Esercizio 15

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$$f(x) = \begin{cases} kx^{-\frac{1}{2}} & 0 < x < 1 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$k = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$= \int_0^1 k x^{-\frac{1}{2}}\,dx$$

$$= k \int_0^1 x^{-\frac{1}{2}}\,dx$$

$$= k \left[2\sqrt{x} \right]_0^1$$

$$= k \left[2 - 0 \right]$$

$$= k\,2$$

$$= k = \frac{1}{2}$$

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② Funzione di Distribuzione

$$F_X = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_0^x k x^{-\frac{1}{2}}\,dx$$

$$= k \int_0^x x^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2} \int \left[2\sqrt{x} \right]_0^x$$

$$= \frac{1}{2} \left[2\sqrt{x} - 0 \right]$$

$= \frac{1}{2} x \sqrt{x}$

$= \sqrt{x}$

$= x^{\frac{1}{2}}$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ x^{\frac{1}{2}} & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Media

$$E(X) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$\int_{0}^{1} \frac{1}{2} x^{-\frac{1}{2}} \, x \, dx$$

$$K\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^6$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}$$

$$= \frac{3}{3}$$

$$E(x^2) = \int_{-\infty}^{+\infty} F(x) \cdot x^2 \, dx$$

$$k \int_0^1 x^{\frac{3}{2}} \, dx$$

$$\frac{1}{2} \left[\frac{2x^2 \sqrt{x}}{5} \right]_0^1$$

$$\frac{1}{2} \left[\frac{2 \cdot 1^2 \sqrt{1}}{5} - \frac{2 \cdot 0^2 \sqrt{0}}{5} \right]$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5}$$

VARIANZA

$$E(x)^2 = (E\,x)^2$$

$$V_{1R}(x) = \frac{1}{5} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{5} - \frac{1}{9}$$

$$= \frac{4}{45}$$

$$= \frac{4}{45}$$

$$Y = |X|^{\frac{1}{2}}$$

$$F_Y = P(Y \leq y)$$

$$F_y = P(X^{\frac{1}{2}} \leq y) = P(X \leq y^2) = (y^2)^{\frac{1}{2}} = y$$

$$F_y(y) = \begin{cases} 0 & y < 0 \\ y & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

Funzione di Densità

$$f(y) = \frac{d}{d(y)} F_y$$

$$= \frac{d}{d(y)} y$$

$= 1$

$$f(y) = \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{altrove} \end{cases}$$