SGN – Assignment #1

Giuseppe Brentino, 221104

1 Periodic orbit

Exercise 1

Consider the 3D Earth–Moon Circular Restricted Three-Body Problem with $\mu = 0.012150$.

1) Find the x-coordinate of the Lagrange point L_1 in the rotating, adimensional reference frame with at least 10-digit accuracy.

Solutions to the 3D CRTBP satisfy the symmetry

$$\mathcal{S} : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t).$$

Thus, a trajectory that crosses perpendicularly the $y = 0$ plane twice is a periodic orbit.

2) Given the initial guess $\mathbf{x}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})$, with

$$\begin{aligned} x_0 &= 1.08892819445324 \\ y_0 &= 0 \\ z_0 &= 0.0591799623455459 \\ v_{x0} &= 0 \\ v_{y0} &= 0.257888699435051 \\ v_{z0} &= 0 \end{aligned}$$

Find the periodic halo orbit that passes through z_0; that is, develop the theoretical framework and implement a differential correction scheme that uses the STM either approximated through finite differences or achieved by integrating the variational equation.

The periodic orbits in the CRTBP exist in families. These can be computed by continuing the orbits along one coordinate, e.g., z_0. This is an iterative process in which one component of the state is varied, while the other components are taken from the solution of the previous iteration.

3) By gradually decreasing z_0 and using numerical continuation, compute the families of halo orbits until $z_0 = 0.034$.

(8 points)

Question 1

The x-coordinate of the Lagrange point L_1 can be computed founding the zeros of the partial derivatives of the scalar potential function U described in Eq. (1), where $r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$ and $r_2 = \sqrt{(x+\mu-1)^2 + y^2 + z^2}$.

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{1}$$

In particular, the libration point L_1 is collinear, so it can be found imposing $y = 0$ and $z = 0$. By doing this, the partial derivatives of U in y and z are equal to zero, and only $\frac{\partial \mathrm{U}}{\partial x}$ expressed in Eq. (2) needs to be investigated.

$$\frac{\partial \mathrm{U}}{\partial x} = x - \frac{(1-\mu)(x+\mu)}{[(x+\mu)^2 + y^2 + z^2]^{3/2}} - \frac{\mu(x+\mu-1)}{[(x+\mu-1)^2 + y^2 + z^2]^{3/2}} \tag{2}$$

As can be shown in Fig. 1, Eq. (2) has three zeros. The one corresponding to L_1 is the one in the middle, which has been computed using the *fsolve* Matlab function. To guarantee convergence to the correct point with 10-digit accuracy, an initial guess between 0 and ~ 0.98 needs to be provided, and the *fsolve* option "OptimalityTolerance" needs to be set equal or smaller than 10^{-10}. An initial guess of 0 and an optimality tolerance value of 10^{-10} have been used, and the x-coordinate of L_1 is found to be 0.8369180073.

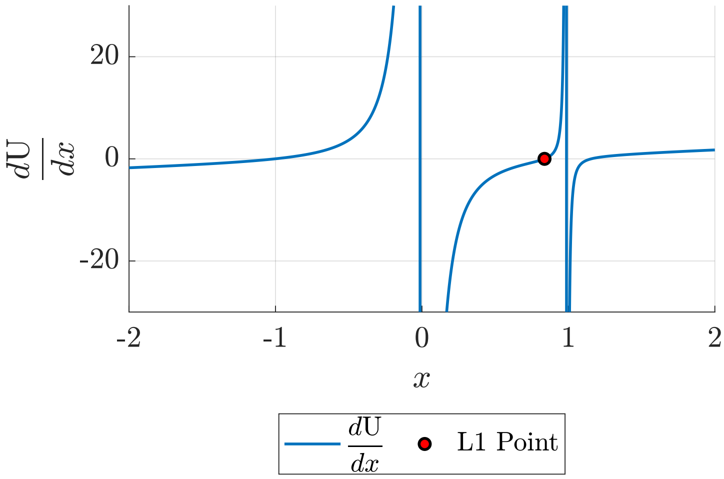

Figure 1: $\dfrac{\partial \mathrm{U}}{\partial x}$ and L_1 point location

Question 2

To find the correct initial state that leads to a periodic halo orbit passing through z_0, a differential correction scheme that changes the values of x_0 and v_y while keeping v_{x0} and v_{z0} equal to zero at the $y = 0$ plane crossing point has been implemented. This is because, as stated in the text in order for the orbit to be periodic, it needs to cross the $y = 0$ plane perpendicularly, i.e. with a velocity parallel to the y axis.

