Esercizio 7

sabato 22 maggio 2021 17:09

$$P(X=-1)=\frac{1}{3}$$

$$P(X=0)=\frac{1}{3}$$

$$P(X=1)=\frac{1}{3}$$

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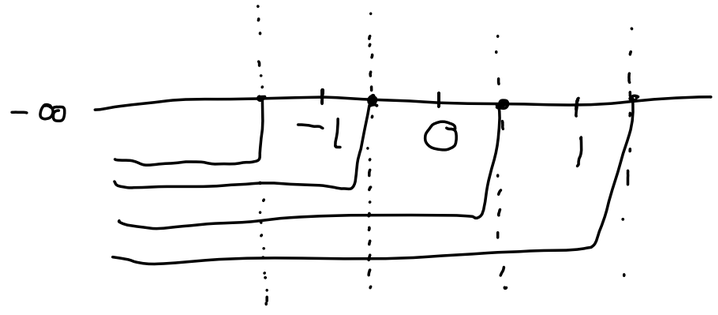
(1)

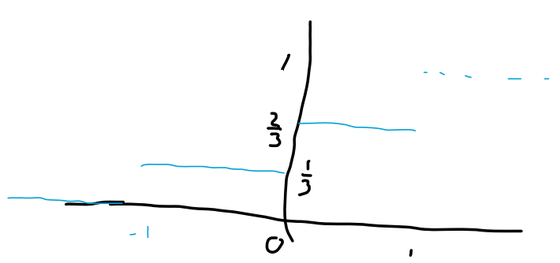
$F_X(x)$

$F_X(x)=P(X\leq x)=P(\omega\in\Omega \cdot X(\omega)\leq x)$ per $\forall x\in\mathbb{R}$

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{3} & -1 \le x < 0 \\ \frac{2}{3} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$



GRAFICO



③

MEDIA

$$E(X) = \sum_{\mu : x_\mu \in S} x_\mu \cdot p_X(x_\mu)$$

$$x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3\ p(x_3)$$

$$-1 \quad \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}$$

$$-\frac{1}{3} + \frac{1}{3} = 0$$

$$-\frac{1}{3} + \frac{1}{3} = 0$$

VARIANZA

$$Var(X) = \sum_{i=1}^{k} (x_i - E[X])^2 \, p_i$$

oppure $\sum_{i=1}^{k} x_i^2 \, p_i - E(x)^2$

$$Var(X) = (-1-0)^2 \cdot \frac{1}{3} + (0-0)^2 \cdot \frac{1}{3} + (1-0)^2 \cdot \frac{1}{3}$$

$$= 1 \cdot \frac{1}{3} + 0 + 1 \cdot \frac{1}{3}$$

$$= \frac{1}{3} + \frac{1}{3}$$

$$= \frac{2}{3}$$

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④

$Y = e^{x^2}$

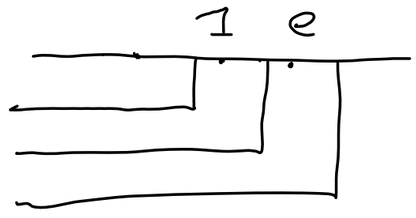
e^{x^2} = è STRETTAMENTE MONOTONA

$$P(Y=e) = P(e^{1^2}) = P(x=-1) = P(x=1) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$P(Y=1) = P(e^{0^2}) = P(x=0) = \frac{1}{3}$$

$$F_Y(y) = P(Y \le y) = P(\omega \in \Omega \cdot X(\omega) \le y) \quad \text{per } \forall y \in \mathbb{R}$$

$$F_Y(y) = \begin{cases} \{\varnothing\} & & y < 1 \\ \{1\} & \frac{1}{3} & 1 \le y < e \\ \Omega & 1 & y \ge e \end{cases}$$



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