Esercizio 14

martedì 15 giugno 2021 16:28

$$f_{XY}(x,y)=\begin{cases}\frac{1}{2} & 0<x<2,\quad 0<y<x\\ 0 & \text{altrove}\end{cases}$$

(1) $$f_X(x)=\int_{-\infty}^{\infty} f_{XY}(x,y)\,dy=\int_0^x \frac{1}{2}\,dy=\frac{1}{2}\Big|_0^x=\frac{1}{2}x$$

$$f_Y(y)=\int_{-\infty}^{\infty} f_{XY}(x,y)\,dx=\int_y^2 \frac{1}{2}\,dx=\frac{1}{2}x\Big|_y^2=1-\frac{1}{2}y$$

$$f_X(x)=\begin{cases}\frac{1}{2}x & 0<x<2\\ 0 & \text{altrove}\end{cases}$$

$$f_Y(y)=\begin{cases}1-\frac{1}{2}y & 0<y<2\\ 0 & \text{altrove}\end{cases}$$

$$E_X=\int^{\infty} x\,f_X(x)=\int^2 \frac{x^2}{2}\,dx=\frac{1}{2}\int_0^2 x^2\,dx=\frac{1}{2}\cdot\frac{x^3}{2}\Big|_0^2=\frac{1}{2}\cdot\frac{8}{3}=\frac{4}{3}$$

$$E x = \int_{-\infty} \ldots = \int_{0} \ldots \Big|_{0}$$

$$E x^2 = \int_0^2 \frac{x^3}{2}\, dx = \frac{1}{2}\, \frac{x^4}{4}\Big|_0^2 = \frac{1}{2} \cdot \frac{16}{4} = 2$$

$$VAR_x = E x^2 - (E x)^2 = 2 - \left(\frac{4}{3}\right)^2 = 2 - \frac{16}{9} = \frac{18 - 16}{9} = \frac{2}{9}$$

$$E y = \int_0^2 y\left(1 - \frac{1}{2}\right) dy \qquad \int_0^2 y - \int_0^2 \frac{1}{2}\, y^2 = \frac{2}{3}$$

$$E y^2 = \int_0^2 y\left(1 - \frac{y}{2}\right) = \int_0^2 y^2 - \int_0^2 \frac{y^3}{2} - \frac{2}{3}$$

$$\mathrm{VAR}\, Y = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$$

(2)

$$f_{XY}(x,y) = \frac{1}{2} \neq f_X(x)\, f_Y(y) = \frac{1}{2}x \quad 1 - \frac{1}{2}y$$

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$$E(XY) = \int_{-\infty}^{\infty} x\, f_{XY}(x,y)\, dx$$

$$= \int_0^2 \frac{1}{2} x\, dx = \frac{1}{2} \int_0^2 x\, dx$$

$$= \frac{1}{2} \frac{x^2}{2} \Big|_0^2 = 1$$

$$Cov(X,Y) = E(XY) - EX\,EY = 1 - \frac{4}{3} \cdot \frac{2}{3} = 1 - \frac{8}{9} = \frac{1}{9}$$

