Esercizio 13

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$$f(x) = \begin{cases} kx^3 & 0 < x < 2 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$K = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$K = \int_0^2 kx^3\,dx = 1$$

$$= \int_0^2 kx^3\,dx$$

$$= k \int_0^2 x^3\,dx$$

$$= k \left[\frac{x^4}{4} \right]_0^2$$

$$= k \left[\frac{2^4}{4} - 0 \right]$$

$$= k \left[\frac{\cancel{16}^{\,4}}{\cancel{4}_{\,1}} \right]$$

$$= k\,4 = 1$$

$$= k = \frac{1}{4}$$

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(2) Funzione di Distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\,dx$$

$$F_x = \int_0^x x^3 k \, dx$$

$$= k \int_0^x x^3 \, dx$$

$$= \frac{1}{4} \left[\frac{x^4}{4} \right]_0^x$$

$$= \frac{1}{4} \left(\frac{x^4}{4} - 0 \right]$$

$$= \frac{1}{4} \left(\frac{x^4}{4} \right)$$

$$= \frac{x^4}{16}$$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^4}{16} & 0 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

MEDIA

$$E(x) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$= \int_{0}^{2} K \cdot x^3 \cdot x \, dx$$

$$= k \int_{0}^{2} x^4 \, dx$$

$$= \frac{1}{4} \left[\frac{x^5}{5} \right]_0^2$$

$$= \frac{1}{4}\left[\frac{32}{5} - 0\right]$$

$$= \frac{1}{\cancel{4}_{1}}\left(\frac{\cancel{32}^{8}}{5}\right)$$

$$= 1 \cdot \frac{8}{5}$$

$$= \frac{8}{5}$$

$$E(x^2) = \int_{-\infty}^{+\infty} F(x) \cdot x^2 \, dx$$

$$= \int^{2} kx^3 \cdot x^2$$

$$= \int_0^2 k x^3 \cdot x^2$$

$$= k \int_0^2 x^5$$

$$= \frac{1}{4} \left[\frac{x^6}{6} \right]_0^2$$

$$= \frac{1}{4} \left[\frac{64}{6} - 0 \right]$$

$$= \frac{1}{\cancel{4}_{1}} \left(\frac{\cancel{64}^{16}}{6} \right)$$

$$= 1 \cdot \frac{\cancel{16}^{8}}{\cancel{6}_{3}}$$

$$= 1 \cdot \frac{8}{3}$$

$= \frac{8}{3}$

VARIANZA

$E(x)^2 - (E x)^2$

$Var(x) = \frac{8}{3} - \left(\frac{8}{5}\right)^2$

$= \frac{8}{3} - \frac{64}{25}$

$$= \frac{8}{75}$$

$$Y = X^2$$

$$F_Y = P(Y \le y)$$

$$= y = P(X \le y) = P(X \le y^{\frac{1}{2}}) = \frac{y^{\frac{4}{2}}}{16} = \frac{y^2}{16}$$

3

$$F_y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^2}{16} & 0 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

Funzione di densità

$$f(y) = \frac{d}{d(y)} F_y$$

$$= \frac{y^2}{16} \, dx$$

$$= \frac{\cancel{2}^{1}\, y}{\cancel{16}\ 8}$$

$$= \frac{y}{8}$$

$$f(y) \begin{cases} \dfrac{y}{8} & 0 \le y \le 2 \\ 0 & \text{altrove} \end{cases}$$