lab14

s302179

December 2022

1 Overview

In this laboratory, we were asked to simulate a Galton-Watson process in which we want to study how the probability of extinction within a generation behaves with respect to the parameter of the Poisson distribution used to generate the number of children for each individual.

1.1 Assumptions

Given X_i the number of children for each individual and Y_i the number of individuals for each generation, let us assume that:

- The X_i's are i.i.d. random variable
- The X_i's are distributed as a Poisson distribution of parameter λ

1.2 Input parameters

- λ: parameter of the Poisson distribution
- RUNS: number of trials
- max_gens: maximum number of individuals per generation after which the tree is going to diverge

1.3 Output metrics

- Probability of extinction within each generation
- for $\lambda = 0.8$ the distribution of the number of individuals in the three

2 Main algorithm

The main algorithm used to evaluate the extinction probability consists in initializing the first generation with one individual and then randomly extract the

number of children from a Poisson distribution of parameter λ. For each following generation, I iterated over the number of individuals of the next generation in order to generate the number of children for each individual and so on. This cycle will go on until extinction occurs or the number of individuals in the current generation is greater than a given threshold. If the number of children of the last detected generation is equal to 0 then we will consider the last recorded generation in order to compute the probability of extinction within the current generation, otherwise, we will return an invalid value that will be considered for the probability of non-extinction for that generation that is printed for each value of λ

