Esercizio 10

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$$f(x) = \begin{cases} kx^2 & 0 < x < 1 \quad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$k = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_0^1 kx^2\,dx = 1$$

$$k \int_0^1 x^2\,dx$$

$$k \left[\frac{x^3}{3} \right]_0^1$$

$$k \left[\frac{1^3}{3} - 0 \right]$$

$$k \cdot \frac{1}{3} = 1$$

$$k = \frac{1}{\frac{1}{3}}$$

$$k = 3$$

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② FUNZIONE di DISTRIBUZIONE

$$F_X = \int_{-\infty}^{x} f(x)\,dx$$

$$F_x = \int_0^x k x^2 \, dx$$

$$= k \int_0^x x^2 \, dx$$

$$= k \left[\frac{x^3}{3} \right]_0^x$$

$$= k \left[\frac{x^3}{3} - 0 \right]$$

$$= k \cdot \frac{x^3}{3}$$

$$= \cancel{3} \cdot \frac{x^3}{\cancel{3}}$$

$$= x^3$$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Media

$$E(X) = \int_{-\infty}^{+\infty} f(x) \cdot x \, dx$$

$$\int_0^1 k x^2 \, x \, dx$$

$$= k \int_0^1 x^3 \, dx$$

$$= 3 \int^1 x^3 \, dx$$

$$= 3\int_0 x^3 \, dx$$

$$= 3\left[\frac{x^4}{4}\right]_0^1$$

$$= 3\left[\frac{1}{4} - 0\right]$$

$$= \frac{3}{4}$$

$$E(x^2) = \int_{-\infty}^{+\infty} F(x) \cdot x^2 \, dx$$

$$= \int_0^1 k x^2 \cdot x^2 \, dx$$

$$= k \int_0^1 x^4 \, dx$$

$$= 3 \left[\frac{x^5}{5} \right]_0^1$$

$$= 3 \left[\frac{1}{5} - 0 \right]$$

$= \frac{3}{5}$

VARIANZA

$E(x)^2 - (E\,x)^2$

$Var(x) = \frac{3}{5} - \left(\frac{3}{5}\right)^2$

$3 \qquad \frac{9}{}$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48 - 45}{80}$$

$$= \frac{3}{80}$$

$$y = x^3$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^3 \leq y) = P(X \leq y^{\frac{1}{3}}) = y$$

$$F_y(y) = \begin{cases} 0 & y < 0 \\ y & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

FUNZIONE DI DENSITÀ

$$f(y) = \frac{d}{d(y)} F_Y$$

$$= f$$

$$f(y) \begin{cases} 1 & 0 \le y \le 1 \\ 0 & \text{altrove} \end{cases}$$