Esercizio 13

martedì 15 giugno 2021 16:27

$$f_{X,Y}(x,y) = \begin{cases} 1 & 0<x<1,\ 0<y<1 \\ 0 & \text{altrove} \end{cases}$$ ← densità congiunta

1

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_0^1 1\,dy = 1\int_0^1 dy = 1(1-0) = 1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_0^1 1\,dx = 1$$

densità MARGINALI

$$f_X(x) = \begin{cases} 1 & x \in (0,1) \quad 0<x<1 \\ 0 & \text{altrove} \end{cases}$$

$$f_Y(y) = \begin{cases} 1 & y \in (0,1) \quad 0<y<1 \\ 0 & \text{altrove} \end{cases}$$

∞

$$E_X = \int_{-\infty} x \, f_X(x)\, dx$$

$$\frac{b+a}{2}$$

$$E_X \quad = \quad = \quad 1 \left. \frac{x^2}{2} \right|_0^1 = 1 \left(\frac{2^1}{2} - \frac{0}{2} \right) = \frac{1}{2}$$

$$E_Y = \int_{-\infty}^{\infty} y \, f_Y(y)\, dy = 1 \, \frac{1}{2}$$

$$E_{X^2} = \int_{-\infty}^{\infty} x^2 \, f_X(x)\, dx = \int_0^1 x^2 \, dx = \left. \frac{x^3}{3} \right|_0^1 = \left(\frac{1}{3} - 0 \right) = \frac{1}{3}$$

$$E_{Y^2} = \int_{-\infty}^{\infty} x^2 \, f_X(x)\, dx = \int_0^1 x^2 \, dx = \left. \frac{x^3}{3} \right|_0^1 = \left(\frac{1}{3} - 0 \right) = \frac{1}{3}$$

Esercizio Blocco 3

$$\text{VAR } x = E x^2 - (E x)^2 = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\text{VAR } y = E y^2 - (E y)^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

2 FUNZIONE DENSITÀ SI FATTORIZZA

$$f_{xy}(x, y) = 1 = f_x(x)\, f_y(y) = 1 \cdot 1 = 1$$

SI SONO INDIPENDENTI

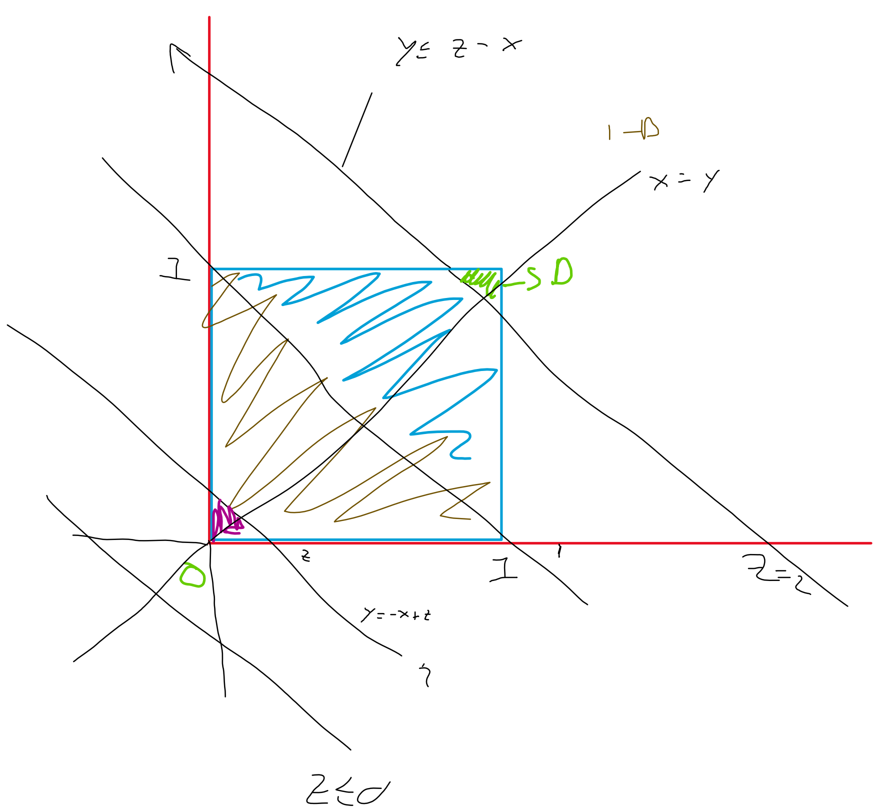
$$\Downarrow$$

$$\text{COV}(X, Y) = 0$$

3

$Z = X + Y$ $\qquad$ $Y \leq z - X$

$F_Z(z) = P(Z \leq z) = P(X+Y \leq z)$



$b\,h$ / 2

$$F_Z(z) = \begin{cases} 0 & z \leq 0 \\ \frac{z^2}{2} & 0 \leq z < 1 \\ 1 - \frac{(z-2)^2}{2} & 1 \leq z < 2 \\ 1 & z \geq 2 \end{cases}$$

AREA $D = \frac{(z-1)}{2}$

$= \frac{[1-(z-1)]^2}{2}$

$= \frac{(1+1-z)}{2}$

$= \frac{(z-2)^2}{2}$

$$f_Z(z) = \begin{cases} 0 & z \leq 0 \\ z & 0 \leq z < 1 \\ 2 - z & 1 \leq z < 2 \\ 0 & z \geq 2 \end{cases}$$

0 $\qquad$ 1 $\qquad$ 2

$$E z = \int_{-\infty} z f_Z(z)\, dz = \int_0 z^2\, dz + \int_1 (2-z) z\, dz$$

$$= \frac{z^3}{3}\Big|_0^1 + \left(z^2 - \frac{z^3}{3}\right)\Big|_1^2$$

$$= \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$$

$$E z^2 = \int_0^1 z^3\, dz + \int_1^2 (2-z) z^2 = \frac{1}{4} + \frac{11}{12} = \frac{7}{6}$$

$$VAR\ z = \frac{7}{6} - \frac{1}{2} = \frac{1}{6}$$