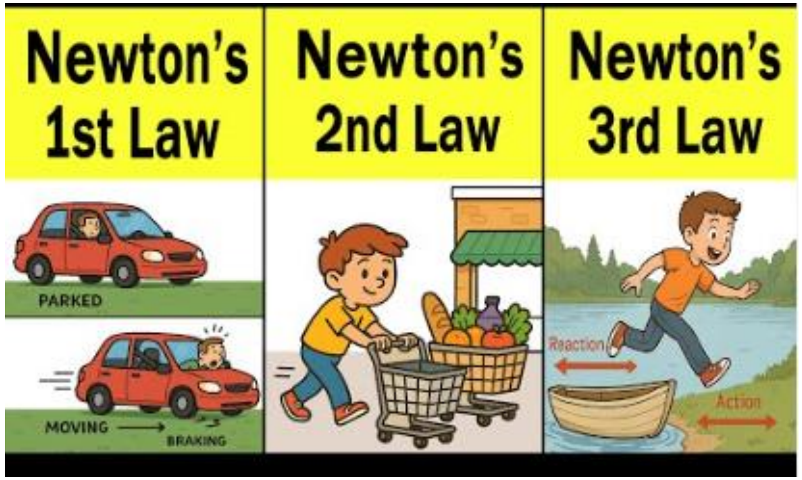
PHYSICS

CHAPTER 4 – Dynamics: Newton’s Laws of Motion

In the **previous chapter (Kinematics)**, we learned how to describe motion in terms of **displacement, velocity, and acceleration** — that is, *how* objects move.
Now we ask a deeper question: **why** do objects move as they do?
What makes an object at rest begin to move?
What causes an object to accelerate or slow down?
What determines motion along a curved path?
In all these situations, the answer involves a **force**.

❖ **What is Dynamics?**

Dynamics is the branch of physics that studies the **relationship between force and motion**.
It connects the **cause of motion (forces)** with the **description of motion (kinematics)**.



4.1 Force

A **force** is any **push or pull** acting on an object. Examples include pushing a car, pulling a cart, or gravity pulling an object downward.
Contact forces occur through physical contact (e.g., pushing, pulling, friction).
Non-contact forces act without contact (e.g., gravity).
A force is needed to **start motion**, **stop motion**, or **change the velocity** (speed or direction) of an object — meaning a force causes **acceleration**.
The **magnitude** of a force can be measured using a **spring scale**, which is commonly used to measure **weight** (the gravitational force on an object).
Because a force has both **magnitude and direction**, it is a **vector quantity** and can be represented by an **arrow** showing its direction and relative strength.

FIGURE 4–1 A force exerted on a grocery cart—in this case exerted by a person.

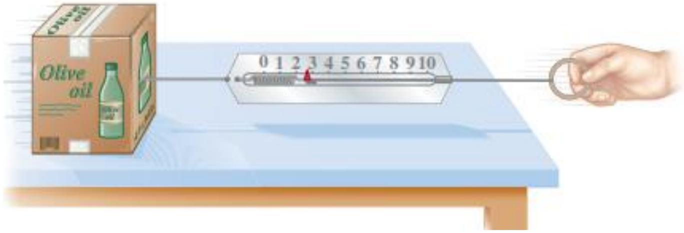
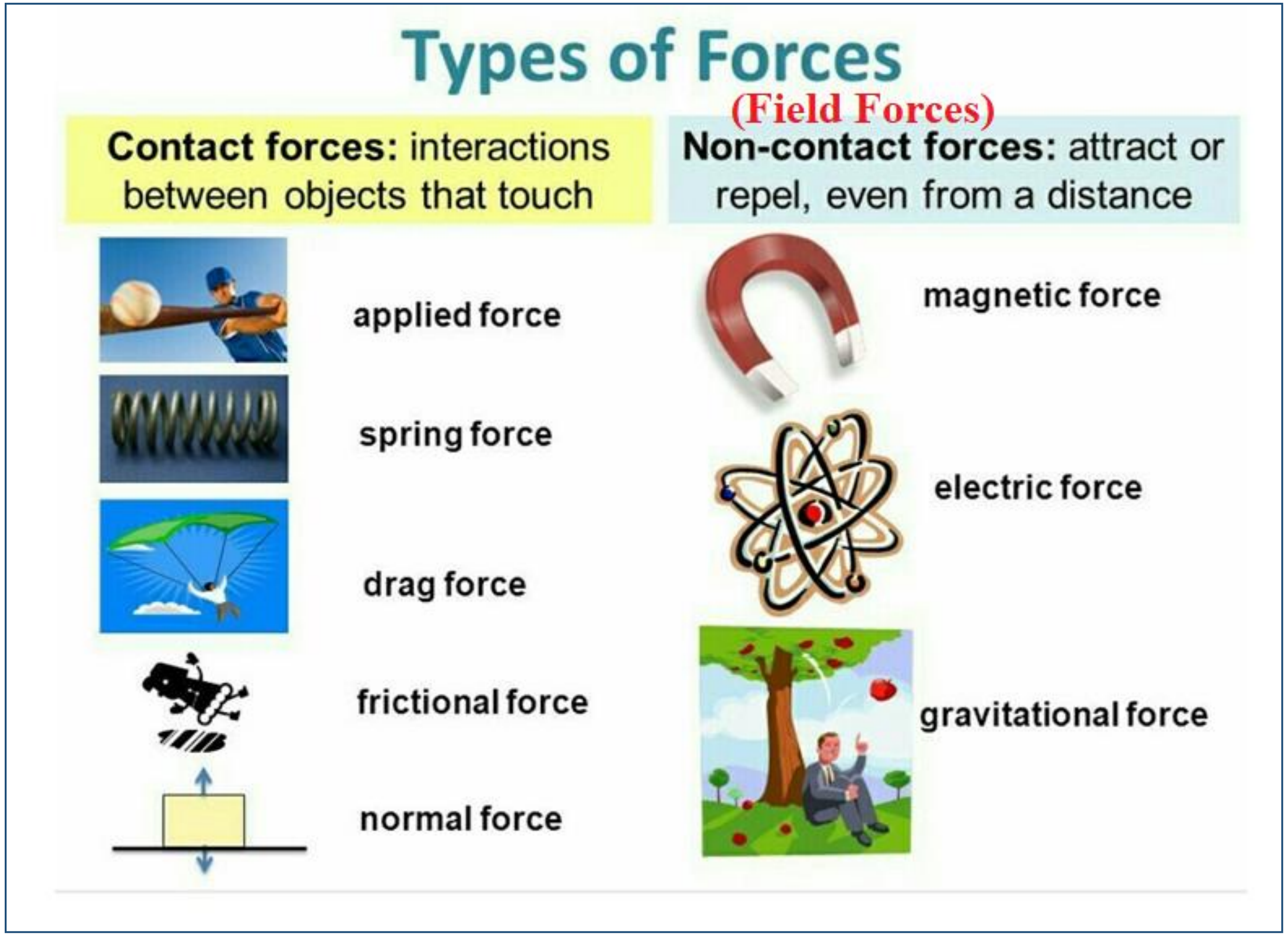


FIGURE 4–2 A spring scale used to measure a force.



Contact Force	Non-Contact Force
A **contact force** involves a force between two objects in contact.	A **non-contact force** involves a force between objects not touching. You can’t ‘see’ anything physically touching, but there is still an attraction or repulsion.
For example, **friction** between your feet and the ground can be present.	For example, **magnetic** forces between two magnets can happen when the magnets are near but not touching.

The Vector Nature of Force

A force can be measured using the deformation of a spring. When a vertical force is applied, the spring stretches, and the scale shows the force value. If one force stretches the spring by 1.00 cm and another twice as strong stretches it by 2.00 cm, their combined (collinear) effect equals the sum of both. However, if the two forces act in different directions (for example, one downward and one horizontal), the total force is found using vector addition. The resulting force has both magnitude and direction — showing that **force is a vector quantity** and must be combined using the rules of vector addition.

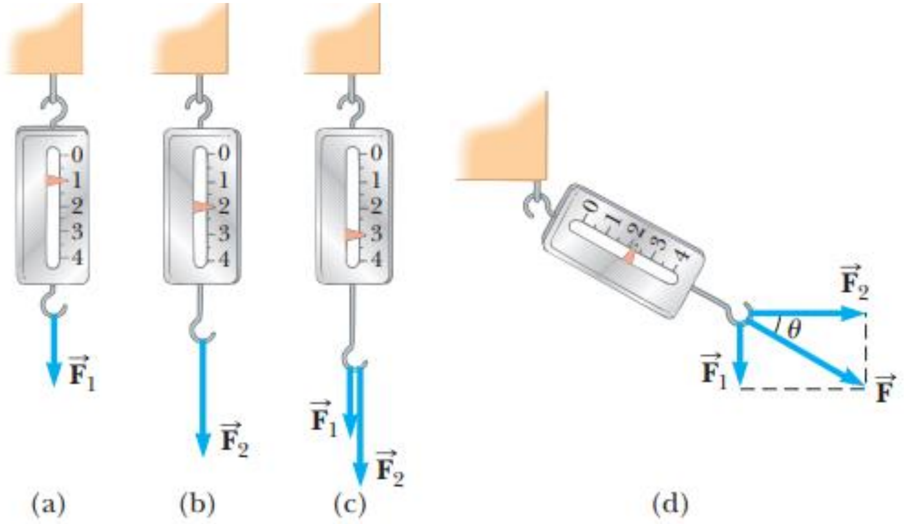


Figure 5.2 The vector nature of a force is tested with a spring scale. (a) A downward force $\vec{F}_1$ elongates the spring 1.00 cm. (b) A downward force $\vec{F}_2$ elongates the spring 2.00 cm. (c) When $\vec{F}_1$ and $\vec{F}_2$ are applied simultaneously, the spring elongates by 3.00 cm. (d) When $\vec{F}_1$ is downward and $\vec{F}_2$ is horizontal, the combination of the two forces elongates the spring 2.24 cm.

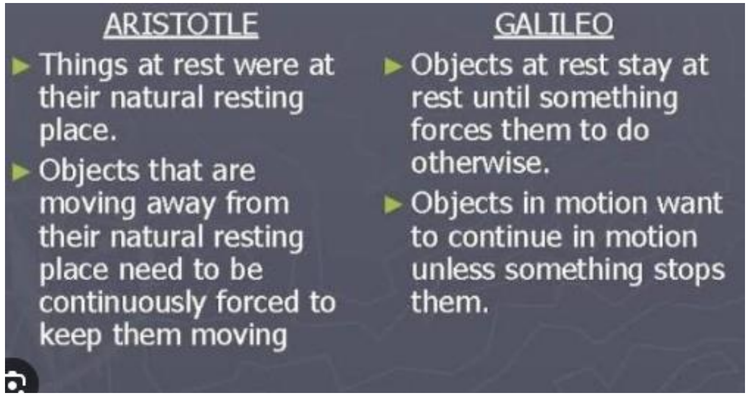
Long ago, **Aristotle** believed that an object needed a **constant force to keep moving** and that if you stopped pushing, it would stop. He thought the natural state of an object was **rest**.

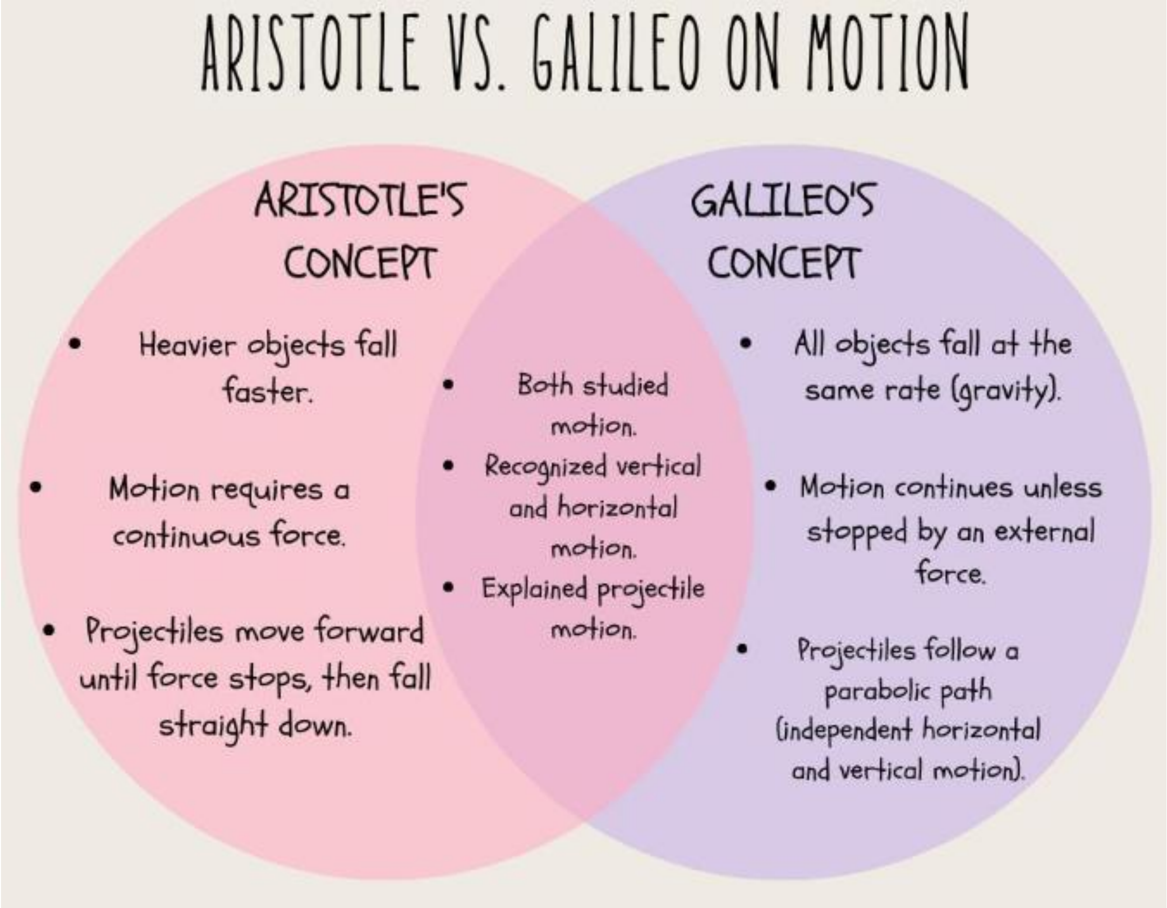
Later, **Galileo** disagreed. He noticed that when friction is reduced (for example, with smooth surfaces or oil), **less force** is needed to keep an object moving. If there were **no friction at all**, an object would **keep moving forever at the same speed and direction** without any force. He realized that **friction** is a force that **slows things down**. Therefore, an object **keeps moving at constant speed** if **no net force** acts on it.

To move an object at a constant speed across a table, your **push must exactly balance the friction force**. These two forces act in **opposite directions** and are **equal in size**, so the **net force is zero**. This agrees with **Galileo's idea** that an object moves with **constant velocity** when **no net force** acts on it.

FIGURE 4–3 $\vec{F}$ represents the force applied by the person and $\vec{F}_{fr}$ represents the force of friction.

Aristotle vs Galileo

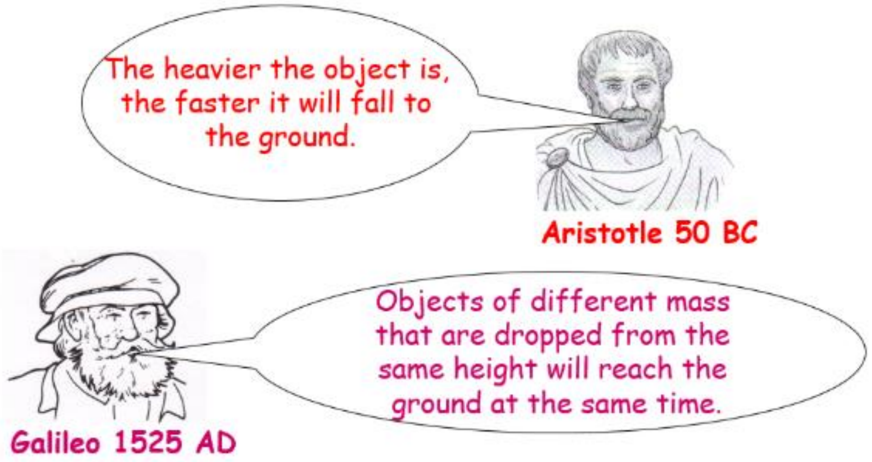




Aristotle vs Galileo

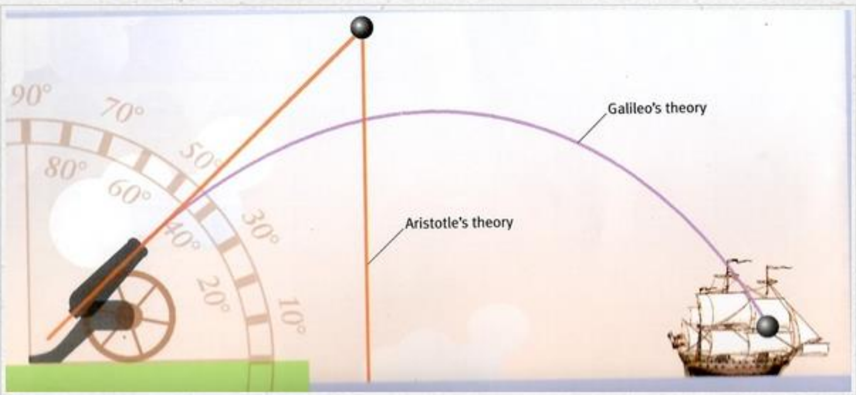
Projectile Motion

Aristotle	Galileo
The motion of a projectile is parallel to the ground until it is the object's time to fall back into the ground.	A. A projectile moves two-dimensional motion in a parabolic path. B. The horizontal motion component has zero acceleration (constant speed horizontally) and vertical acceleration is constant.



