Esercitazione 06-10

giovedì 6 ottobre 2022 14:09

① $\pi_S : S \times T \to S$
$(x,y) \mapsto x$

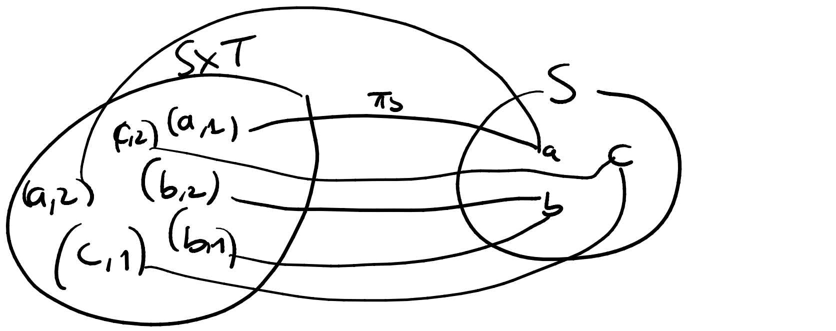
$\pi_T : S \times T \to T$
$(x,y) \mapsto y$

$\forall a \in S$ esiste un elemento di $S \times T$, (s,t) tale che $\pi_S(s,t) = a$. Questo vuol dire $s = a$. Tale coppia (con "a" come prima componente) deve esistere per definizione di prodotto cartesiano.

$S = \{a, b, c\}$
$T = \{1, 2\}$

$S \times T = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

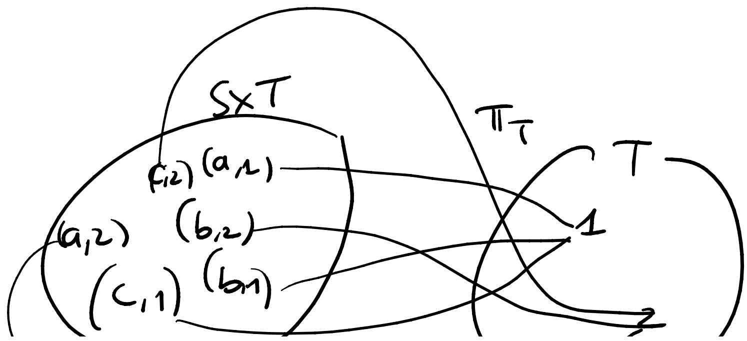
$\pi_S : S \times T \to S$
$(x,y) \mapsto x$

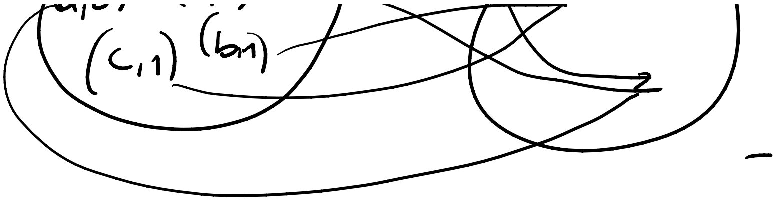


$\forall a, b, c$ $\exists$ una coppia

$\pi_T : S \times T \to T$
$(x,y) \mapsto y$

$\forall a \in T$ $\exists (s,t) \in S \times T$ tc $\underbrace{\pi_T(s,t)}_{t} = a$ $(\Leftrightarrow t = a)$





② $k: 5\mathbb{Z} \to 25\mathbb{Z}$ $\qquad$ $h: 25\mathbb{Z} \to 5\mathbb{Z}$

$z \mapsto z^2$ $\qquad\qquad$ $z \mapsto \frac{z}{5}$

$z \in 5\mathbb{Z} \iff z = 5n \,,\ n \in \mathbb{Z}$

$(5n)^2 = 25n^2 \in 25\mathbb{Z}$

$z \in 25\mathbb{Z} \iff z = 25n \,,\ n \in \mathbb{Z}$

$\frac{z}{5} = \frac{25n}{5} = 5n \in 5\mathbb{Z}$

(a) $k(\{-5, 0, 5, 10\}) = \{k(-5), k(0), k(5), k(10)\} =$

$= \{\underbrace{(-5)^2}_{=25}, 0^2, \underbrace{5^2}_{=25}, (10)^2\} = \{0, 25, 100\}$

$k(50\mathbb{Z}) = \{(50 \cdot s)^2 \mid s \in \mathbb{Z}\} = \{2500 \cdot s^2 \mid s \in \mathbb{Z}\}$

$50\mathbb{Z} = \{50 \cdot s \mid s \in \mathbb{Z}\}$

$k^{-1}(\{0, 25, -25, 50\}) = \{z \in 5\mathbb{Z} \mid k(z) \in \{0, 25, -25, 50\}\}$

$\{k^{-1}(0), k^{-1}(25), \underline{k^{-1}(-25)}, \underline{k^{-1}(50)}\}$

non esistono

$= \{0, \underset{-5}{5}, X, X\} = \{0, 5, -5\}$

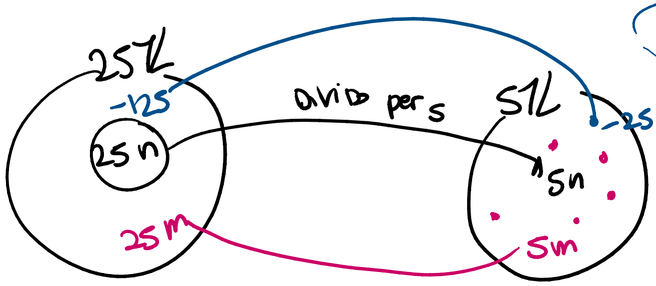
$h^{-1}(5\mathbb{Z}) = 25\mathbb{Z}$ $\qquad$ $h(z) = \frac{z}{5}$

$h^{-1}(5\mathbb{Z}) = \{z \in 25\mathbb{Z} \mid h(z) \in 5\mathbb{Z}\}$

$5n,\ n\in\mathbb{Z}$ esiste $s\in 25\mathbb{Z}$ tc $\underbrace{h(s)=5n}$?

