Esercizio 12

martedì 15 giugno 2021 16:27

$x_1 \sim \mathcal{U}(0, \frac{1}{2})$ $x_2 \sim (\frac{1}{2}, 1)$

$$f_{x_1}(x_1) = \begin{cases} \frac{1}{\frac{1}{2}} & x \in (0, \frac{1}{2}) \\ 0 & \text{altrove} \end{cases}$$

$$f_{x_2}(x_2) = \begin{cases} \frac{1}{\frac{1}{2}} = 2 & x \in (\frac{1}{2}, 1) \\ 0 & \text{altrove} \end{cases}$$

$$F_{x_1}(x_1) = \begin{cases} 0 & x < 0 \\ \int_0^x 2\,dx = 2\int_0^x dx = 2x & 0 \leq x < \frac{1}{2} \\ 1 & x \geq \frac{1}{2} \end{cases}$$

$$F_{x_2}(x_2) = \begin{cases} 0 & x < \frac{1}{2} \\ \int_{\frac{1}{2}}^x 2\,dx = 2\int_{\frac{1}{2}}^x dx = 2x - 1 & \frac{1}{2} \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

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$W = \max(x_1, x_2) = x_2$

$$\begin{cases} 0 & w < \frac{1}{2} \\ - & \frac{1}{2} \leq w < 1 \end{cases}$$

$$F_U(w) = \begin{cases} 2w-1 & \frac{1}{2} \leq w < 1 \\ 1 & w \geq 1 \end{cases}$$

$V = \min(x_1, x_2) = x_1$

$$F_V(v) = \begin{cases} 0 & v < 0 \\ 2v & 0 \leq v < \frac{1}{2} \\ 1 & v \geq 1 \end{cases}$$

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$Z = U + V \Rightarrow Z = x_1 + x_2$

$F_Z(z) = P(Z \leq z) = P(x_2 + x_1 \leq z)$

$$\begin{cases} x_2 = \frac{1}{2} \\ x_1 + x_2 = z \end{cases} \rightarrow \begin{cases} x_2 = \frac{1}{2} \\ x_1 = z - \frac{1}{2} \end{cases}$$

LATO 1 $= z - \frac{1}{2}$

$$\begin{cases} x_2 = 1 \\ x_2 + x_1 = z \end{cases} \Rightarrow \begin{cases} x_2 = 1 \\ x_1 = z - 1 \end{cases}$$

LATO 2 $= \frac{1}{2} - (z-1) = -z + \frac{3}{2}$

$$\text{AREA } 1 = \frac{\left(z - \frac{1}{2}\right)^2}{2} = \frac{4z^2 - 4z + 1}{8}$$

$$\text{AREA } 2 = \frac{\left(\frac{3}{2} - z\right)^2}{2} = \frac{4z^2 - 12z + 9}{2}$$

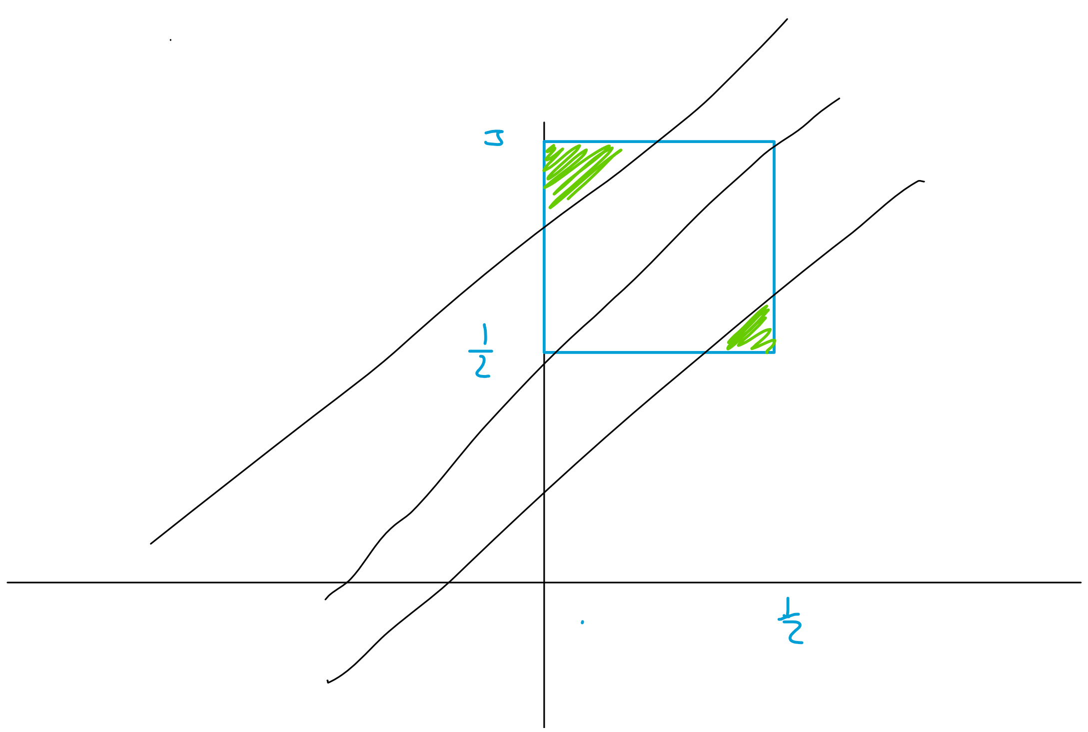
$$F_Z(z) = \begin{cases} 0 & z < \frac{1}{2} \\ \cancel{4}\,\frac{4z^2 - 4z + 1}{\cancel{8}\; 2} = \frac{4z^2 \;\; 4z + 1}{2} & \frac{1}{2} \le z < 1 \\ 4\left(\frac{1}{4} - \frac{4z^2 - 12z + 9}{2}\right) = -8z^2 + 24z - 17 & 1 \le z < \frac{3}{2} \end{cases}$$

$t \geq \frac{3}{2}$

$Y = U - V = X_2 - X_1$

$F_Y(y) = P(Y \leq y) = P(X_2 - X_1 \leq y)$

• Retta $x_2 - x_1 = y \longrightarrow x_2 = x_1 + y$




DEFINITA TRA $\frac{1}{2}, 1$

$$\begin{cases} x_2 = \frac{1}{2} \\ x_2 - x_1 = y \end{cases} \Rightarrow \begin{cases} x_2 = \frac{1}{2} \\ x_1 = \frac{1}{2} - y \end{cases}$$

LATO $\frac{1}{2} - \left(\frac{1}{2} - y\right) = y$

$$\begin{cases} x_2 = 1 \\ x_2 - x_1 = y \end{cases} \Rightarrow \begin{cases} x_2 = 1 \\ x_1 = 1-y \end{cases}$$

cosa $1-(1-y) = y$

Area 1: $\frac{y^2}{2}$

Area 2: $\frac{(1+y)^2}{2} = \frac{y^2+2y+1}{2}$

$$f_{X_1,X_2}(x_1,x_2) = \begin{cases} f_{X_1}(x_1)\, f_{X_2}(x_2) = 4 & x \in (0,1) \text{ e } x(\tfrac{1}{2}, e\, 1) \\ 0 & \text{altrove} \end{cases}$$

$$\begin{cases} 0 & y < 0 \\ 4 \cdot \frac{y^2}{2} = 2y^2 & 0 \le y < \frac{1}{2} \end{cases}$$

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \cancel{4} \cdot \dfrac{y^2}{\cancel{2}} = 2y^2 & 0 \leq y < \frac{1}{2} \\ 4\left(\frac{1}{4} - \dfrac{y^2+2y+1}{2}\right) = 4\left(\dfrac{1-2y^2+4y\cancel{+}2}{4}\right) & \frac{1}{2} \leq y \leq 1 \\ = 4y - 2y^2 - 1 & \\ 1 & y \geq 1 \end{cases}$$