

# Esercizio 1

venerdì 14 maggio 2021 16:43

$X$  v.a. DISCRETA

Valori di  $X = (-1, 0, 1)$

$$p(X = -1) = 0.1$$

$$p(X = 0) = 0.8$$

$$p(X = 1) = c$$

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① Valore di  $c$

$$c = 1 - 0.1 - 0.8$$

$$c = 0.1$$

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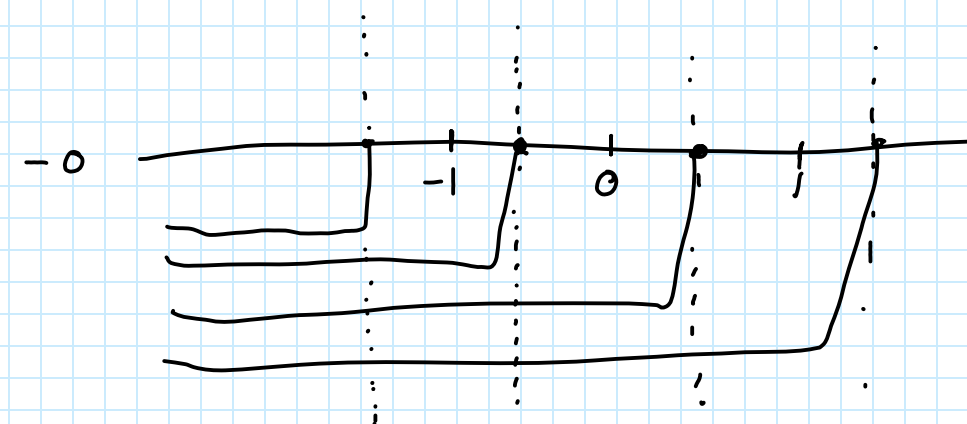
②

$$F_X(x)$$

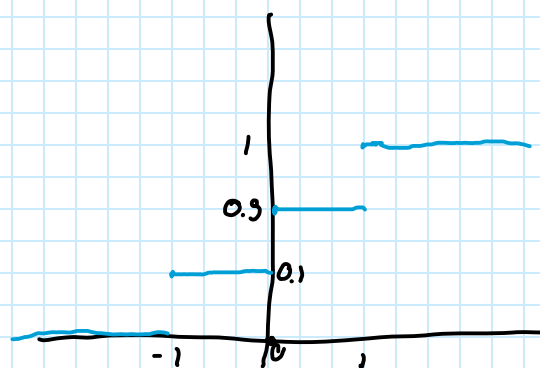
$$F_X(x) = P(X \leq x) = P(\omega \in \Omega : X(\omega) \leq x) \quad \text{per } \forall x \in \mathbb{R}$$

$$F_X(x) = \begin{cases} P(\emptyset) = 0 & x < -1 \\ P(-1) = 0.1 & -1 \leq x < 0 \\ P(-1) + P(0) = 0.9 & 0 \leq x < 1 \\ P(\Omega) = 1 & x \geq 1 \end{cases}$$

$$\begin{aligned} x &< -1 \\ -1 &\leq x < 0 \\ 0 &\leq x < 1 \\ x &\geq 1 \end{aligned}$$



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③

Media

$$E(X) = \sum_{\nu: x_\nu \in S} x_\nu \cdot p_X(x_\nu)$$

$$\begin{aligned}
 & x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \cdot p(x_3) \\
 & = -1 \cdot 0.1 + 0 \cdot 0.8 + 1 \cdot 0.1 \\
 & = -0.1 + 0.1 \\
 & = 0
 \end{aligned}$$

Varianza

$$Var(X) = \sum_{i=1}^K (x_i - E(X))^2 p_i$$

oppure

$$\sum_{i=1}^K x_i^2 p_i - E(X)^2$$

$$= (-1 - 0)^2 \cdot 0.1 + (0 - 0)^2 \cdot 0.8 + (1 - 0)^2 \cdot 0.1$$

$$= 1 \cdot 0.1 + 0 + 0.1$$

$$= 0.1 + 0.1$$

$$= \underline{0.2}$$

④

$$Y = X^4$$

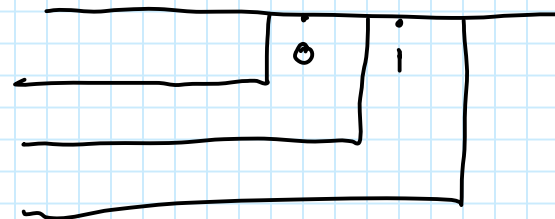
$X^4$  è STRETTAMENTE MONOTONA

$$P(Y=0) = P(X^4=0) = P(X=0) = 0.8$$

$$P(Y=1) = P(X^4=1) = P(X=1) + P(X=-1) = 0.1 + 0.1 = 0.2$$

$$F_Y(y) = P(Y \leq y) = P(\omega \in \Omega : X(\omega) \leq y) \quad \text{per } \forall y \in \mathbb{R}$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ 0.8 & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$



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5

$$E(Y) = 0 \cdot 0.8 + 1 \cdot 0.2$$
$$= 0.2$$

$$\text{Var}(Y) = \sum_{i=1}^y (y_i - E(Y))^2 \cdot p_i$$

$$\text{Var}(Y) = (0 - 0.2)^2 \cdot 0.8 + (1 - 0.2)^2 \cdot 0.2$$
$$= 0.04 \cdot 0.8 + 0.64 \cdot 0.2$$
$$= 0.03 + 0.13$$
$$= 0.16$$