The orbit is propagated starting from the initial condition up to the $y = 0$ point using an event function that stops the integration when $y = 0$. The equations used to propagate the states are the ones of the Circular Restricted Three Body Problem (Eq. (3)).

$$\begin{cases} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{z} &= v_z \\ \dot{v}_x &= 2v_y + x - \dfrac{(1-\mu)(x+\mu)}{[(x+\mu)^2+y^2+z^2]^{3/2}} - \dfrac{\mu(x+\mu-1)}{[(x+\mu-1)^2+y^2+z^2]^{3/2}} \\ \dot{v}_y &= -2v_x + y - \dfrac{1-\mu}{[(x+\mu)^2+y^2+z^2]^{3/2}}y - \dfrac{\mu}{[(x+\mu-1)^2+y^2+z^2]^{3/2}}y \\ \dot{v}_z &= -\dfrac{1-\mu}{[(x+\mu)^2+y^2+z^2]^{3/2}}z - \dfrac{\mu}{[(x+\mu-1)^2+y^2+z^2]^{3/2}}z \end{cases} \tag{3}$$

Implementing the Newton method of Eq. (4), where $\mathbf{x}$ is the vector of unknowns x_0 and v_y, $\mathbf{f}$ is the flow of the ODE at final time $\varphi(x_0, t_0, t_f)$ and $\mathbf{f}'$ is its Jacobian, the state transition matrix Φ.

$$\mathbf{x_{k+1}} = \mathbf{x_k} - \mathbf{f}'(\mathbf{x_k})^{-1}\mathbf{f}(\mathbf{x_k}) \tag{4}$$

It is possible to rewrite the correction therm of Eq. (4) as follows, where δ represent the variation from the nominal state and the 0,f subscript indicate the initial and final state respectively:

$$\begin{pmatrix} \delta x_0 \\ \delta y_0 \\ \delta z_0 \\ \delta v_{x0} \\ \delta v_{y0} \\ \delta v_{z0} \end{pmatrix} = \begin{bmatrix} \dfrac{\partial \varphi_x}{\partial x_0} & \cdots & \dfrac{\partial \varphi_x}{\partial v_{z0}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \varphi_{v_z}}{\partial x_0} & \cdots & \dfrac{\partial \varphi_{v_z}}{\partial v_{z0}} \end{bmatrix}^{-1} \begin{pmatrix} \delta x_f \\ \delta y_f \\ \delta z_f \\ \delta v_{xf} \\ \delta v_{yf} \\ \delta v_{zf} \end{pmatrix} \tag{5}$$

Since the ideal v_{xf} and v_{zf} are equal to zero, their variation is equal to the flows φ_{v_x},φ_{v_z}. These variables are the only ones to be constrained, so it is possible to simplify the equation considering only the rows of Φ corresponding to such variables. Moreover the problem imposes that the variation of the initial states are all equal to zeros except for x and v_y, so only the columns of Φ corresponding to these two variables needs to be used.

The final equation used to compute the correct values of the initial state is then:

$$\begin{pmatrix} x_0 \\ v_{y0} \end{pmatrix}_{k+1} = \begin{pmatrix} x_0 \\ v_{y0} \end{pmatrix}_{k} - \begin{bmatrix} \Phi_{41} & \Phi_{45} \\ \Phi_{61} & \Phi_{65} \end{bmatrix}^{-1} \begin{pmatrix} \varphi_x \\ \varphi_{v_y} \end{pmatrix} \tag{6}$$

To allow faster and more accurate computations, instead of inverting the matrix, the *mldivide* Matlab operator have been used, which solves the linear system $\boldsymbol{\Phi}\boldsymbol{\delta}\mathbf{x_0} = \boldsymbol{\varphi}$.
The iterations stop when the computed v_{xf}, v_{zf} are both less then 10^{-14}.

In order to compute the state transition matrix, the variational approach has been used, since it guarantee a more accurate STM compared to the finite difference method. To do so, the STM dynamics have been propagated solving the following system of ODE:

$$\begin{cases} \dot{\boldsymbol{\Phi}} & = \dfrac{\partial \mathbf{f}}{\partial \mathbf{x}}(t)\boldsymbol{\Phi} \\ \boldsymbol{\Phi_0} & = \boldsymbol{\varphi}(\mathbf{x_0}, t_0, t_0) = \mathbf{I}_{6\times 6} \end{cases} \tag{7}$$

where $\mathbf{f}$ is the the right-hand side of Eq. (3).

The resulting orbit, which initial states are reported in Table 1, in the rotating, adimentionalized reference frame is depicted in Fig. 2.

x_0	1.090278054655005
y_0	0
z_0	0.059179962345546
v_{x0}	0
v_{y0}	0.260349385022455
v_{z0}	0

Table 1: Initial conditions for the required periodic orbit

Question 3

To guarantee fast convergence of the algorithm described hereabove, numerical continuation have been exploited. In particular, a family of five orbits, with equally spaced z_0 between 0.0591799623455459 and 0.034. Starting from the initial guess provided in question 2, the first correct inital state is computed. Then it's solution is used as an initial guess for the following orbit and so on untill the correct initial state for the periodic halo orbit passing through $z_0 = 0.034$ is found.

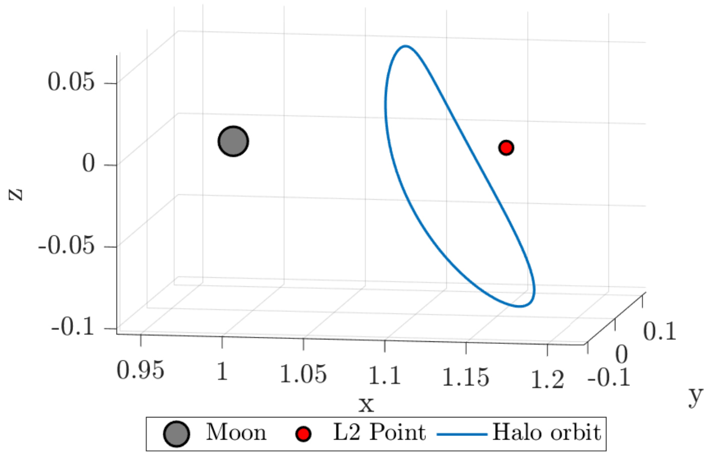

Figure 2: Periodic halo orbit passing through z_0

The resulting orbit, which initial states are reported in Table 2, in the rotating, adimentionalized reference frame is depicted in Fig. 3.

