp} from the SPK kernels. Provide the optimization solution, that is, the optimized departure date, DSM execution epoch and the corresponding $\Delta \mathbf{v}$'s, the spacecraft impact epoch, and time and Distance of Closest Approach (DCA) in Earth radii. Suggestion: try different initial conditions.

(11 points)

Write your answer here

- Develop the exercise in the file `lastname123456_Assign1_Ex2.m`
- Organize the script in sections, one for each point; use local functions if needed.

Launch	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
DSM	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
Impact	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
TCA	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
$\Delta\mathbf{v}_L$ [km/s]	±0000.0000	±0000.0000	±0000.0000
$\Delta\mathbf{v}_{DSM}$ [km/s]	±0000.0000	±0000.0000	±0000.0000
DCA [Re]	±0000.0000		

Table 1: Guidance solution for the impactor mission.

3 Continuous guidance

Exercise 3

A low-thrust option is being considered for an Earth-Venus transfer*. Provide a *time-optimal* solution under the following assumptions: the spacecraft moves in the heliocentric two-body problem, Venus instantaneous acceleration is determined only by the Sun gravitational attraction, the departure date is fixed, and the spacecraft initial and final states are coincident with those of the Earth and Venus, respectively.

1) Using the PMP, write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\boldsymbol{\lambda}_0, t_f\}$ with the appropriate transversality condition.

2) Adimensionalize the problem using as reference length $LU = 1$ AU† and reference mass $MU = m_0$, imposing that $\mu = 1$. Report all the adimensionalized parameters.

3) Solve the problem considering the following data:

   - Launch date: `2023-05-28-14:13:09.000 UTC`
   - Spacecraft mass: $m_0 = 1000$ kg
   - Electric propulsion properties: $T_{\max} = 800$ mN, $I_{sp} = 3120$ s

   To obtain an initial guess for the costate, generate random numbers such that $\lambda_{0,i} \in [-20; +20]$, while $t_f < 2\pi$. Report the obtained solution in terms of $\{\boldsymbol{\lambda}_0, t_f\}$ and the error with respect to the target. Assess your results exploiting the properties of the Hamiltonian in problems that are not time-dependent and time-optimal solutions.

4) Solve the problem for a lower thrust level $T_{\max} = [500]$ mN. Tip: exploit numerical continuation.

(11 points)

Write your answer here

- Develop the exercise in the file `lastname123456_Assign1_Ex3.m`
- Organize the script in sections, one for each point; use local functions if needed.

Fill the tables with the required results.

$\mathbf{r}_0$	±0000.0000	±0000.0000	±0000.0000
$\mathbf{v}_0$	±0000.0000	±0000.0000	±0000.0000
m_0	0000.0000		
I_{sp}	0000.0000		
$T_{\max}$	0000.0000		
g_0	0000.0000		
GM	0000.0000		

Table 2: Adimensionalized quantities ($T_{\max} = 800$ mN).

*Read the necessary gravitational constants and planets positions from SPICE. Use the kernels provided on WeBeep for this assignment.

†Read the value from SPICE

$\boldsymbol{\lambda}_{0,r}$	±0000.0000	±0000.0000	±0000.0000
$\boldsymbol{\lambda}_{0,v}$	±0000.0000	±0000.0000	±0000.0000
$\lambda_{0,m}$	0000.0000		
t_f	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
TOF [days]	0000.0000		

Table 3: Time-optimal Earth-Venus transfer solution ($T_{\max} = 800$ mN).

$\vert\vert\mathbf{r}_f(t_f) - \mathbf{r}_V(t_f)\vert\vert$	[km]	±00.000
$\vert\vert\mathbf{v}_f(t_f) - \mathbf{v}_V(t_f)\vert\vert$	[m/s]	±00.000

Table 4: Final state error with respect to Venus' center ($T_{\max} = 800$ mN).

$\boldsymbol{\lambda}_{0,r}$	±0000.0000	±0000.0000	±0000.0000
$\boldsymbol{\lambda}_{0,v}$	±0000.0000	±0000.0000	±0000.0000
$\lambda_{0,m}$	0000.0000		
t_f	`YYYY-MM-DD-HH:MM:SS.sss UTC`		
TOF [days]	0000.0000		

Table 5: Time-optimal Earth-Venus transfer solution ($T_{\max} = 500$ mN).

$\vert\vert\mathbf{r}_f(t_f) - \mathbf{r}_V(t_f)\vert\vert$	[km]	±00.000
$\vert\vert\mathbf{v}_f(t_f) - \mathbf{v}_V(t_f)\vert\vert$	[m/s]	±00.000

Table 6: Final state error with respect to Venus' center ($T_{\max} = 500$ mN).