

— course overview —

18.06: Linear Algebra

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<http://web.mit.edu/18.06>

Textbook: Strang, *Introduction to Linear Algebra*, 5th edition
+ supplementary notes

Help wanted:

arrive 10 minutes early
and get paid \$10 to erase the boards

(You can forever after put
blackboard monitor, on your resumé
... shout-out to Terry Pratchett fans...)

Administrative Details

Lectures MWF11, 54-100

Tuesday recitations — use Stellar to switch sections

Weekly **psets, due Wednesday 11am in your recitation box.**

- no extensions or makeup, but lowest pset score will be dropped
- **pset 1 to be posted shortly**

Grading: **homework 15%, 3 exams 45%** (9/25, 10/30, & 11/27 in 50-340),
& final exam 40%

Collaboration policy: **talk to anyone** you want, **read anything** you want, but:

- Make an effort on a problem before collaborating.
- **Write up your solutions independently** (from “blank sheet of paper”).
- List your collaborators and external sources (not course materials).

Syllabus and Calendar

- Significant overlap with Strang's OCW video lectures: these are a **useful supplement** but **not a replacement** for attending lecture. Likely topics:
- **Exam 1: Monday 9/25.** Elimination, LU factorization, nullspaces and other subspaces, bases and dimensions, vector spaces, complexity. (Book: 1–3.5, 11.1)
- **Exam 2: Monday 10/30.** Orthogonality, projections, least-squares, QR, Gram–Schmidt, orthogonal functions (Book: 1–4, 10.5).
- **Exam 3: Monday 11/27.** Eigenvectors, determinants, similar matrices, Markov matrices, ODEs, symmetric matrices, definite matrices, matrices from graphs and engineering. (Book: 1–7, 10.1–3.)
- **Other topics:** defective matrices, SVD and principal-components analysis, sparse matrices and iterative methods, complex matrices, symmetric linear operators on functions.
- **Final exam:** all of the above.

What is 18.06 about?

High school:

3 “linear” equations
(only \pm and \times constants)
in 3 unknowns

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 2 \\ 4x_1 + 9x_2 - 3x_3 &= 8 \\ -2x_1 - 3x_2 + 7x_3 &= 10 \end{aligned}$$

Method: eliminate unknowns one at a time.

Equivalent **matrix** problem

$$Ax = b$$

Ax is a “linear operation:”

$$A(x+y) = Ax + Ay$$

$$A(3x) = 3Ax$$

take “dot products” of rows \times columns

$$\begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix}$$

A
matrix of
coefficients

x
vector of
unknowns

b
vector of
right-hand sides

What is 18.06 about?

Linear system of equations,
in matrix form

$$Ax = b$$
$$\begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix}$$

A x b

Will we learn **faster methods to solve this?** **No.** (Except if A is special.) The standard “Gaussian elimination” (and “LU factorization”) matrix methods are **just a slightly more organized** version of the high-school algebra elimination technique.

Will we get better at doing these calculations by hand? Maybe, but **who cares?**
Nowadays, all important matrix calculations are done by computers.

Will we learn **more about the computer algorithms?** **A little.** But mostly the techniques for “serious” numerical linear-algebra are **topics for another course** (e.g. **18.335**).

How do we *think* about linear systems?

(imagine someone gives you a $10^6 \times 10^6$ matrix)

- All the formulas for 2×2 and 3×3 matrices would fit on one piece of paper. They aren't the reason why linear algebra is important (as a class or a field of study).
- Large problems are solved by computers, but must be **understood by human beings**. (And we need to give computers the right tasks!)
- Understand **non-square problems**: #equations > #unknowns or vice versa
- **Break up matrices into simpler pieces**
 - Factorize matrices into **products of simpler matrices**: $A=LU$ (triangular: Gauss), $A=QR$ (orthogonal/triangular), $A=X\Lambda X^{-1}$ (diagonal: eigenvcs/vals), $A=U\Sigma V^*$ (orthogonal/diagonal: SVD)
 - **Submatrices** (matrices of matrices).
- **Break up vectors into simpler pieces**: **subspaces** and basis choices.
- Algebraic **manipulations to turn harder/unfamiliar problems** (e.g. minimization or differential equations) into **simpler/familiar** ones: **algebra on whole matrices at once**

Don't expect a lot of “turn the crank” problems
on psets or exams of the form
“solve this system of equations.”