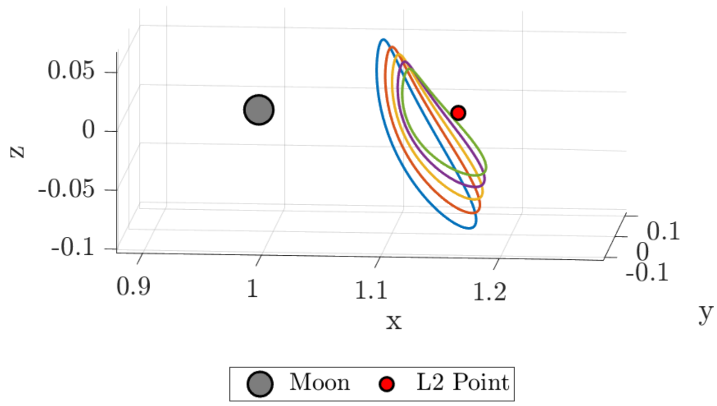

Figure 3: Family of halo orbits

x_0	1.111669365809449
y_0	0
z_0	0.034
v_{x0}	0
v_{y0}	0.201018707751582
v_{z0}	0

Table 2: Initial conditions for the required periodic orbit

2 Impulsive guidance

Exercise 2

The Aphophis close encounter with Earth will occur on April 2029. You shall design a planetary protection guidance solution aimed at reducing the risk of impact with the Earth.

The mission shall be performed with an impactor spacecraft, capable of imparting a $\Delta\mathbf{v} = 0.00005\,\mathbf{v}(t_{\text{imp}})$, where $\mathbf{v}$ is the spacecraft velocity and t_{imp} is the impact time. The spacecraft is equipped with a chemical propulsion system that can perform impulsive manoeuvres up to a total Δv of 5 km/s.

The objective of the mission is to maximize the distance from the Earth at the time of the closest approach. The launch shall be performed between 2024-10-01 (LWO, Launch Window Open) and 2025-02-01 (LWC, Launch Window Close), while the impact with Apophis shall occur between 2028-08-01 and 2029-02-28. An additional Deep-Space Maneuver (DSM) can be performed between LWO+6 and LWC+18 months.

1) Analyse the close encounter conditions reading the SPK kernel and plotting in the time window [2029-01-01; 2029-07-31] the following quantities:
    a) The distance between Apophis and the Sun, the Moon and the Earth respectively.
    b) The evolution of the angle Earth-Apophis-Sun
    c) The ground-track of Apophis for a time-window of 12 hours centered around the time of closest approach (TCA).
2) Formalize an unambiguous statement of the problem specifying the considered optimization variables, objective function, the linear and non-linear equality and inequality constraints, starting from the description provided above. Consider a multiple-shooting problem with $N = 3$ points (or equivalently 2 segments) from t_0 to t_{imp}.
3) Solve the problem with multiple shooting. Propagate the dynamics of the spacecraft considering only the gravitational attraction of the Sun; propagate the post-impact orbit of Apophis using a full n-body integrator. Use an event function to stop the integration at TCA to compute the objective function; read the position of the Earth at t_0 and that of Apophis at t_{imp} from the SPK kernels. Provide the optimization solution, that is, the optimized departure date, DSM execution epoch and the corresponding $\Delta\mathbf{v}$'s, the spacecraft impact epoch, and time and Distance of Closest Approach (DCA) in Earth radii. Suggestion: try different initial conditions.

(11 points)

Question 1

a) Using the kernels provided, the relative position, the relative position between Apophis and the other celestial bodies is quickly computed using the *cspice_ spkpos* function. All the vectors are computed in the same reference frame, the Earth body-fixed rotating frame.
   Fig. 4 shows the norm of the distances over time.

b) The Earth-Apophis-Sun angle is computed as the arc cosine of the dot product between the Earth-Apophis and Sun-Apophis normalized vectors computed before.
   Fig. 5 shows the norm of the distances over time.

c) To compute the groundtrack, the Apophis position vector is first computed in the Earth body-fixed rotating frame. Then the latitude and longitude value are computed using the *cspice_ recgeo* function. The groundtrack is shown in Fig. 6.

