3. LENGUAJES INDEPENDIENTES DEL CONTEXTO

3.1. GRAMÁTICAS INDEPENDIENTES DEL CONTEXTO (GIC)

$G = (N, \Sigma, P, S)$

$P \subseteq N \times (N \cup \Sigma)^*$

$A \to \alpha \qquad A \in N$
$\qquad\qquad \alpha \in (N \cup \Sigma)^*$

3.1.1. SIMPLIFICACIÓN

3.1.1.1. ELIMINACIÓN DE PRODUCCIONES ε

Producciones ε
$A \to \varepsilon \qquad A \in N$

Anulables

$$N\varepsilon = \left\{ A \in N \,/\, A \stackrel{*}{\Rightarrow} \varepsilon \right\}$$

Algoritmo:
$NA = \varnothing$
$N\varepsilon = \{A \in N / A \to \varepsilon \in P\}$
Mientras $NA \neq N\varepsilon$ hacer
    $NA = N\varepsilon$
    $N\varepsilon = NA \cup \{A \in N / A \to \alpha \in P, \alpha \in NA^*\}$
Fin Mientras

Observación:
$S \in N\varepsilon \Rightarrow \varepsilon \in L(G)$

$G = (N, \Sigma, P, S) \Rightarrow G' = (N, \Sigma, P', S)$

$L(G') = L(G) - \{\varepsilon\}$

$$\begin{matrix} A \rightarrow X_1X_2X_3 \cdots X_i \cdots X_n \in P \\ X_1, X_2, X_3, \cdots, X_n \in (N \cup \Sigma) \end{matrix} \quad \Rightarrow \quad \begin{matrix} A \rightarrow Y_1Y_2Y_3 \cdots Y_i \cdots Y_n \in P' \\ Y_1, Y_2, Y_3, \cdots, Y_n \in (N \cup \Sigma) \end{matrix}$$

a) $X_i \notin N\varepsilon \Rightarrow Y_i = X_i$
b) $X_i \in N\varepsilon \Rightarrow Y_i = X_i \vee Y_i = \varepsilon$
c) $A \rightarrow \varepsilon \notin P'$

Ejemplo:
G = ({S, A}, {a}, P, S)
P = {
$S \rightarrow aA$
$A \rightarrow aA \mid \varepsilon$
}

Ejercicio 1:
G = ({S, A, B}, {a, b}, P, S)
P = {
$S \rightarrow AB$
$A \rightarrow aAA \mid \varepsilon$
$B \rightarrow bBB \mid \varepsilon$
}

Ejercicio 2:
G = ({S, X, Y}, {a, b}, P, S)
P = {
$S \rightarrow aXbS \mid bYaS \mid \varepsilon$
$X \rightarrow aXbX \mid \varepsilon$
$Y \rightarrow bYaY \mid \varepsilon$
}

Ejercicio 3:
G = ({S, P, Q}, {x, y, z}, P, S)
P = {
$S \rightarrow zPzQz$
$P \rightarrow xPx \mid Q$
$Q \rightarrow yPy \mid \varepsilon$
}

3.1.1.2. ELIMINACIÓN DE PRODUCCIONES UNITARIAS

Producciones unitarias

$A \rightarrow B \qquad A, B \in N$

$$G = (N, \Sigma, P, S) \quad \Rightarrow \quad G' = (N, \Sigma, P', S)$$

$$\left.\begin{array}{l} A \rightarrow B \\ B \rightarrow \alpha \end{array}\right\} \in P \quad \Rightarrow \quad \left.\begin{array}{l} A \rightarrow \alpha \\ B \rightarrow \alpha \end{array}\right\} \in P'$$

$$U(A) = \left\{ B \in N \,/\, A \overset{*}{\Rightarrow} B \right\} \qquad \forall A \in N$$

Algoritmo:
P’ = $\varnothing$
$\forall$ A $\in$ N
    $\forall$ B $\in$ U(A)
        $\forall$ B $\rightarrow \alpha \in$ P
            Si $\alpha \notin$ N entonces
                P’ = P’ $\cup$ {A $\rightarrow \alpha$}
            Fin Si

Ejemplo:
G = ({S, A, B, C, D}, {a, b}, P, S)
P = {
S $\rightarrow$ A | Aa
A $\rightarrow$ B
B $\rightarrow$ C | b
C $\rightarrow$ D | ab
D $\rightarrow$ b
}

