Esercizio 15

martedì 15 giugno 2021 16:28

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2} & 0<x<y<2 \\ 0 & \text{altrove} \end{cases}$$

1

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dy$$

$$= \int_x^2 \frac{1}{2}\, dy = \frac{1}{2}\int_x^2 dy = \frac{1}{2} y \Big|_x^2 = 1 - \frac{1}{2}x$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\, dx = \int_0^y \frac{1}{2}\, dx = \int_0^y \frac{1}{2}\, dx = \frac{1}{2} x \Big|_0^y = \frac{1}{2} y$$

$$f_X(x) = \begin{cases} 1 - \frac{1}{2}x & 0<x<2 \\ 0 & \text{altrove} \end{cases}$$

$$f_Y(y) = \begin{cases} +\frac{1}{2} y & 0<y<2 \\ 0 & \text{altrove} \end{cases}$$

$$E_X = \int^{\infty} x f_X(x)\, dx = \int^{2} x\left(1 - \frac{1}{2}x\right) dx = \int^{2} x\, dx - \int^{2} \frac{x^2}{2}\, dx = \frac{x^2}{2} - \frac{x^3}{6} \Big|_0^2$$

$$Ex = \int_{-\infty} x f_X(x)\, dx = \int_0 x\left(1 - \frac{1}{2}x\right) dx = \int_0 x\, dx - \int_0 \frac{x^2}{2}\, dx = \frac{x^2}{2} - \frac{x^3}{6} \Big|_0$$

$$= \frac{2^2}{2} - \frac{2^3}{6} - \left(\frac{0^2}{2} - \frac{0^3}{6}\right) = \frac{2}{3}$$

$$Ex^2 = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx = \int_0^2 x^2\left(1 - \frac{1}{2}x\right) = \int_0^2 x^2 - \frac{x^3}{2} = \int_0^2 x^2 - \frac{1}{2}\int_0^2 x^3\, dx$$

$$= \left(\frac{x^3}{8} - \frac{x^5}{8}\right)\Big|_0^2 \quad \frac{2^3}{8} - \frac{2^5}{8} - \left(\frac{0}{8} - \frac{0}{8}\right) = \cancel{\frac{2}{3}}$$

$$0$$

$$E y = \int_0^2 y \left(\frac{1}{2} y\right) dy = +\frac{1}{2}\int_0^2 y^2\, dy = +\frac{1}{2}\, \frac{y^3}{3}\Bigg|_0^2 = +\frac{2^3}{6} - \frac{0^3}{6} = \frac{4}{3}$$

$$E y^2 = \int_0^2 y^2 \left(+\frac{1}{2} y\right) dy = +\frac{1}{2}\int_0^2 y^3 = +\frac{1}{2}\, \frac{y^4}{4}\Bigg|_0^2 = \frac{2^4}{8} - \left(\frac{0^4}{8}\right) = +2$$

$$\text{Var } y.\ +2 - \left(+\frac{4}{3}\right)^2 = +2 - \frac{16}{9} = \frac{2}{9}$$

2

$$f_{XY}(x,y) = \frac{1}{2} \overset{?}{=} f_X(x)\, f_Y(y) = \left(1 - \frac{1}{2}x\right)\left(\frac{1}{2}y\right) =$$

$f_{xy}(x,y) = 2 \neq f_x(x) f_y(y)$

NON INDIPENDENTI

$Cov(X,Y) = E(XY) - E_X E_Y = 1 - \left(\frac{2}{3} \cdot \frac{4}{3}\right) = 1 - \frac{8}{9} = \frac{1}{9}$

