Lab 1

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1 Task 1.0

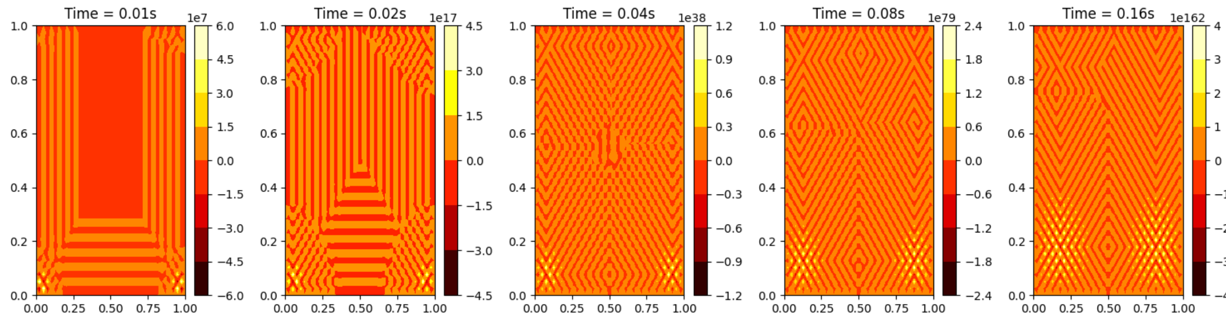
2 Task 1.1

3 Task 1.2

4 Task 1.3

5 Task 1.4

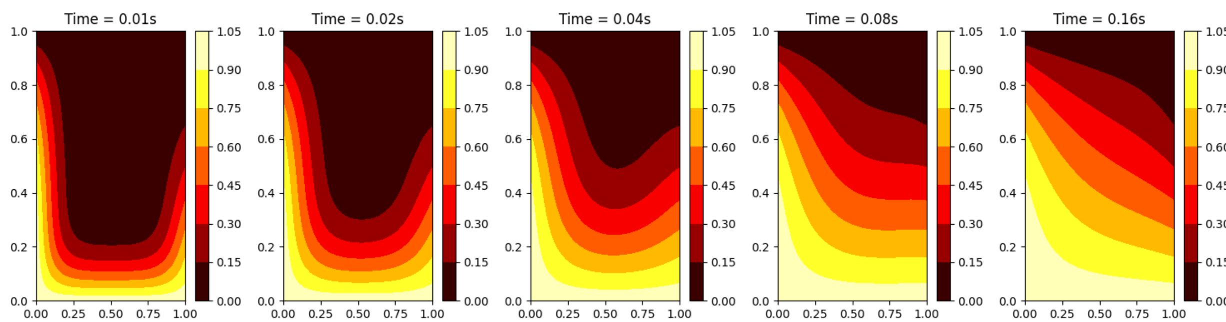
Yes, there are noticeable differences in the contour plots when changing the spatial resolution and/or the time step. Using a fixed spatial resolution of $\Delta x = \Delta y = \frac{1}{40}$ with a number of time steps of 160, we observe that the solution becomes unstable because the time step violates the Von Neumann stability criterion for the explicit Euler method.



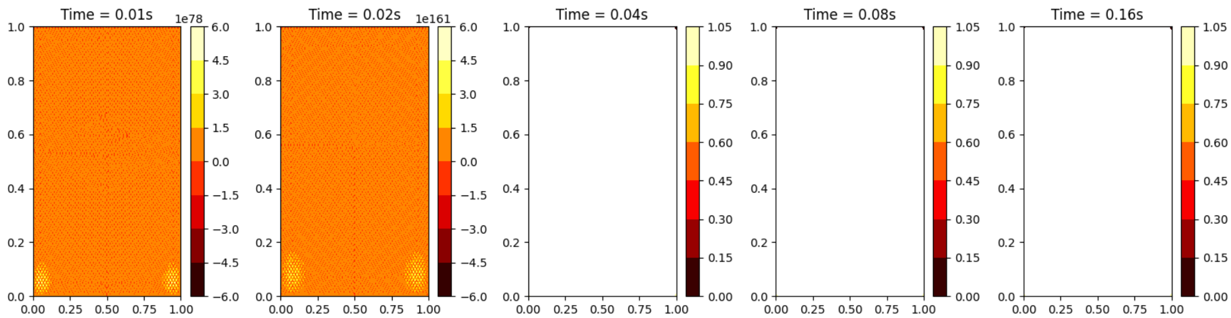
The stability condition ensures that errors introduced at each time step do not grow exponentially as the computation progresses. In particular, for a 2D heat diffusion problem, this condition can be expressed as:

$$\Delta t \leq \frac{\Delta x^2}{4}$$

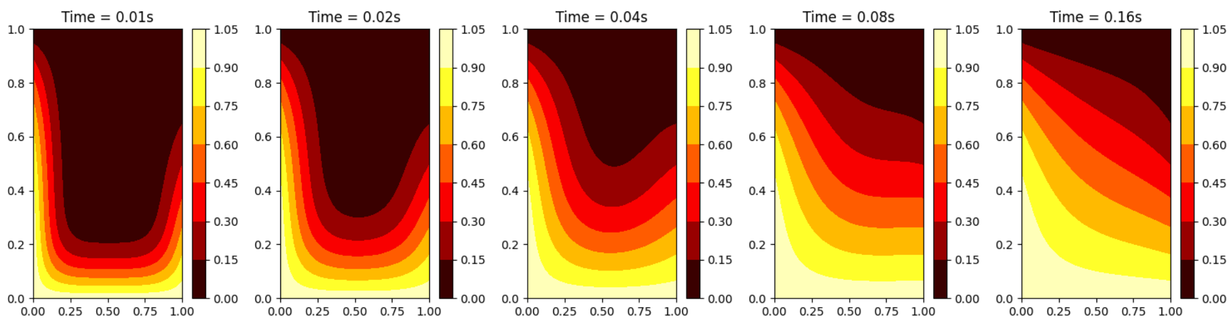
which implies that the minimum required number of time steps must be at least 1024. Infact, using a number of time steps of 1600 the solution becomes stable:



Similarly, using a finer spatial resolution of $\Delta x = \Delta y = \frac{1}{100}$ with a number of time steps of 1600 the solution becomes unstable because the minimum number of time steps for stability must be 6400.



Higher spatial resolution allows for a more accurate representation of temperature but requires smaller time steps to maintain stability. Infact, with a number of time steps of 6400, the solution becomes stable:



The stability of the solution depends on the ratio between Δt and Δx. For finer grids, a smaller Δt is required to avoid instability. The choice of time values that are factors of 2 apart ($t = 0.01, 0.02, 0.04, 0.08, 0.16$) helps us understand how heat diffusion evolves over time, allowing us to observe the progression of the temperature field at exponentially increasing intervals. Using exponentially increasing time steps provides a clear and efficient view of both the rapid initial diffusion and the slower, later stages, without requiring an excessive number of snapshots.

References

[1] *CFD Repository*,
Available at: `https://github.com/GiuseppePisante/CFD.git`

[2] *GitHub Copilot*,
GitHub. Available at: `https://github.com/features/copilot`