

## Esercizio 3

venerdì 14 maggio 2021 18:30

$X$  v.a. DISCRETA

Valori di  $X = (-2, 0, 2)$

$$p(X = -2) = 0.1$$

$$p(X = 0) = c$$

$$p(X = 2) = 0.1$$

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① Valore di  $c$

$$c = 1 - 0.1 - 0.1$$
$$= 0.8$$

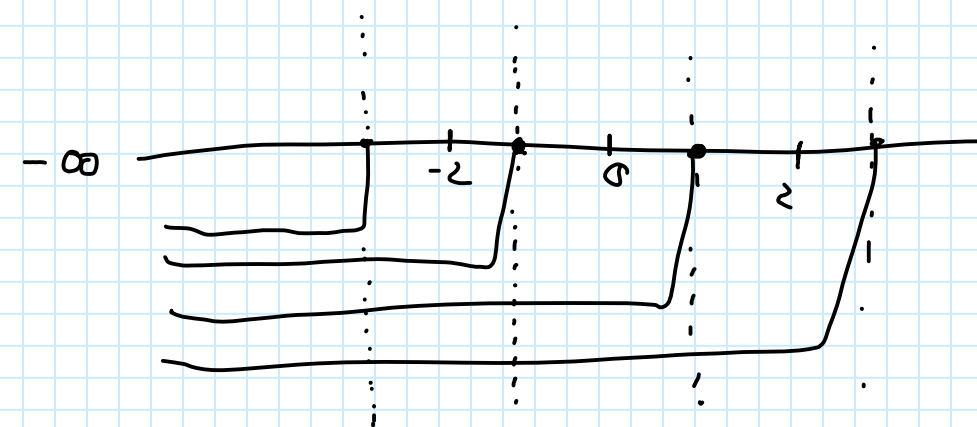
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②

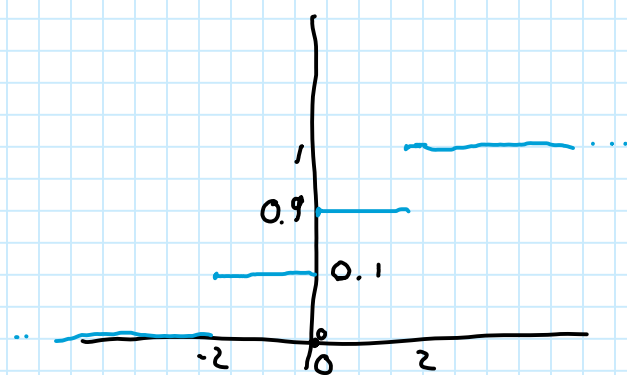
$$F_X(x)$$

$$F_X(x) = P(X \leq x) = P(\omega \in \Omega : X(\omega) \leq x) \quad \text{per } \forall x \in \mathbb{R}$$

$$F_X(x) = \begin{cases} P(\emptyset) = 0 & x < -2 \\ P(-2) = 0.1 & -2 \leq x < 0 \\ P(-2) + P(0) = 0.9 & 0 \leq x < 2 \\ P(\Omega) = 1 & x \geq 2 \end{cases}$$



GRATICO



③

Media

$$E(X) = \sum_{p: x_p \in S} x_p \cdot p_X(x_p)$$

$$\begin{aligned} & x_1 \cdot p(x_1) + x_2 \cdot p(x_2) + x_3 \cdot p(x_3) \\ &= -2 \cdot 0.1 + 0 \cdot 0.8 + 2 \cdot 0.1 \\ &= -0.2 + 0 + 0.2 \end{aligned}$$

$$= 0$$

Varianza

$$\text{Var}(X) = \sum_{i=1}^K (x_i - E(X))^2 p_i$$

oppure

$$\sum_{i=1}^K x_i^2 p_i - E(X)^2$$

$$\text{Var}(X) = (-2 - 0)^2 \cdot 0.1 + (0 - 0)^2 \cdot 0.8 + (2 - 0)^2 \cdot 0.1$$

$$= 4 \cdot 0.1 + 0 + 4 \cdot 0.1$$

$$= 0.4 + 0.4$$

$$= 0.8$$

9

$$Y = X^4$$

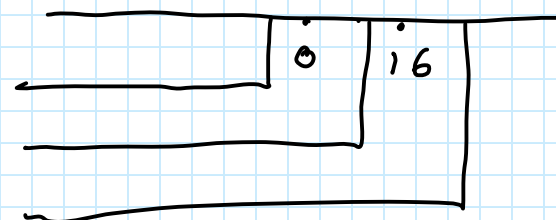
$X^4$  è STRETTAMENTE MONOTONA

$$P(Y=0) = P(X^4=0) = P(X=0) = 0.8$$

$$P(Y=16) = P(X^4=16) = P(X=2) + P(X=-2) = 0.1 + 0.1 = 0.2$$

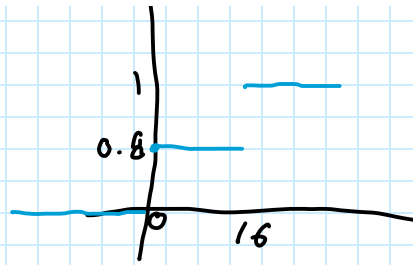
$$F_Y(y) = P(Y \leq y) = P(\omega \in \Omega : X(\omega) \leq y) \quad \text{per } \forall y \in \mathbb{R}$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ 0.8 & 0 \leq y < 16 \\ 1 & y \geq 16 \end{cases}$$



GRAFICO





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⑤

$$E(Y) = 0 \cdot 0.8 + 16 \cdot 0.2$$
$$= 3.2$$

$$\text{Var}(Y) = \sum_{i=1}^2 (y_i - E(Y))^2 \cdot p_i$$

$$\text{Var}(Y) = (0 - 3.2)^2 \cdot 0.8 + (16 - 3.2)^2 \cdot 0.2$$
$$= 10.24 \cdot 0.8 + 163.84 \cdot 0.2$$
$$= 8.192 + 32.768$$
$$= 40.96$$