- Aristotle and Galileo were two of the most important historical figures in Physics. They may had opposing views regarding motion, but they both helps Science progress.

PROJECTILES



Another theory of Aristotle's that Galileo questioned, was regarding the practical motion of projectiles (1). Aristotle believed it traveled in a straight diagonal line until the energy was gone it would fall in a straight line down (1). Through Galileo's studies he was able to prove they moved in curves (1).

Aristotle vs Galileo

Aspect	Aristotle	Galileo
Force and motion	Force is needed to maintain motion	Force is not needed to maintain motion; only to change motion
Falling objects	Heavier objects fall faster	All objects fall at the same rate (ignoring air resistance)
Nature of motion	Motion is natural or violent	Motion is uniform unless acted on by a force
Projectile motion	Objects move along straight lines, then fall straight down; motion requires continuous force	Objects move in a parabolic path due to horizontal inertia and vertical gravity; no continuous force needed horizontally
Method	Based on observation and reasoning	Based on experiments and measurements
Inertia	Objects stop if no force is applied	Objects continue moving unless a force acts

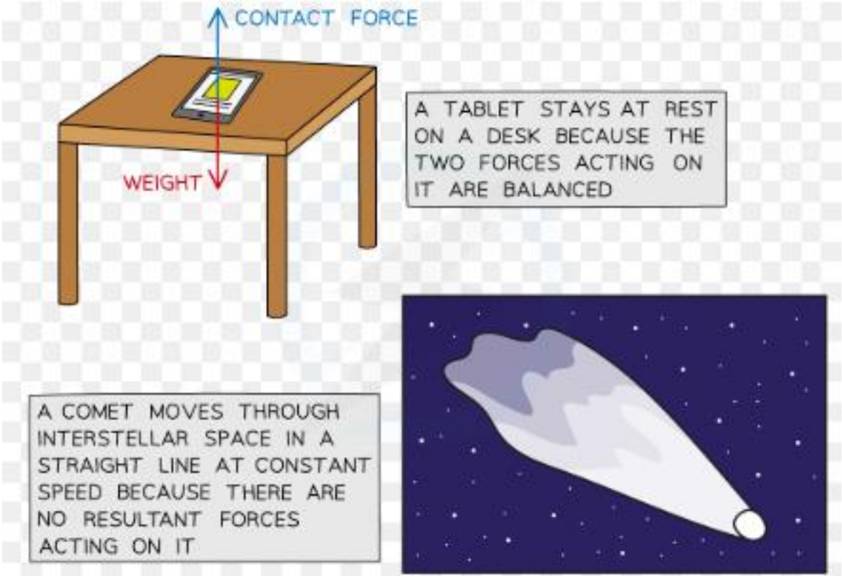
Newton later built on Galileo's ideas and wrote the **First Law of Motion**, also called the **Law of Inertia**:

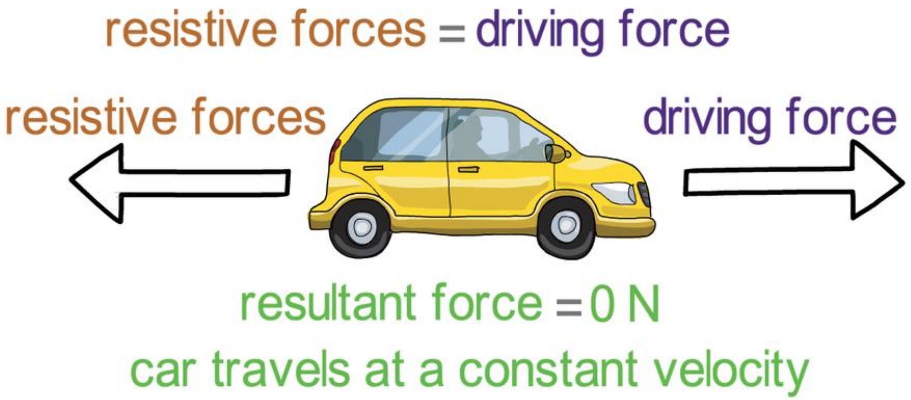
An object will stay at rest or move in a straight line at constant speed unless a net force acts on it.

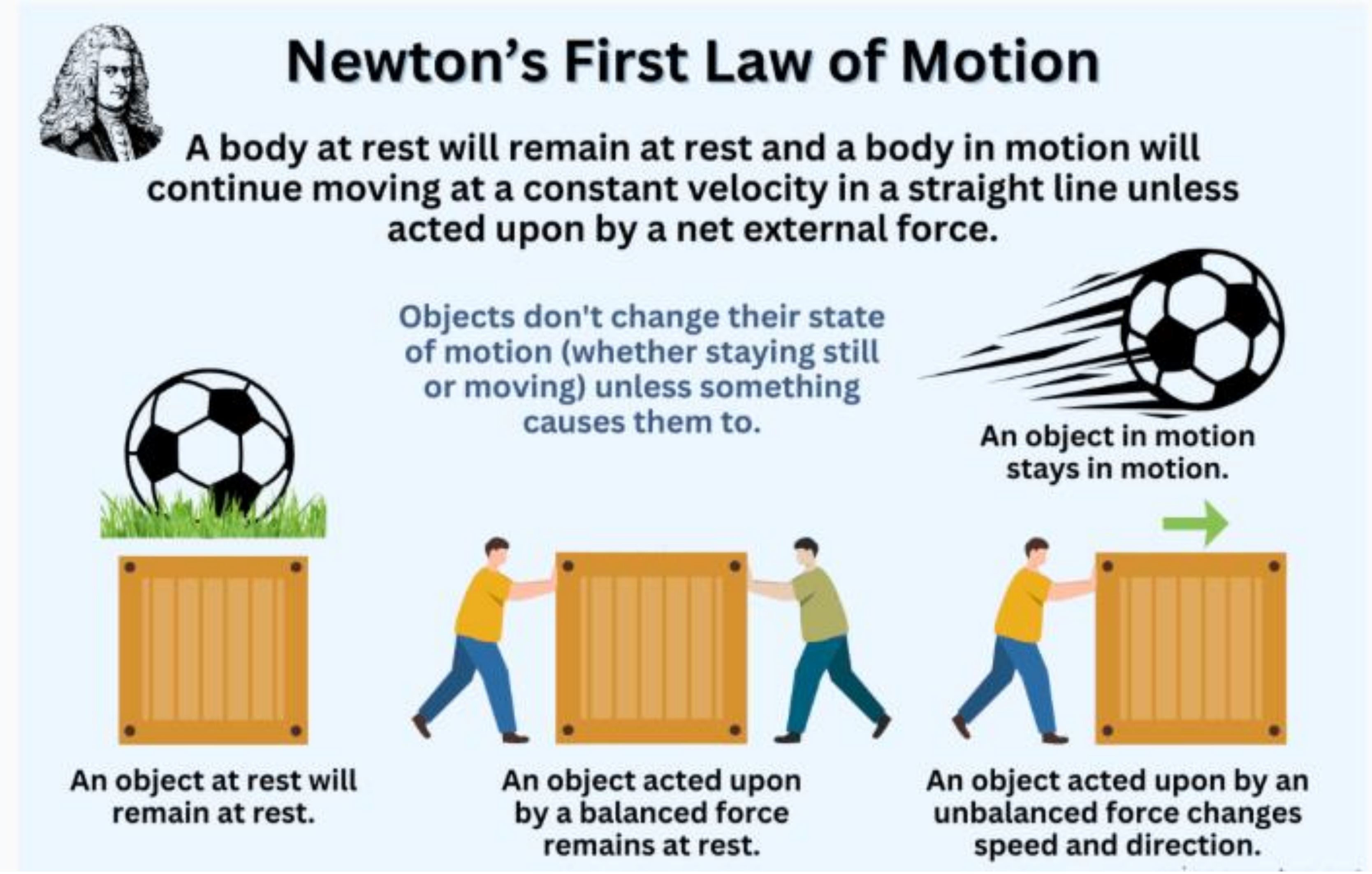
This means that **motion doesn't need a continuous force**—only a force can **change** an object's motion. The tendency of an object to maintain its state of rest or of uniform velocity in a straight line is called inertia. As a result, Newton's first law is often called the law of inertia.

Newton's First Law (Law of Inertia):

When the **net force (F_{net} = 0)** on an object is zero, the object either **remains at rest** or **moves with constant velocity in a straight line**.






Newton's First Law of Motion
A body at rest will remain at rest and a body in motion will continue moving at a constant velocity in a straight line unless acted upon by a net external force.
Objects don't change their state of motion (whether staying still or moving) unless something causes them to.
An object in motion stays in motion.
An object at rest will remain at rest.
An object acted upon by a balanced force remains at rest.
An object acted upon by an unbalanced force changes speed and direction.

Newton's First Law and Galileo's Insight

Common belief (Aristotle): Objects naturally stay at rest; motion requires a force.

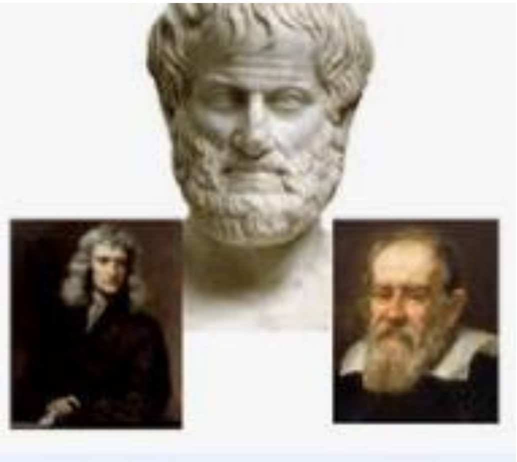
Galileo's discovery: This belief is incorrect; it arises because of friction.

Experiments with reduced friction showed objects maintain their velocity longer.

He concluded that, without friction, an object would keep moving at its initial velocity indefinitely.

Key idea: An isolated object moves in a straight line at constant speed; velocity does not change unless a force acts.

Newton's First Law: Formalizes Galileo's insight: every object continues in its state of rest or uniform motion unless acted upon by an external force.

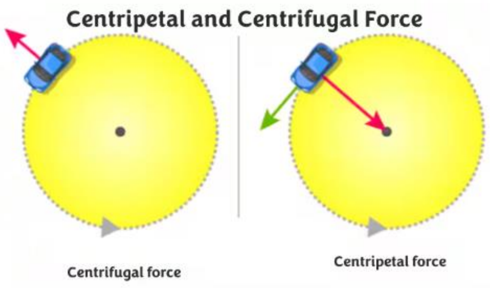


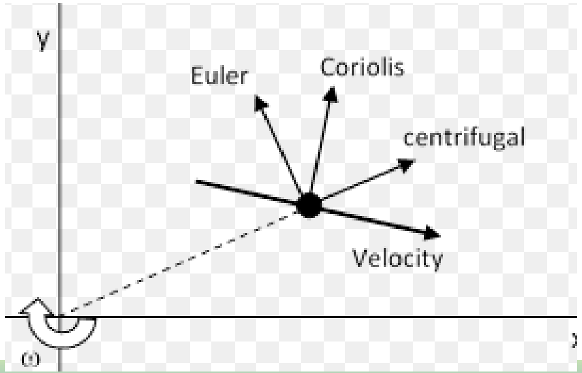
Inertial Reference Frames

Newton's first law works only in certain reference frames called **inertial reference frames**.
So, **Newton's first law defines an inertial frame** — if the law holds true in that frame, it's inertial.

Inertial Frame:
A reference frame that is **at rest or moves with constant velocity**. Newton's laws hold, and **no fictitious forces** appear.
Example: A train moving straight at constant speed.
Non-Inertial Frame:
A reference frame that is **accelerating or rotating**. Newton's laws do not hold directly, and **fictitious forces** appear.

Examples of fictitious forces:
Centrifugal force – felt when a car turns sharply or on a rotating platform.
Coriolis force – affects moving objects on a rotating Earth.
Euler force – appears when the rotation speed of a frame changes.
Example: A car turning a corner (centrifugal force pushes you outward).





A bowling ball is harder to throw or stop than a basketball because it has more **mass**, meaning it resists changes in motion more. **Mass** measures how much an object resists acceleration when a force acts on it — this property is called **inertia**. The greater the mass, the greater the force needed to change its motion.

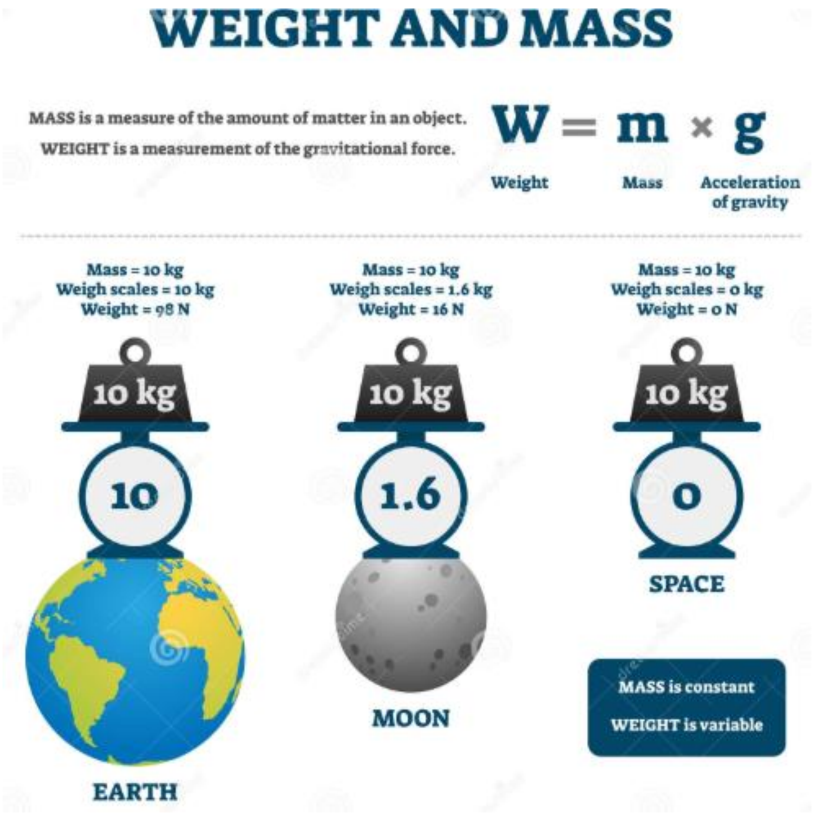
Newton's second law is based on this concept. Newton described mass as the "quantity of matter," but more precisely, mass is a **measure of inertia** — how strongly an object resists a change in its velocity. For example, a truck has much more inertia (and mass) than a baseball, so it takes a much larger force to change its motion.
Mass is a **scalar quantity**, adds normally (e.g., 3 kg + 5 kg = 8 kg), and is measured in **kilograms (kg)** in the SI system.

Mass can be measured by comparing how different objects accelerate under the same force. If a force gives one object (m_1) an acceleration a_1, and another object (m_2) an acceleration a_2 then

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

This means **acceleration is inversely proportional to mass** for a given force. For example, if a 3-kg object accelerates at 4 m/s², the same force would make a 6-kg object accelerate at 2 m/s². By comparing accelerations, we can determine an unknown mass.

Mass should not be confused with weight.
Mass is an inherent property of matter — it stays the same everywhere.
Weight is a **force** caused by gravity and changes with location.
For instance, an object that weighs **800 N** on Earth would weigh only about **130 N** on the Moon because gravity is weaker, but its **mass** remains the same — it still has the same amount of matter and the same inertia.



- Newton's First Law: An object remains at rest or moves with constant velocity if no net force acts on it.
- When a **net force** is applied:

If it acts **in the same direction** as motion → velocity increases (object speeds up).
If it acts **opposite to motion** → velocity decreases (object slows down).
If it acts **sideways** → direction of motion changes.

- Therefore, **a net force always causes acceleration** (a change in velocity's magnitude or direction).

NEWTON'S SECOND LAW OF MOTION

The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

$$\vec{\mathbf{a}} \propto \frac{\sum \vec{\mathbf{F}}}{m}$$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\frac{\mathrm{m}}{\mathrm{s}^2} = \frac{\mathrm{N}}{\mathrm{kg}}$$

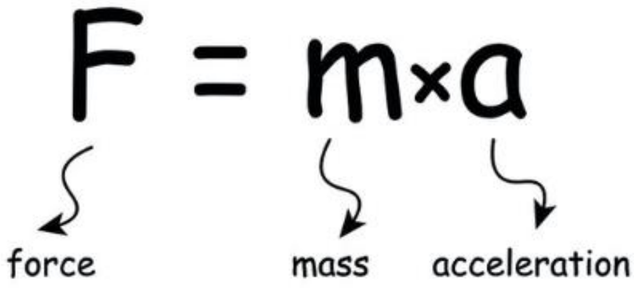


TABLE 4–1
Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) $(= \mathrm{kg \cdot m/s^2})$
cgs	gram (g)	dyne $(= \mathrm{g \cdot cm/s^2})$
British	slug	pound (lb)

Conversion factors: 1 dyne $= 10^{-5}$ N;
1 lb $\approx$ 4.45 N;
1 slug $\approx$ 14.6 kg.

