UNIDAD II. LENGUAJES INDEPENDIENTES DEL CONTEXTO

1. GRAMÁTICAS INDEPENDIENTES DEL CONTEXTO (GIC)

$G = (N, \Sigma, P, S)$

$P \subseteq N \times (N \cup \Sigma)^*$

$A \to \alpha$ $\quad A \in N$
$\quad \alpha \in (N \cup \Sigma)^*$

1.1. SIMPLIFICACIÓN

1.1.1. ELIMINACIÓN DE PRODUCCIONES ε

Producciones ε
$A \to \varepsilon$ $\quad A \in N$

Anulables
$$N\varepsilon = \left\{ A \in N \,/\, A \overset{*}{\Rightarrow} \varepsilon \right\}$$

Algoritmo:
$NA = \varnothing$
$N\varepsilon = \{A \in N / A \to \varepsilon \in P\}$
Mientras $NA \neq N\varepsilon$ hacer
  $NA = N\varepsilon$
  $N\varepsilon = NA \cup \{A \in N / A \to \alpha \in P , \alpha \in NA^+\}$
Fin Mientras

Observación:
$S \in N\varepsilon \Rightarrow \varepsilon \in L(G)$

$$G = (N, \Sigma, P, S) \quad \Rightarrow \quad G' = (N, \Sigma, P', S)$$

$$A \to X_1X_2X_3 \cdots X_i \cdots X_n \in P \quad X_i \notin N\varepsilon \quad \Rightarrow \quad A \to X_1X_2X_3 \cdots X_i \cdots X_n \in P'$$

$$A \to X_1X_2X_3 \cdots X_i \cdots X_n \in P \quad X_i \in N\varepsilon \quad \Rightarrow \quad \left.\begin{array}{l} A \to X_1X_2X_3 \cdots X_i \cdots X_n \\ A \to X_1X_2X_3 \cdots X_{i-1}X_{i+1} \cdots X_n \end{array}\right\} \in P'$$

$L(G') = L(G) - \{\varepsilon\}$

Ejemplo:

G = ({S, A}, {a, b}, P, S)

P = {

$S \rightarrow aAb$

$A \rightarrow aAb \mid \varepsilon$

}

Ejercicio 1:

G = ({S, A, B}, {a, b}, P, S)

P = {

$S \rightarrow AB$

$A \rightarrow aAA \mid \varepsilon$

$B \rightarrow bBB \mid \varepsilon$

}

Ejercicio 2:

G = ({S, X, Y}, {a, b}, P, S)

P = {

$S \rightarrow aXbS \mid bYaS \mid \varepsilon$

$X \rightarrow aXbX \mid \varepsilon$

$Y \rightarrow bYaY \mid \varepsilon$

}

Ejercicio 3:

G = ({S, P, Q}, {x, y, z}, P, S)

P = {

$S \rightarrow zPzQz$

$P \rightarrow xPx \mid Q$

$Q \rightarrow yPy \mid \varepsilon$

}

1.1.2. ELIMINACIÓN DE PRODUCCIONES UNITARIAS

Producciones unitarias

$A \rightarrow B \qquad A, B \in N$

$G = (N, \Sigma, P, S) \quad \Rightarrow \quad G' = (N, \Sigma, P', S)$

$$\left.\begin{matrix} A \rightarrow B \\ B \rightarrow \alpha \end{matrix}\right\} \in P \qquad \Rightarrow \qquad \left.\begin{matrix} A \rightarrow \alpha \\ B \rightarrow \alpha \end{matrix}\right\} \in P'$$

$$U(A) = \left\{ B \in N \ / \ A \overset{*}{\Rightarrow} B \right\} \qquad \forall A \in N$$

Algoritmo:
$P' = \varnothing$
$\forall \ A \in N$
  $\forall \ B \in U(A)$
    $\forall \ B \rightarrow \alpha \in P$
      Si $\alpha \notin N$ entonces
        $P' = P' \cup \{A \rightarrow \alpha\}$
      Fin Si

Ejemplo:
$G = (\{S, T\}, \{0, 1\}, P, S)$
$P = \{$
  $S \rightarrow 0S \mid S1 \mid T$
  $T \rightarrow 01 \mid 0T$
$\}$

