Esercizio 9

$N =$ NUMERO INTERO CASUALE

$$1 \leq N \leq 16$$

$A = \{ N \leq 8 \}$

$B = \{ 8 \leq N \leq 15 \}$

$C = \{ N \text{ PARI} \}$

A B e C sono indipendenti?

$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$|\Omega| = 16$

$A = \{1, 2, 3, 4, 5, 6, 7, 8\} \rightarrow |A| = 8$

$B = \{8, 9, 10, 11, 12, 13, 14, 15\} \rightarrow |B| = 8$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\} \rightarrow |C| = 8$

$$P(A) = \frac{8}{16} = \frac{1}{2}$$

$$P(B) = \frac{8}{16} = \frac{1}{2}$$

$$P(c) = \frac{8}{16} = \frac{1}{2}$$

$$A \cap B \cap C = \{8\} \rightarrow |A \cap B \cap C| = 1$$

$$P(A \cap B \cap c) = \frac{1}{16}$$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(c)$$
$$\frac{1}{16} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$
$$\frac{1}{16} \neq \frac{1}{8}$$

Già a questo punto possiamo concludere
che i 3 eventi non sono indipendenti.

$$A \cap B = \{8\} \rightarrow |A \cap B| = 1$$

$$P(A \cap B) = \frac{1}{16}$$

$$P(A \cap B) = P(A) \cdot P(B)$$
$$\frac{1}{16} = \frac{1}{2} \cdot \frac{1}{2}$$
$$\frac{1}{16} = \frac{1}{4}$$

A e B non sono indipendenti

$A \cap C = \{2, 4, 6, 8\} \rightarrow |A \cap C| = 4$

$P(A \cap C) = \frac{4}{16} = \frac{1}{4}$

$P(A \cap C) = P(A) \cdot P(C)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$

A e C sono indipendenti

$B \cap C = \{8, 10, 12, 14\} \rightarrow |B \cap C| = 4$

$P(B \cap C) = \frac{4}{16} = \frac{1}{4}$

$P(B \cap C) = P(B) \cdot P(C)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$

B e C sono indipendenti

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① $P(A \cap (B \cap \bar{C}))$ ② $P(A \cup (B \cap \bar{C}))$

① $P(A \cap (B \cap \bar{C}))$

$\bar{C} = \{1, 3, 5, 7, 9, 11, 13, 15\}$

$|\bar{C}| = 8$

$B \cap \bar{C} = \{9, 11, 13, 15\}$

$A \cap (B \cap \bar{C}) = \{\emptyset\}$

$|A \cap (B \cap \bar{C})| = 0$

$P(A \cap (B \cap \bar{C})) = \dfrac{|A \cap (B \cap \bar{C})|}{|\Omega|} = \dfrac{0}{16} = 0$

②

$A \cup (B \cap \bar{C}) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15\}$

$|A \cup (B \cap \bar{C})| = 12$

$P(A \cup (B \cap \bar{C})) = \dfrac{|A \cup (B \cap \bar{C})|}{|\Omega|} = \dfrac{12}{16} = \dfrac{3}{4}$