... we will turn it upside-down, give you the
answer and ask the question, ask about
properties of the solution from partial
information, ... the general goal is to **require you
to understand the crank** rather than just turn it.

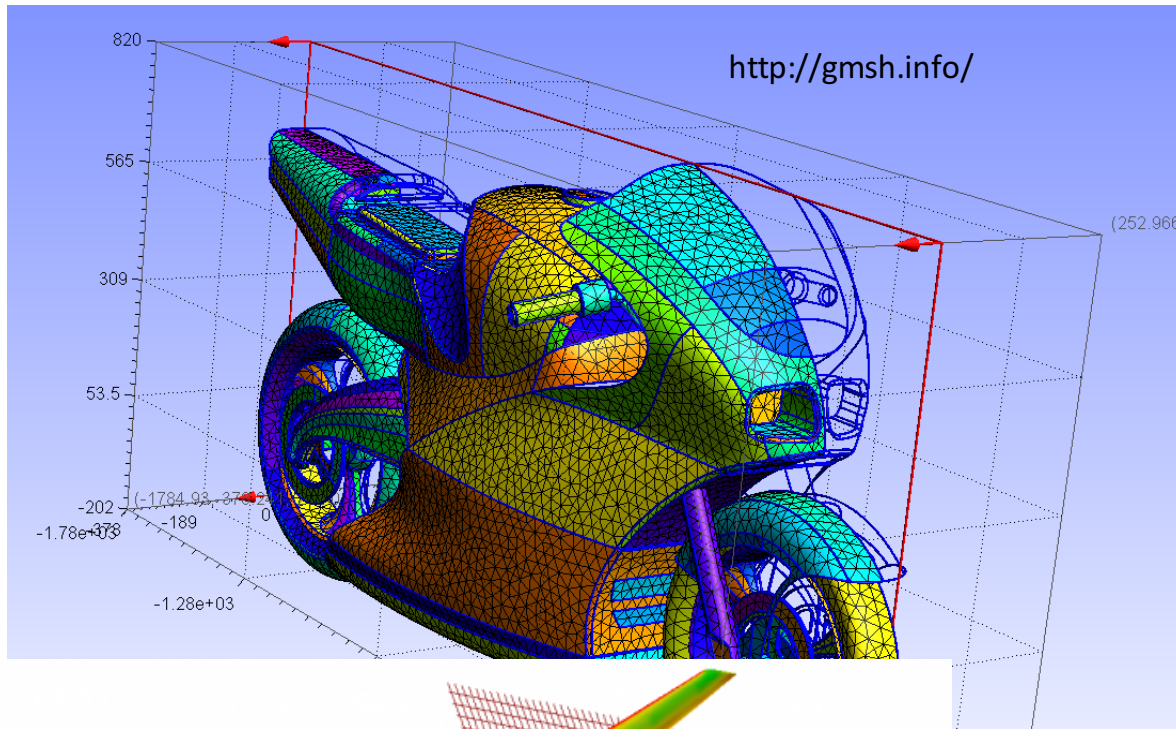
(**Exams might be a bit harder** than in previous
18.06 semesters, but grade cutoffs will be
adjusted accordingly.)

Where do big matrices come from?

Lots of examples in many fields,
but here are a couple that are
relatively easy to understand...

Engineering & Scientific Modeling

[18.303, 18.330, 6.336, 6.339, ...]

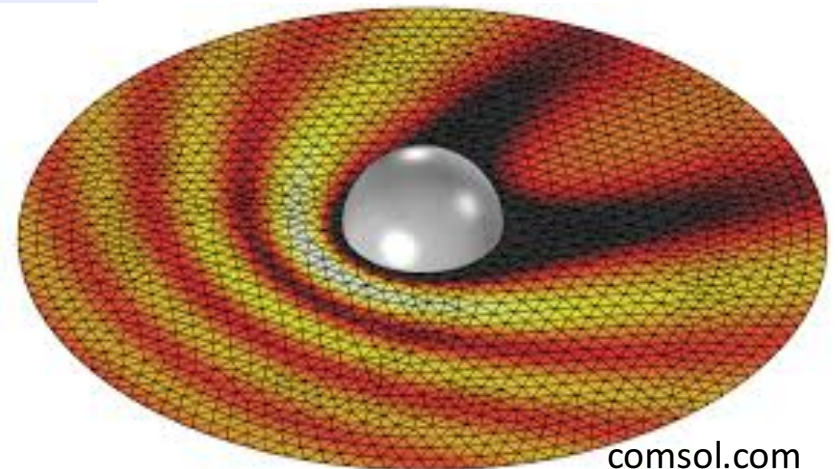
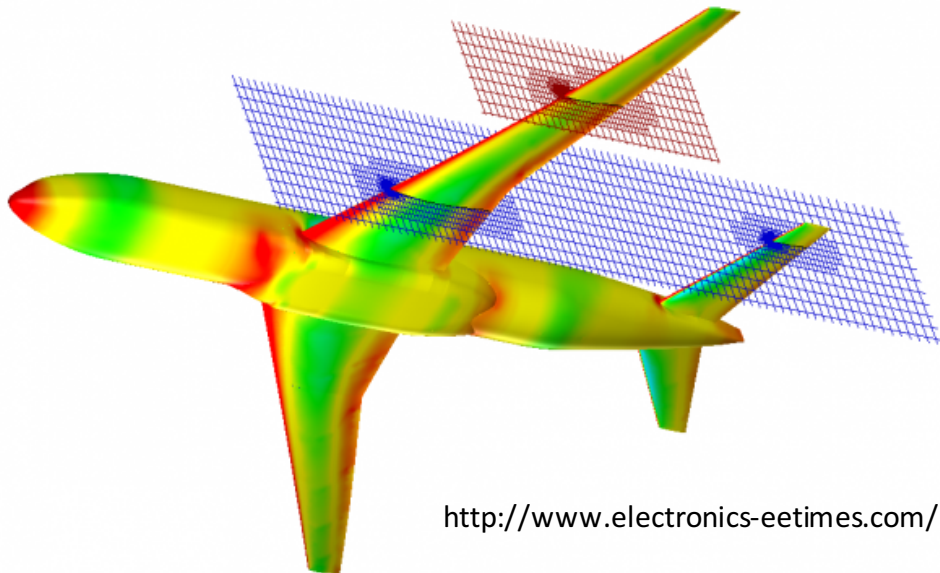


Unknown functions

(fluid flow, mechanical stress, electromagnetic fields, ...)

approximated by values on a discrete mesh/grid

e.g. $100 \times 100 \times 100$ grid
= 10^6 unknowns!



Data analysis

	0.2235	0.1294	Blue	0.4190	0
0.5804	0.2902	0.0627	0.2902	0.2902	0.4824
0.5804	0.0627	0.0627	0.0627	0.2235	0.2588
0.5176	0.1922	0.0627	Green	0.1922	0.2588
0.5176	0.1294	0.1008	0.1294	0.1294	0.2588
0.5176	0.1608	0.0627	0.1608	0.1922	0.2588
0.5490	0.2235	0.5490	Red	0.7412	0.7765
0.5490	0.3882	0.5176	0.5804	0.5804	0.7765
0.2588	0.2902	0.2588	0.2235	0.4824	0.2235
0.2235	0.1608	0.2588	0.2588	0.1608	0.2588
0.3882	0.1608	0.2588	0.2588	0.2588	0



[Mathworks]

Image processing:

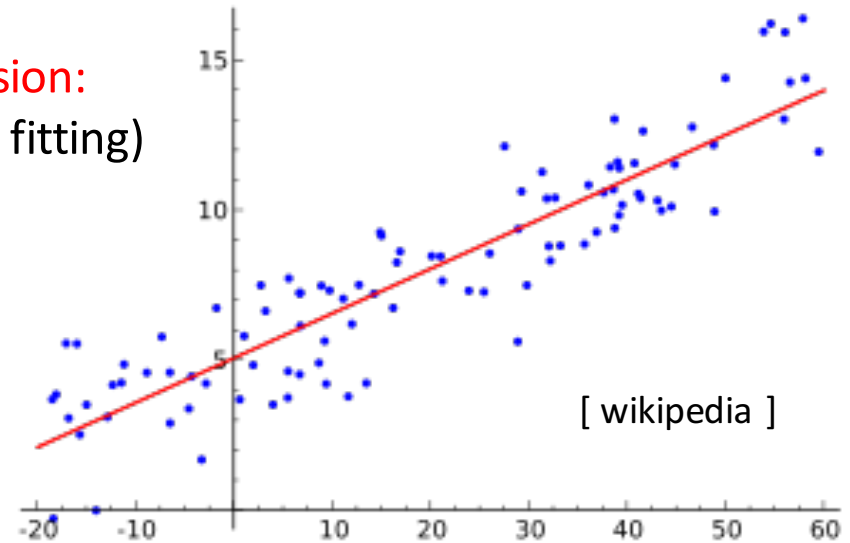
images are just matrices of numbers
(red/green/blue intensity)

Google “page rank” problem (also for gene networks etc.)

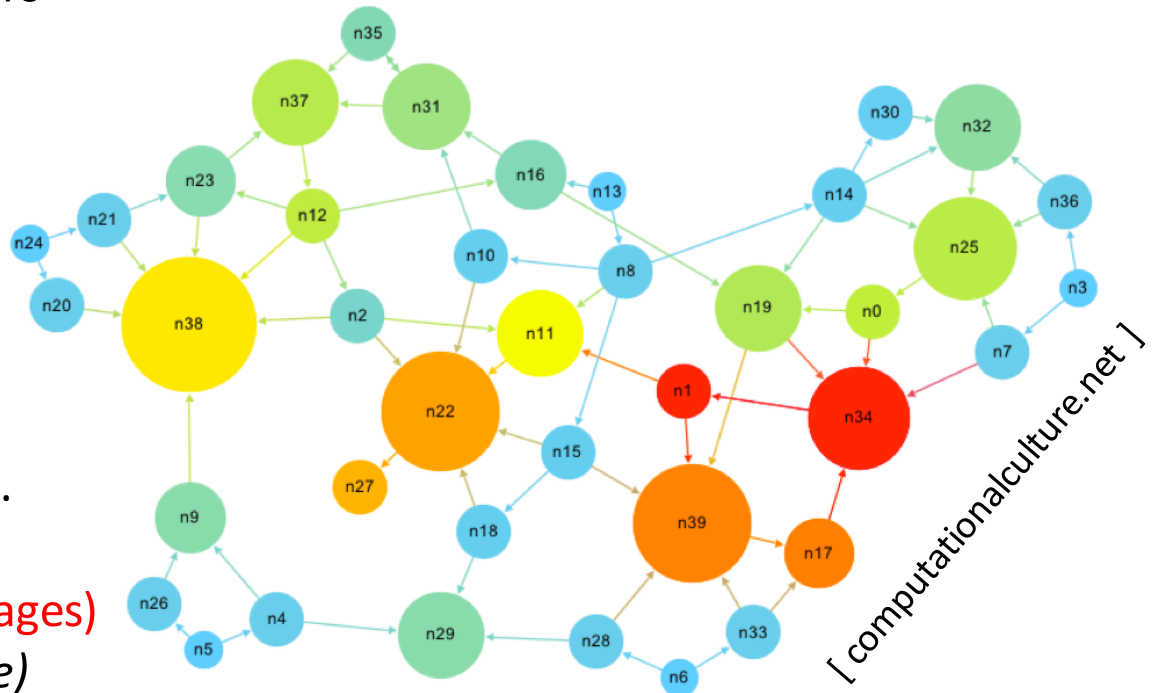
Determine the “most important”
web pages just from how they link.

matrix = (# web pages) × (# web pages)
(entry = 1 if they link, 0 otherwise)

Regression: (curve fitting)



[wikipedia]



[computationalculture.net]

Not just matrices of numbers

- There are lots of **surprising and important generalizations** of the ideas in linear algebra.
- Instead of **vectors** with a finite number of unknowns, **similar ideas apply to functions** with an **infinite number of unknowns**.
- Instead of **matrices** multiplying vectors, we can think about **linear operators on functions**

Poisson's equation
(e.g. 18.303)

$$\nabla^2 u = f$$

The diagram shows three red lines connecting parts of the equation to labels below. A line from ∇^2 points to the label "A", which is above "linear operator" and ∇^2 . A line from u points to the label "x", which is above "unknown function" and $u(x,y,z)$. A line from f points to the label "b", which is above "right-hand side" and $f(x,y,z)$.

