Laboratory$_0$6

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1 Overview

The aim of this laboratory is to compare different behaviours of different random variables generators in terms of time they take. We have generated instances of the following random variables:

- Binomial
- Normal
- Chi-squared
- Poisson
- Rice

2 Binomial

The binomial distribution requires 2 parameters which are p (success probability) and n (number of attempts). For the binomial random variable we tried 3 approaches that we saw during the lectures: convolution, inverse-transform and a third method.

2.1 Convolution

The convolution method consists in generating n instances of a U(0,1) and then count the number of variables that are smaller than success probability and the value of the counter is the generated instance. Such method can be applied only to random variables that represent the sum of other random variables. Such case can be represented for the binomial distribution which represents the sum of N Bernoulli distributions.

2.2 Inverse transform

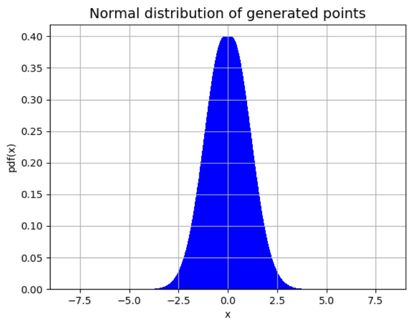
The inverse transform method consists in generating instances from U(0,1) and then, assuming that U = F(X), we can compute the inverse of the cumulative density function in order to get the generated instance. Such method can be theoretically applied to every distribution, but for some of them (like for the Normal distribution) computing the inverse of its cumulative density function, requires a very high computational cost, so it is better to adopt different approaches to generate its instances. In this implementation we generated instances of a Binomial distribution. For lower values of n and p we get the generations in a reasonable amount of time and to reduce the computation cost i decided to store all the values of the factorial of n in a dictionary in order to have them already computed every time we need. Then I decided to use a logarithmic scale so that the different operation are easier to compute and then I computed the exponential of the result. Nevertheless with higher values of n = 10^6 and p = 10^{-5} it seams unfeasible since it has to compute the factorial all "i's" that assumes integer values between 1 and n.

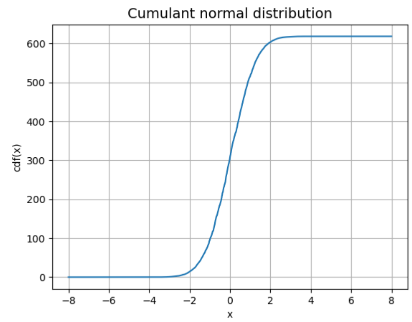
2.3 Method 3

This is computed checking if the instances of the geometric distribution with parameter 1-p assuming values of i+1 corresponds to the i's failed Bernoulli attempts.

3 Normal

The normal distribution accepts 2 parameters: μ which is the mean and the σ which is the standard deviation. In order to generate instances we used the **acceptance-rejection method**. It consists in defining an interval in which I extract the values from a uniform distribution then, if a randomly extracted point between 0 and a given argument c is less than the value of the pdf in the extracted point then it is a valid generation.
In the end I compared the empirical mean and the empirical variance and as we can notice from the plotted graph the mean is around zero and the standard deviation is almost 1.





4 Rice

The rice distribution (which generalizes the Rayleigh distribution) describes the distance of a point P from Q The parameters of this distribution are (ν, σ^2) where ν is the distance of Q from the origin and σ^2 is the variance of the distributions from which p and x are extracted. The instances of the rice distribution are computed as $Z = \sigma\sqrt{x}$ where σ is the standard deviation of the rice distribution and also the standard deviation of the χ-squared distribution that we use in order to generate x. The χ-squared has $2p+2$ degrees of freedom where p is a random variable which follows a Poisson distribution.