Esercizio 15

venerdì 7 maggio 2021 17:12

3 URNE :

LE PRIME due contengono { 6 biglie BIANCHE _ 6 biglie NERE)

LA TERZA contiene { 10 biglie BIANCHE, 2 nere)

Scegliere una biglia da un'urna a CASO

$E_i := \{$ LA BIGLIA è ESTRATTA DALLA URNA i-esima $\}$ tale che $i = \{1, 2, 3\}$

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$

B = LA BIGLIA ESTRATTA è BIANCA

N = LA BIGLIA ESTRATTA è NERA

$P(B)$ e $P(N)$

$P(B) = ?$

a) $P(B|E_1) = P(B)$ → 6 B, 6 N, $\Omega = 12$

$P(B) = \frac{|B|}{\Omega} = \frac{6}{12} = \frac{1}{2}$

b) $P(B|E_2) = P(B)$ → 6 B, 6 N, $\Omega = 12$

$P(B) = \frac{|B|}{\Omega} = \frac{6}{12} = \frac{1}{2}$

c) $P(B|E_3) = P(B)$ → 10 B, 2 N

(c) $P(B|E_3) = P(B)$ $\rightarrow$ 10B
2N
$\Omega = 12$

$P(B) = \frac{|B|}{|\Omega|} = \frac{10}{12} = \frac{5}{6}$

$P(B) = P(B|E_1)\ P(E_1) + P(B|E_2) \cdot P(E_2) + P(B|E_3) \cdot P(E_3)$

$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \frac{5}{6}\ \frac{1}{3}$

$= \frac{1}{6} + \frac{1}{6} + \frac{5}{18}$

$= \frac{3+3+5}{18} = \frac{11}{18}$

$P(N)$

(a) $P(N|E_1) = P(N)$ $\rightarrow$ 6B
6N
$\Omega = 12$

$P(N) = \frac{|N|}{|\Omega|} = \frac{6}{12} = \frac{1}{2}$

(b) $P(N|E_2) = P(N)$ 6B
6N
$\Omega = 12$

$P(N) = \frac{|N|}{|\Omega|} = \frac{6}{12} = \frac{1}{2}$

(c) $P(N|E_3) = P(N)$ 10B
2N
$\Omega = 12$

$P(N) = \frac{|N|}{|\Omega|} = \frac{2}{12} = \frac{1}{6}$

$P(N) = P(N|E_1) \cdot P(E_1) + P(N|E_2) \cdot P(E_2) + P(N|E_2)\ P(E_2)$

$= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{1}{3}$

$$= \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{6}\cdot\frac{1}{3}$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{18}$$

$$= \frac{3+3+1}{18}$$

$$= \frac{7}{18}$$

② $P(E_i \mid B)$ Teorema di Bayes

ⓐ $$P(E_1 \mid B) = \frac{P(B \mid E_1)\cdot P(E_1)}{P(B)} = \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{11}{18}} = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{18}{11} = \frac{1}{\cancel{6}_{1}}\cdot\frac{\cancel{18}^{3}}{11} = \frac{3}{11}$$

ⓑ $$P(E_2 \mid B) = \frac{P(B \mid E_2)\cdot P(E_2)}{P(B)} = \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{11}{8}} = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{18}{11} = \frac{1}{\cancel{6}_{1}}\cdot\frac{\cancel{18}^{3}}{11} = \frac{3}{11}$$

ⓒ $$P(E_3 / B) = \frac{P(B / E_3)\cdot P(E_3)}{P(B)} = \frac{\frac{5}{6}\cdot\frac{1}{3}}{\frac{11}{8}} = \frac{5}{6}\cdot\frac{1}{3}\cdot\frac{18}{11} = \frac{5}{\cancel{18}}\cdot\frac{\cancel{18}}{11} = \frac{5}{11}$$