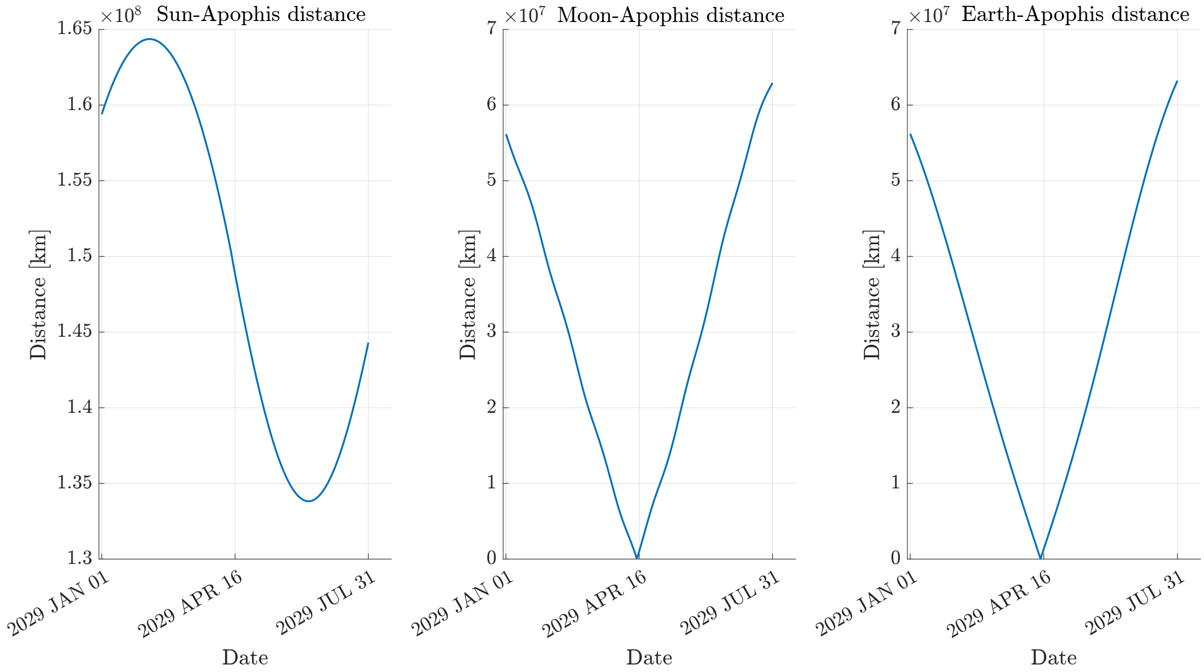

Figure 4: Distance between Apophis and the Sun, the Moon and the Earth

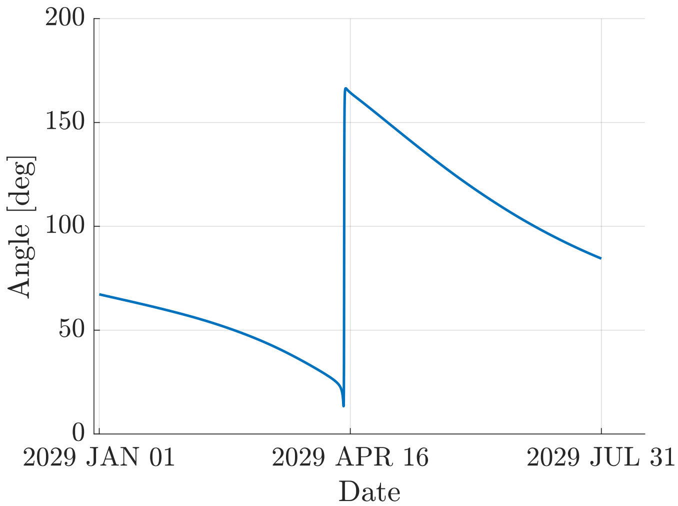

Figure 5: Earth-Apophis-Sun angle

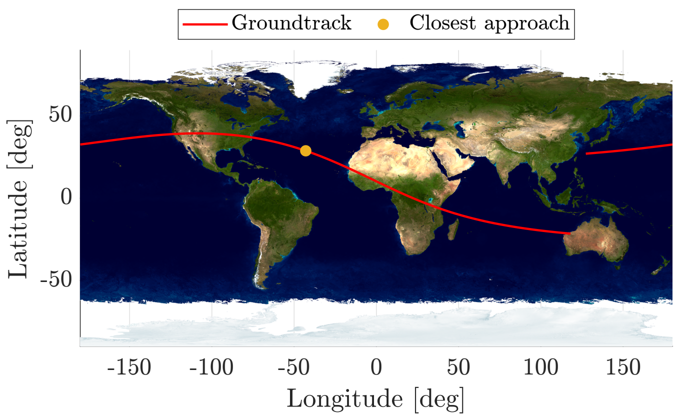

Figure 6: Groundtrack at closest approach ±6 hours.

Question 2

The objective of the problem is to find the maneuvers and their epochs required to maximize the closest approach distance of Apophis with respect to the Earth using the multiple-shooting method. The general impulsive optimization problem requires the solution a NLP as formulated in Eq. (8), where $\mathbf{y}$ is the vector of optimization variables, $\mathbf{g}$ is the set of equality constraints and $\mathbf{h}$ is the vector of inequality constraints.

$$\min_{\mathbf{y}} J \;\; s.t. \;\; \begin{cases} \mathbf{h}(\mathbf{y}) \leq \mathbf{0} \\ \mathbf{g}(\mathbf{y}) = \mathbf{0} \end{cases} \tag{8}$$