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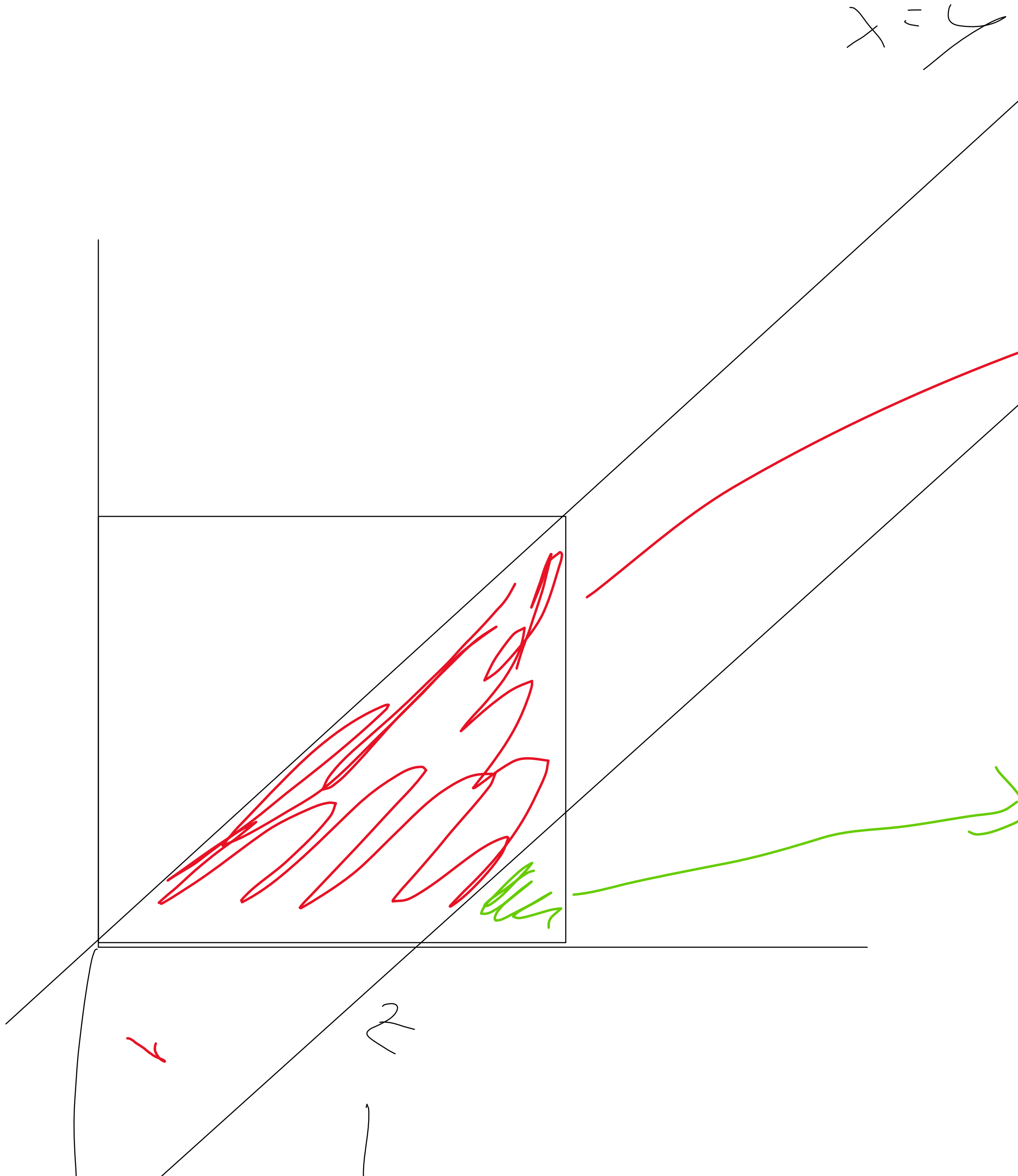
$$Z = X - Y$$

$$F_Z(z) = P(Z \leq z) = P(X - Y \leq z)$$

$$1 - \frac{(2 - z)^2}{2} + \frac{1}{2}\left(1 - \frac{(2-z)^2}{2}\right)$$



$$\text{Area} = \frac{(2-z)^2}{2} = \left(\frac{4 - 2z + z}{2}\right)$$

$0 < z < 2$

$$\begin{cases} y = 0 & x = z \\ y = x - z \end{cases}$$

$$F_Z(z) = \begin{cases} 0 & z < 0 \\ \dfrac{4z - z^2}{4} & 0 < z < 2 \\ 1 & z \geq 2 \end{cases}$$

$$\frac{1 - (2 - z)^2}{2}$$

$$f_Y(z) = \begin{cases} 0 & \text{altrove} \\ 1 - \frac{1}{2} z & 0 < z < 2 \end{cases}$$

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$$E z = \int_0^2 z\, f_z(z) = \int_0^2 z - \frac{1}{2} z^2 \, dz = \int_0^2 z - \int_0^2 \frac{1}{2} z^2$$

$$= \frac{z^2}{2}\Big|_0^2 - \frac{z^3}{6}\Big|_0^2$$

$$= \frac{4}{2} - \frac{8}{6} = \frac{12-8}{6} = \frac{4}{6}$$

$$= \frac{2}{3}$$

$$EZ^2 = \int_0^2 z^2 - \frac{1}{2} z^3 \, dz = \frac{z^3}{3} - \frac{1}{2}\frac{z^4}{4} = \frac{z^3}{3} - \frac{z^4}{8}\Big|_0^2 = \frac{2}{3}$$

$$\mathrm{VAR}\, Z = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$$