3.2. AUTÓMATAS APILADORES (AA)

DEFINICIÓN

$A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

Q : conjunto finito de estados.
Σ : alfabeto de entrada.
Γ : alfabeto de la pila.
δ : función de transición.
$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow Q \times \Gamma^*$
q_0 : estado inicial.
$q_0 \in Q$
z_0 : símbolo inicial de la pila.
$z_0 \in \Gamma$
F : conjunto de estados finales o de aceptación.
$F \subseteq Q$

Ejemplo 1:
$A = (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \{A, B\}, \delta, q_1, A, \{q_4\})$

$\delta(q_1, a, A) = \{(q_2, BA), (q_4, A)\}$
$\delta(q_1, \varepsilon, A) = \{(q_4, \varepsilon)\}$
$\delta(q_2, a, B) = \{(q_2, BB)\}$
$\delta(q_2, b, B) = \{(q_3, \varepsilon)\}$
$\delta(q_3, \varepsilon, A) = \{(q_4, A)\}$
$\delta(q_3, b, B) = \{(q_3, \varepsilon)\}$

Ejemplo 2:
$A = (\{q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b, c\}, \{A, B, Z_0\}, \delta, q_1, Z_0, \{q_6\})$

$\delta(q_1, a, A) = \{(q_1, AA)\}$
$\delta(q_1, a, Z_0) = \{(q_1, AZ_0), (q_2, Z_0)\}$
$\delta(q_1, b, A) = \{(q_3, A)\}$
$\delta(q_2, b, B) = \{(q_2, BB)\}$
$\delta(q_2, a, Z_0) = \{(q_2, Z_0)\}$
$\delta(q_2, b, Z_0) = \{(q_2, BZ_0)\}$
$\delta(q_2, c, B) = \{(q_4, \varepsilon)\}$
$\delta(q_3, b, A) = \{(q_3, A)\}$
$\delta(q_3, c, A) = \{(q_5, \varepsilon)\}$
$\delta(q_4, c, B) = \{(q_4, \varepsilon)\}$
$\delta(q_4, \varepsilon, Z_0) = \{(q_6, \varepsilon)\}$
$\delta(q_5, c, A) = \{(q_5, \varepsilon)\}$
$\delta(q_5, \varepsilon, Z_0) = \{(q_6, \varepsilon)\}$

MOVIMIENTOS

$(q, \alpha) \in \delta(p, a, z) \Rightarrow (p, a\omega, z\beta) \vdash (q, \omega, \alpha\beta)$

$p, q \in Q$
$a \in (\Sigma \cup \{\varepsilon\})$
$\omega \in \Sigma^*$
$Z \in \Gamma$
$\alpha, \beta \in \Gamma^*$

$\vdash$ movimiento en un paso.

$\overset{i}{\vdash}$ movimiento en i pasos.

$\overset{*}{\vdash}$ movimiento en cero o más pasos.

$\overset{+}{\vdash}$ movimiento en uno o más pasos.

3.2.1. AUTÓMATAS APILADORES DETERMINISTAS (AAD)

$A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

1. $\delta(q, \varepsilon, z) \neq \varnothing \Rightarrow \delta(q, \sigma, z) = \varnothing \qquad \forall\, q \in Q\, ,\, \sigma \in \Sigma\, ,\, z \in \Gamma$
2. $\#\, \delta(q, a, z) \leq 1 \qquad \forall\, q \in Q\, ,\, a \in (\Sigma \cup \{\varepsilon\})\, ,\, z \in \Gamma$

Ejemplo:
$A = (\{q_0, q_1\}, \{a, b\}, \{A, B, Z_0\}, \delta, q_0, Z_0, \{q_0\})$

$\delta(q_0, a, Z_0) = \{(q_1, AZ_0)\}$
$\delta(q_0, b, Z_0) = \{(q_1, BZ_0)\}$
$\delta(q_1, a, A) = \{(q_1, AA)\}$
$\delta(q_1, b, B) = \{(q_1, BB)\}$
$\delta(q_1, a, B) = \{(q_1, \varepsilon)\}$
$\delta(q_1, b, A) = \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, Z_0) = \{(q_0, Z_0)\}$

$L(A) = \{\omega \in \{a, b\}^* \,/\, |\omega|_a = |\omega|_b\}$

3.2.2. AUTÓMATAS APILADORES NO DETERMINISTAS (AAN)

Ejemplo 1:
$A = (\{q_0, q_1, q_2\}, \{a, b\}, \{A, B, Z_0\}, \delta, q_0, Z_0, \emptyset)$

