Esercizio 4

venerdì 14 maggio 2021 18:48

X v.a DISCRETA

VALORI di X = (-3, 0, 3)

$p(X=-3) = 0.3$

$p(X=0) = 0.4$

$p(X=3) = c$

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① VALORE di c

$c = 1 - 0.4 - 0.3$

$= 0.3$

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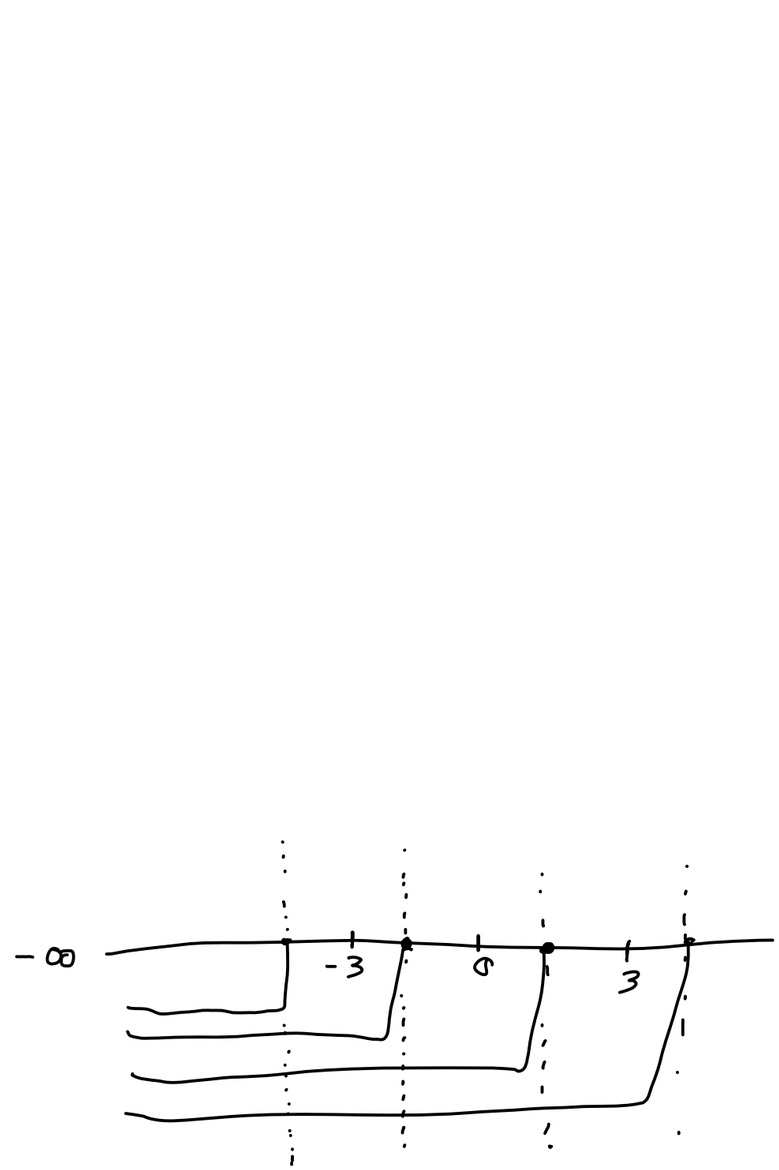
②

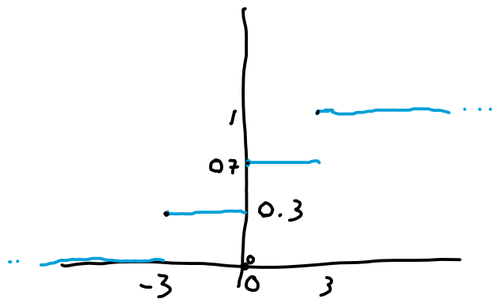
$F_X(x)$

$$F_X(x) = P(X \leq x) = P(\omega \in \Omega : X(\omega) \leq x) \quad \text{per} \quad \forall x \in \mathbb{R}$$

$$F_X(x) = \begin{cases} P(\emptyset) = 0 & x < -3 \\ P(-3) = 0{,}3 & -3 \leq x < 0 \\ P(-3) + P(0) = 0{,}7 & 0 \leq x < 3 \\ P(\Omega) = 1 & x \geq 3 \end{cases}$$



GRAFICO



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③

MEDIA

$$E(X) = \sum_{p:\, x_p \in S} x_n \cdot p_X(x_n)$$

$$= x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3\ p(x_3)$$

$$= -3 \cdot 0.3 + 0 \cdot 0.4 + 3 \cdot 0.3$$

$$= -0.9 + 0 + 0.9$$

$$= 0$$

VARIANZA

$$Var(X) = \sum_{i=1}^{K} (x_i - E[X])^2\ p_i$$

oppure $\sum_{i=1}^{K} x_i^2\ p_i - E(x)^2$

$$Var(X) = (-3-0)^2 0.3 + (0-0)^2 \cdot 0.4 + (3-0)^2 \cdot 0.3$$

$= 9 \cdot 0.3 + 0 + 9 \cdot 0.3$

$= 2.7 + 2.7$

$= 5.4$

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(4)

$Y = X^2$

X^2 è STRETTAMENTE MONOTONA

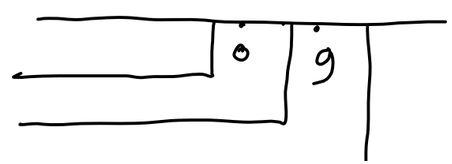
$P(Y=0) = P(X^2=0) = P(X=0) = 0.4$

$P(Y=9) = P(X^2=9) = P(X=-3) = P(X=3) = 0.3+0.3$

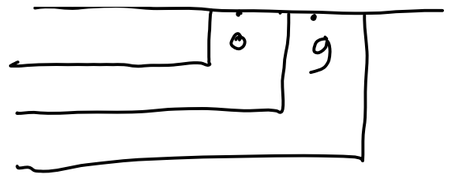
$= 0.6$

$F_Y(y) = P(Y \leq y) = P(\omega \in \Omega \cdot X(\omega) \leq y)$ per $\forall y \in \mathbb{R}$

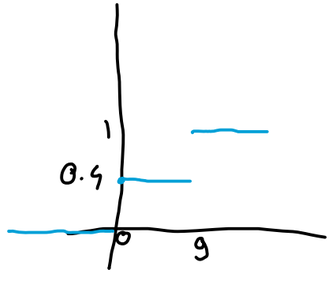
$$F_Y(y) = \begin{cases} \varnothing = & 0 & y < 0 \\ \{0\} = & 0.4 & 0 \leq y < 1 \\ \{0, 9\} & 1 & y \geq 1 \end{cases}$$



$\{0, 9\}$ 1 $y \geq 1$



GRAFICO



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(5)

$$E(Y) = 0 \cdot 0.4 + 9 \quad 0.6$$

$$= 5.4$$

$$Var(Y) = \sum_{i=1}^{y} (y_i - E(Y))^2 \cdot p_i$$

$$VAR(Y) = (0 - 5.4)^2 \cdot 0.4 + (9 - 5.4)^2 \cdot 0.6$$

$$= 29.16 \cdot 0.4 + 12.96 \cdot 0.6$$

$= 11.664 + 7.776$

$= 19.44$