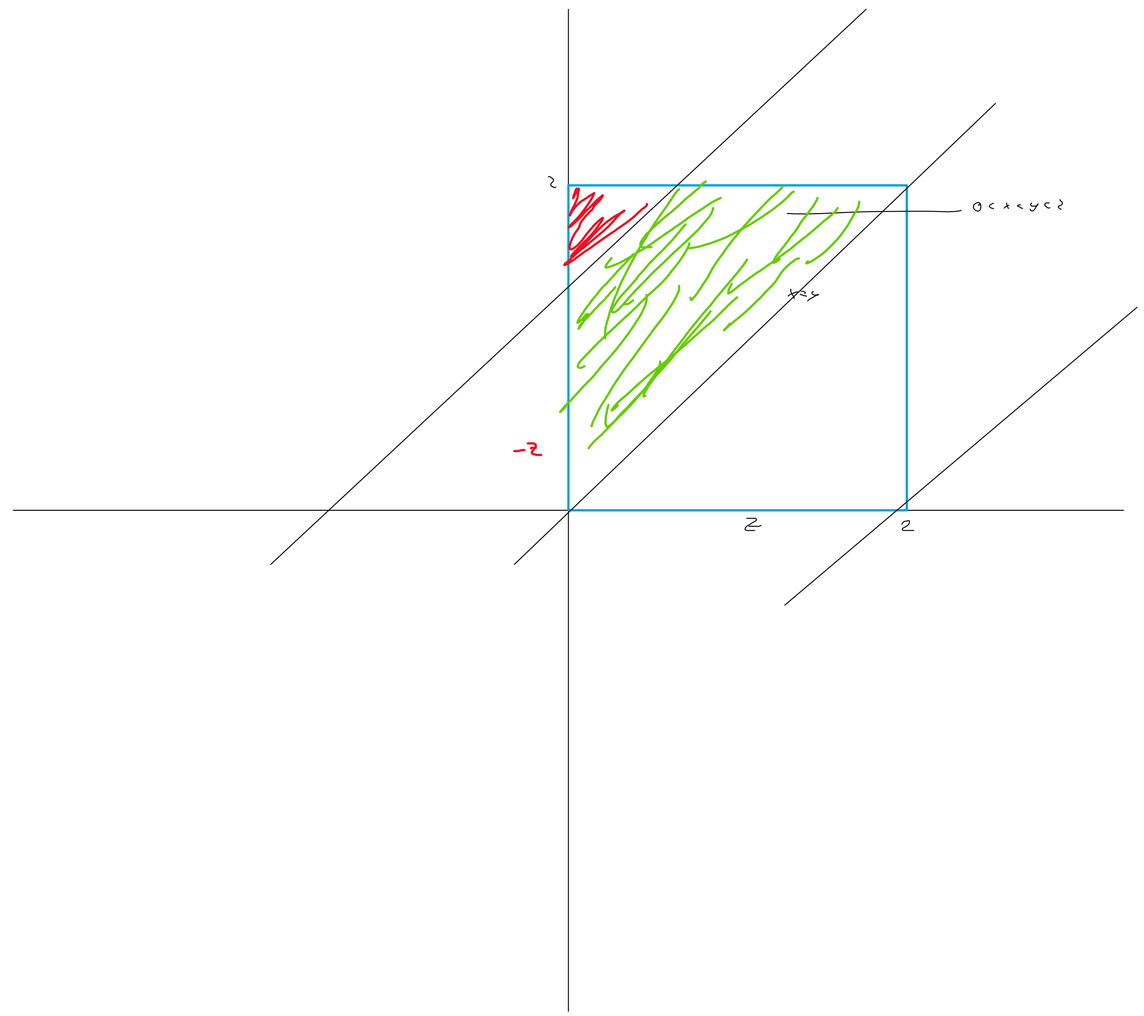
$E_{XY} = \int_0^2 x f_{xy}(x,y)\,dx = \int_0^2 x \frac{1}{2}\,dx = \int_0^2 \frac{x^2}{2}\,dx = 1$

3

$Z = X - Y \qquad 0 < x < y < 2$

$F_Z(z) = P(Z \leq z) = P(X - Y \leq z)$

Esercizio Blocco 3



$$F_Z(z) = \begin{cases} 0 & z < -2 \\ \frac{(2+z)^2}{4} & -2 < z < 0 \\ 1 & z \geq 0 \end{cases}$$

$$f_Z(z) : \begin{cases} \frac{z}{2} + 1 & -2 < z < 0 \\ 0 & \text{altrove} \end{cases}$$

$$EZ : \int_{-\infty}^{\infty} z \, f_Z(z) \, dz \quad \int_{-2}^{0} z\left(\frac{z}{2}+1\right) dz = \int_{-2}^{0} \frac{x^2}{2} + x \, dx = \int_{-2}^{0} \frac{x^2}{2} + \int_{-2}^{0} x$$

$$= \frac{x^3}{6} + \frac{x^2}{2} \Big|_{-2}^{0}$$

$$= \frac{0^3}{6} + \frac{0^2}{2} - \left(\frac{-8}{6} + \frac{4}{2}\right) = -\frac{2}{3}$$

$$E z^2 = \int_{-2}^{0} z^2 \left(\frac{z}{2} + 1\right) dz = \int_{-2}^{0} \frac{z^3}{2} + \int_{-2}^{0} z^2 = \frac{z^4}{8} + \frac{z^3}{3} \Big|_{2}^{0} = -\frac{2}{3}$$

$$VAR\, z = \frac{2}{3} - \left(-\frac{2}{3}\right)^2 = \frac{2}{3} - \frac{4}{9} = \frac{6-4}{9} = \frac{2}{9}$$