$\delta(q_0, a, Z_0) = \{(q_1, AZ_0)\}$
$\delta(q_0, b, Z_0) = \{(q_1, BZ_0)\}$
$\delta(q_0, \varepsilon, Z_0) = \{(q_2, \varepsilon)\}$
$\delta(q_1, a, A) = \{(q_1, AA)\}$
$\delta(q_1, b, B) = \{(q_1, BB)\}$
$\delta(q_1, a, B) = \{(q_1, \varepsilon)\}$
$\delta(q_1, b, A) = \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, Z_0) = \{(q_0, Z_0)\}$

$N(A) = \{\omega \in \{a, b\}^* / |\omega|_a = |\omega|_b\}$

Ejemplo 2:
$A = (\{q_1, q_2\}, \{a, b\}, \{A, B, Z\}, \delta, q_1, Z, \{q_2\})$

$\delta(q_1, \varepsilon, Z) = \{(q_2, Z)\}$
$\delta(q_1, a, Z) = \{(q_1, AZ)\}$
$\delta(q_1, b, Z) = \{(q_1, BZ)\}$
$\delta(q_1, a, A) = \{(q_1, AA)\}$
$\delta(q_1, b, A) = \{(q_1, \varepsilon)\}$
$\delta(q_1, a, B) = \{(q_1, \varepsilon)\}$
$\delta(q_1, b, B) = \{(q_1, BB)\}$

$L(A) = \{\omega \in \{a, b\}^* / |\omega|_a = |\omega|_b\}$

$\omega = abba$

3.2.3. LENGUAJE ACEPTADO MEDIANTE ESTADO FINAL (L(A))

$A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$F \neq \emptyset$

$L(A) = \{\omega \in \Sigma^* / (q_0, \omega, z_0) \overset{*}{\vdash} (q, \varepsilon, \gamma) \text{ , } q \in F \wedge \gamma \in \Gamma^*\}$

Ejemplo 1:
$A = (\{q_0, q_1, q_2\}, \{0, 1, c\}, \{0, 1, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

$\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$
$\delta(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$
$\delta(q_0, 0, 0) = \{(q_0, 00)\}$
$\delta(q_0, 0, 1) = \{(q_0, 01)\}$
$\delta(q_0, 1, 0) = \{(q_0, 10)\}$
$\delta(q_0, 1, 1) = \{(q_0, 11)\}$
$\delta(q_0, c, Z_0) = \{(q_1, Z_0)\}$
$\delta(q_0, c, 0) = \{(q_1, 0)\}$
$\delta(q_0, c, 1) = \{(q_1, 1)\}$
$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$
$\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, Z_0) = \{(q_2, Z_0)\}$

$L(A) = \{\omega c \omega^R / \omega \in \{0, 1\}^*\}$

Ejemplo 2:
$A = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, Z_0\}, \delta, q_0, Z_0, \{q_2\})$

$\delta(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$
$\delta(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$
$\delta(q_0, 0, 0) = \{(q_0, 00)\}$
$\delta(q_0, 0, 1) = \{(q_0, 01)\}$
$\delta(q_0, 1, 0) = \{(q_0, 10)\}$
$\delta(q_0, 1, 1) = \{(q_0, 11)\}$
$\delta(q_0, \varepsilon, Z_0) = \{(q_1, Z_0)\}$
$\delta(q_0, \varepsilon, 0) = \{(q_1, 0)\}$
$\delta(q_0, \varepsilon, 1) = \{(q_1, 1)\}$
$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$
$\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$
$\delta(q_1, \varepsilon, Z_0) = \{(q_2, Z_0)\}$

$L(A) = \{\omega\omega^R / \omega \in \{0, 1\}^*\}$

$\omega = 1111$

3.2.4. LENGUAJE ACEPTADO MEDIANTE AGOTAMIENTO DE PILA (N(A))

$A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, \emptyset)$

$N(A) = \{\omega \in \Sigma^* \ / \ (q_0, \omega, z_0) \overset{*}{\vdash} (q, \varepsilon, \varepsilon) \ , \ q \in Q\}$

Ejemplo 1:
$A = (\{q_1, q_2\}, \{0, 1, c\}, \{R, B, G\}, \delta, q_1, R, \emptyset)$

