Esercizio 19

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$$f(x) = \begin{cases} kx^{\frac{1}{2}} & 0 < x < 3 \qquad k \in \mathbb{R} \\ 0 & \text{altrove} \end{cases}$$

$$k = \int_{-\infty}^{+\infty} F(x)\,dx = 1$$

$$\int_0^3 kx^{\frac{1}{2}}\,dx$$

$$k\int_0^3 x^{\frac{1}{2}}\,dx$$

$$k\left[\frac{x^{\frac{1}{2}+1}}{1+\frac{1}{2}}\right]_0^3$$

$$k\left[\frac{x^{\frac{1}{2}+1}}{1+\frac{1}{2}} - \frac{x^{\frac{1}{2}+1}}{1+\frac{1}{2}}\right]$$

$$k\left[\frac{3^{\frac{3}{2}}}{\frac{3}{2}} - \frac{0^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$k\left[3^{\frac{3}{2}}\cdot\frac{2}{3} - 0^{\frac{3}{2}}\ \frac{2}{3}\right] = 1$$

$$k\left[\sqrt[3]{9}\cdot\frac{2}{3}\right]$$

$$k \cdot 3.46 = 1$$

$$k = \frac{1}{3.46} = \frac{1}{2\sqrt{3}}$$

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(2) Funzione di distribuzione

x

$$F_x = \int_{-\infty} f(x)\, dx$$

$$F_x = \int_0^x k\, x^{\frac{1}{2}}\, dx$$

$$F_x = k \int_0^x x^{\frac{1}{2}}\, dx$$

$$F_x = \frac{1}{3.46} \int_0^x x^{\frac{1}{2}}$$

$$F_x = \frac{1}{3.46} \cdot \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 0 \right]$$

$$= \frac{1}{3.46} \cdot x^{\frac{3}{2}} \; \frac{2}{3}$$

$$= \frac{1}{3.46} \cdot \frac{2\left(x^{\frac{3}{2}}\right)}{3}$$

$$= \frac{1}{\cancel{2}\sqrt{3}} \cdot \frac{\cancel{2}\sqrt[2]{x^3}}{3}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt[2]{x^3}}{3}$$

$$= \frac{\sqrt[2]{x^3}}{3 \cdot \sqrt{3}}$$

$$F_x(x) = \begin{cases} 0 & x < 0 \\ \dfrac{\sqrt[2]{x^3}}{3 \cdot \sqrt{3}} & 0 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

MeDiA

$$E(x) = \int_{-\infty}^{+\infty} F(x) \cdot x \, dx$$

$$\int_0^3 k \, x^{\frac{1}{2}} \, x \, dx$$

$$k \int_0^3 x^{\frac{1}{2}} \cdot x \, dx$$

$$\frac{1}{3{,}46} \int^3 x^{\frac{3}{2}} \, dx$$

$$\frac{1}{3{,}46}\int_0 x^{\frac{3}{2}}\,dx$$

$$\frac{1}{3\,46}\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^3$$

$$\frac{1}{2\sqrt{3}}\left(3^{\frac{5}{2}}\quad\frac{2}{5}\right)$$

$$\frac{1}{2\sqrt{3}}\cdot\sqrt[2]{3^5}\quad\frac{2}{5}$$

$$\frac{1}{2\sqrt{3}}\qquad\frac{2\sqrt[2]{3}^{5}}{}$$

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$\frac{1}{2\sqrt{3}} \qquad \frac{2\sqrt[2]{3^5}}{5}$

$\frac{1}{\cancel{\sqrt{3}}} \qquad \frac{\cancel{\sqrt{243}}}{5}$

$\frac{1}{1} \qquad \frac{9}{5}$

$E(x) = \frac{9}{5}$

$+\infty$

$$E(x^2) = \int_{-\infty} F(x) \cdot x^2 \, dx$$

$$= \int_0^3 k \, x^{\frac{1}{2}} \, x^2 \, dx$$

$$= k \int_0^3 x^{\frac{5}{2}} \, dx$$

$$= k \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^3$$

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$$= k \cdot \frac{3^{\frac{7}{2}}}{\frac{7}{2}}$$

$$= k \cdot \frac{\sqrt[2]{3^7}}{\frac{7}{2}}$$

$$= \frac{1}{2\sqrt{3}} \cdot \frac{2\sqrt[3]{3^7}}{7}$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt[3]{3^7}}{}$$

$$= \frac{\sqrt{3}^{7}}{7\sqrt{3}}$$

$$= \frac{27\cancel{\sqrt{3}}}{7\cancel{\sqrt{3}}}$$

$$= \frac{27}{7}$$

VARIANZA

$$E(x)^2 - (E x)^2$$

$$V_{1R}(x) = \frac{27}{7} - \left(\frac{9}{5}\right)^2$$

$$= \frac{27}{7} - \frac{81}{25}$$

$$= \frac{675 - 567}{175}$$

$$= \frac{108}{175}$$

$$Y = X^{\frac{1}{2}}$$

$$F_Y = P(Y \le y)$$

$$F_Y = P(X^{\frac{1}{2}} \le y) = P(X \le y^2) = \left(\frac{\sqrt[2]{y^6}}{3 \cdot \sqrt{3}} \right) = \frac{y^3}{3 \cdot \sqrt{3}}$$

$$F_y(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^3}{3 \cdot \sqrt{3}} & 0 \leq y < 9 \\ 1 & y \geq 9 \end{cases}$$

Funzione di densità

$$f_{(y)} = \frac{d}{d(y)} F_y$$

$$= \frac{y^3}{3 \cdot \sqrt{3}} \quad \frac{d}{dx}$$

$$= \frac{\sqrt{3}\, y^3}{9} \quad \frac{d}{dx}$$

9

$$= \frac{\sqrt{3}}{9} \cdot \left(y^3\right) \frac{d}{dx}$$

$$= \frac{\sqrt{3}}{\cancel{9}} \cancel{3}\, y^2$$

$$= \frac{\sqrt{3}}{3}\, y^2$$

$$f(y) \begin{cases} \frac{\sqrt{3}}{3}\, y^2 & 0 \leq y \leq 3 \\ 0 & \text{altrove} \end{cases}$$