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1. COMPANY BACKGROUND

Pizza Pilgrims is a Limited Liability Company established by Thom and James Elliot and began as a market stall in Soho, London in 2012. After a year and a half of trading at various markets and events throughout the United Kingdom (UK), the company launched its first permanent pizzeria on Dean Street in Soho. This original outlet was committed to serving high-quality Neapolitan pizzas, using premium ingredients, and fostering a welcoming atmosphere for both staff and patrons. As of 2022, Pizza Pilgrims has grown into a modestly-sized chain with 19 locations in London, strategically situated near busy business areas, tourist hotspots and universities. Additionally, the chain has expanded beyond London, opening new branches in Brighton, Cambridge, Oxford and Nottingham, marking a shift towards national growth. The company has received industry acclaim, being nominated as one of the top five in the "Most Admired Brand" category at the RROTY Awards for three consecutive years. They currently offer the following selection of pizzas, each at its respective price point:

- Margherita at £10.5
- Margherita extra at £12.75
- Smokey aubergine parm at £11.95
- Mushroom & truffle at £13.95
- Double pepperoni & hot honey at £14.25
- Nduja at £12.95
- The 8 cheese at £12.95
- You've got maiale at £15.25
- Pizz& love at £12.95
- Carbonara at £13.95

Despite its success, the company currently faces financial challenges stemming from a confluence of external factors, including soaring inflation rates, increase in wage costs following rises in the national living wages, as it applies to a significant proportion of staff, rising costs for energy and raw materials and the resultant reduction in consumers' purchasing power. Additionally, the profound economic disruption wrought by the pandemic has added to their woes. In a bid to navigate these challenges and revitalise their financial health, Pizza Pilgrims has enlisted the expertise of GADH, a renowned British hospitality consultancy firm. They are jointly undertaking a comprehensive strategy to boost the company's profit. This plan involves adjusting pizza prices across all their branches to effectively attract and retain customers, both new and existing, while also maximising revenue. GADH aims to achieve this objective by:

- Developing a dynamic pricing strategy, differentiating pizza prices between peak and non-peak days
 of the week
- Introducing a tailored pricing structure for students and non-students
- Crafting the ideal assortment of pizzas during peak and non-peak days of the week
- Guide them in optimising prices of their seasonal special pizzas

After consulting with Pizza Pilgrims, GADH identified the peak and non-peak days of the week to inform our analysis. According to their insights, Sunday through Wednesday are considered non-peak days, while Thursday to Saturday are peak days.

2. SUMMARY OF THE TECHNIQUES EMPLOYED AND ANALYSIS OF THE RESULTS OBTAINED

2.1. USING PRICING OPTIMIZATION WITH CONSUMER CHOICE USING SIMULATED SURVEY DATA OF CUSTOMERS'S WILLINGNESS TO PAY (WTP) FOR PEAK AND NON-PEAK PRICING STRATEGIES

The initial phase of our strategy focused on developing a dynamic pricing model for each pizza, tailored to consumer preferences and distinguishing between peak and non-peak periods. In the absence of real data, we employed a parametric demand model using simulated survey data on the WTP of 1000 customers across both peak and non-peak periods for each of the 10 different pizzas. This empirical data was directly integrated into our analysis. Next, we used linear approximations for the demand model.

To model customer behaviour, we explored a range of prices from 1 to 20 in increments of 1, resulting in 20 price points for both non-peak and peak periods. Our objective was to comprehend demand relative to WTP prices. To facilitate our analysis, we created a new data frame to store these prices alongside their corresponding demand levels. This approach was adopted to enhance the efficiency and accuracy of our forthcoming regression analysis. Specifically, we performed two separate linear regression models to predict non-peak and peak demand as a function of their respective prices. Subsequently, the coefficients from both regression models were extracted and employed in nonlinear optimization model to determine the prices that maximise revenue for each peak period. The optimised non-peak and peak prices of each pizza, are illustrated in Table 1. As shown in Figure 1, these optimised pizza prices are projected to generate the highest possible revenue.