3 Results

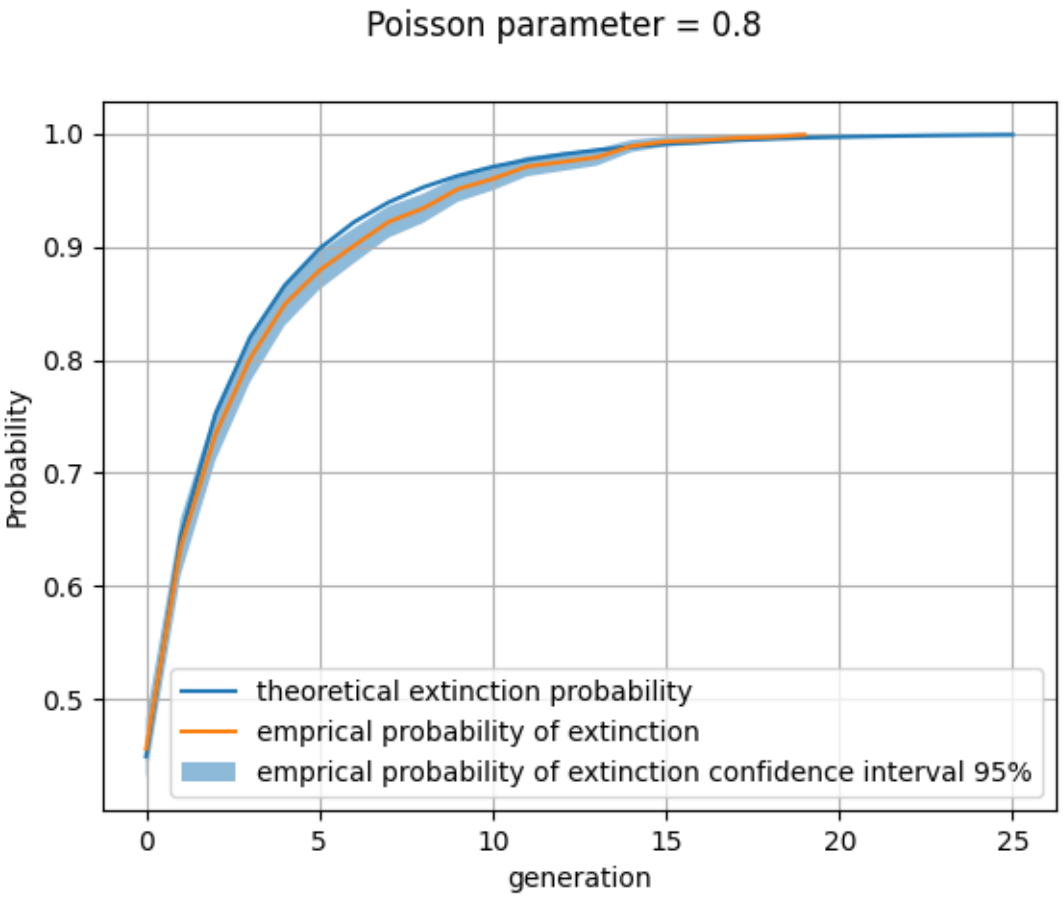

Figure 1: Extinction probability within the i^{th} generation with λ= 0.9

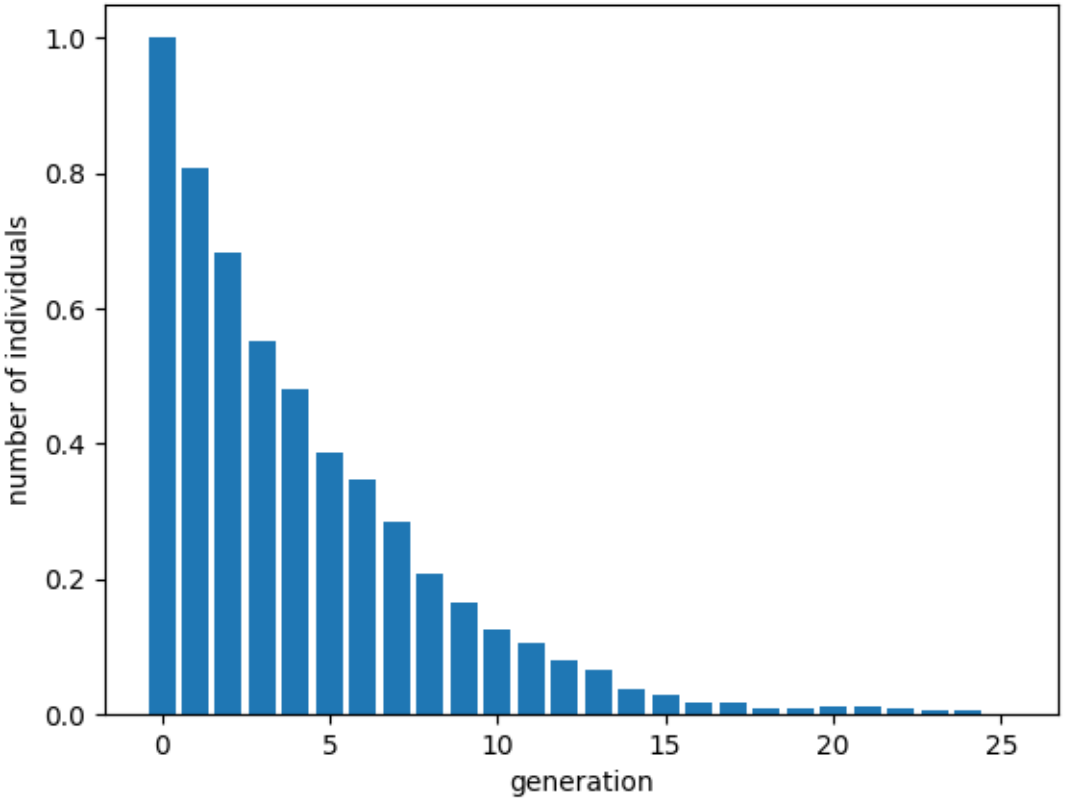

Figure 2: Extinction probability within the i^{th} generation with λ= 0.9

This figure 1 shows the probability of extinction (y-axis) in the i^{th} generation: the blue curve is the theoretical one and the orange curve is the experimental one. We can notice that the trend of the experimental curve almost follows the theoretical one that almost falls into the confidence interval computed at a confidence level of 95%. It is noteworthy to notice that the empirical curve grows at the beginning and after some generations, it becomes stable at a value that is almost the same as the one of the theoretical curve. In the end, in figure 2 we can see see a histogram of the number of individuals for each generation which reaches a maximum value for the first generation. This is verified also in theory because with $\lambda = 0.8$ since also in figure 1 we can notice how the extinction probability is almost equal to 1 then the distribution of the individuals for high values of generation tends to 0