Newton's Third Law of Motion

Forces always arise from interactions between objects; one object exerts a force on another.

Example: A hammer exerts a force on a nail, and the nail exerts an equal force back on the hammer.

Law statement: *Whenever one object exerts a force on a second object, the second object exerts an equal and opposite force on the first.*

Key point: Action and reaction forces act on **different objects**, not on the same object.

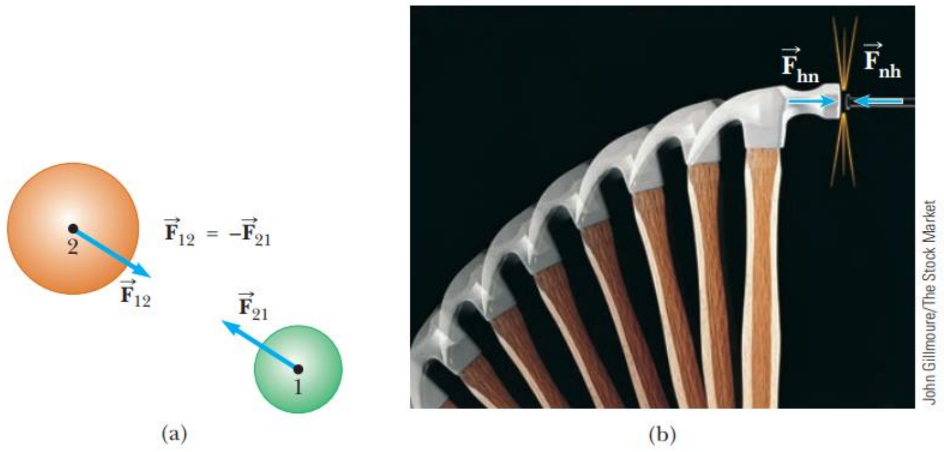
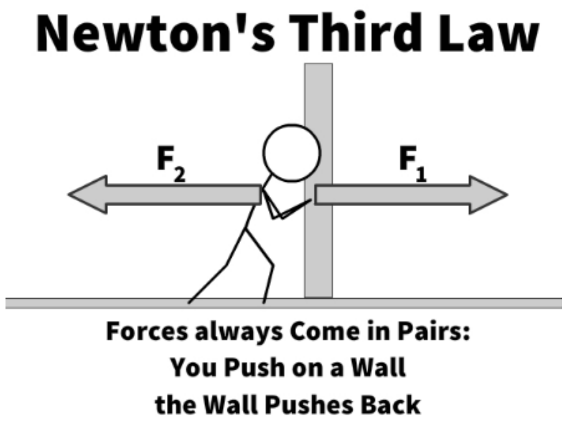
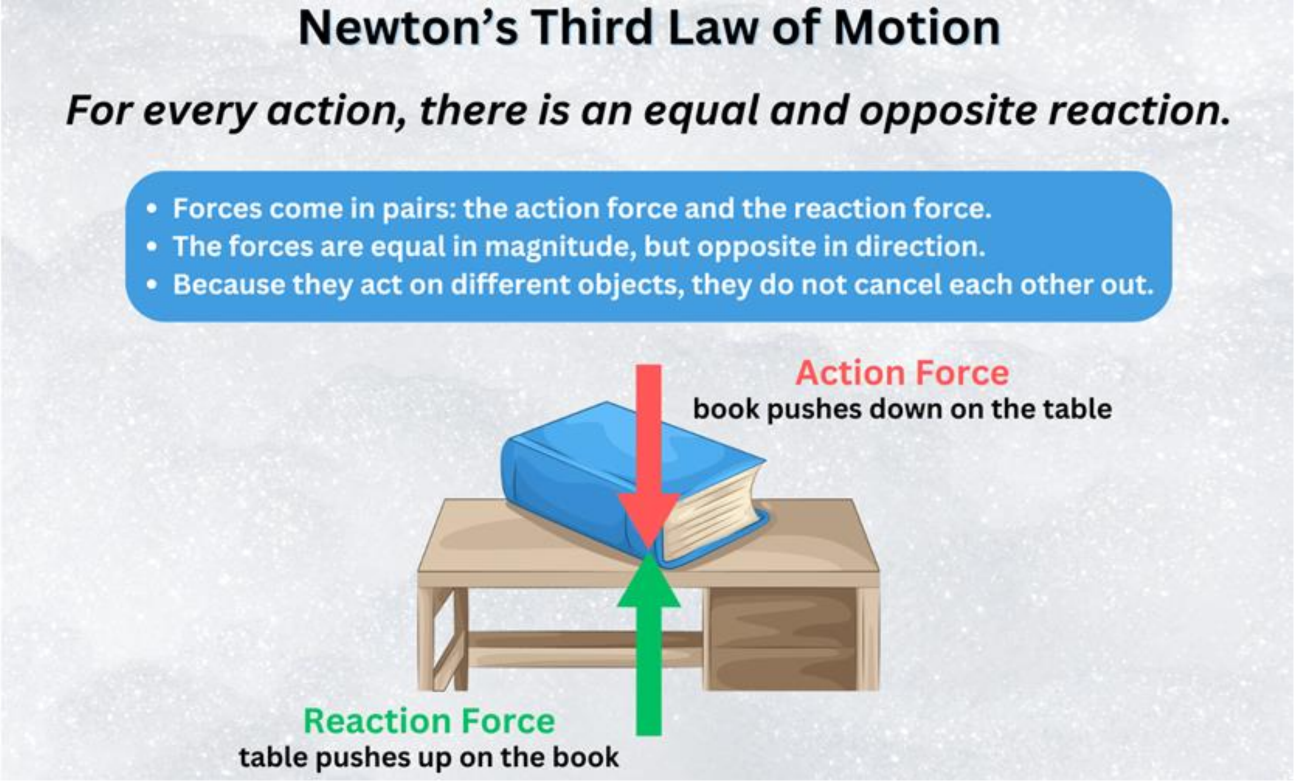


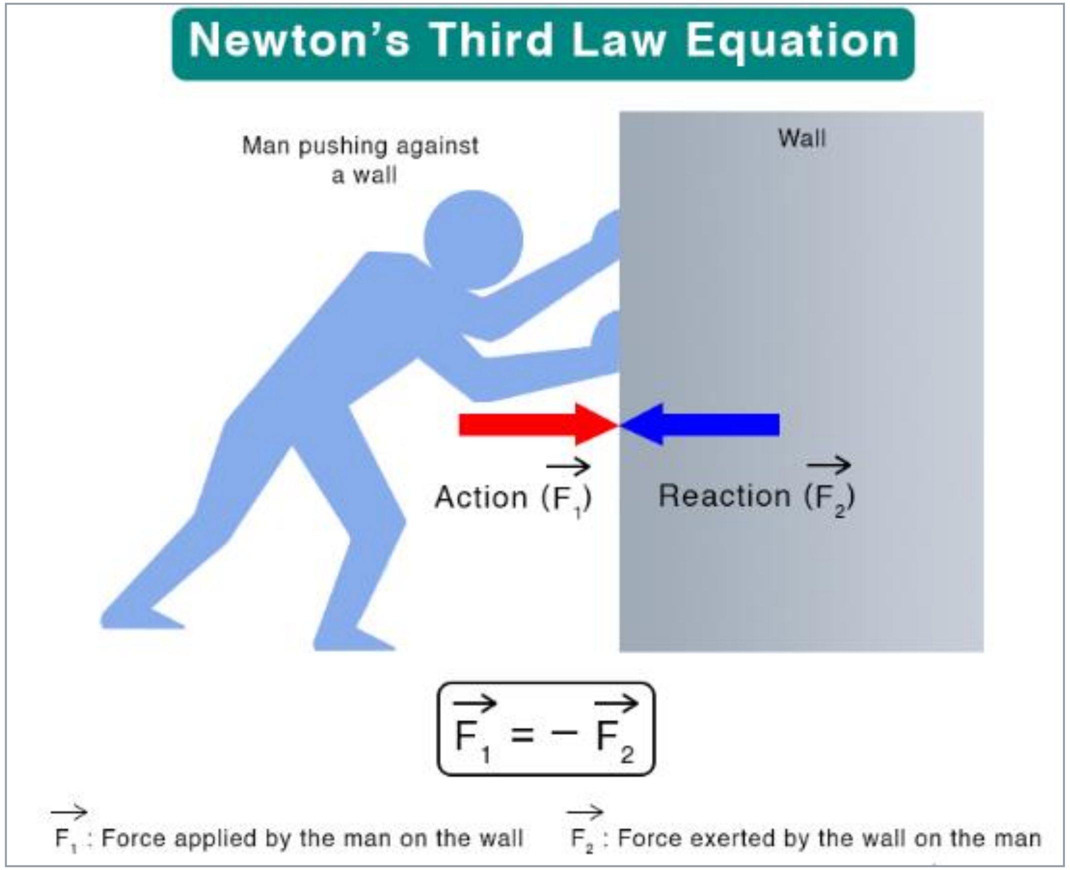
Figure 5.5 Newton's third law. (a) The force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1. (b) The force $\vec{F}_{hn}$ exerted by the hammer on the nail is equal in magnitude and opposite to the force $\vec{F}_{nh}$ exerted by the nail on the hammer.

If two objects interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1:

$$\vec{F}_{12} = -\vec{F}_{21} \tag{5.7}$$




Newton's Third Law of Motion
For every action, there is an equal and opposite reaction.
• Forces come in pairs: the action force and the reaction force.
• The forces are equal in magnitude, but opposite in direction.
• Because they act on different objects, they do not cancel each other out.
Action Force
book pushes down on the table
Reaction Force
table pushes up on the book


Newton's Third Law Equation
Man pushing against a wall
Wall
Action ($\vec{F}_1$)
Reaction ($\vec{F}_2$)
$\vec{F}_1 = -\vec{F}_2$
$\vec{F}_1$: Force applied by the man on the wall
$\vec{F}_2$: Force exerted by the wall on the man

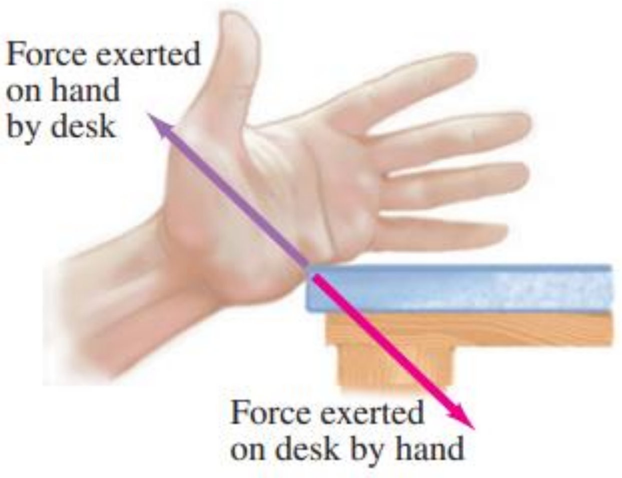
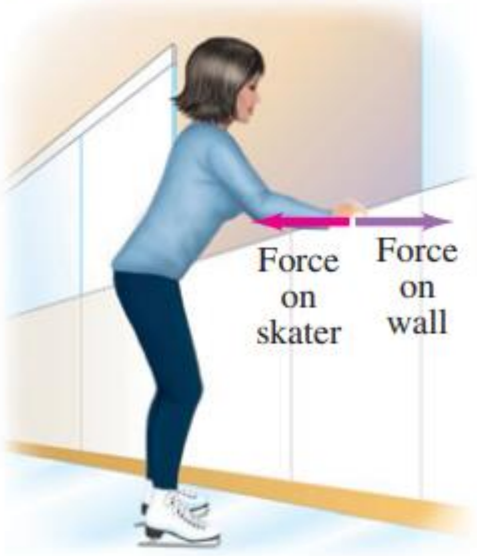


FIGURE 4–8 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, violet, to remind us that this force acts on a different object).

FIGURE 4–9 An example of Newton's third law: when an ice skater pushes against the wall, the wall pushes back and this force causes her to accelerate away.

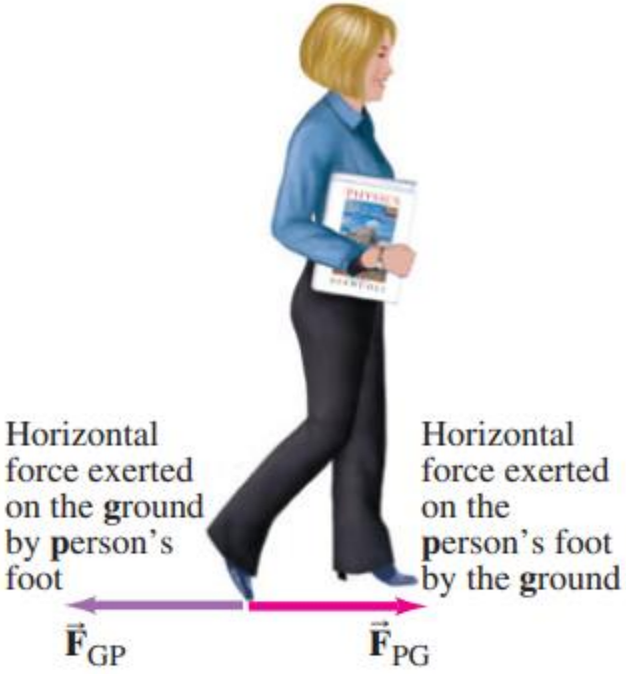


A key to the correct application of the third law is that *the forces are exerted on different objects*. Make sure you don't use them as if they were acting on the *same* object.

4.5 Newton's Third Law of Motion

FIGURE 4–10 Another example of Newton's third law: the launch of a rocket. The rocket engine pushes the gases downward, and the gases exert an equal and opposite force upward on the rocket, accelerating it upward. (A rocket does *not* accelerate as a result of its expelled gases pushing against the ground.)

FIGURE 4–11 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot (Newton's third law). The two forces shown *act on different objects.*

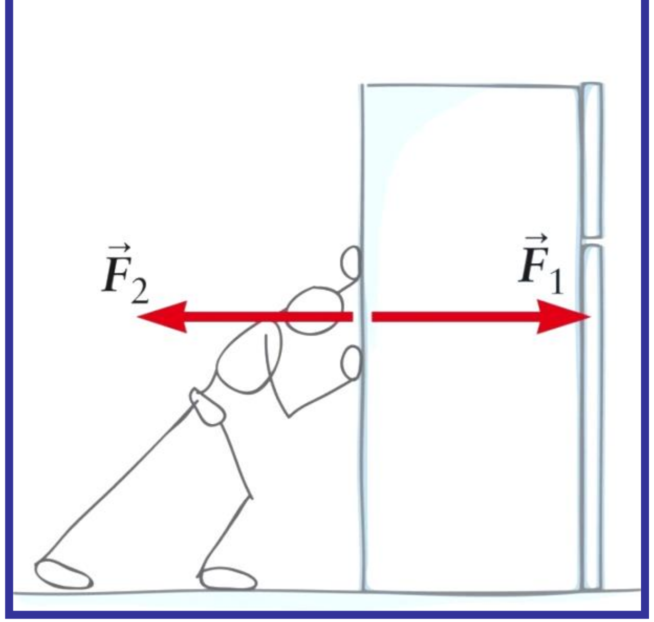


$$\vec{F}_{GP} = -\vec{F}_{PG}.$$

Newton's 3rd Law: Consequences

- **Newton's 3rd Law** tells us that
  FORCES COME IN PAIRS
- The 2 forces are
  always equal in magnitude
  & opposite in direction.
- ***Important!!*** The 2 forces act on
  different objects

The forces in an action–reaction pair act on different objects.