$\mathbf{y}$ is a vector of 21 optimization variables: position and velocity of the spacecraft at the initial ($\mathbf{y_1}$), deep space maneuver ($\mathbf{y_2}$) and impact ($\mathbf{y_3}$) time. These three time instants are also the last three optimization variables, since they are not fixed a priori.
$J = -\text{DCA}$ is the cost function to minimize, where DCA is the distance of closest approach, computed propagating the asteroid dynamics until it reaches the minimum distance from the Earth.
Let $\boldsymbol{\psi_0}(t_0)$, $\boldsymbol{\psi_f}(t_{imp})$ be the boundary conditions of the problem, i.e. the position of the Earth at departure and of Apophis at impact respectively. Let $\boldsymbol{\varphi_1}(\mathbf{y_1}, t_0, t_{dsm})$ as the flow of the spacecraft dynamics from the initial time to the time of the second maneuver and $\boldsymbol{\varphi_2}(\mathbf{y_2}, t_{dsm}, t_{imp})$ as the flow of the spacecraft dynamics from the deep space maneuver to impact. Then the equality constraints vector $\mathbf{g(y)}$ is defined as follows:

$$\mathbf{g}(\mathbf{y}) = \begin{cases} \mathbf{y_{1p}} - \boldsymbol{\psi_{0p}} \\ \boldsymbol{\varphi_{1p}} - \mathbf{y_{2p}} \\ \boldsymbol{\varphi_2} - \mathbf{y_3} \\ \mathbf{y_{3p}} - \boldsymbol{\psi_{fp}} \end{cases} \tag{9}$$

The non linear constraint are the ones related to the maximum Δv and the time windows for launch, deep space maneuver and impact. Defining IMPO, IMPC as the opening and closing epochs for impact and DSMO, DSMC as the opening and closing epochs between which the deep space maneuver shall be executed, $\mathbf{h}(\mathbf{y})$ is defined as follows:

$$h(\mathbf{y}) = \begin{cases} \|\mathbf{y_{1v}} - \boldsymbol{\psi_{0v}}(t_0)\| + \|\boldsymbol{\varphi_{1v}} - \boldsymbol{y_{2v}}\| - 5 \\ t_0 - \text{LWC} \\ \text{LWO} - t_0 \\ t_{dsm} - \text{DSMC} \\ \text{DSMO} - t_{dsm} \\ t_{imp} - \text{IMPC} \\ \text{IMPO} - t_{imp} \end{cases} \tag{10}$$

In both Eq. (9) and Eq. (10), the subscripts p and v indicate the position and velocity part of the state vector respectively.

Question 3

To solve the problem the initial guess of the optimization variable must be provided.
For the three epochs, a good guess that was found to guarantee relatively fast convergence was chosen as the mean value between the opening and closing of the windows, minus a constant term defined as $0.4(\text{LWC} - \text{LWO})$.
The initial guess of $\mathbf{y_1}$ is chosen to be equal to the state vector of the Earth at the chosen initial time; the first guesses of $\mathbf{y_2}$ and $\mathbf{y_3}$ are computed propagating the spacecraft dynamics up to

the maneuver and impact times respectively.
To reduce the search space, the lower and upper bounds on $\mathbf{y_1}$, $\mathbf{y_2}$, $\mathbf{y_3}$ are set to be equal to the minimum and maximum values of position and velocity of the Earth and Apophis during the whole transfer time. Moreover, an offset of ±1000km on the position and ±10km/s has been added.

In order to reduce the computational time, *fmincon* is allowed to compute the gradients in parallel taking advantage of the multi-core configuration of the CPU. It was not possible to provide analytical gradients because the linearity assumption made to compute the STM is not valid over such long propagation times.

Table 3 summarizes the results obtained solving the NLP. In Fig. 7, the distance between the Earth and the asteroid is shown. The nominal distance is in blue, while the red dashed line shows the distance after the impact.

Launch	2024 OCT 08 00:23:55.1 UTC		
DSM	2025 SEP 29 20:44:32.2 UTC		
Impact	2028 OCT 31 09:44:14.8 UTC		
TCA	2029 APR 13 23:10:32.6 UTC		
$\Delta \mathbf{v}_L$ [km/s]	−1.2861	−0.7353	0.2505
$\Delta \mathbf{v}_{DSM}$ [km/s]	−2.5635	−1.0501	−2.1350
DCA [Re]	14.6456		

Table 3: Guidance solution for the impactor mission.

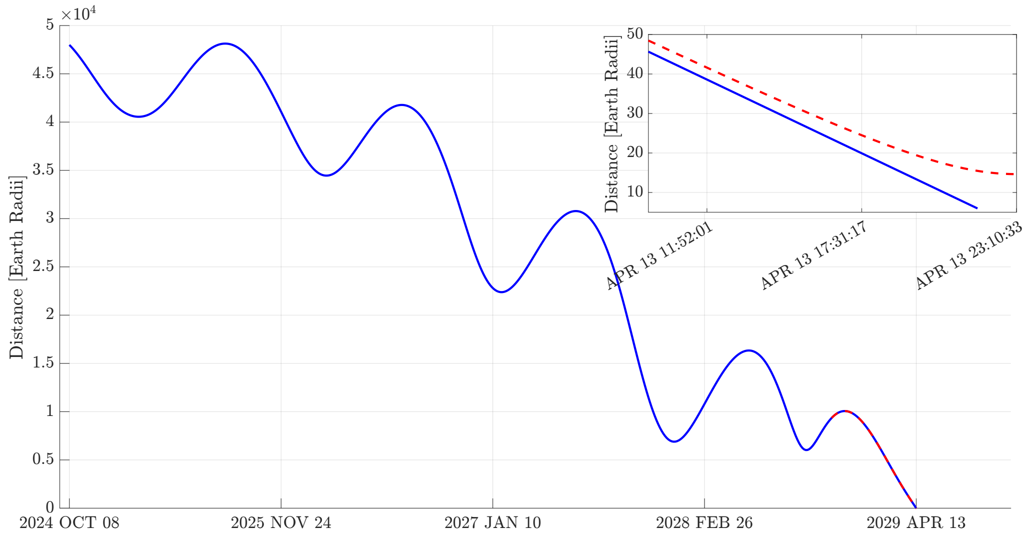

Figure 7: Apophis distance from Earth, in Earth radii.

