Esercizio 11
sabato 12 giugno 2021 12:47

$X_1, X_2 \sim \mathcal{U}(1,2)$

$$f_{X_2}(x_2) = f_{X_1}(x_1) = \begin{cases} 1 & x \in (1,2) \\ 0 & \text{altrimenti} \end{cases}$$

$$F_{X_2}(x_2) = F_{X_1}(x_1) = \begin{cases} 0 \\ \int_1^x dx \\ 1 \end{cases} = \begin{cases} 0 & x < 1 \\ x-1 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

- $U = \max(X_1, X_2)$

$F_U(w) = P(\max(X_1, X_2) \le w) = P(X_1 \le w, X_2 \le w) =$

SONO INDIPENDENTI

indipendenti $= P(X_1 \le w)\, P(X_2 \le w)$

SONO IDENTICAMENTE DISTRIBUITE quindi è equivalente $X_1 = X_2$

i.d. $= P[(F_X(w))^2] = \begin{cases} 0 & w < 1 \\ (w-1)^2 & 1 \le w < 2 \\ 1 & w \ge 2 \end{cases}$

- $V = \min(X_1, X_2)$

evento complementare

$F_V(v) = P(\min(X_1, X_2) \le v) = 1 - P(\min(X_1, X_2) > v)$

ind. e i.d.

$= 1 - [1 - F_X(v)]^2$

$$= \begin{cases} 0 \\ 1-(1-(v-1))^2 \\ 1 \end{cases} = \begin{cases} 0 \\ 1-(2-v)^2 \\ 1 \end{cases} \begin{cases} 0 & v < 1 \\ -v^2+4v-3 & 1 \le v < 2 \\ 1 & v \ge 2 \end{cases}$$

- $Y = U - V$

$F_Y(y) = \underbrace{P(\max(X_1, X_2) - \min(X_1, X_2) \le y)}_{A}$

NO INDIP

Si usa il teorema Alternativo

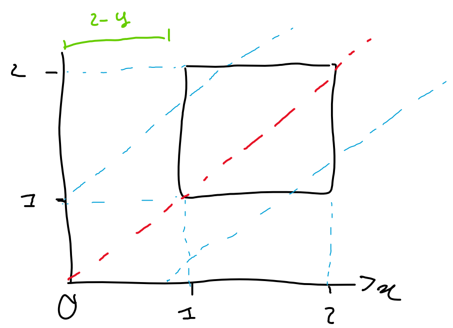
$= P(\max(X_1, X_2) - \min(X_1, X_2) \le y, \underbrace{X_1 < X_2}_{B})$

$+ P(\max(X_1, X_2) - \min(X_1, X_2) \le y, X_1 \ge X_2)$

$= P(X_2 - X_1 \le y, X_1 < X_2) + P(X_1 - X_2 \le y, X_1 \ge X_2)$

$$= P(X_2 - X_1 \leq \underset{1}{y},\ X_1 < X_2) + P(X_1 - X_2 \leq \underset{2}{y},\ X_1 \geq X_2)$$

$X_2 - X_1 = Y$



$0 \quad Y < 0$

$1 \quad Y \geq 1$

$$= 1 - 2\,\frac{(1-Y)^2}{2}$$

$$= 1 - (1-Y)^2$$

$$F_Y(y) = \begin{cases} 0 & Y < 0 \\ 1 - (1-Y)^2 & 0 \leq Y < 1 \\ 1 & Y \geq 1 \end{cases}$$