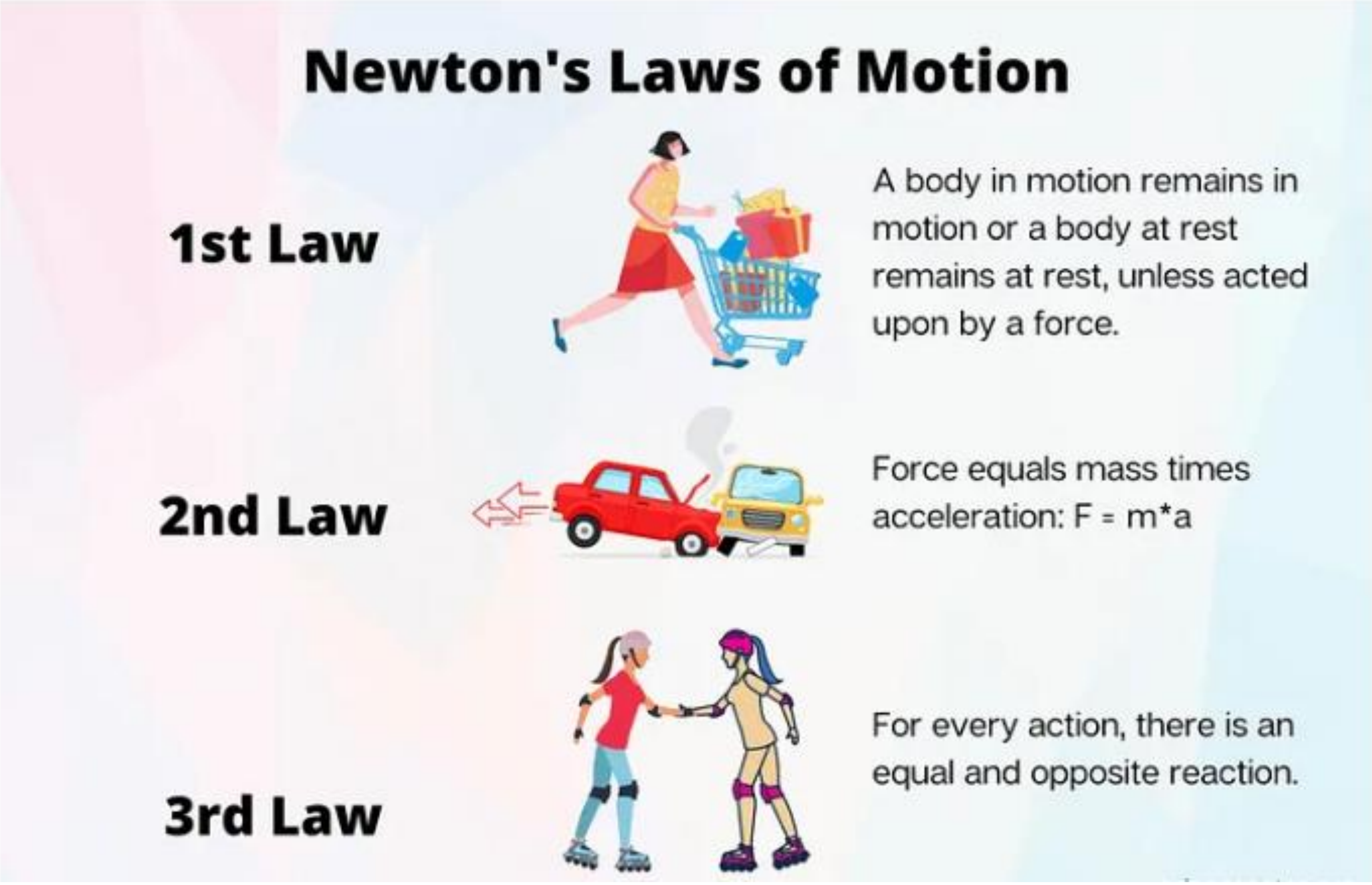


- **Figure:** The person exerts force $\mathbf{F_1}$ ***on the refrigerator***.
- The refrigerator exerts force $\mathbf{F_2}$ ***on the person***.

Newton's 3rd Law: $F_2 = -F_1$

Newton's 3rd Law: Alternative Statements

1. Forces ***ALWAYS*** occur in pairs
2. A single isolated force ***CANNOT*** exist
3. The "action force" is equal in magnitude to the "reaction force" & opposite in direction.
4. One of the forces is the "action force", the other is the "reaction force"
5. It doesn't matter which is considered the "action" & which the "reaction"
6. The action & reaction forces ***MUST ACT ON DIFFERENT OBJECTS*** & be of the same type.



$$\vec{F}_{\text{net}} = 0 \implies \vec{a} = 0$$

$$\vec{F}_{\text{net}} = m\,\vec{a}$$

$$\vec{F}_{\text{AB}} = -\vec{F}_{\text{BA}}$$

Galileo observed that all objects near Earth's surface fall with the same acceleration if air resistance is negligible. This acceleration is caused by the gravitational force, which is exerted by the Earth and acts vertically downward toward its center. Using Newton's second law, the gravitational force on an object of mass m is:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}.$$

where g is the acceleration due to gravity. The magnitude of this force is called the object's weight. On Earth, a 1.0-kg mass weighs about 9.8 N, but weight varies on other planets or moons—for example, the same mass weighs only 1.6 N on the Moon due to lower gravity.

Gravity acts on objects even when they are at rest. An object on a table doesn't move because the gravitational force is balanced by an upward contact force from the table, called the **normal force**, which acts perpendicular to the surface. This ensures the net force on the object is zero, in accordance with Newton's second law.

The two forces on a resting object—gravity pulling down and the table pushing up—are equal and opposite, but they are **not** the action-reaction pair of Newton's third law because they act on the same object. The true reaction forces are: the statue pushes down on the table in response to the table pushing up, and the statue pulls on the Earth in response to Earth's gravity pulling it down.

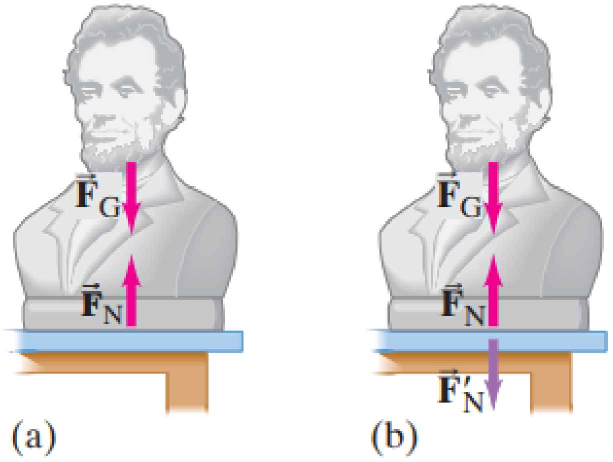
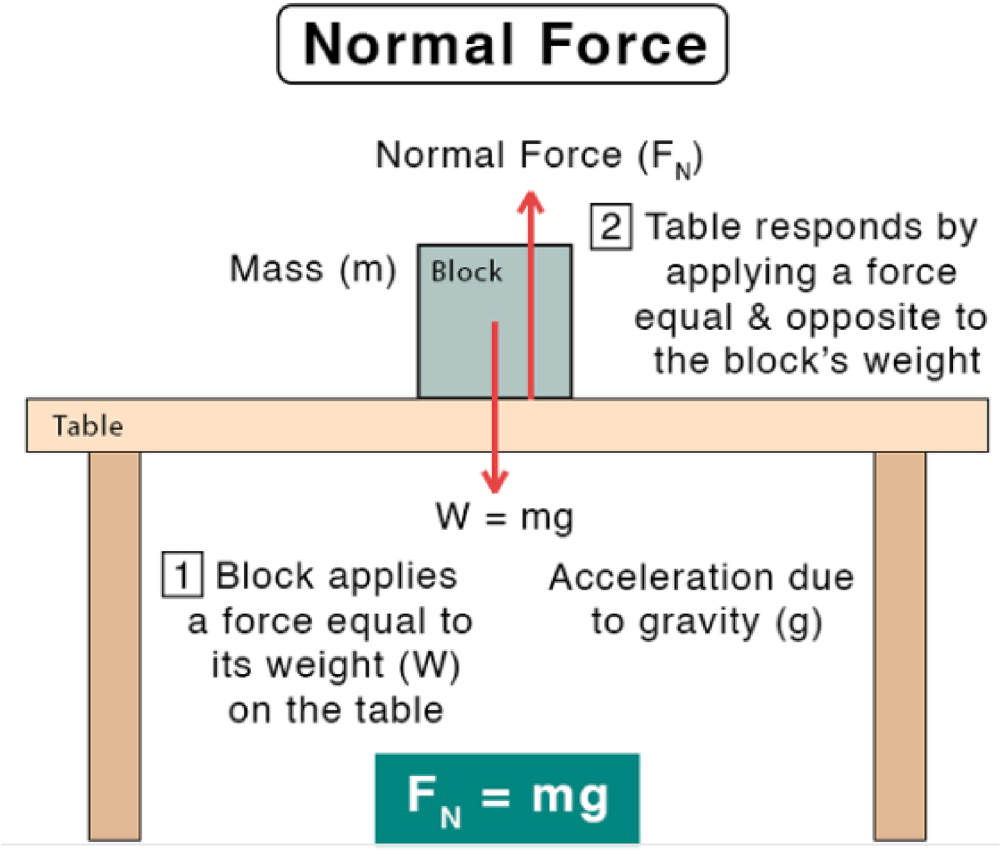


FIGURE 4–14 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity ($\vec{\mathbf{F}}_G$) on an object at rest must be balanced by an upward force (the normal force $\vec{\mathbf{F}}_N$) exerted by the table in this case. (b) $\vec{\mathbf{F}}'_N$ is the force exerted on the table by the statue and is the reaction force to $\vec{\mathbf{F}}_N$ by Newton's third law. ($\vec{\mathbf{F}}'_N$ is shown in a different color to remind us it acts on a different object.) The reaction force to $\vec{\mathbf{F}}_G$ is not shown.



Normal Force

Definition:

The **normal force** (F_N) is the contact force a surface exerts on an object, **perpendicular** to the surface.

Key Points:

1. **Direction:** Always perpendicular to the surface.
2. **Relation to Weight:**
   - At rest on a horizontal surface with no extra forces:

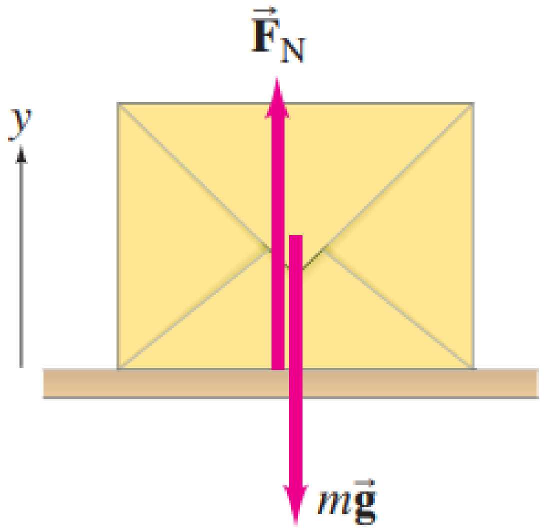
$$F_N = mg$$

   - With extra vertical forces:
     - Downward push: $F_N = mg + F_{\text{push}}$
     - Upward pull: $F_N = mg - F_{\text{pull}}$
3. **Net Force:**

$$\sum F_y = F_N + F_{\text{up}} - F_{\text{down}} = ma_y$$

At rest, $a_y = 0$.

Example: A 10.0 kg box rests on a frictionless table.
(a) Find its weight and the normal force.
(b) If a 40.0 N downward force is applied, what is the normal force?
(c) If a 40.0 N upward force is applied, what is the normal force?



(a) $\Sigma F_y = F_{\mathrm{N}} - mg = 0$

(a)

The weight of the box acts downward:

$$F_{\text{gravity}} = mg = 10.0\,\mathrm{kg} \times 9.8\,\mathrm{m/s}^2 = 98.0\,\mathrm{N}$$

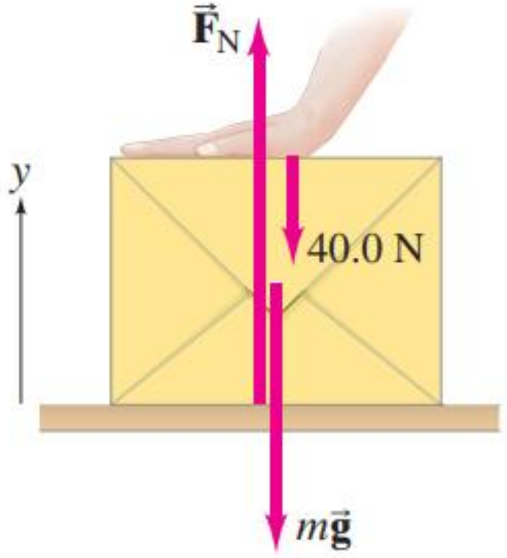
The table exerts an upward normal force F_N. Since the net force is zero:

$$\sum F_y = F_N - F_{\text{gravity}} = 0$$

$$F_N = F_{\text{gravity}} = 98.0\,\mathrm{N}$$

The normal force is 98 N upward.

A 10.0 kg box rests on a frictionless table.
(a) Find its weight and the normal force.
(b) If a 40.0 N downward force is applied, what is the normal force?
(c) If a 40.0 N upward force is applied, what is the normal force?



(b) $\Sigma F_y = F_N - mg - 40.0\ \text{N} = 0$

(b)

A 40.0 N downward force is applied. The net force in the vertical direction is:

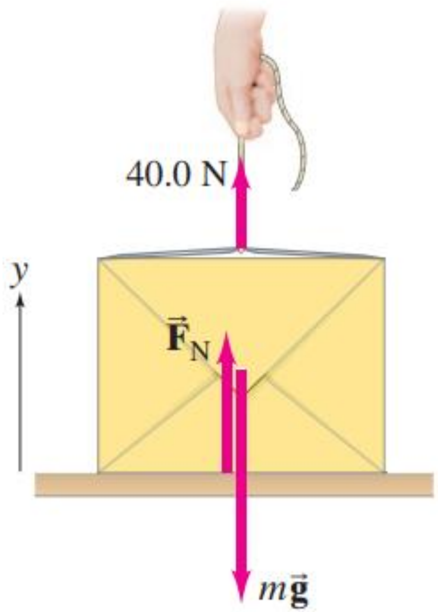
$$\sum F_y = F_N - F_{\text{gravity}} - F_{\text{push}} = ma_y$$

Since $a_y = 0$:

$$F_N - 98.0 - 40.0 = 0 \quad \Rightarrow \quad F_N = 138.0\,\text{N}$$

The normal force is 138 N upward.

A 10.0 kg box rests on a frictionless table.
(a) Find its weight and the normal force.
(b) If a 40.0 N downward force is applied, what is the normal force?
(c) If a 40.0 N upward force is applied, what is the normal force?



(c) $\Sigma F_y = F_N - mg + 40.0\text{ N} = 0$

(c)

A 40.0 N upward force is applied. The forces acting on the box are:

- Weight: $F_{\text{gravity}} = mg = 98.0\,\text{N}$ (downward)
- Friend's pull: $F_{\text{pull}} = 40.0\,\text{N}$ (upward)
- Normal force: F_N (upward)

Using Newton's second law in the vertical direction:

$$\sum F_y = F_N + F_{\text{pull}} - F_{\text{gravity}} = ma_y$$

Since the box is at rest, $a_y = 0$:

$$F_N + 40.0 - 98.0 = 0$$

$$F_N = 98.0 - 40.0 = 58.0\,\text{N}$$

Answer: The normal force is **58 N upward**.

Accelerating the box:
What happens when a person pulls upward on a box with a force equal to, or greater than, the box's weight? For example, suppose the pull is Fp=100 N.

The net upward force is

$$\Sigma F_y = F_P - mg = 100.0\ \text{N} - 98.0\ \text{N} = 2.0\ \text{N}$$

Since the pull is greater than the weight, the normal force becomes zero and the box accelerates upward. Applying Newton's second law:

$$a_y = \frac{\Sigma F_y}{m} = \frac{2.0\ \text{N}}{10.0\ \text{kg}} = 0.20\ \text{m/s}^2$$

Thus, the box moves upward with an acceleration of $0.20\ \text{m/s}^2$.

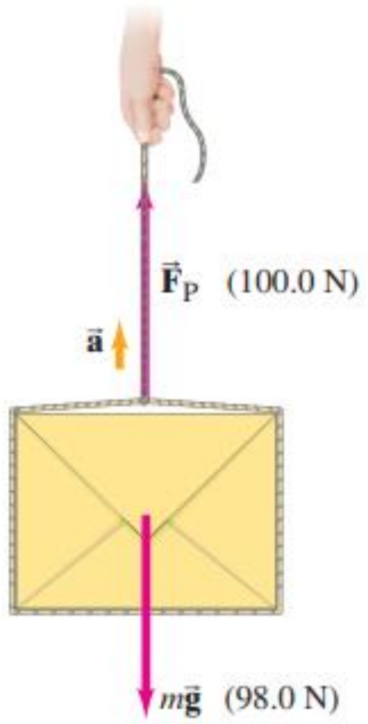
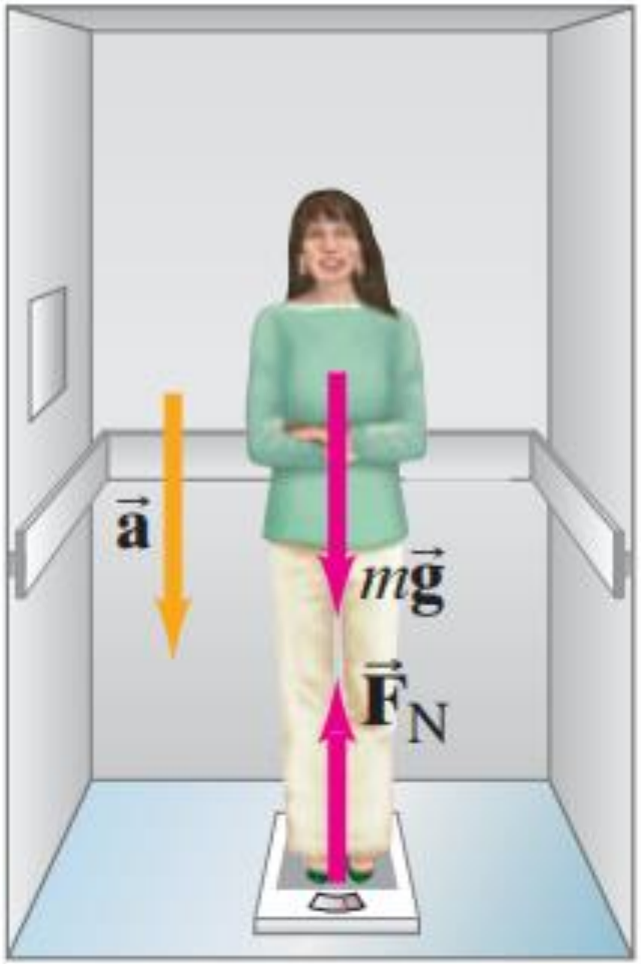


FIGURE 4–16 Example 4–7. The box accelerates upward because $F_P > mg$.