PIZZAS	OPTIMAL PEAK PRICES	OPTIMAL NON-PEAK PRICES
MARGHERITA	£8.60	£10.30
MARGHERITA EXTRA	£11.40	£13.20
NDUJA	£11.20	£13.70
DOUBLE PEPPERONI & SPICY HONEY	£12.70	£15.30
MUSHROOM & TRUFFLE	£11.70	£14.50
SMOKY AUBERGINE PARM	£10.10	£11.85
PIZZ'& LOVE	£10.80	£12.35
CHEESE	£10.80	£12.60
CARBONARA	£11.35	£14.00
YOU'VE GOT MAIALE	£13.50	£16.40

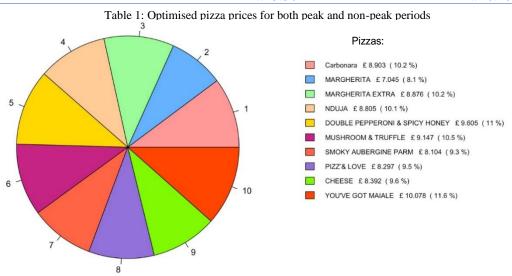


Figure 1: Total optimal maximised revenue

Total Revenue: £ 87.252

2.2. USING MULTINOMIAL LOGIT (MNL) MODEL USING SIMULATED SURVEY DATA OF CUSTOMERS'S WTP TO OBTAIN THE BEST PRICE TO CHARGE FOR STUDENTS AND NON-STUDENTS DURING BOTH PEAK PERIODS

In this analysis, our focus was on determining optimal prices for various pizzas during both non-peak and peak hours, exclusively for students. We employed MNL model considering statistical insights and student's WTP for each pizza during peak and non-peak periods. Statistical analysis, including averages and standard deviations, provided valuable insights into the overall preferences and variability of choices within this student subset. Critical parameters, including μ peak and μ non-peak, were estimated based on the calculated variances. These parameters captured the essence of student preferences during different times of the day. Our objective function was crafted to maximise expected revenue and incorporate the attraction values, which quantify the appeal of pizzas at different prices during the time of purchase. For non-peak period, the attraction value for a pizza was computed by exponentiating the difference between the average WTP for that pizza and the proposed non-peak price. We applied the same approach for the peak period.

We used nonlinear optimisation technique, to determine the optimal non-peak and peak prices for pizzas for students. The optimisation process began with initial price guesses, defined lower and upper bounds, and incorporated additional constraints and rules to guide the search for optimal prices. The result from this analysis are reported in Table 2. These results carefully balance the task of attracting student customers while optimising profitability during both peak periods. This strategic adjustment is designed to not only cater to our regular customers but also accommodate students, ensuring that our pricing structure is well-suited for generating optimal revenue.

PIZZAS	STUDENTS NON-PEAK PRICES	STUDENTS PEAK PRICES	STUDENTS ATTRACTION VALUES PEAK PERIOD	STUDENTS ATTRACTION VALUES NON- PEAK PERIOD
MARGHERITA	£8.2	£9.4	2.41	0.09
MARGHERITA EXTRA	£11.7	£11.8	4	0.08
NDUJA	£11.9	£11.9	2.61	0.08
DOUBLE PEPPERONI &				
SPICY HONEY	£11.4	£12.55	2.4	0.13
MUSHROOM & TRUFFLE	£11	£12.3	2.58	0.07
SMOKY AUBERGINE PARM	£10.7	£11	3.79	0.08
PIZZ'& LOVE	£10.7	£11.1	2.94	0.16
CHEESE	£10.7	£11.3	2.73	0.14
CARBONARA	£10.8	£12.1	2.5	0.08
YOU'VE GOT MAIALE	£11.9	£13.4	2.23	0.13

Table 2: Student prices and attraction values

2.3. MENU ASSORTMENT OPTIMIZATION USING MARKOV CHAIN CHOICE MODEL

We optimised the pizza menu for different customer segments and periods by using a Markov Chain model and the MNL. Initially, we determined the product attraction values for all pizzas during both peak and non-peak periods for regular customers and students. The attraction values for students were already available from the previous MNL analysis. However, given the absence of direct empirical data for the 'outside option'

attraction value in each market segment, we conducted a sensitivity analysis. This helped estimate the range of possible outcomes and the impact of the 'outside option' value ('v0') on the optimal product assortment. For instance, pizza attractions ('v') varied between 0.98 and 2.39 ror regular customers during peak times. We selected a 'v0' of 1.1, representing a moderate outside option attractiveness. We then normalised the attraction values so their sum equaled one, enabling us to calculate each pizza's first choice probability. We created a transition matrix to estimate the likelihood of customers switching between pizza options.

