Esercizio 5

venerdì 14 maggio 2021 19:02

X v.a. DISCRETA

Valori di $X = (2, 4, 6)$

$p(x=2) = 0.5$

$p(x=4) = 0.4$

$p(x=6) = c$

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① VALORE di c

$c = 1 - 0.4 - 0.5$

$= 0.1$

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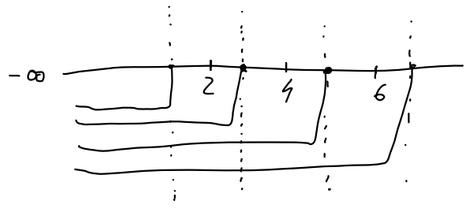
②

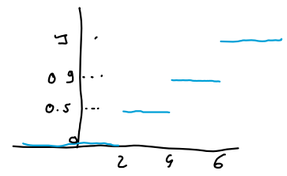
$F_X(x)$

$F_X(x) = P(X \leq x) = P(\omega \in \Omega : X(\omega) \leq x)$ per $\forall x \in \mathbb{R}$

$$F_X(x) = \begin{cases} P(\emptyset) = 0 & x < 2 \\ P(2) = 0.5 & 2 \leq x < 4 \\ P(2) + P(4) = 0.9 & 4 \leq x < 6 \\ P(\Omega) = 1 & x \geq 6 \end{cases}$$



GRAFICO



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③

MEDIA

$$E(X) = \sum_{\mu : x_\mu \in S} x_\mu \cdot p_X(x_\mu)$$

$$= x_1\, p(x_1) + x_2 \cdot p(x_2) + x_3\, p(x_3)$$

$$= 2 \cdot 0.5 + 4 \cdot 0.4 + 6 \cdot 0.1$$

$$= 1 + 1.6 + 0.6$$

$$= 3.2$$

VARIANZA

$$Var(X) = \sum_{i=1}^{K} (x_i - EM)^2\, p_i$$

oppure $\sum_{i=1}^{K} x_i^2\, p_i - E(x)^2$

$$Var(X) = (2-3.2)^2 \cdot 0.5 + (4-3.2)^2 \cdot 0.4 + (6-3.2)^2 \cdot 0.1$$

$$= 1.44 \cdot 0.5 + 0.64\ 0.4 + 7.84 \cdot 0.1$$

$$= 0.72 + 0.256 + 0.784$$

$$= 1.76$$

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④

$Y = X + b$

$X + b$ = è STRETTAMENTE MONOTONA

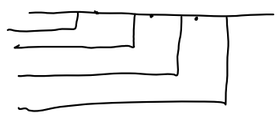
$$P(2+b) = P(x+b) = P(x=2)$$

$$P(4+b) = P(x+b) = P(x=4)$$

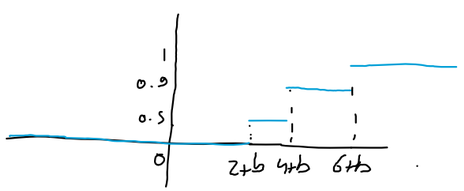
$$P(6+b) = P(x+b) = P(x=6)$$

$$F_Y(y) = P(Y \le y) = P(\omega \in \Omega \cdot X(\omega) \le y) \quad \forall y \in \mathbb{R}$$

$$F_Y(y) = \begin{cases} \varnothing & 0 & y < 2+b \\ \{2+b\} & 0.5 & 2+b \le y < 4+b \\ \{2+b, 4+b\} & 0.9 & 4+b \le y < 6+b \\ \Omega & 1 & y \ge 6+b \end{cases}$$



GRAFICO



$$E(Y) = (2+b)\cdot 0.5 + (4+b)\,0.4 + (6+b)\cdot 0.1$$
$$= 1 + 0.5b + 1.6 + 0.4b + 0.6 + 0.1b$$
$$= 3.2 + b$$

$$\text{Var}(Y) = (4 + b^2 + 4b)\,0.5 + (16 + b^2 + 8b)\cdot 0.4 + (36 + b^2 + 12b)\,0.1$$
$$- (10{,}25 + b^2 + 6{,}4\,b)$$

$$= 2 + 0.5\,b^2 + 2b + 6.4 + 0.4b^2 + 3.2\,b + 3.6 + 0.1\,b^2 + 1.2\,b - 10.25 + b^2 - 6.5\,b$$

$$= 1.76$$