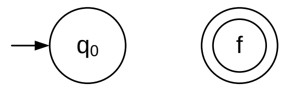
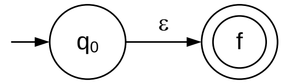
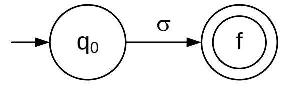
5. EQUIVALENCIAS

5.1. ER $\Rightarrow$ AFN-ε

Tabla 5.1. Construcción de Thompson.

ER	**AFN-ε**
$\varnothing$	
ε	
$\sigma \in \Sigma$	

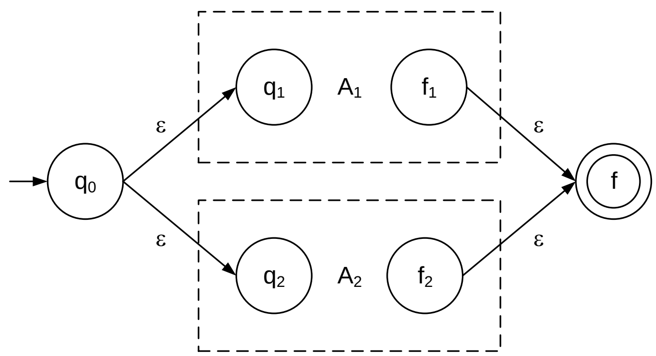
ER

$r_1 + r_2$

$A_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$
$A_2 = (Q_2, \Sigma_2, \delta_2, q_2, \{f_2\})$ $\qquad Q_1 \cap Q_2 = \emptyset$

$L(A_1) = L(r_1)$
$L(A_2) = L(r_2)$

AFN-ε



$$A = (Q, \Sigma, \delta, q_0, F) \text{ donde } \begin{cases} Q = Q_1 \cup Q_2 \cup \{q_0, f\} & q_0, f \notin (Q_1 \cup Q_2) \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ \delta: \begin{cases} \delta(q_0, \varepsilon) = \{q_1, q_2\} \\ \delta(q, a) = \delta_1(q, a) & \forall q \in (Q_1 - \{f_1\}), a \in (\Sigma_1 \cup \{\varepsilon\}) \\ \delta(q, a) = \delta_2(q, a) & \forall q \in (Q_2 - \{f_2\}), a \in (\Sigma_2 \cup \{\varepsilon\}) \\ \delta(f_1, \varepsilon) = \{f\} \\ \delta(f_2, \varepsilon) = \{f\} \end{cases} \\ F = \{f\} \end{cases}$$

$L(A) = L(A_1) \cup L(A_2)$
$L(A) = L(r_1) \cup L(r_2)$

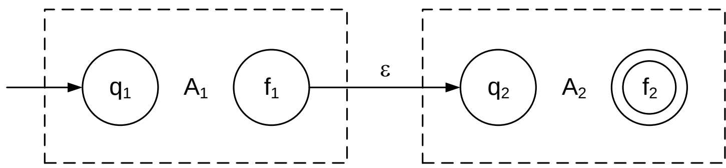
ER

$r_1 r_2$

$A_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$
$A_2 = (Q_2, \Sigma_2, \delta_2, q_2, \{f_2\})$ $\qquad Q_1 \cap Q_2 = \emptyset$

$L(A_1) = L(r_1)$
$L(A_2) = L(r_2)$

AFN-ε



$$A = (Q, \Sigma, \delta, q_0, F) \text{ donde } \begin{cases} Q = Q_1 \cup Q_2 \\ \Sigma = \Sigma_1 \cup \Sigma_2 \\ \delta\text{:} \begin{cases} \delta(q,a) = \delta_1(q,a) & \forall q \in (Q_1 - \{f_1\}), a \in (\Sigma_1 \cup \{\varepsilon\}) \\ \delta(f_1, \varepsilon) = \{q_2\} \\ \delta(q,a) = \delta_2(q,a) & \forall q \in Q_2, a \in (\Sigma_2 \cup \{\varepsilon\}) \end{cases} \\ q_0 = q_1 \\ F = \{f_2\} \end{cases}$$

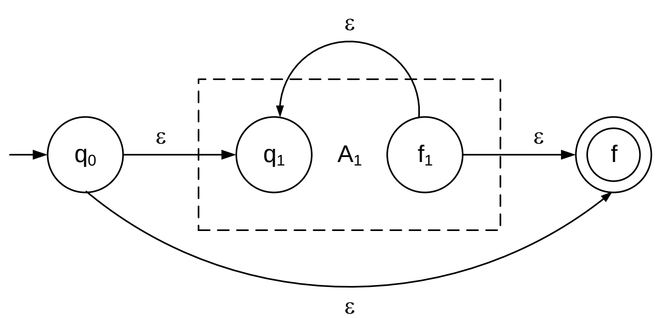
$L(A) = L(A_1)L(A_2)$
$L(A) = L(r_1)L(r_2)$

