Esercizio 18

sabato 22 maggio 2021 17:12

$$f(x) = \begin{cases} k x^3 & -1 < x < 0 \qquad k > 0 \\ 0 & \text{altrove} \end{cases}$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

(1)

$$\int_{-1}^{0} k x^3\,dx = 1$$

$$k \int_{-1}^{0} x^3\,dx = 1$$

$$k \left[\frac{x^4}{4} \right]_{-1}^{0} = 1$$

$$k\left[\frac{0}{0}-\frac{1}{5}\right]=1$$

$$-\frac{1}{5}k=1$$

$$k=-\frac{1}{\frac{1}{5}}$$

$$k=-\frac{1}{1}\cdot\frac{5}{1}=-5$$

$$k=-5$$

---

(2)

$$F_x=\int_{-\infty}^{x} f(x)\,dx$$

$$F_x=\int_{-1}^{x} kx^3\,dx$$

$$F_x=k\int_{-1}^{x} x^3\,dx$$

$$Fx = k\left[\frac{x^5}{5}\right]_{-1}^{x}$$

$$Fx = k\left[\frac{x^5}{5} - \frac{1}{5}\right]$$

$$Fx = k\left[\frac{x^5-1}{5}\right]$$

$$Fx = -5\left[\frac{x^5-1}{5}\right]$$

$$Fx = -x^5+1$$

$$F_x(X) = \begin{cases} 0 & x < -1 \\ -x^4+1 & -1 \leq x < 0 \\ 1 & x \geq 0 \end{cases}$$

MEDIA

$+\infty$

$$\int_{-\infty} F(x) \cdot x \, dx$$

$$\int_{-1}^{0} -4x^3 \cdot x \, dx$$

$$-4 \int_{-1}^{6} x^4 \, dx$$

$$-4 \left[\frac{x^5}{5} \right]_{-1}^{6}$$

$$-4 \left[0 - \left(-\frac{1}{5} \right) \right]$$

$$-4 \cdot \frac{1}{5} = \frac{-4}{5}$$

Media x^2

$$\int_{-1}^{0} -4x^3 \cdot x^2 \, dx$$

$$-4 \int_{-1}^{0} x^5 \, dx$$

$$-4 \left[\frac{x^6}{6} \right]_{-1}^{0}$$

$$-4\left[0-\frac{1}{6}\right]$$

$$-4\cdot-\frac{1}{6}=\frac{2}{3}$$

VARIANZA

$$Var(x)=\sum_{i=1}^{n}(x_i-E[x])^2\,p_i$$

oppure

$$\sum_{i=1}^{n}x_i^2\,p_i-E[x]^2$$

$$E(X)^2 - (EX)^2$$

$$\text{VAR}(X) = \frac{2}{3} - \left(\frac{4}{5}\right)^2$$

$$= \frac{2}{3} - \frac{16}{25}$$

$$= \frac{50 - 48}{75}$$

$$= \frac{2}{75}$$

$$\|v\| = |v|^{\frac{1}{2}}$$

$$Y = |X|^{\frac{1}{2}}$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^{\frac{1}{2}} \leq y) = P(X \leq y^2) = (-y^8 + 1)$$

$$F_Y(Y) = \begin{cases} 0 & y < 0 \\ -y^8 + 1 & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

Funzione Di Distribuzione

$f(y) = \frac{d}{d(y)} \quad Fy =$

$= -8y^7$

$f(y) \begin{cases} -8y^7 & -1 \leq y < 0 \\ 0 & \text{altrove} \end{cases}$