3 Continuous guidance

Exercise 3

A low-thrust option is being considered for an Earth-Venus transfer*. Provide a *time-optimal* solution under the following assumptions: the spacecraft moves in the heliocentric two-body problem, Venus instantaneous acceleration is determined only by the Sun gravitational attraction, the departure date is fixed, and the spacecraft initial and final states are coincident with those of the Earth and Venus, respectively.

1) Using the PMP, write down the spacecraft equations of motion, the costate dynamics, and the zero-finding problem for the unknowns $\{\boldsymbol{\lambda}_0, t_f\}$ with the appropriate transversality condition.

2) Adimensionalize the problem using as reference length $LU = 1$ AU† and reference mass $MU = m_0$, imposing that $\mu = 1$. Report all the adimensionalized parameters.

3) Solve the problem considering the following data:

   – Launch date: `2023-05-28-14:13:09.000 UTC`
   – Spacecraft mass: $m_0 = 1000$ kg
   – Electric propulsion properties: $T_{\max} = 800$ mN, $I_{sp} = 3120$ s

   To obtain an initial guess for the costate, generate random numbers such that $\lambda_{0,i} \in [-20; +20]$, while $t_f < 2\pi$. Report the obtained solution in terms of $\{\boldsymbol{\lambda}_0, t_f\}$ and the error with respect to the target. Assess your results exploiting the properties of the Hamiltonian in problems that are not time-dependent and time-optimal solutions.

4) Solve the problem for a lower thrust level $T_{\max} = [500]$ mN. Tip: exploit numerical continuation.

(11 points)

Question 1

The time-optimal problem for a spacecraft moving in the two-body problem that needs to match the position and velocity of a target body at an undetermined final time t_f can be formalized as follows:

$$\min_{(\hat{\boldsymbol{\alpha}}, u)\in\Omega} \int_{t_0}^{t_f} 1\,\mathrm{d}t \;\; s.t. \begin{cases} \dot{\mathbf{x}} & = \mathbf{f}(\mathbf{x}, \hat{\boldsymbol{\alpha}}, t) \\ \mathbf{x}(t_0) & = \mathbf{x_0} \\ \mathbf{r}(t_f) & = \mathbf{r_f} \\ \mathbf{v}(t_f) & = \mathbf{v_f} \\ \lambda_m(t_f) & = 0 \end{cases} \tag{11}$$

Ω is the set of admissible control actions, where $\hat{\alpha}$ is the thrust direction unit vector and u is the throttle factor:

$$\Omega = \{(u,\ \hat{\boldsymbol{\alpha}})\ :\ u \in [0, 1],\ \|\hat{\boldsymbol{\alpha}}\| = 1\}$$

*Read the necessary gravitational constants and planets positions from SPICE. Use the kernels provided on WeBeep for this assignment.

†Read the value from SPICE

The dynamics of a spacecraft moving in the two-body problem while generating thrust with its engines can be described by Eq. (12).

$$\begin{cases} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= -\dfrac{\mu}{r^3}\mathbf{r} + u\dfrac{T_{max}}{m}\hat{\boldsymbol{\alpha}} \\ \dot{m} &= -u\dfrac{T_{max}}{I_{sp}g_0} \end{cases} \tag{12}$$

Defining the Hamiltonian H as $\mathrm{H} = 1 + \boldsymbol{\lambda} \cdot \mathbf{f}$, the costate dynamics can be described ad the partial derivative of H with respect to the state vector $\mathbf{x}$:

$$\begin{cases} \dot{\boldsymbol{\lambda}}_{\boldsymbol{r}} &= -\dfrac{3\mu}{r^5}(\mathbf{r} \cdot \boldsymbol{\lambda}_v)\mathbf{r} + \dfrac{\mu}{r^3}\boldsymbol{\lambda}_{\boldsymbol{v}} \\ \dot{\boldsymbol{\lambda}}_{\boldsymbol{v}} &= -\boldsymbol{\lambda}_{\boldsymbol{r}} \\ \dot{\lambda}_m &= -u\dfrac{T_{max}}{m^2}\boldsymbol{\lambda}_v \cdot \hat{\boldsymbol{\alpha}} \end{cases} \tag{13}$$