ER
r_1^*

$A_1 = (Q_1, \Sigma_1, \delta_1, q_1, \{f_1\})$

$L(A_1) = L(r_1)$

AFN-ε



$$A = (Q, \Sigma, \delta, q_0, F) \text{ donde } \begin{cases} Q = Q_1 \cup \{q_0, f\} & q_0, f \notin Q_1 \\ \Sigma = \Sigma_1 & \\ \delta: \begin{cases} \delta(q_0, \varepsilon) = \{q_1, f\} \\ \delta(q, a) = \delta_1(q, a) \\ \delta(f_1, \varepsilon) = \{q_1, f\} \end{cases} & \forall q \in (Q_1 - \{f_1\}), a \in (\Sigma_1 \cup \{\varepsilon\}) \\ F = \{f\} & \end{cases}$$

$L(A) = L(A_1)^*$
$L(A) = L(r_1)^*$

Ejemplo:
$01^* + 1$

Ejercicio:
$(0 + 1)^*1(0 + 1)$

5.2. AFD ⇒ ER

AFD
A = (Q, Σ, δ, q_1, F)

Q = {q_1, q_2, q_3, …, q_n}

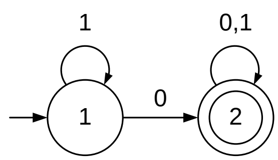
ER

$$L(A) = \bigcup_{q_j \in F} {R_{1j}}^n$$

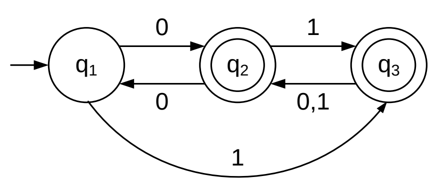
$$ {R_{ij}}^0 = \begin{cases} \{\sigma \in \Sigma \ / \ \delta(q_i, \sigma) = q_j\} & \text{si} \quad i \neq j \\ \{\sigma \in \Sigma \ / \ \delta(q_i, \sigma) = q_j\} \cup \{\varepsilon\} & \text{si} \quad i = j \end{cases}$$

$${R_{ij}}^k = {R_{ij}}^{k-1} \cup {R_{ik}}^{k-1}({R_{kk}}^{k-1})^* {R_{kj}}^{k-1}$$

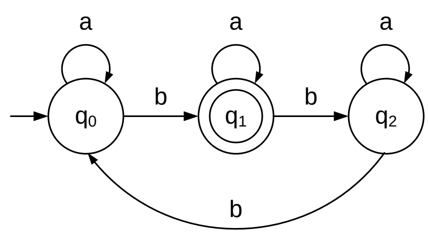
Ejemplo:



Ejercicio propuesto 1:



Ejercicio propuesto 2:



5.3. AFN $\Rightarrow$ ER

Ecuaciones Lineales

AFN
$A = (Q, \Sigma, \delta, q_0, F)$

ER
$X_i = \{\omega \in \Sigma^* / \delta(q_i, \omega) \cap F \neq \emptyset\}$

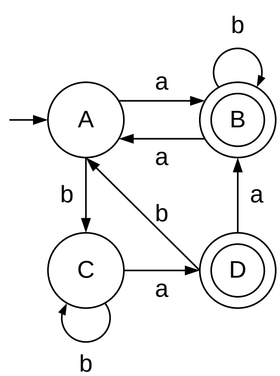
$$X_i = \bigcup_{\sigma \in \Sigma} \{\sigma X_j \ / \ q_j \in \delta(q_i, \sigma)\} \qquad \forall q_i \in (Q - F)$$

$$X_i = \bigcup_{\sigma \in \Sigma} \{\sigma X_j \ / \ q_j \in \delta(q_i, \sigma)\} \cup \{\varepsilon\} \quad \forall q_i \in F$$

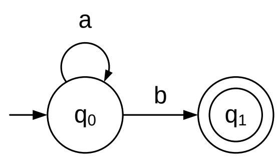
$L(A) = X_0$

Lema de Arden
$X = AX + B$, $\varepsilon \notin A \Rightarrow X = A^*B$

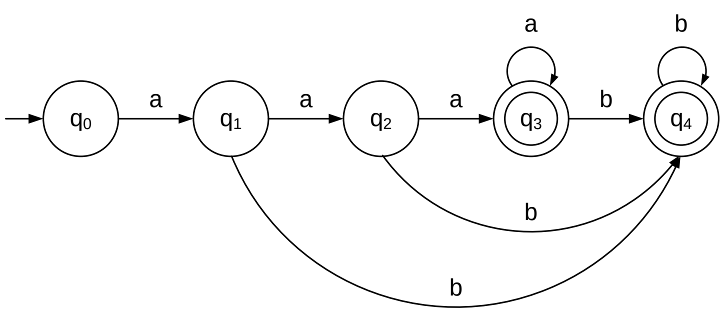
Ejemplo:



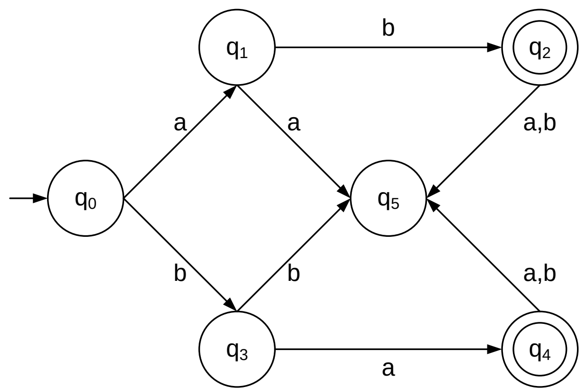
Ejercicio 1:



Ejercicio 2:



Ejercicio 3:



5.4. GRLD $\Rightarrow$ AFN-ε

GRLD
G = (N, Σ, P, S)

AFN-ε
A = (Q, Σ, δ, S, F)

$Q = N \cup \{F\}$ $\quad F \notin N$
δ:
Si $A \to \sigma_1\sigma_2\sigma_3\ldots\sigma_n \in P$ entonces
$\delta = \delta \cup \delta(A, \sigma_1\sigma_2\sigma_3\ldots\sigma_n) = \delta(q_1, \sigma_2\sigma_3\ldots\sigma_n) = \delta(q_2, \sigma_3\ldots\sigma_n) = \ldots = \delta(q_{n-1}, \sigma_n) = \{F\}$
$Q = Q \cup \{q_1, q_2, q_3, \ldots, q_{n-1}\}$
Fin Si
Si $A \to \sigma_1\sigma_2\sigma_3\ldots\sigma_n B \in P$ entonces
$\delta = \delta \cup \delta(A, \sigma_1\sigma_2\sigma_3\ldots\sigma_n) = \delta(q_1, \sigma_2\sigma_3\ldots\sigma_n) = \delta(q_2, \sigma_3\ldots\sigma_n) = \ldots = \delta(q_{n-1}, \sigma_n) = \{B\}$
$Q = Q \cup \{q_1, q_2, q_3, \ldots, q_{n-1}\}$
Fin Si
F = {F}

Ejemplo:
G = ({S, A, B}, {a, b}, P, S)
P = {
$S \to aB \mid bA \mid \varepsilon$
$A \to abaS$
$B \to babS$
}

5.5. GRLD $\Rightarrow$ AFN

GRLD **AFN**

$G = (N, \Sigma, P, S)$ en forma normal $\Rightarrow$ $A = (Q, \Sigma, \delta, q_0, F)$

$$\text{donde} \begin{cases} Q = N \\ \delta(A, \sigma) = \{B \in N \ / \ A \to \sigma B \in P\} \quad \forall A \in N, \sigma \in \Sigma \\ q_0 = S \\ F = \{A \in N \ / \ A \to \varepsilon \in P\} \end{cases}$$

Ejemplo:

$G = (\{S, A, B, D_1, D_2, D_3, D_4\}, \{a, b\}, P, S)$

$P = \{$

$S \to aB \mid bA \mid \varepsilon$
$A \to aD_1$
$D_1 \to bD_2$
$D_2 \to aS$
$B \to bD_3$
$D_3 \to aD_4$
$D_4 \to bS$

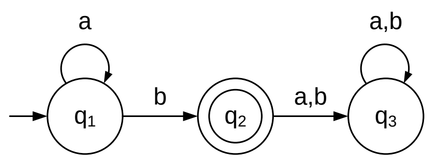
$\}$

5.6. AFN $\Rightarrow$ GRLD

AFN **GRLD**

$$A = (Q, \Sigma, \delta, q_0, F) \quad \Rightarrow \quad G = (N, \Sigma, P, S) \text{ donde } \begin{cases} N = Q \\ P = \{q \to \sigma p \;/\; p \in \delta(q, \sigma)\} \cup \{q \to \varepsilon \;/\; q \in F\} \\ S = q_0 \end{cases}$$

Ejemplo:



Ejercicio: