Esercizio 17

venerdì 7 maggio 2021 17:47

URNA contiene 9 biglie numerate da 1 a 9

Si estrae UNA BIGLIA, e
- Se PARI → RIMBUSSOLATA
- Se DISPARI → ELIMINATA

A_i { i-esima BIGLIA ESTRATTA PARI } con $i = \{1,2\}$

B_i { i-esima BIGLIA ESTRATTA DISPARI } con $i = \{1,2\}$

$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A_1 = \{2, 4, 6, 8\}$

$B_1 = \{1, 3, 5, 7, 9\}$

$\Omega = \text{UGUALE}$

$\rightarrow A_2 = \{2, 4, 6, 8\}$

$B_2 = \{1, 3, 5, 7, 9\}$

$n = n - 1$

$\rightarrow A_2 = \{4, 4, 6, 8\}$

$\rightarrow B_2 = \{1, 3, 5, 7, 9\} - 1$

$P(A_1) = \frac{|A_1|}{|\Omega|} = \frac{4}{9} \quad P(B_1) = \frac{|B_1|}{|\Omega|} = \frac{5}{9}$

LEGGE ALTERATIVE

$A_2 = P(A_2|A_1)\,P(A_1) + P(A_2|B_1) \cdot P(B_1)$

$= \frac{4}{9} \cdot \frac{4}{9} + \frac{4}{8} \cdot \frac{5}{9}$

$= \frac{4}{9} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{5}{9}$

$= \frac{16}{81} + \frac{5}{18}$

$= \frac{32+45}{162} = \frac{77}{162}$

$B_2 = P(B_2|A_1) \cdot P(A_1) + P(B_2|B_1) \cdot P(B_1)$

$= \frac{5}{9} \cdot \frac{4}{9} + \frac{4}{8} \cdot \frac{5}{9}$

$= \frac{5}{9} \cdot \frac{4}{9} + \frac{1}{2} \cdot \frac{5}{9}$

$= \frac{20}{81} + \frac{5}{18}$

$= \frac{40+45}{162} = \frac{85}{162}$

LEGGE DI BAYES

$P(A_1|A_2) = \frac{P(A_2|A_1) \cdot P(A_1)}{P(A_2)} = \frac{\frac{16}{81}}{\frac{77}{162}} = \frac{16}{81_1} \cdot \frac{162^2}{77} = \frac{32}{77}$

$P(A_1|B_2) = \frac{P(B_2|A_1) \cdot P(A_1)}{P(B_2)} = \frac{\frac{20}{81}}{\frac{85}{162}} = \frac{\cancel{20}^{5}}{\cancel{81}_{1}} \cdot \frac{162^{2}}{\cancel{85}_{17}} = \frac{8}{17}$