Esercizio 11

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$$f(x) = \begin{cases} kx^2 & -1 < x < 0 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$k = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_{-1}^{0} kx^2 \, dx$$

$$k \int_{-1}^{0} x^2 \, dx$$

$$k \left[\frac{x^3}{3} \right]_{-1}^{0}$$

$$k \left[0 - \frac{-1^3}{3} \right]$$

$$k \left[\frac{1}{3} \right]$$

$$k \frac{1}{3} = 1$$

$$k = \frac{1}{\frac{1}{3}}$$

$$k = 3$$

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② Funzione di Distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\,dx$$

$$F_x = \int_{-1}^{x} k x^2 \, dx$$

$$= k \int_{-1}^{x} x^2 \, dx$$

$$= 3 \left[\frac{x^3}{3} \right]_{-1}^{x}$$

$$= 3 \left[\frac{x^3}{3} - \left(\frac{-1^3}{3} \right) \right]$$

$$= 3 \left[\frac{x^3}{3} + \frac{1}{3} \right]$$

$$= \cancel{3} \left(\frac{x^3 + 1}{\cancel{3}} \right)$$

$$= x^3 + 1$$

$$F_x(x) = \begin{cases} 0 & x < -1 \\ x^3 + 1 & -1 \leq x < 0 \\ 1 & x \geq 0 \end{cases}$$

MEDIA

$$E(x) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$= \int_{-1}^{0} kx^2 \cdot x \, dx$$

$$= k \int_{-1}^{1} x^3 \, dx$$

$$= k \left[\frac{x^4}{4} \right]_{-1}^{0}$$

$$= 3\left[0 - \frac{1}{4}\right]$$

$$= -\frac{3}{4}$$

$$E(x^2) = \int_{-\infty}^{+\infty} f(x) \cdot x^2 \, dx$$

$$= \int_{-1}^{0} kx^2 \cdot x^2$$

$$= k \int_{-1}^{0} x^4$$

$$= k \left[\frac{x^5}{5} \right]_{-1}^{0}$$

$$= 3 \left[0 - \left(-\frac{1}{5} \right) \right]$$

$$= 3 \cdot \frac{1}{5}$$

$$= \frac{3}{5}$$

VARIANZA

$$E(x)^2 - (E\,x)^2$$

$$V_{1R}(x) = \frac{3}{5} - \left(-\frac{3}{4}\right)^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80}$$

$$= \frac{3}{80}$$

$V \quad x^3$

3

$$Y = X^3$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^3 \leq y) = P(X \leq y^{\frac{1}{3}}) = y + 1$$

$$F_y(y) = \begin{cases} 0 & y < 0 \\ y+1 & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

Funzione di densità

$$f(y) = \frac{d}{d(y)} F_y$$

$$= \frac{d}{dy}\, y+1$$

$$= 1$$

$$f(y) \begin{cases} \underline{1} & -1 \leq y \leq 1 \\ 0 & \text{altrove} \end{cases}$$