Esercizio 14

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$$f(x) = \begin{cases} kx^3 & 0 < x < 4 \quad x \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$K = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_0^4 kx^3 \, dx$$

$$k \int_0^4 x^3 \, dx$$

$$k \left[\frac{x^4}{4} \right]_0^4 dx$$

$$k \left[\frac{256}{4} - 0 \right]$$

$$k \cdot 64 = 1$$

$$k = \frac{1}{64}$$

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② Funzione di Distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\,dx$$

$$F_x = \int_0^x kx^3 \, dx$$

$$= k \int_0^x x^3 \, dx$$

$$= k \left[\frac{x^4}{4} \right]_0^x$$

$$= \frac{1}{} \left[\frac{x^4}{} - 0 \right]$$

$$= \frac{1}{64}\left[\frac{x^4}{4} - 0\right]$$

$$= \frac{1}{64} \cdot \frac{x^4}{4}$$

$$= \frac{x^4}{256}$$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ \frac{x^4}{256} & 0 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$$

MediA

$$E(X) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

4

$$= \int_0 kx^3 \cdot x$$

$$= k \int_0^4 x^4$$

$$= k \left[\frac{x^5}{5} \right]_0^4$$

$$= k \left[\frac{1024}{5} - 0 \right]$$

$$= \frac{1}{\cancel{64}_{\,1}} \cdot \frac{\cancel{1024}^{\,16}}{5}$$

$$= 1 \cdot \frac{16}{5}$$

$$Ex = \frac{16}{5}$$

$$E(x^2) = \int_{-\infty}^{+\infty} f(x) \cdot x^2 \, dx$$

$$= \int_0^4 k x^3 \cdot x^2 \, dx$$

$$= k \int_0^4 x^5 \, dx$$

$$= k \left[\frac{x^6}{6} \right]_0^4$$

$$= \frac{1}{64}\left[\frac{4096}{6} - 0\right]$$

$$= \frac{1}{\cancel{64}_{\,1}} \cdot \frac{\cancel{4096}^{\,64}}{6}$$

$$= \frac{64}{6}$$

$$= \frac{32}{3}$$

VARIANZA

$$E(x)^2 - (E\,x)^2$$

$$V_{1r}(x) = \frac{32}{3} - \left(\frac{16}{5}\right)^2$$

$$= \frac{32}{3} - \frac{256}{25}$$

$$= \frac{800 - 768}{75}$$

$$= \frac{32}{75}$$

$$Y = x^2$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^2 \leq y) = P(X \leq y^{\frac{1}{2}}) = \frac{y^{4/2}}{256} = \frac{y^2}{256}$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^2}{256} & 0 \leq y < 16 \\ 1 & y \geq 16 \end{cases}$$

Funzione di densità

$$f_{(y)} = \frac{d}{d(y)} F_y$$

$$= \frac{d}{dy} \frac{y^2}{256}$$

$$= \frac{\cancel{2}y}{\cancel{256}}$$

$$= \frac{y}{128}$$

$$f(y) \begin{cases} \dfrac{y}{128} & 0 \le y \le 5 \\ 0 & \text{above} \end{cases}$$