Lastly, we applied Linear Programming (LP) to solve for the optimal pizza assortment, using the pizzas' prices and a zero matrix for the objective function in our model, which included 20 decision variables. The analysis revealed that during peak periods, the most profitable pizza assortment for regular customers included Margherita Extra, Nduja, Double Pepperoni and Spicy Honey, Mushroom and Truffle, Carbonara and You've Got Maiale. For students, adding the 8 Cheese pizza maximised revenue. During non-peak periods, the most profitable selection for regular customers was Double Pepperoni and Spicy Honey and You've Got Maiale, while for students, adding Margherita Extra was optimal. This tailored approach to menu planning aims to reduce operational costs, including those for raw materials and labor, and to improve inventory management, ultimately boosting revenue.

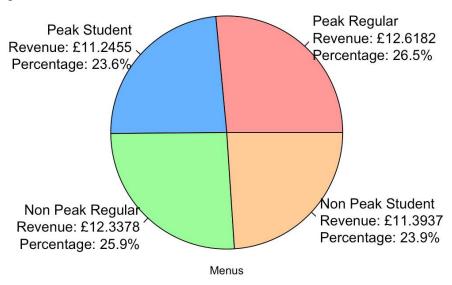


Figure 2: Total revenue of the menu assortment optimised

2.4. IMPLEMENTING MARKDOWN OPTIMIZATION STRATEGY WITH PREDETERMINED PRICES FOR REVENUE AND INVENTORY OPTIMIZATION

We employed a markdown optimization strategy to price Pizza Pilgrims' seasonal special pizzas, including Venison, Wild Mushroom, Cod and Asparagus pizzas. Our goal was to manage the limited inventory of special ingredients effectively while maximising revenue. The challenge was to set an optimal initial price for each pizza, given the scarcity of ingredients and a fixed expiration date for unsold inventory. Our approach began with setting an optimal list price, intending to reduce it twice monthly by 20% to maximise total revenue, including salvage. We'll illustrate the process using the Venison pizza, but the same method applies to the other special pizzas.

We first estimated demand for the Venison pizza at various prices, ranging from £1 to £24. This involved assessing the surplus for each potential customer and determining the purchase likelihood at each price point. Subsequently, we fitted a linear regression model (Demand~Price) to this data to understand the price-demand relationship. For the initial pricing, we used nonlinear optimization, assuming a cost of £4 per unit of Venison pizza. With a limited supply of 100 Kg of venison (each pizza requiring 100 grams), the maximum production was capped at 1000 pizzas. The objective function aimed to maximise profit, considering price, cost and demand from the regression model, with a constraint to keep demand within the 1000-pizza limit. This yielded an optimal initial price of £14.40. The price-response function was determined as 471.4 - 17.9p. We then estimated weekly demand at various discounted prices, setting a salvage price of £7.25 for unsold pizzas at season's end. Next, we formulated an LP model to maximise revenue, considering the initial inventory, total selling horizon and price levels.

Our analysis concluded that the best strategy was to maintain the initial price of £14.40 throughout the season and sell any remaining stock (estimated at 360 units) at the salvage price. This markdown optimization strategy is expected to effectively balance inventory management with revenue maximisation for Pizza Pilgrims' seasonal offerings.

3. CONCLUSION

To conclude, GADH yielded a comprehensive optimisation pricing strategy to navigate financial challenges and drive sustained growth of Pizza Pilgrims. The dynamic pricing strategy, informed by simulated WTP survey data and linear regression models, is capable to differentiate between peak and non-peak periods. The student-centric pricing structure, crafted through MNL models, aims to strike an optimal balance between attracting this market segment and maintaining profitability. The menu assortment optimization, utilising Markov Chain and MNL models, reflects a strategic alignment of offerings with customer preferences during peak and non peak-periods, fostering operational efficiency and cost-effectiveness. The markdown strategy for seasonal specials showcases a thoughtful approach, leveraging nonlinear optimization to set initial prices that maximise revenue while prudently managing limited inventory. This holistic strategy, anchored in customer-centric adjustments to pricing and menu offerings, positions Pizza Pilgrims for sustained financial success. By integrating these optimisation techniques, Pizza Pilgrims aims not only to weather economic uncertainties but also to enhance its competitive edge, ensuring a robust and adaptive business model in the dynamic landscape of the hospitality industry.