$$Z = U + V$$

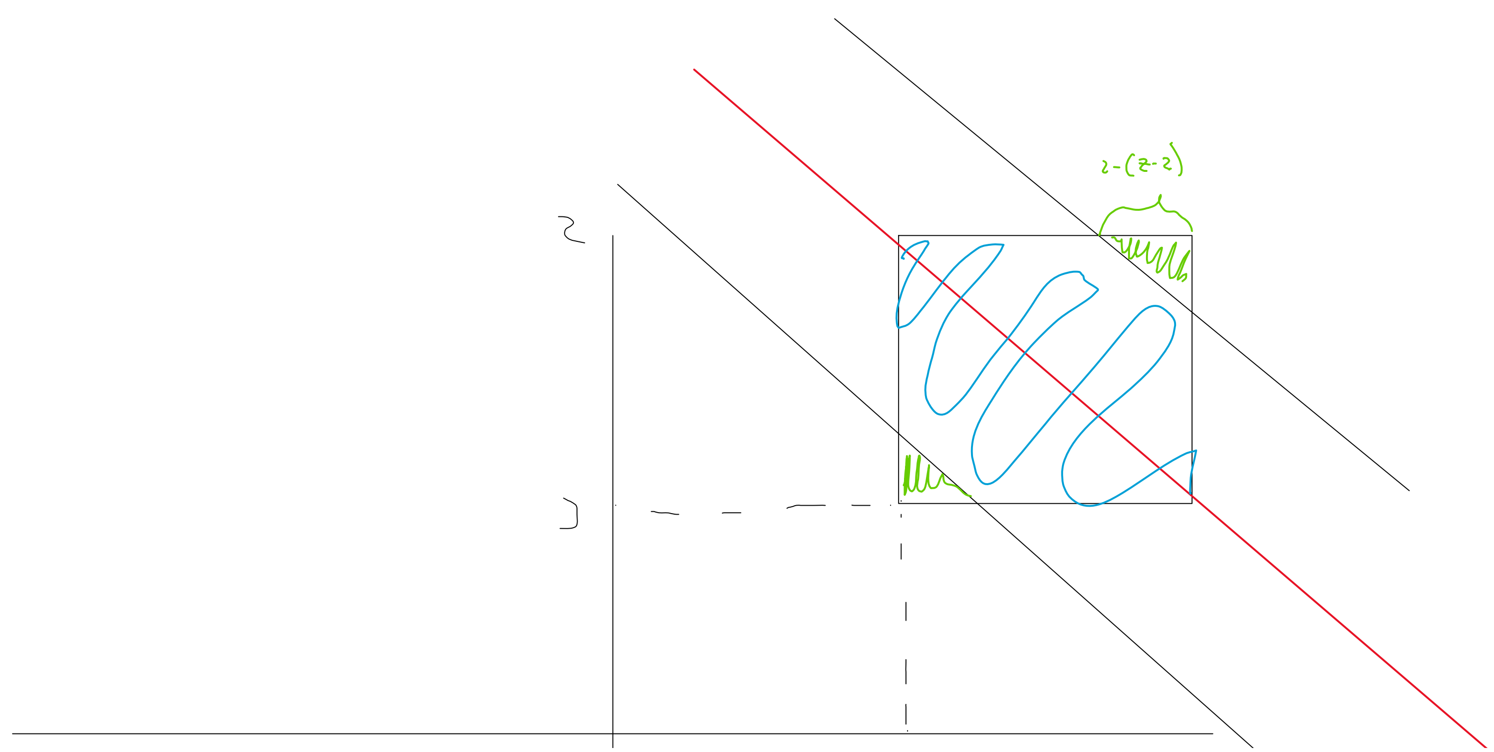
$$P_Z(z) = P\left(\max(X_1, X_2) + \min(X_1, X_2) \le z\right)$$

$$= P\left(\max(X_1, X_2) + \min(X_1, X_2) < z,\ X_1 > X_2\right) +$$

$$P\left(\max(X_1, X_2) + \min(X_1, X_2) < z,\ X_2 \geq X_1\right)$$

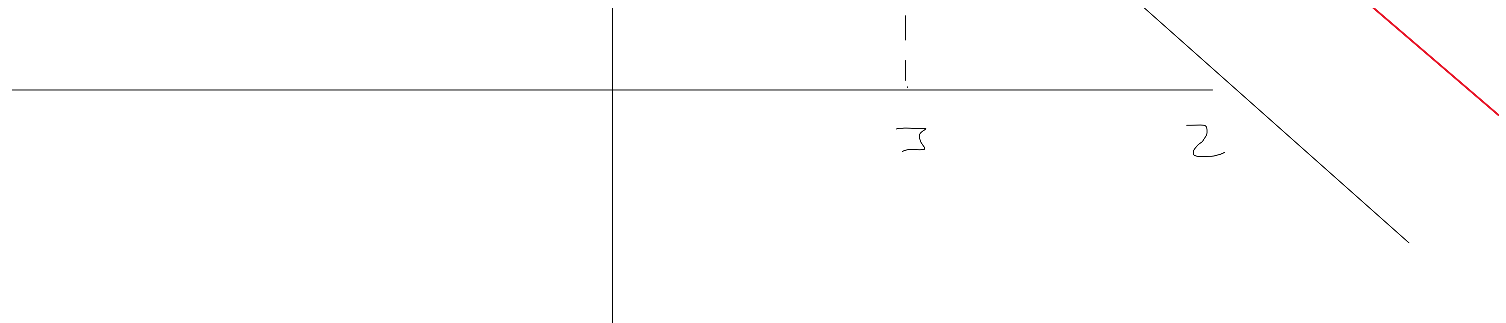
$$= P(X_1 + X_2 < z,\ X_1 > X_2) + P(X_1 + X_2 < z,\ X_2 \geq X_1)$$

$$= P\left[X_1 + X_2 \le z\right]$$



$$x = \begin{cases} x_2 = 2 \\ x_2 = z - x_1 \end{cases} = \begin{cases} x_2 = 2 \\ x_1 = z - x_2 \end{cases} \quad \begin{cases} x_2 = 2 \\ x_1 = z - 2 \end{cases}$$

$$\text{Area} = \frac{(2-(z-2))^2}{2} = \frac{(4-z)^2}{2}$$



$\int_0^{\cdot} 2-z \rightarrow \frac{(2-z)^2}{2}$ quando $2 \le z < 3$

$$F_Z(z) = \begin{cases} 0 & z < 2 \\ \frac{(z-2)^2}{2} & 2 \le z < 3 \\ 1 - \frac{(4-z)}{2} = \frac{-z^2+8z-14}{2} & 3 \le z < 4 \\ 1 & z \ge 4 \end{cases}$$