"A"
linear operator
 ∇^2

"x"
unknown function
 $u(x,y,z)$

"b"
right-hand side
 $f(x,y,z)$

18.06 vs. 18.700

“applied” vs. “pure” math

few proofs vs. formal proofs expected

(deduce patterns
from examples,
informal arguments)

(definitions to
lemmas to theorems
... training in proof writing)

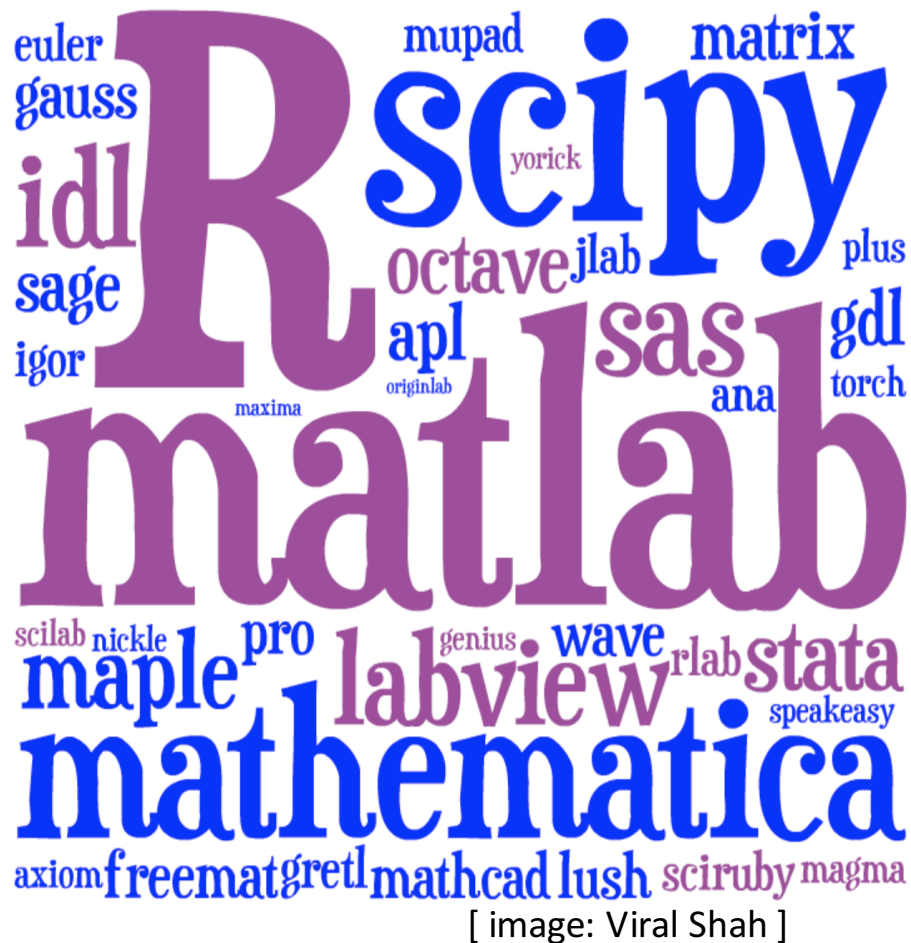
more applications vs. more theorems

more concrete vs. more abstract

some computers vs. only pencil-and-paper

Computer software

Lots of choices:



This semester: a relatively new language
that scales better to real problems.



No programming required for 18.06,
just a “glorified calculator” to turn the crank.

Use it online: log in at juliabox.com
see “Julia” link on Stellar

Optional tutorial: Monday 5pm 32-xxx