Esercizio 6

X v.a   DISCRETA

VALORi di $X = (2, 4, 6)$

$P(X = 2) = 0.5$

$P(X = 4) = 0.4$

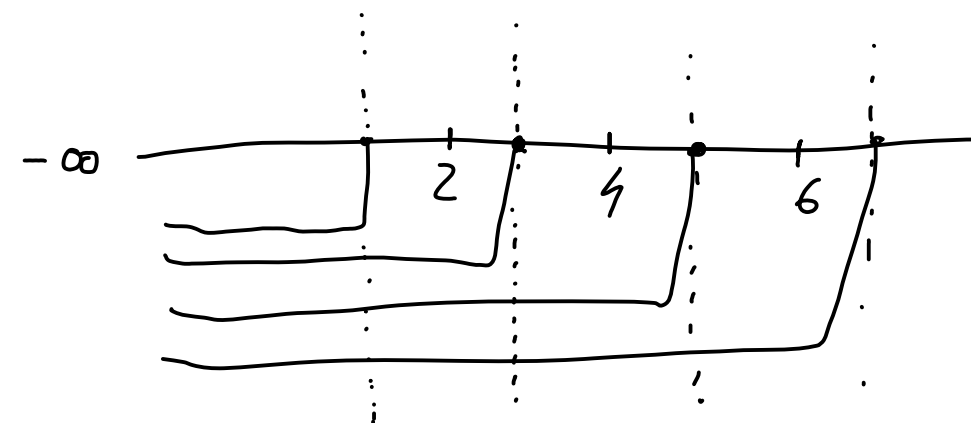$P(X = 6) = c$

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① VALORE di c

$c = 1 - 0.4 - 0.5$

$= 0.1$
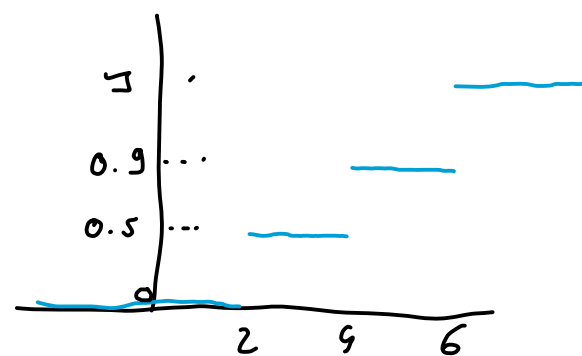
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②

$F_X(x)$

$F_X(x) = P(X \leq x) = P(\omega \in \Omega : X(\omega) \leq x)$ per $\forall x \in \mathbb{R}$

$$F_X(x) = \begin{cases} P(\emptyset) = 0 & x < 2 \\ P(2) = 0 \, 5 & 2 \leq x < 4 \\ P(2) + P(4) = 0 \, 9 & 4 \leq x \leq 6 \\ P(\Omega) = 1 & x \geq 6 \end{cases}$$

GRAFICO

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③

MEDIA

$$E(X) = \sum_{\mu : x_\mu \in S} x_N \cdot p_X(x_N)$$

$$x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \, p(x_3)$$

$$= 2 \cdot 0.5 + 4 \cdot 0.4 + 6 \cdot 0.1$$
$$= 1 + 1.6 + 0.6$$
$$= 3.2$$

VARIANZA

$$Var(X) = \sum_{i=1}^{k} (x_i - E(x))^2 \, p_i$$

appure

$$\sum_{i=1}^{k} x_i^2 \, p_i - E(x)^2$$

$$Var(X) = (2 - 3.2)^2 \cdot 0.5 + (4 - 3.2)^2 \cdot 0.4 + (6 - 3.2)^2 \cdot 0.1$$
$$= 1.44 \cdot 0.5 + 0.64 \cdot 0.4 + 7.84 \cdot 0.1$$
$$= 0.72 + 0.256 + 0.784$$
$$= 1.76$$

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④

$$Y = -3X + b$$

$-3X + b$ = è STRETTAMENTE MONOTONA

$$P(Y = -6 + b) = P(-3X + b) = P(x = 2) = 0.5$$

$$P(Y = -12 + b) = P(-3X+b) = P(X=4) = 0.9$$

$$P(Y = -18 + b) = P(-3X+b) = P(X=6) = 0.1$$

$$F_Y(y) = P(Y \leq y) = P(\omega \in \Omega : X(\omega) \leq y) \quad \text{per} \quad \forall y \in \mathbb{R}$$

$$F_Y(y) = \begin{cases} \{\emptyset\} & 0 & y < -18+b \\ \{-18+b\} & 0.1 & -18 \leq y < b - -12+b \\ \{-18+b, -12+b\} & 0.5 & -1 \leq y < b - -6+b \\ \Omega & 3 & \geq -6+b \end{cases}$$

$$E(y) = (-6 + b) \cdot 0.5 + (-12 + b) \cdot 0.4 + (-18 + b) \cdot 0.1$$

$$= -3 + 0.5b - 4.8 + b \cdot 0.4 - 1.8 + 0.1$$

$$= -9.6 + b$$

$$Var(y) = \sum_{i=1}^{y} (y_i - E(y))^2 \cdot p_i$$

$$Var(y) = (-6 + b - 9.6 + b) \cdot 0.5 + (-12 + b - 9.6 + b)^2 \cdot 0.4 + (-18 + b - 9.6 + b) \cdot 0.1$$

$$\left[\left(-15.6^2\right) + \frac{1}{2}b^2\right] \cdot 0.5 + \left[(-21.6^4) \cdot (2b)^2\right] \cdot 0.4 + \left[(-27.6)^2 + (b)^2\right] \cdot 0.1$$

$$6,48 + 1,304 = 7,056$$

$= 15.84$