Ejercicio:
G = ({E, T, F, I}, {+, *, (,), a, b, 0, 1}, P, E)
P = {
E $\rightarrow$ E+T | T
T $\rightarrow$ T*F | F
F $\rightarrow$ (E) | I
I $\rightarrow$ a | b | Ia | Ib | I0 | I1
}

3.1.1.3. ELIMINACIÓN DE SÍMBOLOS INÚTILES

Símbolo útil

$S \overset{*}{\Rightarrow} \alpha X \beta \overset{*}{\Rightarrow} \omega$ $\qquad \alpha, \beta \in (N \cup \Sigma)^*$, $X \in (N \cup \Sigma)$, $\omega \in \Sigma^*$

a) $G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S)$

$L(G) \neq \varnothing$

$A \overset{*}{\Rightarrow} \omega$ $\qquad A \in N$, $\omega \in \Sigma^*$

Algoritmo:
NA = $\varnothing$
N' = $\{A \in N / A \to \omega \in P , \omega \in \Sigma^*\}$
Mientras NA $\neq$ N' hacer
  NA = N'
  N' = $NA \cup \{A \in N / A \to \alpha \in P , \alpha \in (\Sigma \cup NA)^*\}$
Fin Mientras
P' = $\{A \to \alpha \in P / A \in N' , \alpha \in (N' \cup \Sigma)^*\}$

Ejemplo:
G = ({S, A, B}, {a, b}, P, S)
P = {
  S $\to$ AB | a
  A $\to$ b
}

Ejercicio:
G = ({S, A, B}, {a, b}, P, S)
P = {
  S $\to$ AB | a
  A $\to$ BA
  B $\to$ b
}

b) $G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma', P', S)$

$S \overset{*}{\Rightarrow} \alpha X \beta \qquad \alpha, \beta \in (N \cup \Sigma)^* \quad X \in (N \cup \Sigma)$

Algoritmo:
$N' = \{S\}$
$\Sigma' = \emptyset$
$P' = \emptyset$
$\forall\ A \in N'$
$\quad \forall\ A \to \gamma \in P$
$\quad \{$
$\quad\quad N' = N' \cup \{X_i \in N \,/\, \gamma = X_1 X_2 X_3 \ldots X_i \ldots X_n,\ X_1, X_2, X_3, \ldots, X_n \in (N \cup \Sigma)\}$
$\quad\quad \Sigma' = \Sigma' \cup \{X_i \in \Sigma \,/\, \gamma = X_1 X_2 X_3 \ldots X_i \ldots X_n,\ X_1, X_2, X_3, \ldots, X_n \in (N \cup \Sigma)\}$
$\quad\quad P' = P' \cup \{A \to \gamma\}$
$\quad \}$

Ejemplo:
$G = (\{S, A, B\}, \{a, b\}, P, S)$
$P = \{$
$\quad S \to AB \mid a$
$\quad A \to b$
$\}$

Ejercicio:
$G = (\{S, A, B\}, \{a, b\}, P, S)$
$P = \{$
$\quad S \to AB \mid a$
$\quad A \to BA$
$\quad B \to b$
$\}$

3.1.2. FORMA NORMAL DE CHOMSKY

$G = (N, \Sigma, P, S)$

$$\left.\begin{array}{l} A \rightarrow BC \\ A \rightarrow \sigma \end{array}\right\} \in P \qquad \begin{array}{c} A, B, C \in N \\ \sigma \in \Sigma \end{array}$$

a) Simplificar

b)

$$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S)$$

$$A \rightarrow X_1X_2X_3 \cdots X_i \cdots X_n \in P \quad X_i = \sigma \quad n \geq 2 \quad \Rightarrow \quad \begin{array}{l} \left.\begin{array}{l} A \rightarrow X_1X_2X_3 \cdots C_\sigma \cdots X_n \\ C_\sigma \rightarrow \sigma \end{array}\right\} \in P' \\ N' = N \cup \{C_\sigma\} \end{array}$$

c)

$$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S)$$

$$A \rightarrow B_1B_2B_3 \cdots B_n \in P \quad n \geq 3 \quad \Rightarrow \quad \begin{array}{l} \left.\begin{array}{l} A \rightarrow B_1D_1 \\ D_1 \rightarrow B_2D_2 \\ D_2 \rightarrow B_3D_3 \\ D_3 \rightarrow B_4D_4 \\ \cdots \\ D_{n\text{-}3} \rightarrow B_{n\text{-}2}D_{n\text{-}2} \\ D_{n\text{-}2} \rightarrow B_{n\text{-}1}B_n \end{array}\right\} \in P' \\ N' = N \cup \{D_1, D_2, D_3, \cdots, D_{n\text{-}2}\} \end{array}$$

