Esercizio 7

CROCE   TESTA

Ω	X	Y
T T	2	0
C T	1	1
T C	1	1
C C	0	2

SOLA DIAGONALE

①

Y \ X	0	1	2	
0	0	0	$\frac{1}{4}$	$\frac{1}{4}$
1	0	$\frac{1}{2}$	0	$\frac{1}{2}$
2	$\frac{1}{4}$	0	0	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	✓

②

$$P(X=0) = \frac{1}{4}$$
$$P(X=1) = \frac{1}{2}$$
$$P(X=2) = \frac{1}{4}$$

$$\longrightarrow \quad P_X(x) = \begin{cases} \frac{1}{4} & x = 0 \\ \frac{1}{2} & x = 1 \\ \frac{1}{4} & x = 2 \end{cases}$$

$$P(y=0) = \frac{1}{4}$$

$$P(y=1) = \frac{1}{2}$$

$$P(y=2) = \frac{1}{4}$$

$$\longrightarrow \qquad P_Y(y) = \begin{cases} \frac{1}{4} & y=0 \\[2mm] \frac{1}{2} & y=1 \\[2mm] \frac{1}{4} & y=2 \end{cases}$$

NON INDIPENDENTI

$$P(0) \cdot P(0) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(0) \cdot P(1) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(0) \cdot P(2) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(1) \cdot P(0) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$P(1) \cdot P(1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(1) \cdot P(2) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

$$P(z) \cdot P(c) = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$$P(z) \cdot P(1) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(z) \cdot P(z) = \frac{1}{4} \quad \frac{1}{4} = \frac{1}{16}$$

MEDIA e VARIANZA

$$Ex = \frac{1}{4} \cdot 0 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= \boxed{1}$$

$$Ey = \frac{1}{4} \cdot 0 + \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= \boxed{1}$$

$$Ex^2 = \frac{1}{4} \cdot 0^2 + \frac{1}{2} \cdot 1^2 + \frac{1}{4} \cdot 2^2$$

$$= 0 + \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

$$Ey^2 = \frac{1}{4} \cdot 0^2 + \frac{1}{2} \cdot 1^2 + \frac{1}{4} \cdot 2^2$$
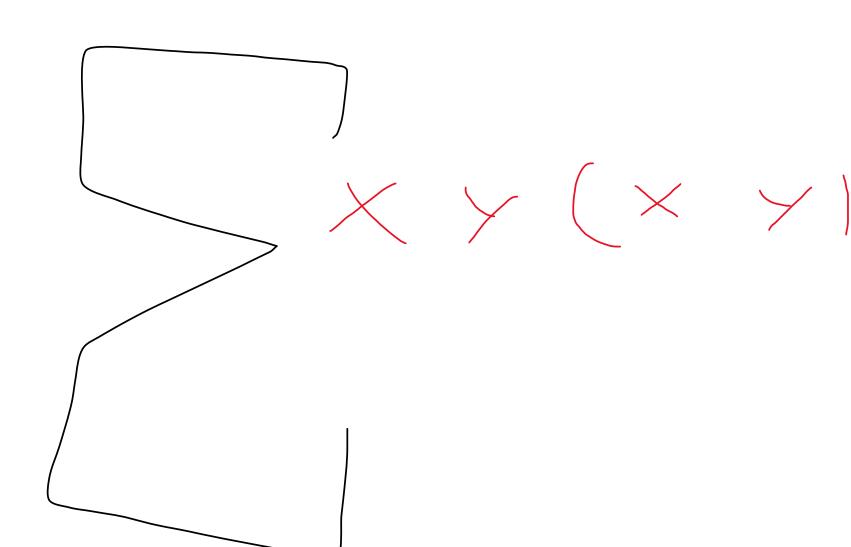
$$= 0 + \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

$$Var\,x = Ex^2 - (Ex)^2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\text{VARY} = Ey^2 - (Ey)^2 = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$

$$\text{Cov}(xy) = E(xy) - Ex \, Ey$$

$$=$$

$$E(x,y) = \sum xy(x \; y)$$

$$F\left(0, 0 \right) = 0$$

$$E\left(0, 1 \right) = 0$$

$$E\left(0, 2 \right) = 0 \quad \frac{1}{4} = 0$$

$$E\left(1, 0 \right) = 0$$

$$E\left(1, 1 \right) = 1 \cdot 1 \quad \frac{1}{2} = \frac{1}{2}$$

$$E\left(1, 2 \right) = 1 \quad 2 \quad 0 = 0$$

$$F\left(2, 0 \right) = 2 \cdot 0 \quad \frac{1}{4} = 0$$

$$E\left(2, 0\right) = 2 \cdot 0 \cdot \frac{1}{4} = 0$$

$$E\left(2, 1\right) = 2 \cdot 1 \cdot 0 = 0$$

$$E\left(2, 2\right) = 2 \cdot 2 \cdot 0 = 0$$

$$E\left(x, y\right) = \frac{1}{2}$$

$$Cov\left(xy\right) = E\left(xy\right) - Ex \, Ey$$

$$= \frac{1}{2} - 1 \quad 1$$

$$= - \frac{1}{2}$$