$\delta(q_1, 0, R) = \{(q_1, BR)\}$
$\delta(q_1, 1, R) = \{(q_1, GR)\}$
$\delta(q_1, 0, B) = \{(q_1, BB)\}$
$\delta(q_1, 1, B) = \{(q_1, GB)\}$
$\delta(q_1, 0, G) = \{(q_1, BG)\}$
$\delta(q_1, 1, G) = \{(q_1, GG)\}$
$\delta(q_1, c, R) = \{(q_2, R)\}$
$\delta(q_1, c, B) = \{(q_2, B)\}$
$\delta(q_1, c, G) = \{(q_2, G)\}$
$\delta(q_2, 0, B) = \{(q_2, \varepsilon)\}$
$\delta(q_2, 1, G) = \{(q_2, \varepsilon)\}$
$\delta(q_2, \varepsilon, R) = \{(q_2, \varepsilon)\}$

$N(A) = \{\omega c \omega^R \ / \ \omega \in \{0, 1\}^*\}$

Ejemplo 2:
$A = (\{q_1, q_2\}, \{0, 1\}, \{R, B, G\}, \delta, q_1, R, \emptyset)$

$\delta(q_1, 0, R) = \{(q_1, BR)\}$
$\delta(q_1, 1, R) = \{(q_1, GR)\}$
$\delta(q_1, 0, B) = \{(q_1, BB), (q_2, \varepsilon)\}$
$\delta(q_1, 0, G) = \{(q_1, BG)\}$
$\delta(q_1, 1, B) = \{(q_1, GB)\}$
$\delta(q_1, 1, G) = \{(q_1, GG), (q_2, \varepsilon)\}$
$\delta(q_2, 0, B) = \{(q_2, \varepsilon)\}$
$\delta(q_2, 1, G) = \{(q_2, \varepsilon)\}$
$\delta(q_1, \varepsilon, R) = \{(q_2, \varepsilon)\}$
$\delta(q_2, \varepsilon, R) = \{(q_2, \varepsilon)\}$

$N(A) = \{\omega \omega^R \ / \ \omega \in \{0, 1\}^*\}$

$\omega = 001100$

3.2.5. EQUIVALENCIAS

$A = (\{q_1, q_2, q_3\}, \{a, b\}, \{a, z\}, \delta, q_1, z, \{q_3\})$

$\delta(q_1, a, z) = \{(q_1, az)\}$
$\delta(q_1, b, z) = \{(q_2, \varepsilon)\}$
$\delta(q_1, a, a) = \{(q_3, a)\}$

$L(A) = \{aa\}$
$N(A) = \{b\}$

3.2.5.1. L(A) ⇒ N(A')

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F) \Rightarrow A' = (Q', \Sigma, \Gamma', \delta', q_0', X_0, \emptyset)$$

$$\text{donde} \begin{cases} Q' = Q \cup \{q_0', q_e\} \\ \Gamma' = \Gamma \cup \{X_0\} \\ \delta' \begin{cases} \delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0X_0)\} & \\ \delta'(q, a, Z) = \delta(q, a, Z) & \forall q \in (Q - F), a \in (\Sigma \cup \{\varepsilon\}), Z \in \Gamma \\ \delta'(q, \sigma, Z) = \delta(q, \sigma, Z) & \forall q \in F, \sigma \in \Sigma, Z \in \Gamma \\ \delta'(q, \varepsilon, Z) = \delta(q, \varepsilon, Z) \cup \{(q_e, \varepsilon)\} & \forall q \in F, Z \in \Gamma \\ \delta'(q, \varepsilon, X_0) = \{(q_e, \varepsilon)\} & \forall q \in F \\ \delta'(q_e, \varepsilon, Z) = \{(q_e, \varepsilon)\} & \forall Z \in \Gamma' \end{cases} \end{cases}$$

3.2.5.2. N(A) ⇒ L(A')

$$A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \emptyset) \Rightarrow A' = (Q', \Sigma, \Gamma', \delta', q_0', X_0, F')$$

$$\text{donde} \begin{cases} Q' = Q \cup \{q_0', q_f\} \\ \Gamma' = \Gamma \cup \{X_0\} \\ \delta' \begin{cases} \delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0X_0)\} & \\ \delta'(q, a, Z) = \delta(q, a, Z) & \forall q \in Q, a \in (\Sigma \cup \{\varepsilon\}), Z \in \Gamma \\ \delta'(q, \varepsilon, X_0) = \{(q_f, \varepsilon)\} & \forall q \in Q \end{cases} \\ F' = \{q_f\} \end{cases}$$

Ejemplo:
$A = (\{q\}, \{i, e\}, \{Z\}, \delta, q, Z, \emptyset)$

$\delta(q, i, Z) = \{(q, ZZ)\}$
$\delta(q, e, Z) = \{(q, \varepsilon)\}$