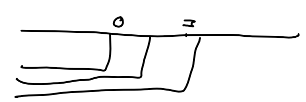
Esercizio 10

sabato 12 giugno 2021 12:47

$X_1, X_2 \sim \mathcal{U}(0,1)$

$$f_{X_2}(x_2) = f_{X_1}(x_1) = \begin{cases} 1 & x \in (0,1) \\ 0 & \text{altrimenti} \end{cases}$$

$$F_{X_2}(x_2) = F_{X_1}(x_1) = \begin{cases} 0 & x < 0 \\ x & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$



$P(U<0) -$

$- U = \max(X_1, X_2)$

$F_U(w) = P(\max(X_1, X_2) \le w) = P(X_1 \le w, X_2 \le w) =$

SONO INDIPENDENTI

indipendenti $= P(X_1 \le w)\, P(X_2 \le w)$

SONO IDENTICAMENTE DISTRIBUITE

quindi è equivalente $X_1 = X_2$

v.d. $= P[(X \le w)^2] = \begin{cases} 0 & w < 0 \\ w^2 & 0 \le w < 1 \\ 1 & w \ge 1 \end{cases}$

$- V = \min(X_1, X_2)$

evento complementare

$F_V(v) = P(\min(X_1, X_2) < v) = 1 - P(\min(X_1, X_2) > v)$

ind. e v.d.

$= 1 - [1 - F_X(v)]^2$

$$= \begin{cases} 1 - (1-0) = 0 & v < 0 \\ 1 - (1-v)^2 & 0 \le v < 1 \quad v^2 - 2v \\ 1 - (1-1)^2 = 1 & v \ge 1 \end{cases}$$

$- Y = U - V$

$F_Y(y) = P(\underbrace{\max(X_1, X_2) - \min(X_1, X_2) \le y}_{A})$

$= P(\max(X_1, X_2) - \min(X_1, X_2) \le y, \underbrace{X_1 < X_2}_{B})$

$+ P(\max(X_1, X_2) - \min(X_1, X_2) \le y, X_1 \ge X_2)$

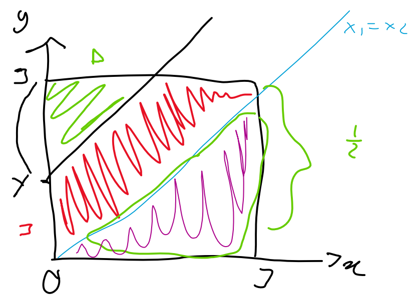
NO INDIP

Simile teorema Attesa(?)

$= P(X_2 - X_1 \le y, X_1 < X_2) + P(X_1 - X_2 \le y, X_1 \ge X_2)$

① ②

$X_2 - X_1 = Y$



$0 \quad y < 0$

$1 \quad y \geq 1$

$$= 1 - 2\,\frac{(1-y)^2}{2}$$

$$= 1 - (1-y)^2$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ 1-(1-y)^2 & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases} \qquad 2y - y^2$$

$$Z = U + V$$

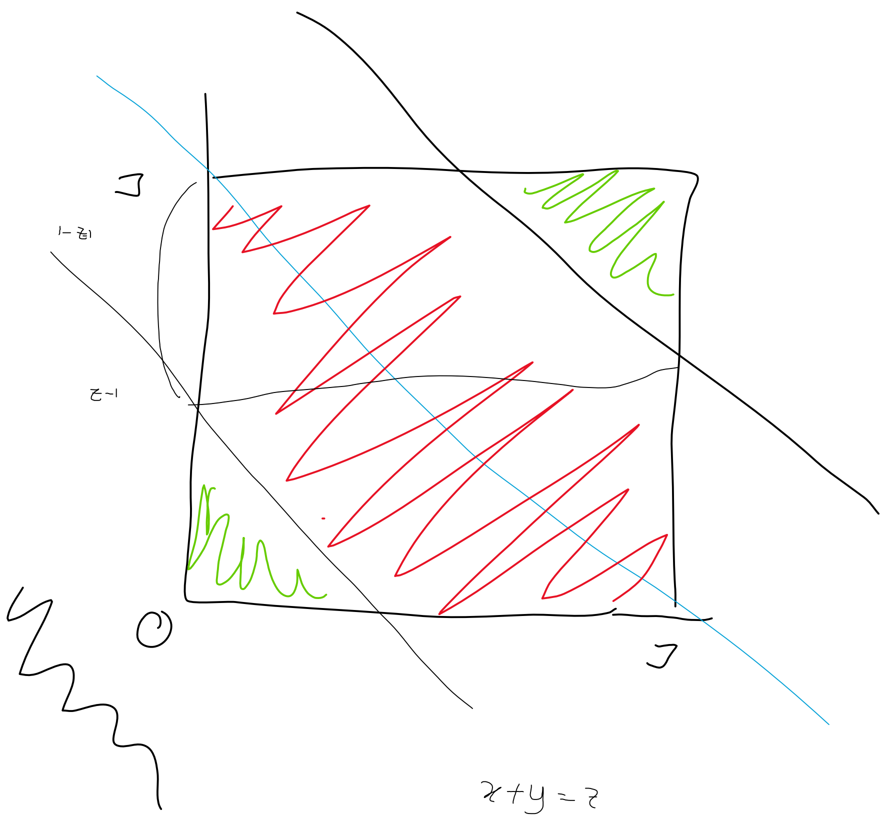
$$P_Z(z) : P\left(\max(x_1, x_2) + \min(x_1, x_2) \leq z\right)$$

$$= P\left(\max(x_1, x_2) + \min(x_1, x_2) < z, \; x_1 > x_2 \right) +$$

$$P(\max(x_1, x_2) + \min(x_1, x_2) < z\ ,\ x_2 \geq x_1)$$

$$= P(x_1 + x_2 < z,\ x_1 > x_2) + P(x_1 + x_2 < z\ ,\ x_2 \geq x_1)$$

$$= P[x_1 + x_2 \leq z]$$



$$= \begin{cases} 0 & w < 0 \\ \dfrac{z^2}{2} & 0 \leq w < 1 \\ 1 - \dfrac{[1-(z-1)]^2}{2} = 1 - \dfrac{(2-z)^2}{2} & 1 \leq w < 2 \\ 1 & w \geq 2 \end{cases}$$

Dez Poligoni