Since the control action is constrained, the following transversality condition also needs to be satisfied:

$$\mathrm{H}(t_f) - [\boldsymbol{\lambda}_r(t_f) \cdot \dot{\boldsymbol{\psi}}_r(t_f) + \boldsymbol{\lambda}_v(t_f) \cdot \dot{\boldsymbol{\psi}}_v(t_f)] = 0$$

where $\boldsymbol{\psi}_r$ $\boldsymbol{\psi}_v$ are the position and velocity vector of Venus, since these are the six constraints on the final states of the spacecraft.
According to the Pointryagin Maximum Principle the optimal u, $\hat{\boldsymbol{\alpha}}$ are the ones that minimize H. In this particular case:

$$\begin{cases} u^* &= 1 \\ \hat{\boldsymbol{\alpha}}^* &= \dfrac{\boldsymbol{\lambda}_v}{\lambda_v} \end{cases}$$

In order to compute the optimal thrust direction, the TPBVP of Eq. (12) and Eq. (13) needs to be solved simultaneously. To do so, the final time and the initial costates needs to be computed first. After providing an initial guess of the variables of interest, the zero-finding problem described in algorithm 1 is solved using *fsolve*, where Fun is the function to analyze. A new guess of the sought after variables is generated in case the algorithm did not converge.

Algorithm 1

$\boldsymbol{\lambda}_0 \leftarrow$ guess$(1:7)$
$t_f \leftarrow$ guess(8)
$[\mathbf{x}_f; \boldsymbol{\lambda}_f] \leftarrow$ ODE propagation
$[\mathbf{V}_r; \mathbf{V}_v] \leftarrow$ *cspice_spkezr*(t_f)
$\mathrm{H} \leftarrow 1 + \boldsymbol{\lambda}_f \cdot \mathbf{f}(tf)$
$\dot{\boldsymbol{\psi}}_{\boldsymbol{r}} \leftarrow \mathbf{V}_v$
$\dot{\boldsymbol{\psi}}_{\boldsymbol{v}} \leftarrow -\dfrac{\mu}{V_r^3}\mathbf{V}_r$
$\mathrm{Fun}(1:3) \leftarrow \mathbf{r}_f - \mathbf{V}_r$
$\mathrm{Fun}(4:6) \leftarrow \mathbf{v}_f - \mathbf{V}_v$
$\mathrm{Fun}(7) \leftarrow \lambda_{mf}$
$\mathrm{Fun}(8) \leftarrow \mathrm{H} - [\boldsymbol{\lambda}_{rf} \cdot \dot{\boldsymbol{\psi}}_r + \boldsymbol{\lambda}_{vf} \cdot \dot{\boldsymbol{\psi}}_v]$

Question 2

Besides the unit length and unit mass provided by the text, in order to impose $\mu = 1$, the new time unit needs to be computed. From the unit of measurement of μ, $[L]^3[T]^{-2}$, the new time unit can be quickly computed as $\mathrm{TU} = \sqrt{\dfrac{\mathrm{LU}^3}{\mu}}$, with LU expressed in km and μ in km^3/s^2. The values of the adimentionalized quantities are reported in Table 4.

$\mathbf{r}_0$	-0.4009	-0.9306	0.00005
$\mathbf{v}_0$	0.9023	-0.3992	0.00004
m_0	1		
I_{sp}	$6.2119 \cdot 10^{-4}$		
$T_{\max}$	0.1349		
g_0	1654.28		
GM	1		

Table 4: Adimensionalized quantities ($T_{\max} = 800$ mN).

Question 3

Solving the problem described above, the initial lambdas and time of flight are summarized in Table 5. As initial guess, random values for $\boldsymbol{\lambda}_{0,r}$, $\boldsymbol{\lambda}_{0,v}$ are chosen to be between ± 20 and the time of flight is randomly generated to be between 0 and 2π. Since for this type of problem λ_m is always positive, the initial guess is generated as a random number between 0 and 20.

$\boldsymbol{\lambda}_{0,r}$	0.4978	-13.819	0.1082
$\boldsymbol{\lambda}_{0,v}$	5.8136	-10.4004	1.4851
$\lambda_{0,m}$	1.7514		
t_f	`2023 OCT 17 04:05:14.660 UTC`		
TOF [days]	141.5778		

Table 5: Time-optimal Earth-Venus transfer solution ($T_{\max} = 800$ mN).

The final errors in position and velocity between the spacecraft and Venus are shown in Table 6; Fig. 8 shows the orbits of Venus and the spacecraft during the transfer.

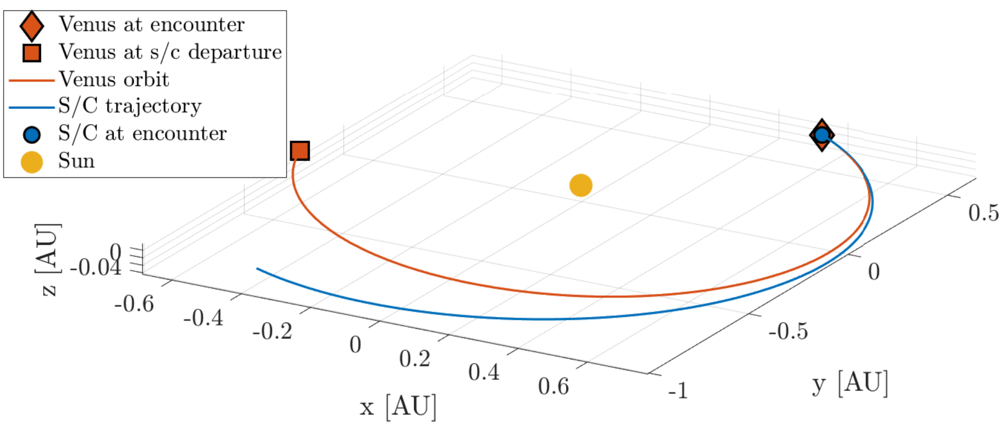

Figure 8: Spacecraft and Venus orbit during the transfer

In problems that are not time-dependent and time-optimal solutions the Hamiltonian is constant. To check that the computations are performed correctly, the Hamiltonian at t_0 is