EXAMPLE 4–8 **Apparent weight loss.** A 65-kg woman descends in an elevator that briefly accelerates at $0.20g$ downward. She stands on a scale that reads in kg. (*a*) During this acceleration, what is her weight and what does the scale read? (*b*) What does the scale read when the elevator descends at a constant speed of 2.0 m/s?

APPROACH Figure 4–17 shows all the forces that act on the woman (and *only* those that act on her). The direction of the acceleration is downward, so we choose the positive direction as down (this is the opposite choice from Examples 4–6 and 4–7).



SOLUTION (*a*) From Newton's second law,

$$\begin{aligned} \Sigma F &= ma \\ mg - F_N &= m(0.20g). \end{aligned}$$

We solve for F_N:

$$\begin{aligned} F_N &= mg - 0.20mg \\ &= 0.80mg, \end{aligned}$$

and it acts upward. The normal force $\vec{F}_N$ is the force the scale exerts on the person, and is equal and opposite to the force she exerts on the scale: $F'_N = 0.80mg$ downward. Her weight (force of gravity on her) is still $mg = (65\text{ kg})(9.8\text{ m/s}^2) = 640\text{ N}$. But the scale, needing to exert a force of only $0.80mg$, will give a reading of $0.80m = 52\text{ kg}$.

(*b*) Now there is no acceleration, $a = 0$, so by Newton's second law, $mg - F_N = 0$ and $F_N = mg$. The scale reads her true mass of 65 kg.

A free-body diagram (FBD) is a simple sketch of an object showing **all external forces** acting on it.

How to draw an FBD:

Isolate the object (draw as a box or point)

Draw arrows for all forces

Label each force

Choose x-y axes

Common Forces:

Weight (mg) (downward)

Normal force, F_N (upward from surface)

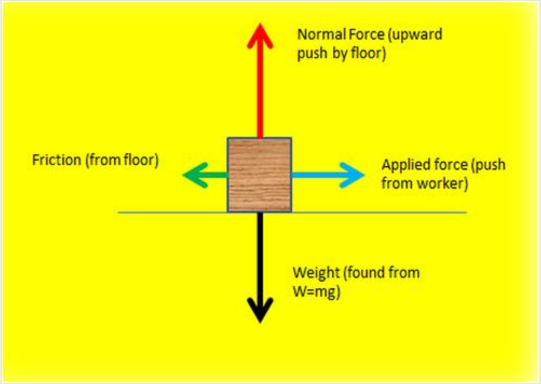
Applied force (push/pull)

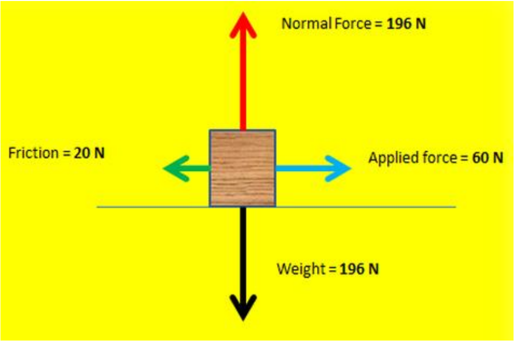
Friction (opposes motion)

Tension (in rope/string)

A **20 kg box** is at rest on a flat surface. A worker pushes it to the right with **60 N**, while friction **opposes the motion with 20 N**.

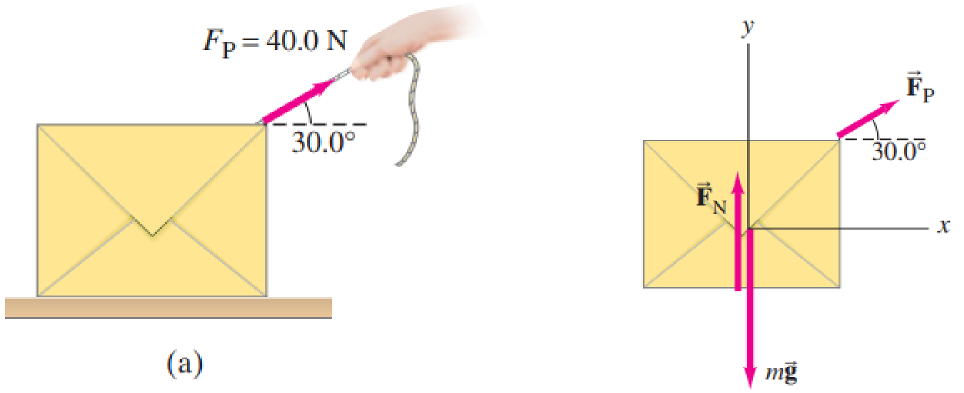
Draw the Free Body Diagram





Question: A **10.0 kg box** is pulled along a smooth table with a force of $F_P = 40.0\,\mathrm{N}$ at a **30° angle**. There is **no friction**. Find the **acceleration** of the box and the **upward force** (normal force) from the table.

Free-Body Diagram



(a) (b)

Choose axes and resolve vectors: We expect the motion to be horizontal, so we choose the x axis horizontal and the y axis vertical. The pull of 40.0 N has components

$$F_{Px} = (40.0\,\mathrm{N})(\cos 30.0°) = (40.0\,\mathrm{N})(0.866) = 34.6\,\mathrm{N},$$
$$F_{Py} = (40.0\,\mathrm{N})(\sin 30.0°) = (40.0\,\mathrm{N})(0.500) = 20.0\,\mathrm{N}.$$

In the horizontal (x) direction, $\vec{\mathbf{F}}_N$ and $m\vec{\mathbf{g}}$ have zero components. Thus the horizontal component of the net force is F_{Px}.

(a) **Apply Newton's second law** to get the x component of the acceleration:

$$F_{Px} = ma_x.$$

(a) **Solve**:

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\,\mathrm{N})}{(10.0\,\mathrm{kg})} = 3.46\,\mathrm{m/s^2}.$$

The acceleration of the box is $3.46\,\mathrm{m/s^2}$ to the right.

(b) Next we want to find F_N.

(b) **Apply Newton's second law** to the vertical (y) direction, with upward as positive:

$$\Sigma F_y = ma_y$$
$$F_N - mg + F_{Py} = ma_y.$$

(b) **Solve**: We have $mg = (10.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 98.0\,\mathrm{N}$ and, from point 3 above, $F_{Py} = 20.0\,\mathrm{N}$. Furthermore, since $F_{Py} < mg$, the box does not move vertically, so $a_y = 0$. Thus

$$F_N - 98.0\,\mathrm{N} + 20.0\,\mathrm{N} = 0,$$

so

$$F_N = 78.0\,\mathrm{N}.$$

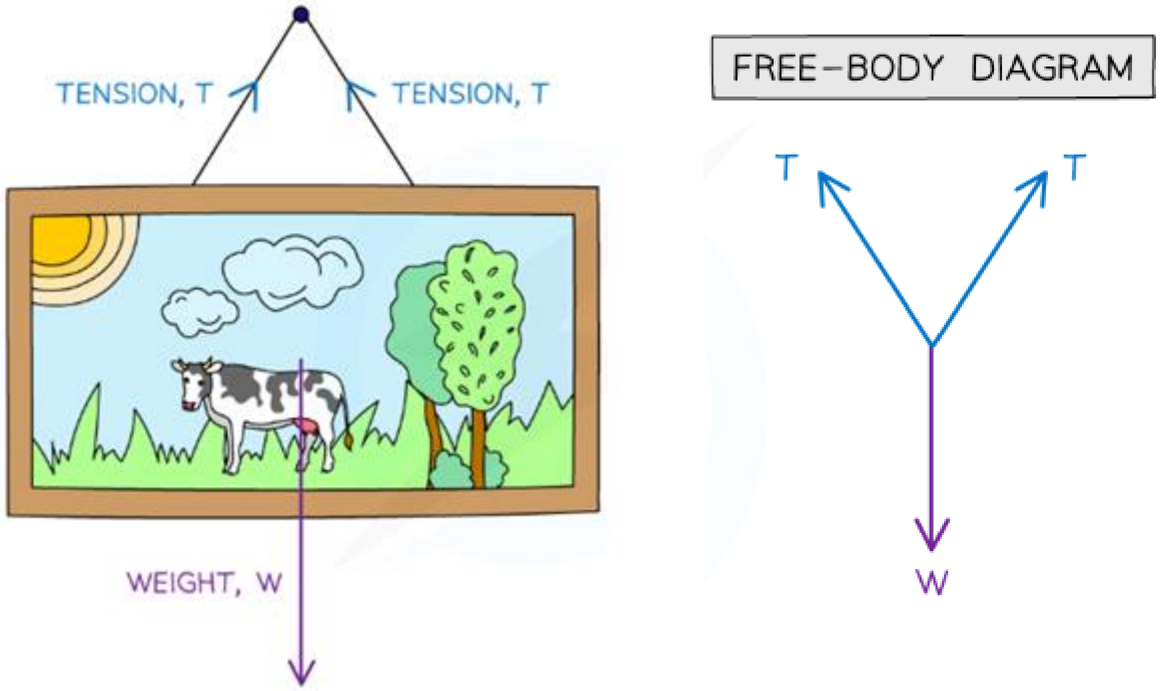
Tension in a Cord

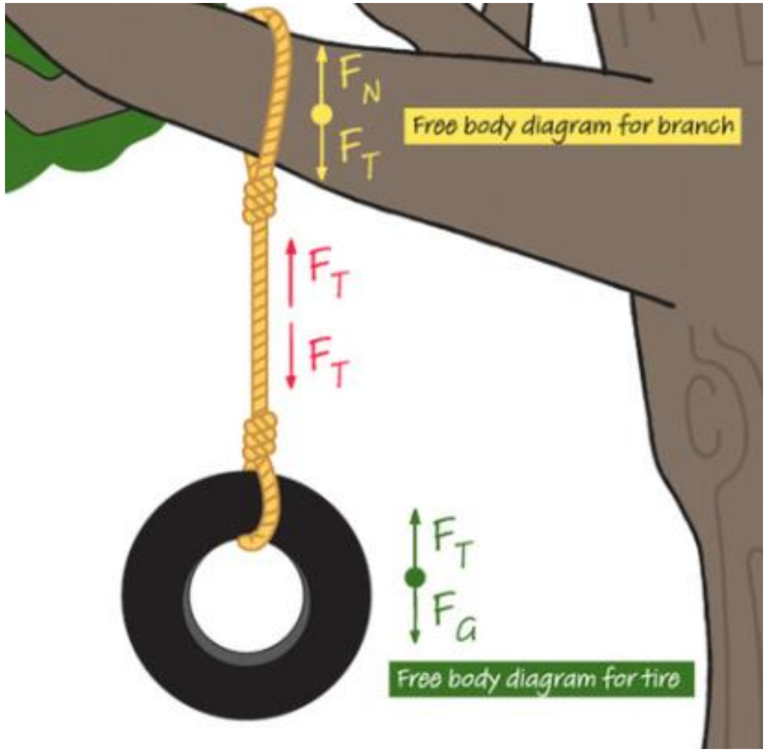
When a flexible cord pulls on an object, it is said to be under tension.

The force the cord exerts on the object is called the tension F_T

If the cord has negligible mass, the force applied at one end is transmitted undiminished along the entire cord to the other end.

Flexible cords and strings can only pull, not push, because they bend.



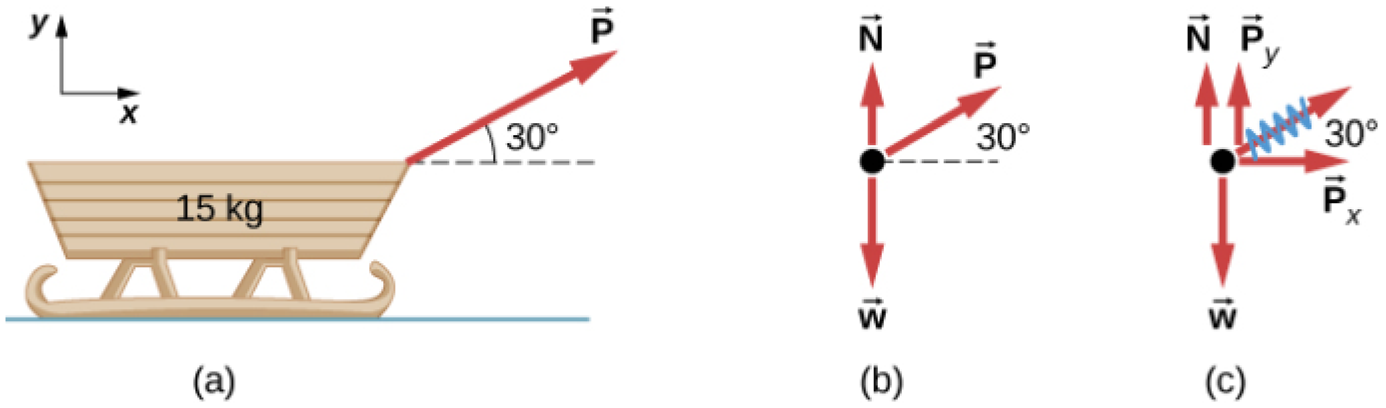


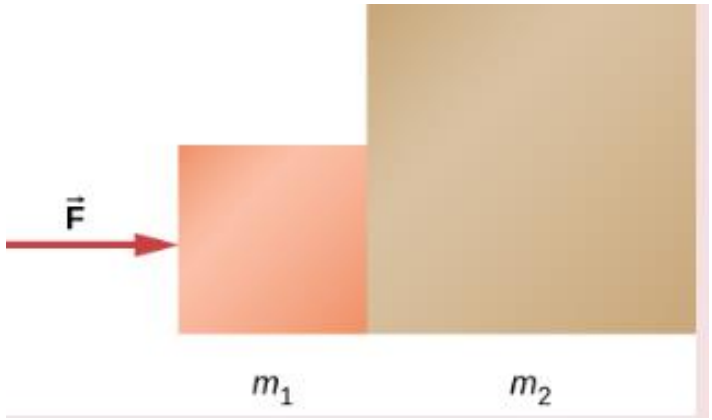
Rules for Constructing a Free-Body Diagram (FBD)

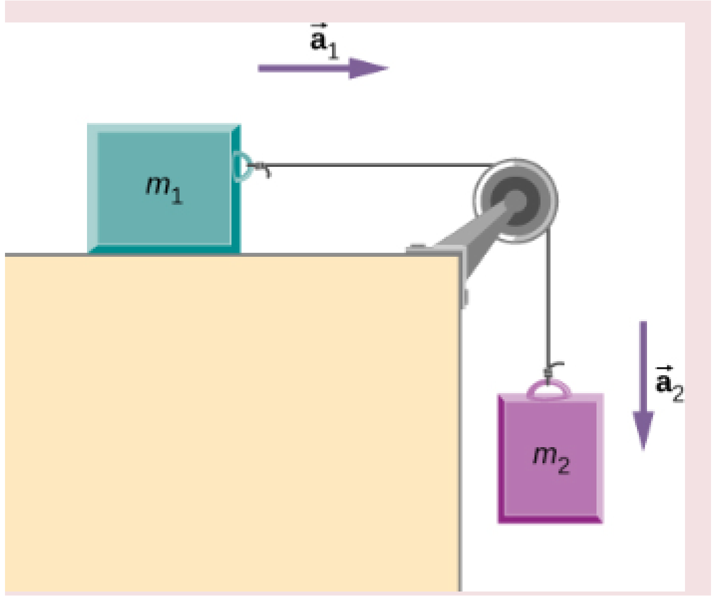
Represent the object as a **point** or simple shape; optionally, circle it to focus on forces.
Include **all external forces** acting on the object (weight, normal, friction, tension, spring, applied), but **do not include the net force**.
Only show forces **acting on the object**, not forces it exerts on others; **do not include both action-reaction pairs**.
Break forces into **x– and y–components** when needed, replacing the original vector.

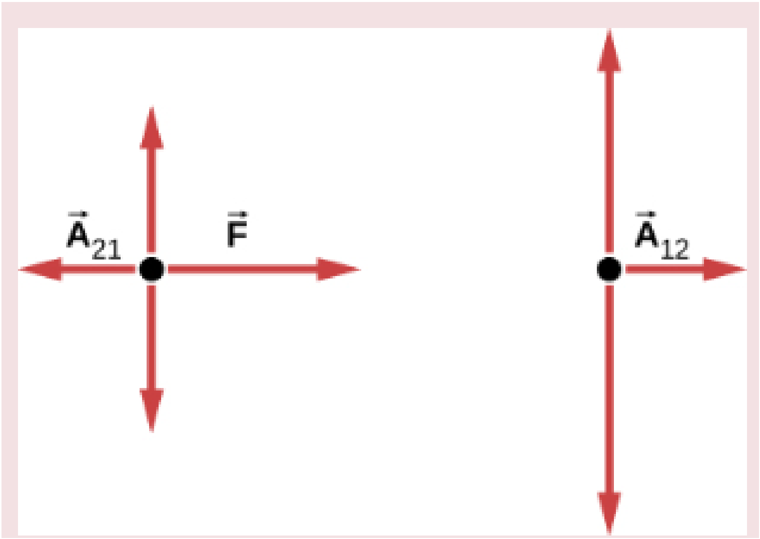
Draw a **separate FBD for each object** in multi-body problems.

