Esercizio 9
sabato 12 giugno 2021 12:17

$X \sim U(-1, 1)$

essendo equamente distribuita analizziamo la funzione nell'intervallo a, b

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \int_{-1}^{x} \frac{1}{2}\, dx = \underline{\frac{1}{2}x + \frac{1}{2}} & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

$$\frac{x+1}{2}$$

(1)

$a = -1$
$b = 1$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in (a, b) \\ 0 & \text{altrove} \end{cases} = \begin{cases} \frac{1}{2} & x \in (-1, 1) \\ 0 & \text{altrimenti} \end{cases}$$

$$P(a \le X \le b) = \int_a^b F_X(x)\, dx$$

$$F_X = \int_{-1}^{1} \frac{1}{2} x \, dx$$

$$= \frac{1}{2} \frac{x^2}{2} \Big|_{-1}^{1}$$

$$= \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right)$$

$$= \frac{1}{2} \cdot 0$$

$$= \emptyset$$

$$E(x^2) = \int_{-1}^{1} \frac{1}{2} x^2 \, dx$$

$$= \frac{1}{2} \int_{-1}^{1} x^2 \, dx$$

$$= \frac{1}{2} \left. \frac{x^3}{3} \right|_{-1}^{1}$$

$$= \frac{1}{2} \left(\frac{1}{3} - \left(-\frac{1}{3} \right) \right)$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}$$

$$= \frac{1}{3}$$

$$VAR(x) = Ex^2 - (Ex)^2$$

$$= \frac{1}{} - 0$$

$= \frac{1}{3} - 0$

$= \frac{1}{3}$

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(2)

$$z = \frac{X - E(x)}{\sqrt{Var(x)}} = \frac{x}{\sqrt{\frac{1}{3}}} = x \cdot \sqrt{3}$$

$$F_Z(z) = P(Z \leq z) = P(x\sqrt{3} \leq z) = P\left(x \leq \frac{z}{\sqrt{3}}\right)$$

$$\frac{z}{\sqrt{3}} < 1 \rightarrow z < \sqrt{3}$$

$\{ \quad 0$

$$F_Z(z) = \begin{cases} \dfrac{z}{2\sqrt{3}} + \dfrac{1}{2} & -\sqrt{3} \le z < \sqrt{3} \\ 1 & z \geq \sqrt{3} \end{cases}$$

densità

$\downarrow$

$$f_Z(z) = \begin{cases} \dfrac{1}{2\sqrt{3}} & -\sqrt{3} \le z \le \sqrt{3} \\ 0 & \text{altrove} \end{cases}$$

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(3)

$Y = \max(X, 0)$

$$F_Y(y) = P(Y \leq y)$$

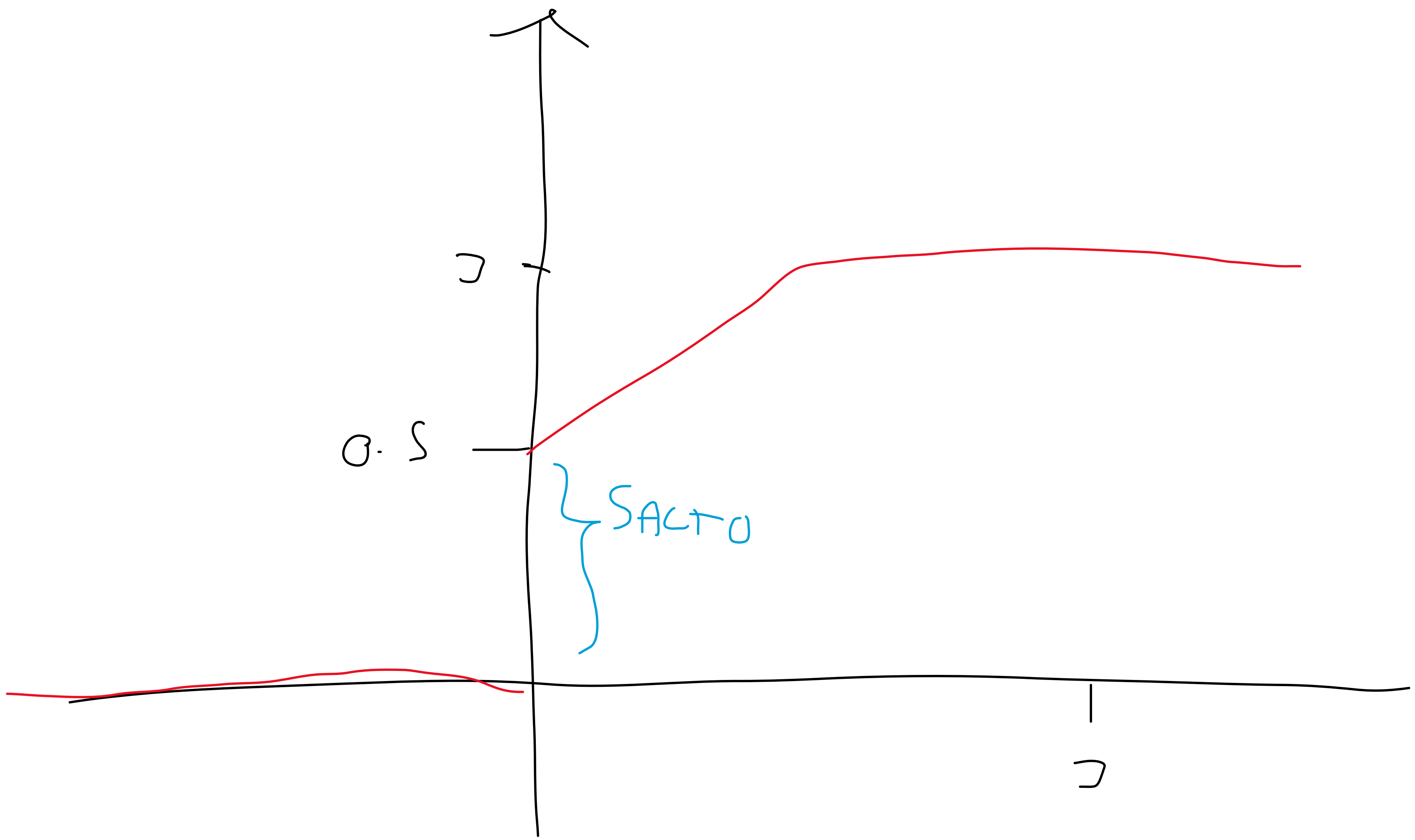
$$= P(\max(X, 0) \leq y)$$

$$= P(X \leq y, 0 \leq y)$$

$$= P(X \leq y)\, P(0 \leq y)$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{2}y + \frac{1}{2} & 0 \le y < 1 \\ 1 & y \ge 1 \end{cases}$$

$\frac{1}{2}y + 1$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