$\|\|\mathbf{r}_f(t_f) - \mathbf{r}_V(t_f)\|\|$	[km]	0.017
$\|\|\mathbf{v}_f(t_f) - \mathbf{v}_V(t_f)\|\|$	[m/s]	$2.6 \cdot 10^{-6}$

Table 6: Final state error with respect to Venus' center ($T_{\max} = 800$ mN).

compared with the values computed during the whole integration. As shown in Fig. 9, the relative error between the initial Hamiltonian and the values computed at each time step is in the order of 10^{-12}, which is caused by numerical approximations.

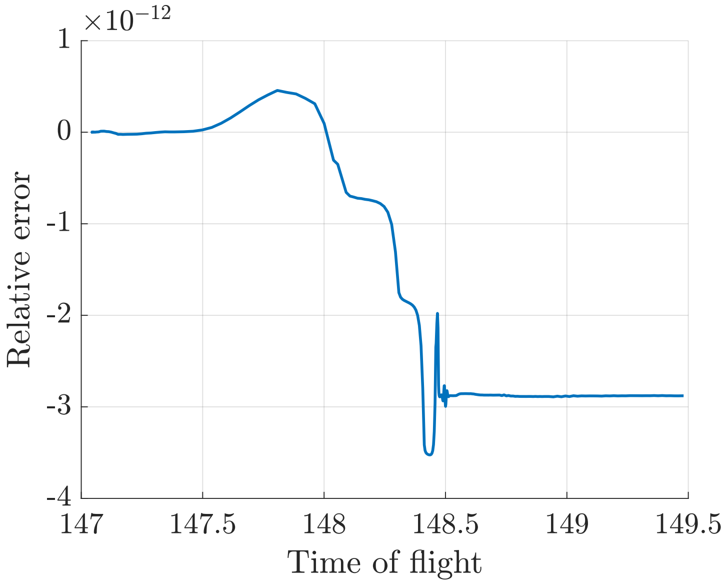

Figure 9: Relative error of the Hamiltonian

Question 4

To guarantee fast convergence also for the 500mN thrust case, numerical continuation has been exploited. Starting from the initial guess found for the 800mN case, the new initial conditions are computed for a thrust of 700mN. The solution found is used as initial guess for the next iteration, decreasing the maximum thrust value. The process is repeated 5 times, with maximum thrust values of 700, 650, 600, 550 and 500 mN.
Results are summarized in Table 7 and Table 8; Fig. 10 shows the orbits of Venus and the spacecraft during the transfer.

$\boldsymbol{\lambda}_{0,r}$	-0.5635	-24.5358	-0.4751
$\boldsymbol{\lambda}_{0,v}$	14.1809	-18.9084	1.9970
$\lambda_{0,m}$	2.7281		
t_f	`2024 JAN 01 04:20:47.412 UTC`		
TOF [days]	217.5886		

Table 7: Time-optimal Earth-Venus transfer solution ($T_{\max} = 500$ mN).

$\|\|\mathbf{r}_f(t_f) - \mathbf{r}_V(t_f)\|\|$	[km]	0.01
$\|\|\mathbf{v}_f(t_f) - \mathbf{v}_V(t_f)\|\|$	[m/s]	$2.3 \cdot 10^{-6}$

Table 8: Final state error with respect to Venus' center ($T_{\max} = 500$ mN).

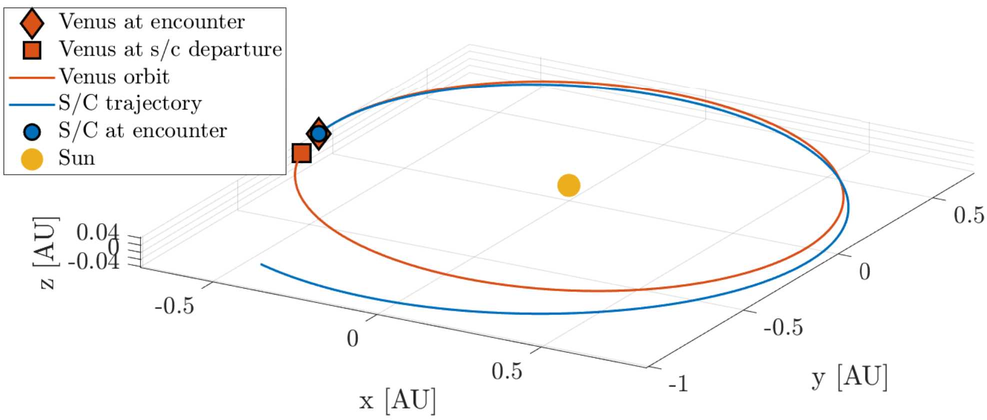

Figure 10: Spacecraft and Venus orbit during the transfer