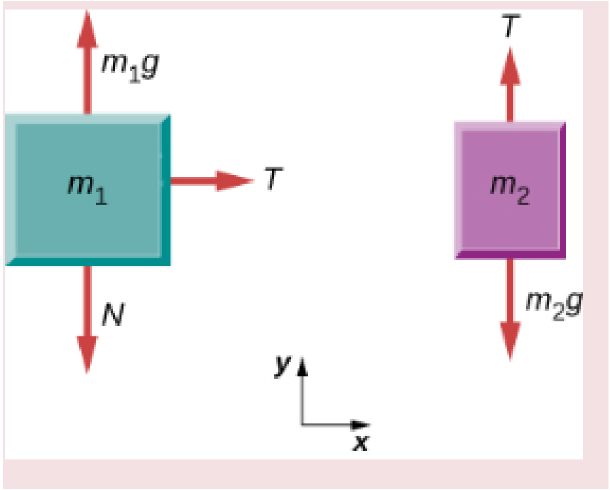
(a) A moving sled is shown as (b) a free-body diagram and (c) a free-body diagram with force components.

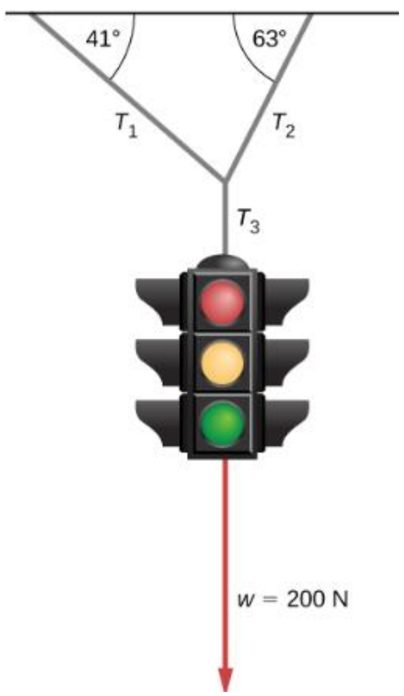


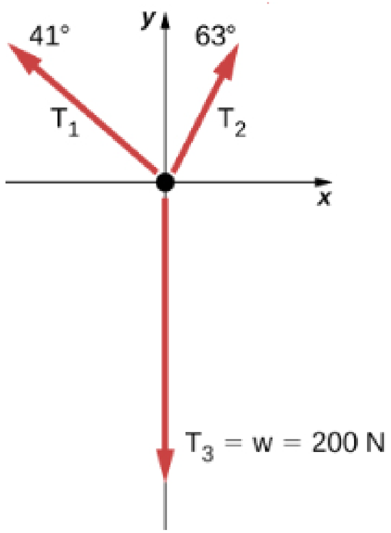

F
m1
m2

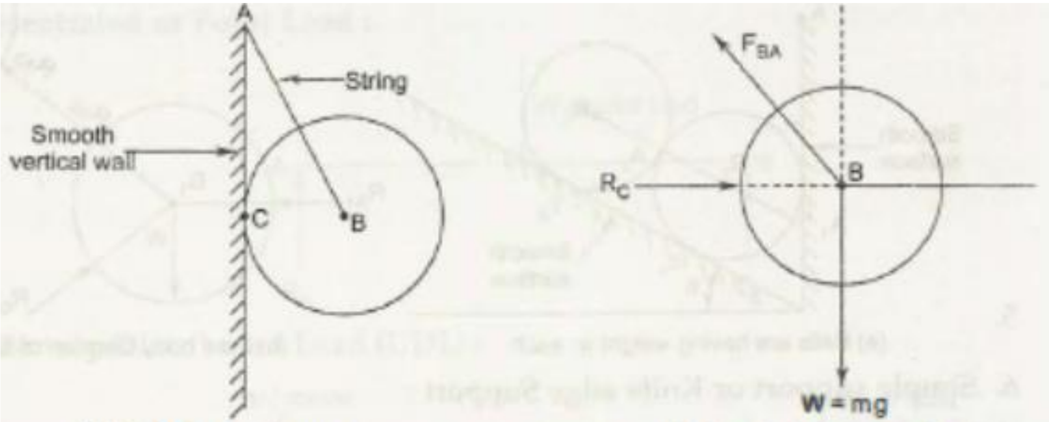

a1
m1
a2
m2


A21
F
A12

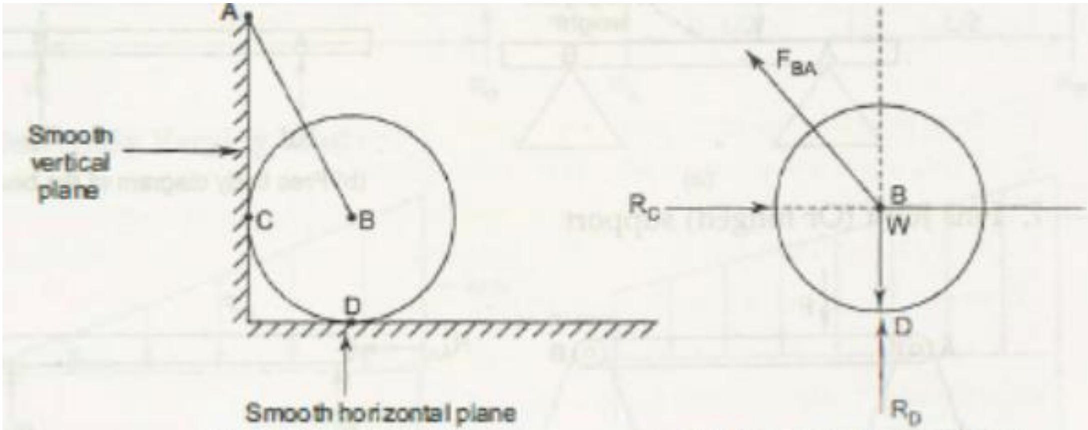

m1g
T
m1
T
m2
N
m2g
y
x





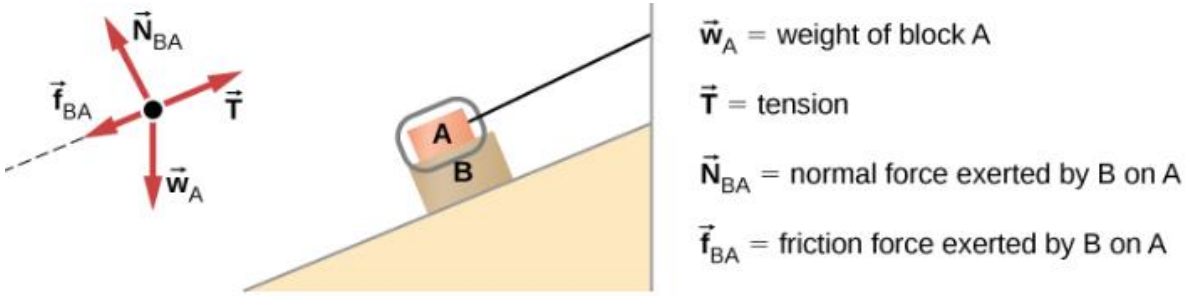


(a) Ball has a weight w = mg
(b) Free body diagram of ball

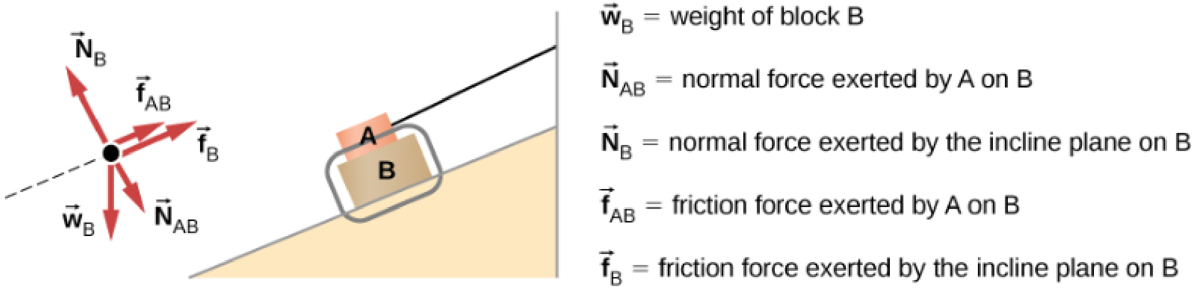


(a) Ball has a weight w
(b) Free body diagram of ball

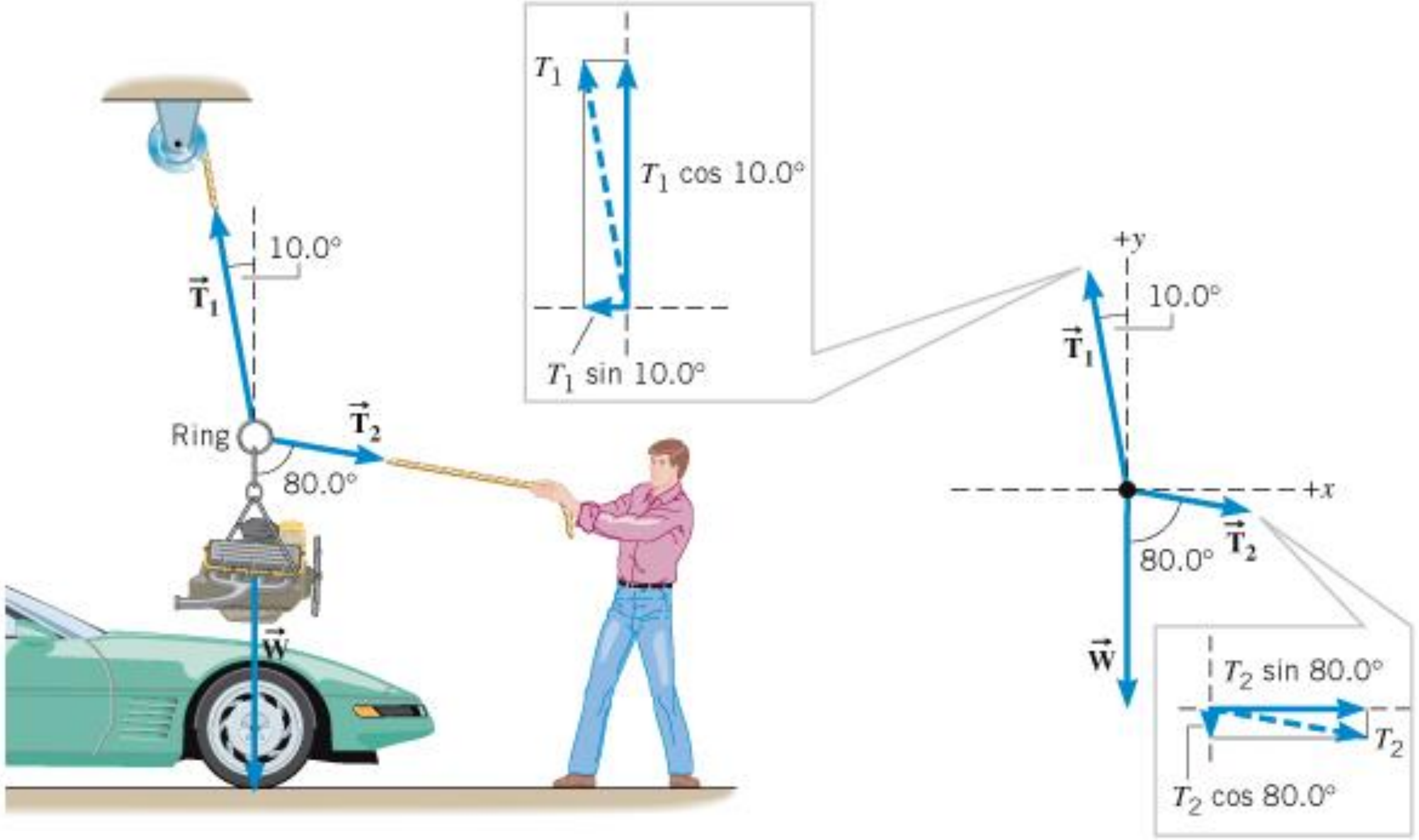
(a) The free-body diagram for isolated object A. (b) The free-body diagram for isolated object B. Comparing the two drawings, we see that friction acts in the opposite direction in the two figures. Because object A experiences a force that tends to pull it to the right, friction must act to the left. Because object B experiences a component of its weight that pulls it to the left, down the incline, the friction force must oppose it and act up the ramp. Friction always acts opposite the intended direction of motion.



(a)



(b)



(a)

(b) Free-body diagram for the ring

EXAMPLE 4–12 **Two boxes connected by a cord.** Two boxes, A and B, are connected by a lightweight cord and are resting on a smooth (frictionless) table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord connecting the boxes.

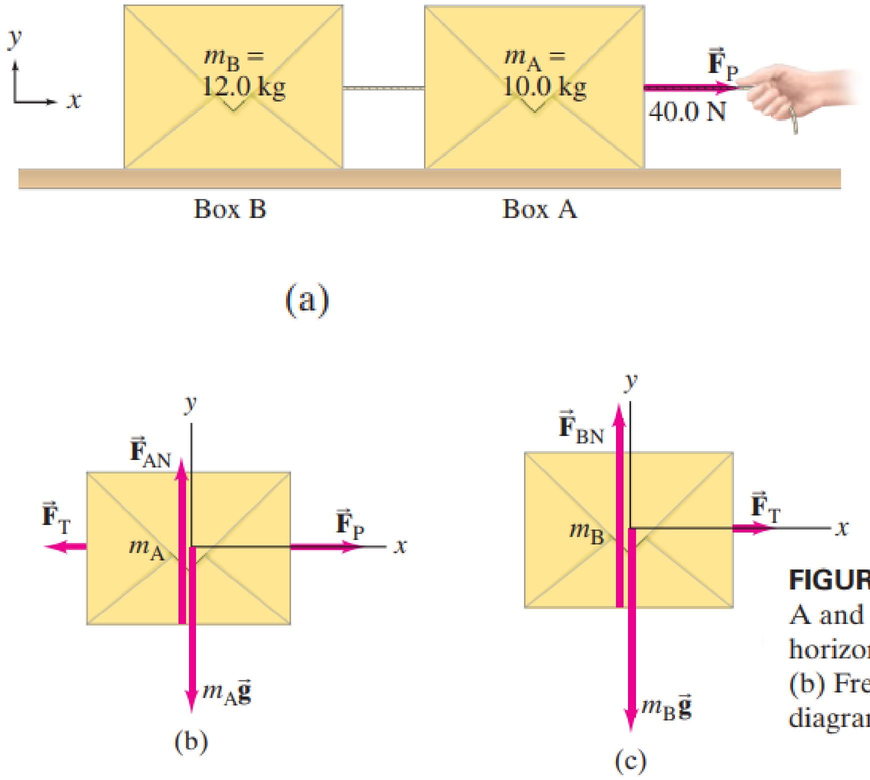


FIGURE 4–22 Example 4–12. (a) Two boxes, A and B, are connected by a cord. A person pulls horizontally on box A with force $F_P = 40.0$ N. (b) Free-body diagram for box A. (c) Free-body diagram for box B.

SOLUTION (*a*) We apply $\Sigma F_x = ma_x$ to box A:

$$\Sigma F_x = F_P - F_T = m_A a_A. \qquad \text{[box A]}$$

For box B, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_B a_B. \qquad \text{[box B]}$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_A = a_B = a$. We are given $m_A = 10.0\,\text{kg}$ and $m_B = 12.0\,\text{kg}$. We can add the two equations above to eliminate an unknown (F_T) and obtain

$$(m_A + m_B)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_A + m_B} = \frac{40.0\,\text{N}}{22.0\,\text{kg}} = 1.82\,\text{m/s}^2.$$

This is what we sought.

(*b*) From the equation for box B above $(F_T = m_B a_B)$, the tension in the cord is

$$F_T = m_B a = (12.0\,\text{kg})(1.82\,\text{m/s}^2) = 21.8\,\text{N}.$$

Thus, $F_T < F_P\ (= 40.0\,\text{N})$, as we expect, since F_T acts to accelerate only m_B.

EXAMPLE 4–13 **Elevator and counterweight (Atwood machine).** A system of two objects suspended over a pulley by a flexible cable, as shown in Fig. 4–23a, is sometimes referred to as an *Atwood machine*. Consider the real-life application of an elevator (m_E) and its counterweight (m_C). To minimize the work done by the motor to raise and lower the elevator safely, m_E and m_C are made similar in mass. We leave the motor out of the system for this calculation, and assume that the cable's mass is negligible and that the mass of the pulley, as well as any friction, is small and ignorable. These assumptions ensure that the tension F_T in the cable has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_C = 1000\text{ kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_E = 1150\text{ kg}$. For the latter case $(m_E = 1150\text{ kg})$, calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

SOLUTION (*a*) To find F_T as well as the acceleration a, we apply Newton's second law, $\Sigma F = ma$, to each object. We take upward as the positive y direction for both objects. With this choice of axes, $a_C = a$ because m_C accelerates upward, and $a_E = -a$ because m_E accelerates downward. Thus

$$F_T - m_E g = m_E a_E = -m_E a$$
$$F_T - m_C g = m_C a_C = +m_C a.$$

We can subtract the first equation from the second to get

$$(m_E - m_C)g = (m_E + m_C)a,$$

where a is now the only unknown. We solve this for a:

$$a = \frac{m_E - m_C}{m_E + m_C}g = \frac{1150\text{ kg} - 1000\text{ kg}}{1150\text{ kg} + 1000\text{ kg}}g = 0.070g = 0.68\text{ m/s}^2.$$

The elevator (m_E) accelerates downward (and the counterweight m_C upward) at $a = 0.070g = 0.68\text{ m/s}^2$.

