Esercizio 16

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$$f(x) = \begin{cases} kx^{\frac{1}{2}} & 0 < x < 1 \\ 0 & \text{altrove} \end{cases} \qquad k \in \mathbb{R}$$

$$k = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$= \int_0^1 k\, x^{\frac{1}{2}}\,dx$$

$$= k \int_0^1 x^{\frac{1}{2}}\,dx$$

$$= k \left[ \frac{2\,x\sqrt{x}}{3} \right]_0^1$$

$$= k \left[ \frac{2 \cdot 1 \cdot \sqrt{1}}{3} - \frac{2 \cdot 0 \cdot \sqrt{0}}{3} \right]$$

$$= k \left[ \frac{2}{3} - 0 \right]$$

$$k\,\frac{2}{3} = 1$$

$$k = 1\,\frac{3}{2}$$

$$\boxed{k = \frac{3}{2}}$$

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② Funzione di distribuzione

$$F_x = \int_{-\infty}^{x} f(x)\,dx$$

$$F_x = \int_0^x k\, x^{\frac{1}{2}}\, dx$$

$$= k \int_0^x x^{\frac{1}{2}}\, dx$$

$$= k \left[ \frac{2x \cdot \sqrt{x}}{3} \right]_0^x$$

$$= k \left[ \frac{2x\sqrt{x}}{3} - 0 \right]$$

$$= \frac{3}{2} \cdot \frac{2x \cdot \sqrt{x}}{3}$$

$$= x\sqrt{x}$$

$$= x^{\frac{3}{2}}$$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ x^{\frac{3}{2}} & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$$

Media

$$E(x) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$\int_0^1 k \, x^{\frac{1}{2}} \cdot x \, dx$$

$$k \int_0^1 x^{\frac{1}{2}} \cdot x \, dx$$

$$\frac{3}{2} \int x^{\frac{3}{2}} \, dx$$

$$\frac{3}{2}\left[\frac{2x^2\sqrt{x}}{5}\right]_0^1$$

$$\frac{3}{2}\left[\frac{2\cdot 1\cdot\sqrt{1}}{5} - 0\right]$$

$$\frac{3}{2}\cdot\frac{2}{5}$$

$$= \frac{3}{5}$$

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$$E(x^2) = \int_{-\infty}^{+\infty} f(x)\cdot x^2\,dx$$

$$\int_{\quad}^{\quad} x^{\frac{1}{2}}\, x^2\, dx$$

$$= \int_0^{\cdot} k \cdot x^{\frac{1}{2}} \cdot x^{\cdot} \, dx$$

$$= k \int_0^{\cdot} x^{\frac{5}{2}} \, dx$$

$$= \frac{3}{2} \left[ \frac{2 x^3 \sqrt{x}}{7} \right]_0^{\cdot}$$

$$= \frac{3}{2} \left[ \frac{2 \cdot 1 \sqrt{1}}{7} - 0 \right]$$

$$= \frac{3}{2} \cdot \frac{2}{7}$$

$$= \frac{3}{7} \cdot \frac{7}{7}$$

$$= \frac{3}{7}$$

VARIANZA

$$E(x)^2 - (Ex)^2$$

$$Var(x) = \frac{3}{7} - \left(\frac{3}{5}\right)^2$$

$$= \frac{3}{7} - \frac{9}{25}$$

$$= \frac{75-63}{175}$$

$$= \frac{12}{175}$$

$$V_1 = X^2$$

$$F_Y = P(Y \leq y)$$

$$F_Y = P(X^{\frac{1}{2}} \leq y) = P(X \leq y^2) = (y^2)^{\frac{3}{2}} = y^3$$

$$F_y(y) = \begin{cases} 0 & y < 0 \\ y^3 & 0 \le y < 1 \\ 1 & y \ge 1 \end{cases}$$

Funzione di Densità

$$f(y) = \frac{d}{d(y)} F_y$$

$$= \frac{d}{d(y)} y^3$$

$= 3y^2$

$f(y) \begin{cases} 3y^2 & 0 \le y \le 1 \\ 0 & \text{altove} \end{cases}$