Esercizio 16

venerdì 7 maggio 2021 17:38

3 URNE :

LE PRIME {4 biglie BIANCHE _ 4 biglie NERE}

LA SECONDA {10 BIANCHE, 2 NERE}

LA TERZA {5 BIANCHE, 5 NERE

Si estrae una biglia da un urna a CASO

E_i := { LA BIGLIA è ESTRATTA DALLA URNA i-esima } tale che $i = \{1,2,3\}$

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$

B = LA Biglia ESTRATTA è BIANCA

N = LA Biglia ESTRATTA è NERA

P(B) e P(N)

P(B)

ⓐ $P(B|E_1) = P(B)$ → 4B, 4N, $\Omega = 8$

$P(B) = \frac{|B|}{|\Omega|} = \frac{4}{8} = \frac{1}{2}$

ⓑ $P(B|E_2) = P(B)$ → 10B, 2N, $\Omega = 12$

ⓑ $P(B|E_2) = P(B)$ → 2N, Ω = 12

$P(B) = \frac{|B|}{|\Omega|} = \frac{10}{12} = \frac{5}{6}$

ⓒ $P(B|E_3) = P(B)$ → 5B, 5N, Ω = 10

$P(B) = \frac{|B|}{|\Omega|} = \frac{5}{10} = \frac{1}{2}$

$P(B) = P(B|E_1)\ P(E_1) + P(B|E_2) \cdot P(E_2) + P(B|E_3) \cdot P(E_3)$

$= \frac{1}{2} \cdot \frac{1}{3} + \frac{5}{6} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{6} + \frac{5}{18} + \frac{1}{6}$

$= \frac{3+5+3}{18}$

$= \frac{11}{18}$

$\overline{P(N)}$

ⓐ $P(N|E_1) = P(N)$ → 4B, 4N, Ω = 8

$P(N) = \frac{|N|}{|\Omega|} = \frac{4}{8} = \frac{1}{2}$

ⓑ $P(N|E_2) = P(N)$ → 10B, 2N, Ω = 12

$P(N) = \frac{|N|}{|\Omega|} = \frac{2}{12} = \frac{1}{6}$

ⓒ $P(N|E_3) = P(N)$ → 5B, 5N

$P(N) = \frac{|N|}{\quad} = \frac{5}{\quad} = \frac{1}{\quad}$

ⓒ $P(N|E_3) = P(N)$

$P(N) = \frac{|N|}{|\Omega|} = \frac{5}{10} = \frac{1}{2}$ → S B, S N, $\Omega = 10$

$P(N) = P(N|E_1) \cdot P(E_1) + P(N|E_2) \cdot P(E_2) + P(N|E_2) P(E_2)$

$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3}$

$= \frac{1}{6} + \frac{1}{18} + \frac{1}{6}$

$= \frac{3+1+3}{18}$

$= \frac{7}{18}$

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② $P(E_i|B)$ Teorema di Bayes

ⓐ $P(E_2|B) = \frac{P(B|E_1) \cdot P(E_1)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{11}{18}} = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{18}{11} = \frac{1}{\cancel{6}_1} \cdot \frac{\overset{3}{\cancel{18}}}{11} = \frac{3}{11}$

ⓑ $P(E_2|B) = \frac{P(B|E_2) \cdot P(E_2)}{P(B)} = \frac{\frac{5}{6} \cdot \frac{1}{3}}{\frac{11}{8}} = \frac{5}{6} \cdot \frac{1}{3} \cdot \frac{18}{11} = \frac{5}{\cancel{18}} \cdot \frac{\cancel{18}}{11} = \frac{5}{11}$

ⓒ $P(E_3|B) = \frac{P(B|E_3) \cdot P(E_3)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{11}{8}} = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{18}{11} = \frac{1}{\cancel{6}} \cdot \frac{\overset{3}{\cancel{18}}}{11} = \frac{3}{11}$