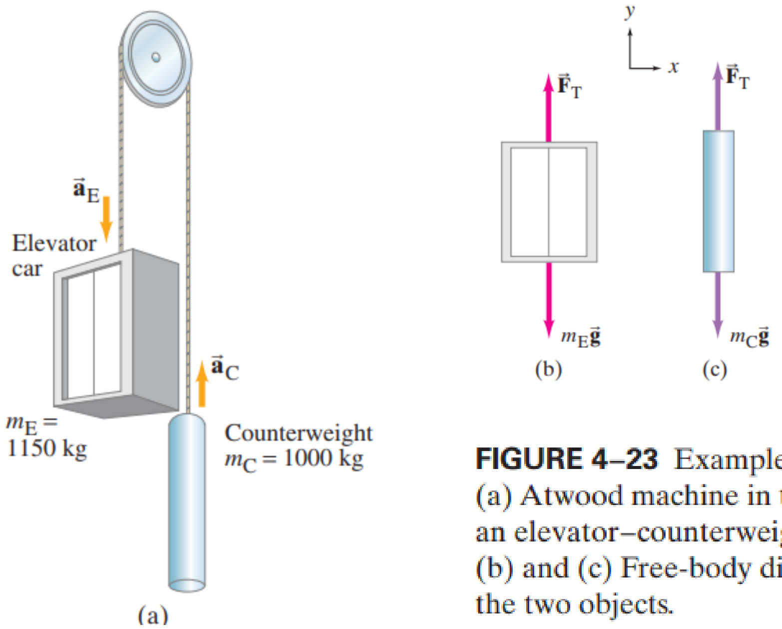


FIGURE 4–23 Example 4–13. (a) Atwood machine in the form of an elevator–counterweight system. (b) and (c) Free-body diagrams for the two objects.

(*b*) The tension in the cable F_T can be obtained from either of the two $\Sigma F = ma$ equations at the start of our solution, setting $a = 0.070g = 0.68\text{ m/s}^2$:

$$F_T = m_E g - m_E a = m_E(g - a)$$
$$= 1150\text{ kg}\left(9.80\text{ m/s}^2 - 0.68\text{ m/s}^2\right) = 10{,}500\text{ N},$$

or

$$F_T = m_C g + m_C a = m_C(g + a)$$
$$= 1000\text{ kg}\left(9.80\text{ m/s}^2 + 0.68\text{ m/s}^2\right) = 10{,}500\text{ N},$$

Example: A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?

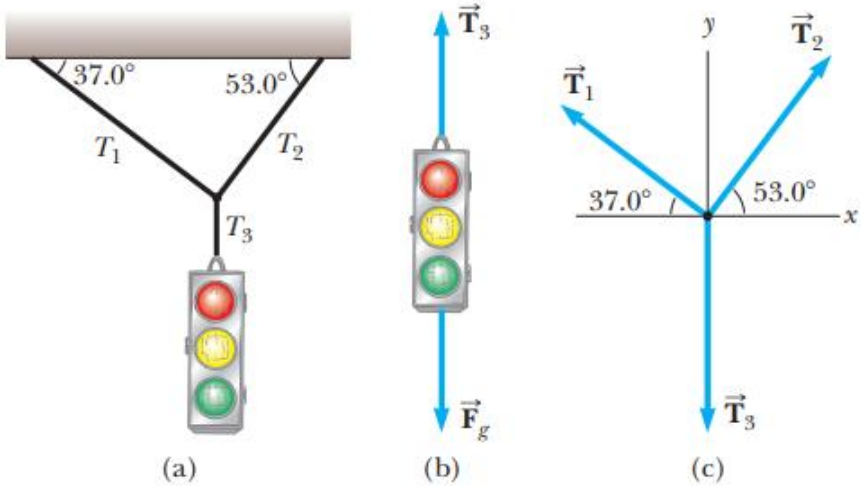


Figure 5.10 (Example 5.4) (a) A traffic light suspended by cables. (b) The free-body diagram for the traffic light. (c) The free-body diagram for the knot where the three cables are joined.

$$\sum F_y = 0 \quad \rightarrow \quad T_3 - F_g = 0$$

$$T_3 = F_g = 122 \text{ N}$$

Force	*x* Component	*y* Component
$\vec{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\vec{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\vec{T}_3$	0	-122 N

(1) $\sum F_x = -T_1 \cos 37.0° + T_2 \cos 53.0° = 0$

(2) $\sum F_y = T_1 \sin 37.0° + T_2 \sin 53.0° + (-122 \text{ N}) = 0$

Equation (1) shows that the horizontal components of $\vec{T}_1$ and $\vec{T}_2$ must be equal in magnitude, and Equation (2) shows that the sum of the vertical components of $\vec{T}_1$ and $\vec{T}_2$ must balance the downward force $\vec{T}_3$, which is equal in magnitude to the weight of the light.

Solve Equation (1) for T_2 in terms of T_1:

(3) $T_2 = T_1\left(\frac{\cos 37.0°}{\cos 53.0°}\right) = 1.33T_1$

Substitute this value for T_2 into Equation (2):

$$T_1 \sin 37.0° + (1.33T_1)(\sin 53.0°) - 122 \text{ N} = 0$$

$$T_1 = 73.4 \text{ N}$$

$$T_2 = 1.33T_1 = 97.4 \text{ N}$$

Both values are less than 100 N (just barely for T_2), so the cables will not break.

Example: A car of mass m is on an icy driveway inclined at an angle u as in Figure 5.11a. (A) Find the acceleration of the car, assuming that the driveway is frictionless.

$$(1)\quad \sum F_x = mg\sin\theta = ma_x$$

$$(2)\quad \sum F_y = n - mg\cos\theta = 0$$

$$(3)\quad a_x = \boxed{g\sin\theta}$$

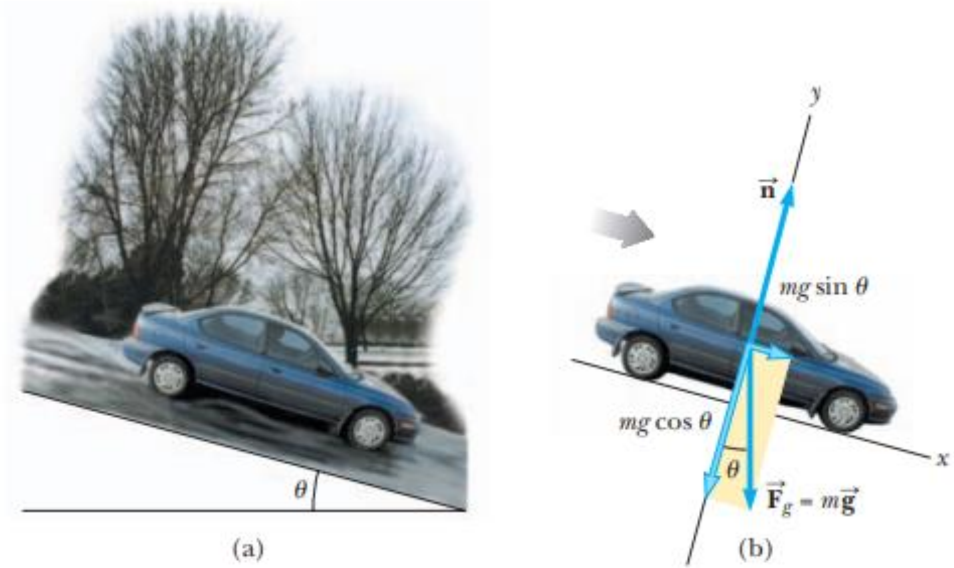


Figure 5.11 (Example 5.6) (a) A car of mass m on a frictionless incline. (b) The free-body diagram for the car.

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

$$d = \tfrac{1}{2}a_x t^2$$

Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, apply Equation 2.16, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

Solve for t:

$$(4)\quad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

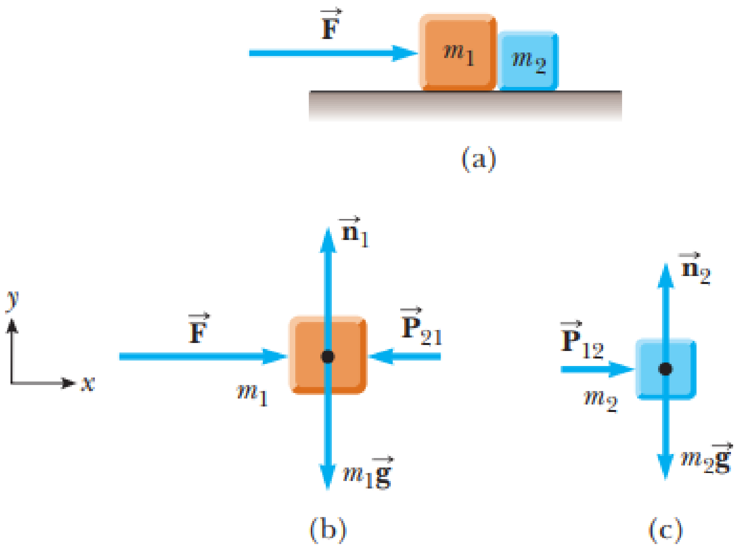
$$v_{xf}^2 = 2a_x d$$

$$(5)\quad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

Example:

Two blocks of masses m_1 and m_2, with $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as in Active Figure 5.12a. A constant horizontal force $\vec{\mathbf{F}}$ is applied to m_1 as shown.

(A) Find the magnitude of the acceleration of the system.



ACTIVE FIGURE 5.12

(Example 5.7) A force is applied to a block of mass m_1, which pushes on a second block of mass m_2. (b) The free-body diagram for m_1. (c) The free-body diagram for m_2.

Analyze First model the combination of two blocks as a single particle. Apply Newton's second law to the combination:

$$\sum F_x = F = (m_1 + m_2)a_x$$

$$(1) \quad a_x = \boxed{\frac{F}{m_1 + m_2}}$$

(B) Determine the magnitude of the contact force between the two blocks.

Apply Newton's second law to m_2:

$$(2) \quad \sum F_x = P_{12} = m_2 a_x$$

Solve for P_{12} and substitute the value of a_x from Equation (1):

$$(3) \quad P_{12} = m_2 a_x = \boxed{\left(\frac{m_2}{m_1 + m_2}\right)F}$$

Apply Newton's second law to m_1:

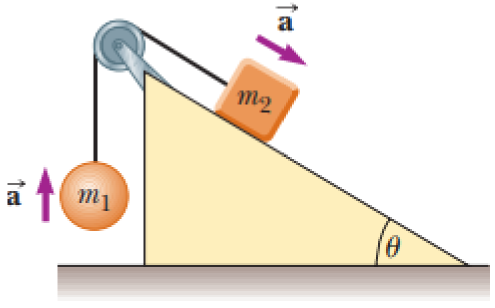
$$(4) \quad \sum F_x = F - P_{21} = F - P_{12} = m_1 a_x$$

Solve for P_{12} and substitute the value of a_x from Equation (1):

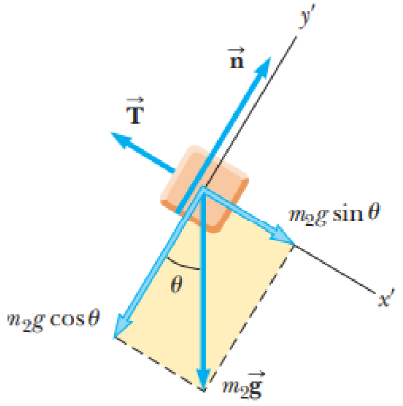
$$P_{12} = F - m_1 a_x = F - m_1\left(\frac{F}{m_1 + m_2}\right) = \left(\frac{m_2}{m_1 + m_2}\right)F$$

Example:

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass as in Figure 5.15a. The block lies on a frictionless incline of angle θ. Find the magnitude of the acceleration of the two objects and the tension in the cord.



(a)



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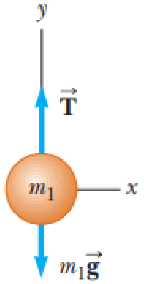


(b)

Figure 5.15 (Example 5.10) (a) Two objects connected by a lightweight cord strung over a frictionless pulley. (b) The free-body diagram for the ball. (c) The free-body diagram for the block. (The incline is frictionless.)

Apply Newton's second law in component form to the ball, choosing the upward direction as positive:

$$(1) \quad \sum F_x = 0$$

$$(2) \quad \sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

Apply Newton's second law in component form to the block:

$$(3) \quad \sum F_{x'} = m_2 g \sin\theta - T = m_2 a_{x'} = m_2 a$$

$$(4) \quad \sum F_{y'} = n - m_2 g \cos\theta = 0$$

Solve Equation (2) for T: $$(5) \quad T = m_1(g + a)$$

Substitute this expression for T into Equation (3):

$$m_2 g \sin\theta - m_1(g + a) = m_2 a$$

Solve for a:

$$(6) \quad a = \boxed{\frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}}$$

Substitute this expression for a into Equation (5) to find T:

$$(7) \quad T = \boxed{\frac{m_1 m_2 g (\sin\theta + 1)}{m_1 + m_2}}$$

Friction happens when two surfaces touch and try to move past each other. Even smooth-looking surfaces are rough under a microscope, and tiny bumps can stick together for a moment like tiny bonds. This sticking makes it harder for objects to slide, causing movement to feel jerky. Sliding friction is called **kinetic friction**. There is also rolling friction when objects roll, but it is usually much smaller. Friction is important to consider in real-life problems, especially on ramps or inclined surfaces.

Kinetic friction acts in the opposite direction of an object's motion. Its size depends on the types of surfaces touching each other and is roughly proportional to the normal force (F_n). For many hard surfaces, friction does not depend much on the contact area. This relationship is described by the equation

$$F_{fr} = \mu_k F_n$$

where μ_k is the coefficient of kinetic friction, which depends on the materials. This value can change if surfaces are wet, rough, or polished, but it does not depend much on speed or contact area. This equation is based on experiments, not a fundamental law of physics.

Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 4–26.

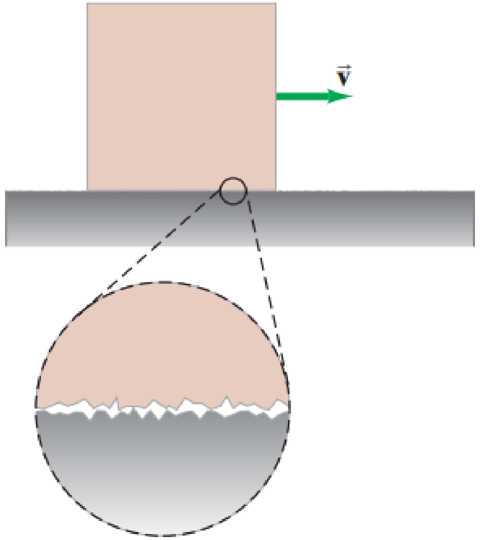
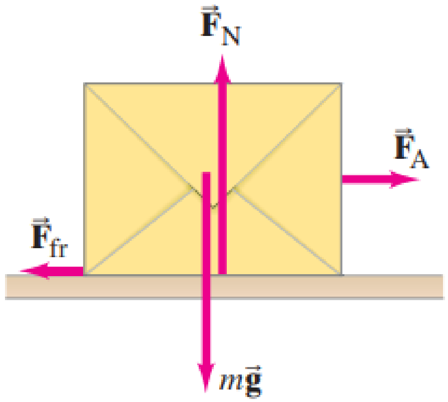


FIGURE 4–26 An object moving to the right on a table. The two surfaces in contact are assumed smooth, but are rough on a microscopic scale.

FIGURE 4–27 When an object is pulled along a surface by an applied force ($\vec{F}_A$), the force of friction $\vec{F}_{fr}$ opposes the motion. The magnitude of $\vec{F}_{fr}$ is proportional to the magnitude of the normal force (F_N).



Static friction acts between two surfaces that are in contact but not sliding relative to each other. When no external horizontal force is applied, the static friction force is zero. As you begin to push an object, like a desk on the floor, static friction increases to oppose your push and prevent motion. The object remains at rest as long as the static friction force can match your applied force, keeping the net force zero. However, static friction has a limit. The maximum static friction force is given by:

$$F_{\text{fr}} \leq \mu_s F_N$$

where μ_S is the coefficient of static friction and F_N is the normal force. If your applied force exceeds this maximum value, the object will start to move. Once motion begins, kinetic friction replaces static friction, and because $\mu_S > \mu_K$ for most surfaces, it is typically harder to start an object moving than to keep it sliding

TABLE 4–2 Coefficients of Friction[†]

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

[†] Values are approximate and intended only as a guide.

EXAMPLE 4–16 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

APPROACH We don't know, right off, if we are dealing with static friction or kinetic friction, nor if the box remains at rest or accelerates. We need to draw a free-body diagram, and then determine in each case whether or not the box will move: the box starts moving if F_A is greater than the maximum static friction force (Newton's second law). The forces on the box are gravity $m\vec{\mathbf{g}}$, the normal force exerted by the floor $\vec{\mathbf{F}}_N$, the horizontal applied force $\vec{\mathbf{F}}_A$, and the friction force $\vec{\mathbf{F}}_{fr}$, as shown in Fig. 4–27.

SOLUTION The free-body diagram of the box is shown in Fig. 4–27. In the vertical direction there is no motion, so Newton's second law in the vertical direction gives $\Sigma F_y = ma_y = 0$, which tells us $F_N - mg = 0$. Hence the normal force is

$$F_N = mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}.$$

(*a*) Because $F_A = 0$ in this first case, the box doesn't move, and $F_{fr} = 0$.