1.1.3. ELIMINACIÓN DE SÍMBOLOS INÚTILES

Símbolo útil

$S \overset{*}{\Rightarrow} \alpha X \beta \overset{*}{\Rightarrow} \omega \qquad \alpha, \beta \in (N \cup \Sigma)^*$
$X \in (N \cup \Sigma)$
$\omega \in \Sigma^*$

a) $G = (N, \Sigma, P, S) \qquad L(G) \neq \varnothing \qquad \Rightarrow \qquad G' = (N', \Sigma, P', S)$

$A \overset{*}{\Rightarrow} \omega \qquad A \in N$
$\omega \in \Sigma^*$

Algoritmo:
$NA = \varnothing$
$N' = \{A \in N \,/\, A \rightarrow \omega \in P \,,\, \omega \in \Sigma^*\}$
Mientras $NA \neq N'$ hacer
$NA = N'$
$N' = NA \cup \{A \in N \,/\, A \rightarrow \alpha \in P \,,\, \alpha \in (\Sigma \cup NA)^*\}$
Fin Mientras
$P' = \{A \rightarrow \alpha \in P \,/\, A \in N' \,,\, \alpha \in (N' \cup \Sigma)^*\}$

Ejemplo:
$G = (\{S, A, B, C, D\}, \{a, b, c\}, P, S)$
$P = \{$
$S \rightarrow aAAA$
$A \rightarrow aAb \mid aC$
$B \rightarrow BD \mid Ac$
$C \rightarrow b$
$\}$

b) $G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma', P', S)$

$S \overset{*}{\Rightarrow} \alpha X \beta \qquad \alpha, \beta \in (N \cup \Sigma)^*$
$\qquad\qquad\qquad X \in (N \cup \Sigma)$

Algoritmo:
$NA = \varnothing$
$N' = \{S\}$
$\Sigma' = \varnothing$
Mientras $NA \neq N'$ hacer
$\quad NA = N'$
$\quad N' = NA \cup \{B \in N \,/\, A \rightarrow \alpha B \beta \in P,\ A \in NA,\ \alpha,\beta \in (N \cup \Sigma)^*\}$
$\quad \Sigma' = \Sigma' \cup \{\sigma \in \Sigma \,/\, A \rightarrow \alpha \sigma \beta \in P,\ A \in NA,\ \alpha,\beta \in (N \cup \Sigma)^*\}$
Fin Mientras
$P' = \{A \rightarrow \alpha \in P \,/\, A \in N',\ \alpha \in (N' \cup \Sigma')^*\}$

Ejemplo:
$G = (\{S, A, B, C, D\}, \{a, b, c\}, P, S)$
$P = \{$
$\quad S \rightarrow aAAA$
$\quad A \rightarrow aAb \mid aC$
$\quad B \rightarrow BD \mid Ac$
$\quad C \rightarrow b$
$\}$

1.2. FORMA NORMAL DE CHOMSKY

G = (N, Σ, P, S)

$$\left.\begin{array}{l} A \rightarrow BC \\ A \rightarrow \sigma \end{array}\right\} \in P \qquad \begin{array}{c} A, B, C \in N \\ \sigma \in \Sigma \end{array}$$

a) Simplificar

b) $G = (N, \Sigma, P, S) \quad \Rightarrow \quad G' = (N', \Sigma, P', S)$

$$A \rightarrow X_1X_2X_3 \cdots X_i \cdots X_n \in P \quad X_i = \sigma \quad n \geq 2 \quad \Rightarrow \quad \left.\begin{array}{l} A \rightarrow X_1X_2X_3 \cdots C_\sigma \cdots X_n \\ C_\sigma \rightarrow \sigma \end{array}\right\} \in P'$$

$$N' = N \cup \{C_\sigma\}$$

c) $G = (N, \Sigma, P, S) \quad \Rightarrow \quad G' = (N', \Sigma, P', S)$

$$A \rightarrow B_1B_2B_3 \cdots B_n \in P \quad n \geq 3 \quad \Rightarrow \quad \left.\begin{array}{l} A \rightarrow B_1D_1 \\ D_1 \rightarrow B_2D_2 \\ D_2 \rightarrow B_3D_3 \\ D_3 \rightarrow B_4D_4 \\ \cdots \\ D_{n-3} \rightarrow B_{n-2}D_{n-2} \\ D_{n-2} \rightarrow B_{n-1}B_n \end{array}\right\} \in P'$$

$$N' = N \cup \{D_1, D_2, D_3, \cdots, D_{n-2}\}$$

Ejemplo:
G = ({S, A, B}, {0, 1}, P, S)
P = {
S → BA
A → 01AB0 | 0
B → 1
}

Ejercicio:
G = ({S, A, B}, {a, b}, P, S)
P = {
S → bA | aB
A → bAA | aS | a
B → aBB | bS | b
}