$\frac{s}{5}=5n \Rightarrow s=25n$

h è suriettiva



$$\boxed{h(25\mathbb{Z})=5\mathbb{Z}}$$

$h(\{0,25,-50\})=\{h(0),h(25),h(-50)\}=\{0,\frac{25}{5},\frac{-50}{5}\}=\{0,5,-10\}$

Diviso per 5

$h^{-1}(\{0,25,50\})=\{0,125,250\}$

Moltiplicato per 5

(b) k non è iniettiva : $k(5)=k(-5)=25$

k non è suriettiva : ad esempio $50\notin Im(k)$ (non è quadrato) ma $50\in 25\mathbb{Z}$

oppure $-25\in 25\mathbb{Z}$ ma -25 non è quadrato

h è suriettiva (visto sopra)

h è iniettiva : $\forall\ x_1\neq x_2 \Rightarrow h(x_1)\neq h(x_2)$

$$\boxed{\forall\ h(x_1)=h(x_2)\Rightarrow x_1=x_2}$$

Sia $25n,\ 25m\in 25\mathbb{Z}$ $\quad h(25n)=h(25m)\Rightarrow$

$\Rightarrow \frac{25n}{\cancel{5}}=\frac{25m}{\cancel{5}} \Rightarrow 25n=25m$

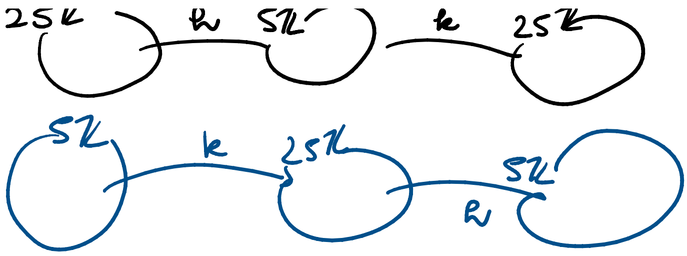
(c) l'inversa di h sarà $h^{-1}: 5\mathbb{Z}\to 25\mathbb{Z}$

$z\mapsto 5\cdot z$

(d) $k\circ h: 25\mathbb{Z}\to 25\mathbb{Z}$

$h\circ k: 5\mathbb{Z}\to 5\mathbb{Z}$

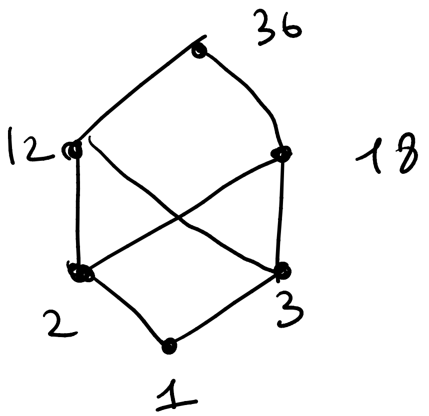
$h \circ k : 5\mathbb{Z} \to 5\mathbb{Z}$



$$(k \circ h)(z) = k(h(z)) = k\left(\frac{z}{5}\right) = \left(\frac{z}{5}\right)^2 = \frac{z^2}{25}$$

$$(h \circ k)(z) = h(k(z)) = h(z^2) = \frac{z^2}{5}$$

(7.1) $A = \{1, 2, 3, 12, 18, 36\}$ $\qquad (A, |)$



$\inf(a,b) = MCD(a,b)$

$\sup(a,b) = mcm(a,b)$

Non è un reticolo perché non esiste $\sup\{2,3\}$ $(2 \vee 3)$

$2 \vee 3 = \min(\text{maggioranti})$

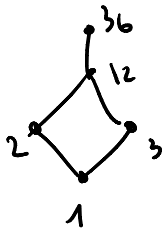
Maggioranti di $\{2,3\} = \{12, 18, 36\}$

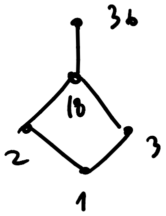
Poiché 12 e 18 non sono confrontabili, non esiste $\min\{12, 18, 36\}$

Vediamo anche che non esiste $12 \wedge 18$

minoranti $\{12, 18\} = \{2, 3, 1\}$. Poiché 2 e 3 non sono confrontabili

non esiste $\max\{2, 3, 1\}$.


36
12
2
3
1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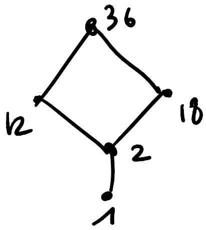
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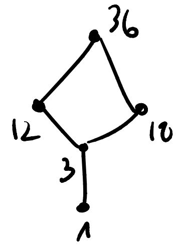
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