2. LENGUAJES REGULARES

2.1. EXPRESIONES REGULARES (ER)

Notación

r, s, t

Definición

Tabla 2.1. Expresiones Regulares.

Expresión Regular	Lenguaje Representado
$\varnothing$	$L(\varnothing) = \varnothing$
ε	$L(\varepsilon) = \{\varepsilon\}$
$\sigma \in \Sigma$	$L(\sigma) = \{\sigma\}$
$r + s$	$L(r + s) = L(r) \cup L(s)$
rs	$L(rs) = L(r)L(s)$
r^*	$L(r^*) = L(r)^*$
(r)	$L((r)) = L(r)$

Ejemplos:
1. 00
2. $(0 + 1)^*011$
3. $(0 + 1)^*00(0 + 1)^*$

Precedencia

1. Clausura
2. Concatenación
3. Unión

Ejemplo:
$((0(1^*)) + 0) = 01^* + 0$

Ejemplos

1. $\varnothing$
2. ε
3. 0
4. 1
5. 0^*
6. 01
7. $0 + 1$
8. $(0 + 1)^*$
9. $(0 + 1)(0 + 1)^*$
10. $(0 + 1)(0 + 1)$
11. $01(0 + 1)^*$
12. $(0 + 1)^*01$
13. $(0 + 1)^*01(0 + 1)^*$

Ejercicios resueltos

Considere el alfabeto $\Sigma = \{a, b, c\}$:

1. El lenguaje de las palabras que contienen la subpalabra abc.

   $(a + b + c)^*abc(a + b + c)^*$

2. El lenguaje de las palabras de longitud múltiplo de tres.

   $((a + b + c)(a + b + c)(a + b + c))^*$

3. El lenguaje de las palabras que tienen un número par de a.

   $(b + c)^* + ((b + c)^*a(b + c)^*a(b + c)^*)^*$

Equivalencia

"Dado un lenguaje regular, siempre existen infinitas expresiones regulares que lo describen" (Cases, 2002, p. 132).

$L(r) = L(s) \Rightarrow r \equiv s$[1]

Ejemplo:
$(b + c)^*(a(b + c)^*a(b + c)^*)^* = (b + c + a(b + c)^*a)^*$

Propiedades

1. $r + s = s + r$
2. $r + \varnothing = r = \varnothing + r$
3. $r + r = r$
4. $(r + s) + t = r + (s + t)$
5. $r\varepsilon = \varepsilon r = r$
6. $r\varnothing = \varnothing r = \varnothing$
7. $(rs)t = r(st)$
8. $r(s + t) = rs + rt$
9. $(r + s)t = rt + st$
10. $r^* = r^{**} = r^*r^* = (\varepsilon + r)^* = r^*(r + \varepsilon) = (r + \varepsilon)r^* = \varepsilon + rr^*$
11. $(r + s)^* = (r^* + s^*)^* = (r^*s^*)^* = (r^*s)^*r^* = r^*(sr^*)^*$
12. $r(sr)^* = (rs)^*r$
13. $(r^*s)^* = \varepsilon + (r + s)^*s$
14. $(rs^*)^* = \varepsilon + r(r + s)^*$
15. $s(r + \varepsilon)^*(r + \varepsilon) + s = sr^*$
16. $rr^* = r^*r$

[1] Se escribe r = s.

2.1.1. SIMPLIFICACIÓN

Ejemplos:

1. $\varepsilon + \varepsilon(\varepsilon)^*\varepsilon = \varepsilon + \varepsilon = \varepsilon$
2. $0 + \varepsilon(\varepsilon)^*0 = 0 + 0 = 0$
3. $1 + \varepsilon(\varepsilon)^*1 = 1 + 1 = 1$
4. $0 + 0(\varepsilon)^*\varepsilon = 0 + 0 = 0$
5. $\varepsilon + 0(\varepsilon)^*0 = \varepsilon + 00$
6. $1 + 0(\varepsilon)^*1 = 1 + 01 = (\varepsilon + 0)1$
7. $\varnothing + \varnothing(\varepsilon)^*\varepsilon = \varnothing + \varnothing = \varnothing$
8. $0 + 1 + \varnothing(\varepsilon)^*0 = 0 + 1 + \varnothing = 0 + 1$
9. $\varepsilon + \varnothing(\varepsilon)^*1 = \varepsilon + \varnothing = \varepsilon$

Ejercicios:

1. $\varepsilon + 0(\varepsilon + 00)^*0$
2. $0 + 0(\varepsilon + 00)^*(\varepsilon + 00)$
3. $1 + 0(\varepsilon + 00)^*(\varepsilon + 0)1$
4. $0 + (\varepsilon + 00)(\varepsilon + 00)^*0$
5. $\varepsilon + 00 + (\varepsilon + 00)(\varepsilon + 00)^*(\varepsilon + 00)$
6. $(\varepsilon + 0)1 + (\varepsilon + 00)(\varepsilon + 00)^*(\varepsilon + 0)1$
7. $\varnothing + (0 + 1)(\varepsilon + 00)^*0$
8. $0 + 1 + (0 + 1)(\varepsilon + 00)^*(\varepsilon + 00)$
9. $\varepsilon + (0 + 1)(\varepsilon + 00)^*(\varepsilon + 0)1$
10. $0(00)^* + 0^*1(\varepsilon + (0 + 1)0^*1)^*(0 + 1)(00)^*$
11. $0^*1 + 0^*1(\varepsilon + (0 + 1)0^*1)^*(\varepsilon + (0 + 1)0^*1)$
12. $1 + \varepsilon + (1 + \varepsilon)(1 + \varepsilon)^*(1 + \varepsilon)$
13. $0 + (1 + \varepsilon)(1 + \varepsilon)^*0$
14. $\varnothing + \varnothing(1 + \varepsilon)^*(1 + \varepsilon)$
15. $0 + 1 + \varepsilon + \varnothing(1 + \varepsilon)^*0$
16. $1^*0 + 1^*0(0 + 1 + \varepsilon)^*(0 + 1 + \varepsilon)$

Tarea:

1. $a + \varepsilon + (a + \varepsilon)(a + \varepsilon)^*(a + \varepsilon)$
2. $b + (a + \varepsilon)(a + \varepsilon)^*b$
3. $\varnothing + (a + \varepsilon)(a + \varepsilon)^*\varnothing$
4. $\varnothing + \varnothing(a + \varepsilon)^*(a + \varepsilon)$
5. $a + \varepsilon + \varnothing(a + \varepsilon)^*b$
6. $b + \varnothing(a + \varepsilon)^*\varnothing$
7. $b + b(a + \varepsilon)^*(a + \varepsilon)$
8. $\varnothing + b(a + \varepsilon)^*b$
9. $a + \varepsilon + b(a + \varepsilon)^*\varnothing$
10. $a^* + a^*b(a + \varepsilon)^*\varnothing$
11. $a^*b + a^*b(a + \varepsilon)^*(a + \varepsilon)$
12. $\varnothing + a^*b(a + \varepsilon)^*b$
13. $ba^* + ba^*b(a + \varepsilon)^*\varnothing$
14. $ba^*b + ba^*b(a + \varepsilon)^*(a + \varepsilon)$
15. $a + \varepsilon + ba^*b(a + \varepsilon)^*b$
16. $a^*ba^* + a^*ba^*b(a + \varepsilon + ba^*ba^*b)^*ba^*ba^*$