Esercizio 12

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$$f(x) = \begin{cases} kx^{\frac{1}{2}} & -1 < x < 1 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$K = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_{-1}^{1} kx^2 \; dx$$

$$k \int_{-1}^{1} x^2 \; dx$$

$$k \left[\frac{x^3}{3} \right]_{-1}^{1}$$

$$k \left[\frac{1^3}{3} - \frac{-1^3}{3} \right]$$

$$k \left[\frac{1}{3} + \frac{1}{3} \right]$$

$$k \left[\frac{2}{3} \right]$$

$$k \, \frac{2}{3} = 1$$

$$k = \frac{1}{\frac{2}{3}}$$

$$k = 1 \cdot \frac{3}{2}$$

$$k = \frac{3}{2}$$

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② Funzione di Distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\,dx$$

x

$$F_x = \int_{-1} k\, x^2 \, dx$$

$$= k \int_{-1}^{x} x^2 \, dx$$

$$= k \left[\frac{x^3}{3} \right]_{-1}^{x}$$

$$= k \left[\frac{x^3}{3} + \frac{1}{3} \right]$$

$$= \frac{\cancel{3}}{2} \left[\frac{x^3 + 1}{\cancel{3}} \right]$$

$$= \frac{x^3 + 1}{2}$$

$$F_x(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x^3+1}{2} & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Media

$$E(x) = \int_{-\infty}^{+\infty} f(x) \cdot x \, dx$$

$$= \int_{-1}^{1} kx^2 \cdot x \, dx$$

$$= k \int_{-1}^{1} x^3 \, dx$$

$$= k \left[\frac{x^4}{} \right]^1$$

$$= k\left[\frac{x^4}{4}\right]_{-1}$$

$$= \frac{3}{2}\left[\frac{1}{4} - \frac{1}{4}\right]$$

$$= \frac{3}{2} \cdot 0$$

$$= 0$$

$$E(x^2) = \int_{-\infty}^{+\infty} F(x) \cdot x^2 \, dx$$

$$= \int_{-1}^{1} k x^2 \; x^2$$

$$= k \int x^4$$

$$= \frac{3}{2} \left[\frac{x^5}{5} \right]_{-1}^{1}$$

$$= \frac{3}{2} \left[\frac{1}{5} + \frac{1}{5} \right]$$

$$= \frac{3}{2} \cdot \left(\frac{2}{5} \right)$$

$$= \frac{3}{2} \cdot \left(\frac{2}{5} \right)$$

$$= \frac{3}{5}$$

VARIANZA

$$E(x)^2 - (E\,x)^2$$

$$Var(x) = \frac{3}{5} - (\theta)^2$$

$$= \frac{3}{5}$$

$$Y = X^2$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^2 \le y) = P(X \le y^{\frac{1}{2}}) = \frac{(y^{\frac{1}{2}}) + 1}{2} = \frac{y^{\frac{3}{2}} + 1}{2}$$

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ y^{\frac{3}{2}} + 1 & \\ 1 & y \ge 1 \end{cases}$$

Funzione di densità

$$f(y) = \frac{d}{} F_Y$$

$$f(y) = \frac{d}{d(y)} F_Y$$

$$= y^{\frac{3}{2}} + 1 \; dx$$

$$= \frac{3}{4} y^{\frac{1}{2}}$$

$$f(y) \begin{cases} \frac{3}{4} y^{\frac{1}{2}} & -1 \le y \le 1 \\ 0 & \text{altrove} \end{cases}$$