Ejemplo:
G = ({S, A, B}, {a, b}, P, S)
P = {
S → bA | aB
A → bAA | aS | a
B → aBB | bS | b
}

Ejercicio:
G = ({E, T, F, I}, {+, *, (,), a, b, 0, 1}, P, E)
P = {
E → E+T | T
T → T*F | F
F → (E) | I
I → a | b | Ia | Ib | I0 | I1
}

3.1.3. OPERACIONES

3.1.3.1. UNIÓN

$$\begin{array}{l} G_1 = (N_1, \Sigma_1, P_1, S_1) \\ G_2 = (N_2, \Sigma_2, P_2, S_2) \end{array} \quad N_1 \cap N_2 = \emptyset \Rightarrow G = (N, \Sigma, P, S) \text{ donde } \begin{cases} N = N_1 \cup N_2 \cup \{S\} \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ P = P_1 \cup P_2 \cup \{S \to S_1 \mid S_2\} \end{cases}$$

$L(G) = L(G_1) \cup L(G_2)$

Ejemplo:
$L(G) = \{a^i b^j \,/\, i \neq j\}$
$L(G) = \{a^i b^j \,/\, i > j \vee i < j\}$
$L(G) = \{a^i b^j \,/\, i > j\} \cup \{a^i b^j \,/\, i < j\}$

$L(G_1) = \{a^i b^j \,/\, i > j\}$
$L(G_2) = \{a^i b^j \,/\, i < j\}$

$G_1 = (\{A\}, \{a, b\}, \{A \to aA \mid aAb \mid a\}, A)$
$G_2 = (\{B\}, \{a, b\}, \{B \to Bb \mid aBb \mid b\}, B)$

3.1.3.2. CONCATENACIÓN

$$\begin{array}{l} G_1 = (N_1, \Sigma_1, P_1, S_1) \\ G_2 = (N_2, \Sigma_2, P_2, S_2) \end{array} \quad N_1 \cap N_2 = \emptyset \Rightarrow G = (N, \Sigma, P, S) \text{ donde } \begin{cases} N = N_1 \cup N_2 \cup \{S\} \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ P = P_1 \cup P_2 \cup \{S \to S_1 S_2\} \end{cases}$$

$L(G) = L(G_1)L(G_2)$

Ejemplo:
$L(G) = \{a^i b^j c^k \,/\, i, j, k \geq 0 \wedge j = i + k\}$
$L(G) = \{a^i b^i b^k c^k \,/\, i, k \geq 0\}$
$L(G) = \{a^i b^i \,/\, i \geq 0\}\ \{b^k c^k \,/\, k \geq 0\}$

$L(G_1) = \{a^i b^i \,/\, i \geq 0\}$
$L(G_2) = \{b^k c^k \,/\, k \geq 0\}$

$G_1 = (\{X\}, \{a, b\}, \{X \to aXb \mid \varepsilon\}, X)$
$G_2 = (\{Y\}, \{b, c\}, \{Y \to bYc \mid \varepsilon\}, Y)$

3.1.3.3. CLAUSURA

$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S')$ donde $\begin{cases} N' = N \cup \{S'\} \\ P' = P \cup \{S' \rightarrow SS' \mid \varepsilon\} \end{cases}$

$L(G') = L(G)^*$

Ejemplo:
$L(G) = \{ba^n \ / \ n \geq 0\}$

$G = (\{S_0\}, \{a, b\}, \{S_0 \rightarrow S_0\ a \mid b\}, S_0)$

3.1.3.4. CLAUSURA POSITIVA

$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S')$ donde $\begin{cases} N' = N \cup \{S'\} \\ P' = P \cup \{S' \rightarrow SS' \mid S\} \end{cases}$

$L(G') = L(G)^+$

Ejemplo:
$L(G) = \{ba^n \ / \ n \geq 0\}$

$G = (\{S_0\}, \{a, b\}, \{S_0 \rightarrow S_0\ a \mid b\}, S_0)$

3.1.3.5. TRANSPOSICIÓN

$G = (N, \Sigma, P, S) \Rightarrow G' = (N, \Sigma, P', S)$ donde $P' = \{A \rightarrow \alpha^R \ / \ A \rightarrow \alpha \in P\}$

$L(G') = L(G)^R$

Ejemplo:
$G = (\{S\}, \{+, \times, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{S \rightarrow +SS \mid \times SS \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9\}, S)$