1.3. OPERACIONES

1.3.1. UNIÓN

$$\begin{matrix} G_1 = (N_1, \Sigma_1, P_1, S_1) \\ G_2 = (N_2, \Sigma_2, P_2, S_2) \end{matrix} \quad N_1 \cap N_2 = \emptyset \Rightarrow G = (N, \Sigma, P, S) \quad S \notin (N_1 \cup N_2)$$

$$\text{donde} \begin{cases} N = N_1 \cup N_2 \cup \{S\} \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ P = P_1 \cup P_2 \cup \{S \to S_1 \mid S_2\} \end{cases}$$

$L(G) = L(G_1) \cup L(G_2)$

Ejemplo:
$L(G) = \{a^i b^j \ / \ i \neq j\} = \{a^i b^j \ / \ i > j \lor i < j\} = \{a^i b^j \ / \ i > j\} \cup \{a^i b^j \ / \ i < j\}$

$L(G_1) = \{a^i b^j \ / \ i > j\}$
$L(G_2) = \{a^i b^j \ / \ i < j\}$

$G_1 = (\{A\}, \{a, b\}, \{A \to aA \mid aAb \mid a\}, A)$
$G_2 = (\{B\}, \{a, b\}, \{B \to Bb \mid aBb \mid b\}, B)$

1.3.2. CONCATENACIÓN

$$\begin{matrix} G_1 = (N_1, \Sigma_1, P_1, S_1) \\ G_2 = (N_2, \Sigma_2, P_2, S_2) \end{matrix} \quad N_1 \cap N_2 = \emptyset \Rightarrow G = (N, \Sigma, P, S) \quad S \notin (N_1 \cup N_2)$$

$$\text{donde} \begin{cases} N = N_1 \cup N_2 \cup \{S\} \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ P = P_1 \cup P_2 \cup \{S \to S_1 S_2\} \end{cases}$$

$L(G) = L(G_1)L(G_2)$

Ejemplo:
$L(G) = \{a^i b^j c^k \ / \ i, j, k \geq 0 \land j = i + k\} = \{a^i b^i b^k c^k \ / \ i, k \geq 0\} = \{a^i b^i \ / \ i \geq 0\} \ \{b^k c^k \ / \ k \geq 0\}$

$L(G_1) = \{a^i b^i \ / \ i \geq 0\}$
$L(G_2) = \{b^k c^k \ / \ k \geq 0\}$

$G_1 = (\{X\}, \{a, b\}, \{X \to aXb \mid \varepsilon\}, X)$
$G_2 = (\{Y\}, \{b, c\}, \{Y \to bYc \mid \varepsilon\}, Y)$

1.3.3. CLAUSURA

$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S')$ donde $\begin{cases} N' = N \cup \{S'\} \\ P' = P \cup \{S' \rightarrow SS' \mid \varepsilon\} \end{cases}$

$L(G') = L(G)^*$

1.3.4. CLAUSURA POSITIVA

$G = (N, \Sigma, P, S) \Rightarrow G' = (N', \Sigma, P', S')$ donde $\begin{cases} N' = N \cup \{S'\} \\ P' = P \cup \{S' \rightarrow SS' \mid S\} \end{cases}$

$L(G') = L(G)^+$

1.3.5. TRANSPOSICIÓN

$G = (N, \Sigma, P, S) \Rightarrow G' = (N, \Sigma, P', S)$ donde $P' = \{A \rightarrow \alpha^R / A \rightarrow \alpha \in P\}$

$L(G') = L(G)^R$

Ejemplo:
$G = (\{S\}, \{+, \times, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{S \rightarrow +SS \mid \times SS \mid 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9\}, S)$