Esercizio 17

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$$f(x) = \begin{cases} kx^3 & -2 < x < 0 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$K = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int^{0} kx^3 \quad dx$$

$$k \int_{-2}^{0} x^3 \quad dx$$

$$k \left[\frac{x^4}{4} \right]_{0}^{-2}$$

$$k \left[\frac{x^4}{4} - \frac{x^4}{4} \right]_{-2}^{0}$$

$$k \left[\frac{0^4}{4} - \frac{(-2)^4}{4} \right]$$

$$k \left[-4 \right]$$

$$k \cdot -4 = 1$$

$$k \cdot 4 = 1$$

$$k = -\frac{1}{4}$$

② Funzione di Distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\, dx$$

$$F_x = \int_{-2}^{x} k x^3 \, dx$$

$$F_x = k \int_{-2}^{x} x^3 \, dx$$

$$F_x = -\frac{1}{4} \int_{-2}^{x} x^3$$

$$F_x = -\frac{1}{4} \cdot \left[\frac{x^4}{4} - \frac{x^4}{4} \right]_{-2}^{x}$$

$$= -\frac{1}{4} \quad \frac{x^4}{4} - 4$$

$$= -\frac{1}{4} \quad \frac{x^4 - 16}{4}$$

$$= \frac{16 - x^4}{16}$$

$$F_x(x) = \begin{cases} 0 & x < -2 \\ \dfrac{16 - x^4}{16} & -2 \leq x \leq 0 \\ 1 & x \geq -2 \end{cases}$$

MediA

$$E(x) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$\int_{-2}^{0} k \, x^{3} \, x \, dx$$

$$k \int_{-2}^{0} x^{5} \, dx$$

$$k \int_{-2}^{0} x^{5} \, dx$$

$$-\frac{1}{4}\left[\frac{x^5}{5}-\frac{x^5}{5}\right]_{-2}^{0}$$

$$-\frac{1}{4}\left[0-\left(-\frac{32}{5}\right)\right]$$

$$-\frac{1}{4}\quad\frac{32}{5}$$

$$\cancel{37}\quad 8$$

$$-\frac{\cancel{32}^{\,8}}{\cancel{20}_{\,5}}$$

$$E(x) = -\frac{8}{5}$$

$$E(x^2) = \int_{-\infty}^{+\infty} f(x)\cdot x^2\,dx$$

0

$$= \int_{-2} k\, x^{3}\, x^{2}\, dx$$

$$= k \int_{-2}^{0} x^{5}\, dx$$

$$= k \left[\frac{x^{6}}{6} \right]_{-2}^{0}$$

$$= -\frac{1}{4} \left[0 - \frac{64}{6} \right]$$

$$= -\frac{1}{4} \cdot \left(-\frac{64}{6}\right)$$

$$= \frac{64}{24}$$

$$= \frac{8}{3}$$

VARIANZA

$$E(x)^2 - (E x)^2$$

$$Var(x) = \frac{8}{3} - \left(-\frac{8}{5}\right)^2$$

$$= \frac{8}{3} - \frac{65}{25}$$

$$= \frac{8}{75}$$

$$y = x^{2}$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^2 \leq y) = P(X \leq y^{\frac{1}{2}}) = \frac{16 - \left(y^{\frac{1}{2}}\right)^4}{16} = \frac{16 - y^2}{16}$$

$$F_Y(y) = \begin{cases} 0 & y \leq 0 \\ \dfrac{16 - y^2}{16} & 0 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

FUNZIONE DI DENSITÀ

$$f(y) = \frac{d}{d(y)} F_Y$$

$$= \frac{d}{d(y)} \frac{16 - y^2}{16}$$

$$= -\frac{1}{8} y$$

$$f(y) \begin{cases} & 0 < y \leq 3 \\ 0 & \text{altrove} \end{cases}$$