(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98.0\,\text{N}) = 39\,\text{N}.$$

When the applied force is $F_A = 10\,\text{N}$, the box will not move. Newton's second law gives $\Sigma F_x = F_A - F_{fr} = 0$, so $F_{fr} = 10\,\text{N}$.

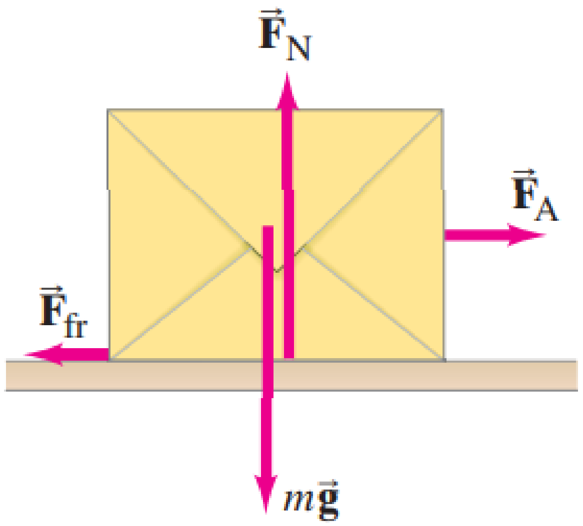


FIGURE 4–27 Repeated for Example 4–16.

(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\text{ N}$ to balance the applied force.

(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\text{ N}) = 39\text{ N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98.0\text{ N}) = 29\text{ N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\text{ N} - 29\text{ N} = 11\text{ N}$, so the box will accelerate at a rate

$$a_x = \frac{\Sigma F}{m} = \frac{11\text{ N}}{10.0\text{ kg}} = 1.1\text{ m/s}^2$$

as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

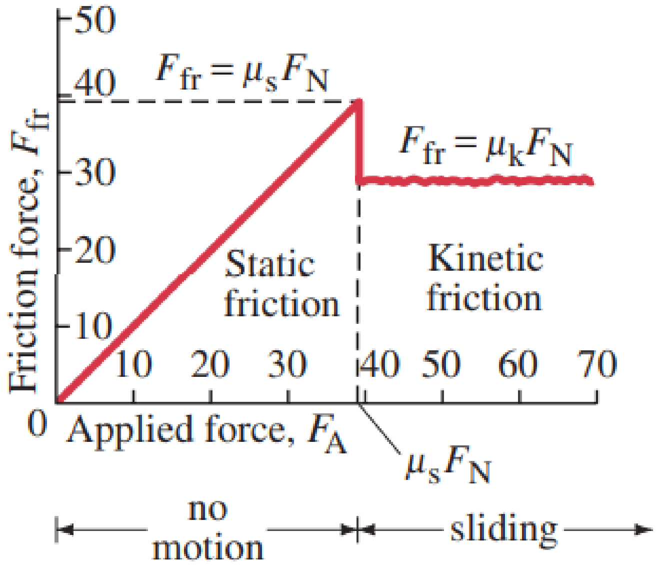


FIGURE 4–28 Example 4–16. Magnitude of the force of friction as a function of the external force applied to an object initially at rest. As the applied force is increased in magnitude, the force of static friction increases in proportion until the applied force equals $\mu_s F_N$. If the applied force increases further, the object will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.

Example:

A hockey puck on a frozen pond is given an initial speed of 20.0 m/s. If the puck always remains on the ice and slides 115 m before coming to rest, determine the coefficient of kinetic friction between the puck and ice.

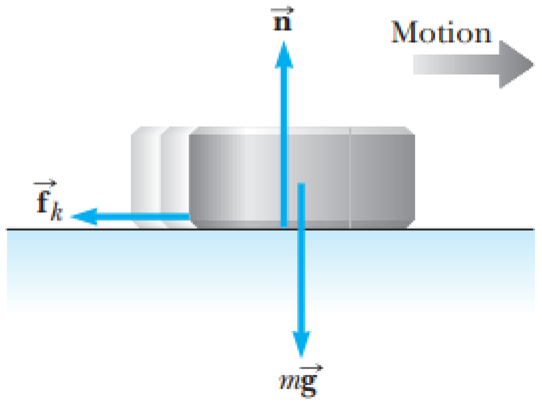


Figure 5.19 (Example 5.12) After the puck is given an initial velocity to the right, the only external forces acting on it are the gravitational force $m\vec{g}$, the normal force $\vec{n}$, and the force of kinetic friction $\vec{f}_k$.

Solution:

Apply the particle under a net force model in the x direction to the puck:

$$(1)\quad \sum F_x = -f_k = ma_x$$

Apply the particle in equilibrium model in the y direction to the puck:

$$(2)\quad \sum F_y = n - mg = 0$$

Substitute $n = mg$ from Equation (2) and $f_k = \mu_k n$ into Equation (1):

$$-\mu_k n = -\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

Apply the particle under constant acceleration model to the puck, using Equation 2.17, $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$, with $x_i = 0$ and $v_f = 0$:

$$0 = v_{xi}^2 + 2a_x x_f = v_{xi}^2 - 2\mu_k g x_f$$

$$\mu_k = \frac{v_{xi}^2}{2gx_f}$$

$$\mu_k = \frac{(20.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)(115\text{ m})} = 0.117$$

Inclines

Now we consider what happens when an object slides down an incline, such as a hill or ramp. Such problems are interesting because gravity is the accelerating force, yet the acceleration is not vertical. Solving problems is usually easier if we choose the xy coordinate system so the x axis points along the incline (the direction of motion) and the y axis is perpendicular to the incline, as shown in Fig. 4–33. Note also that the normal force is not vertical, but is perpendicular to the sloping surface of the plane, along the y axis in Fig. 4–33.

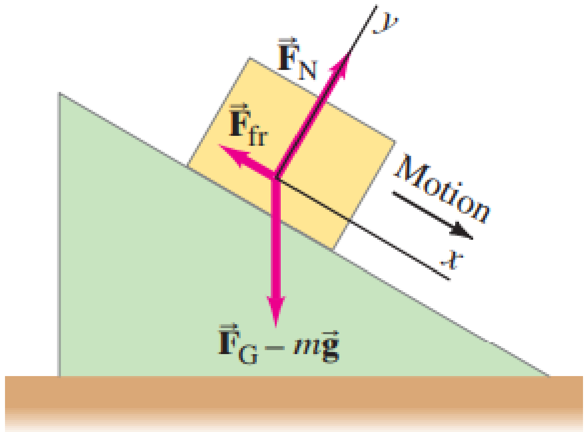
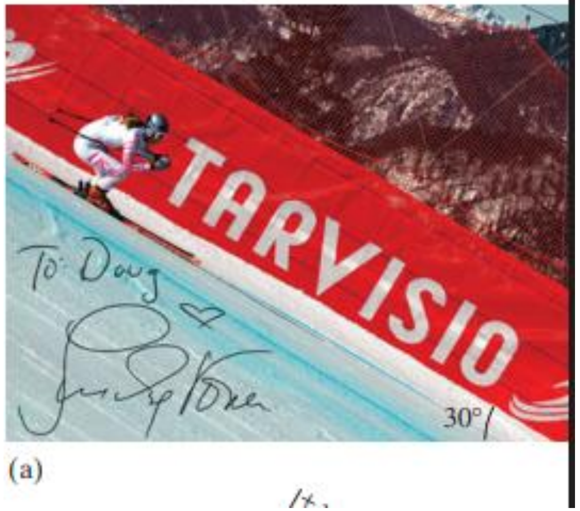


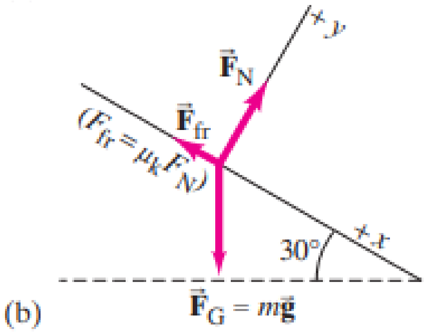
FIGURE 4–33 Forces on an object sliding down an incline.

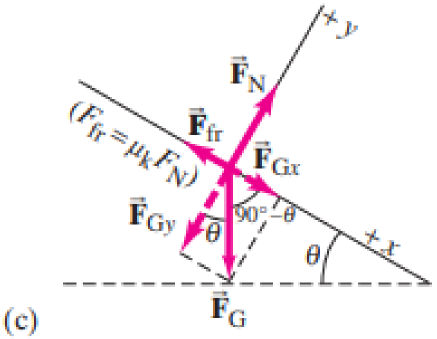
EXAMPLE 4–21 **The skier.** The skier in Fig. 4–34a has begun descending the 30° slope. If the coefficient of kinetic friction is 0.10, what is her acceleration?

APPROACH We choose the x axis along the slope, positive downslope in the direction of the skier’s motion. The y axis is perpendicular to the surface. The forces acting on the skier are gravity, $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$, which points vertically downward (*not* perpendicular to the slope), and the two forces exerted on her skis by the snow—the normal force perpendicular to the snowy slope (*not* vertical), and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–34b, which is our free-body diagram for the skier.

FIGURE 4–34 Example 4–21. Skier descending a slope; $\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}$ is the force of gravity (weight) on the skier.







SOLUTION We have to resolve only one vector into components, the weight $\vec{\mathbf{F}}_G$, and its components are shown as dashed lines in Fig. 4–34c. To be general, we use θ rather than 30° for now. We use the definitions of sine (“side opposite”) and cosine (“side adjacent”) to obtain the components:

$$F_{Gx} = mg\sin\theta,$$
$$F_{Gy} = -mg\cos\theta$$

where F_{Gy} is in the negative y direction. To calculate the skier’s acceleration down the hill, a_x, we apply Newton’s second law to the x direction:

$$\Sigma F_x = ma_x$$
$$mg\sin\theta - \mu_k F_N = ma_x$$

where the two forces are the x component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don’t yet know F_N in the last equation. Let’s see if we can get F_N from the y component of Newton’s second law:

$$\Sigma F_y = ma_y$$
$$F_N - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$a_x = g\sin 30° - \mu_k g\cos 30°$$
$$= 0.50g - (0.10)(0.866)g = 0.41g.$$

The skier’s acceleration is 0.41 times the acceleration of gravity, which in numbers† is $a = (0.41)(9.8\ \mathrm{m/s^2}) = 4.0\ \mathrm{m/s^2}$.

Summary

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the **law of inertia**) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad (4\text{–}1)$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one object exerts a force on a second object, the second object always exerts a force on the first object which is equal in magnitude but opposite in direction:

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA} \qquad (4\text{–}2)$$

where $\vec{\mathbf{F}}_{BA}$ is the force on object B exerted by object A.

The tendency of an object to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of an object.

Weight refers to the **gravitational force** on an object, and is equal to the product of the object's mass m and the acceleration of gravity $\vec{\mathbf{g}}$:

$$\vec{\mathbf{F}}_G = m\vec{\mathbf{g}}. \qquad (4\text{–}3)$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on that object.

When two objects slide over one another, the force of friction that each object exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each object exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the objects are at rest relative to each other, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} < \mu_s F_N$, where μ_s is the coefficient of **static friction**.

For solving problems involving the forces on one or more objects, it is essential to draw a **free-body diagram** for each object, showing all the forces acting on only that object. Newton's second law can be applied to the vector components for each object.

EXERCISE 1

Figure 1 shows a block (m_A) on a smooth horizontal surface, connected by a thin cord that passes over a pulley to a second block (m_B) which hangs vertically. **(a)** Draw a free-body diagram for each block, showing the force of gravity on each, the force (tension) exerted by the cord, and any normal force. **(b)** Apply Newton's second law to find formulas for the acceleration of the system and for the tension in the cord. Ignore friction and the masses of the pulley and cord.

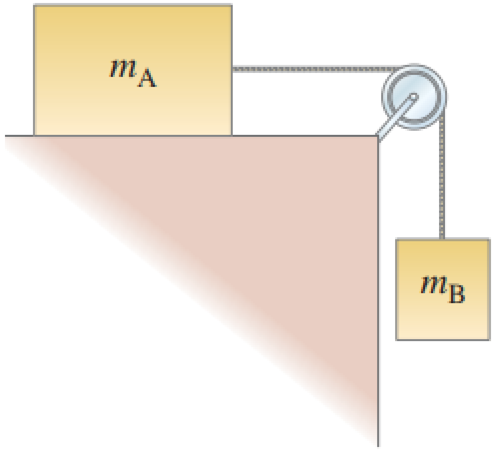
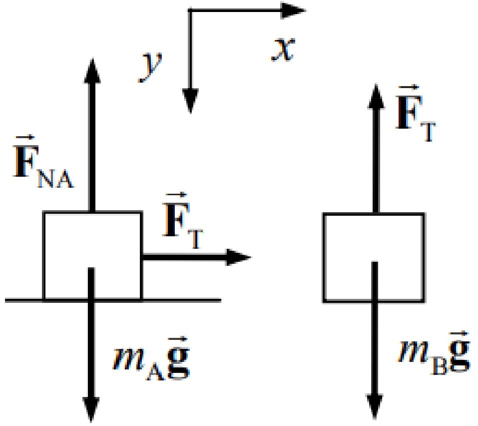


Figure 1

SOLUTION

(a)



(b)

For block A, since there is no motion in the vertical direction, we have $F_{NA} = m_A g$. We write Newton's second law for the x direction: $\sum F_{Ax} = F_T = m_A a_{Ax}$. For block B, we only need to consider vertical forces: $\sum F_{By} = m_B g - F_T = m_B a_{By}$. Since the two blocks are connected, the magnitudes of their accelerations will be the same, so let $a_{Ax} = a_{By} = a$. Combine the two force equations from above, and solve for a by substitution.

$$F_T = m_A a \quad m_B g - F_T = m_B a \;\rightarrow\; m_B g - m_A a = m_B a \;\rightarrow$$

$$m_A a + m_B a = m_B g \;\rightarrow\; \boxed{a = g\frac{m_B}{m_A + m_B} \quad F_T = m_A a = g\frac{m_A m_B}{m_A + m_B}}$$

EXERCISE 2

A girl walks on a rope between two buildings. The buildings are 10 m apart. When she stands in the middle, the rope drops at a 10° angle on each side. Her mass is 50 kg. What is the tension in the rope?

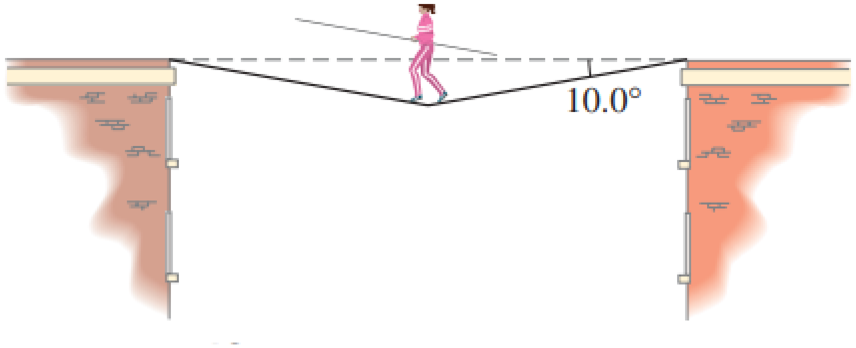
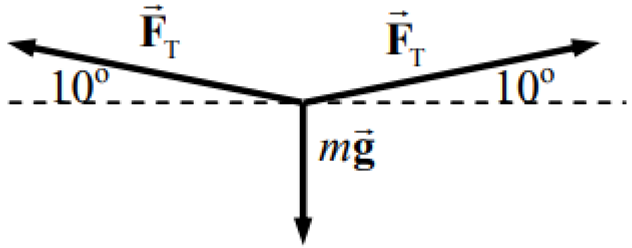


Figure 2

SOLUTION



$$\sum F = F_T \sin 10.0^\circ + F_T \sin 10.0^\circ - mg = 0 \quad \rightarrow$$

$$F_T = \frac{mg}{2 \sin 10.0^\circ} = \frac{(50.0\ \text{kg})(9.80\ \text{m/s}^2)}{2 \sin 10.0^\circ} = \boxed{1410\ \text{N}}$$

EXERCISE 3

One 3.2-kg paint bucket is hanging by a massless cord from another 3.2-kg paint bucket, also hanging by a massless cord, as shown in Figure 3. **(a)** If the buckets are at rest, what is the tension in each cord? **(b)** If the two buckets are pulled upward with an acceleration of by the upper cord, calculate the tension in each cord.

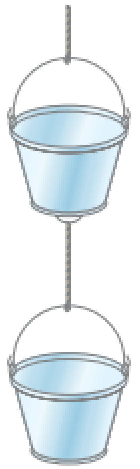
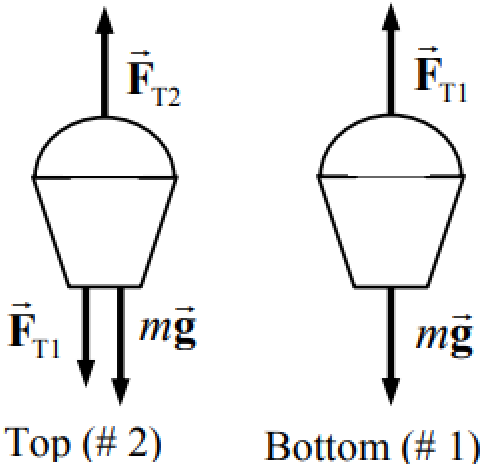


Figure 3

Free Body Diagrams



SOLUTION

(a) Since the buckets are at rest, their acceleration is 0. Write Newton's second law for each bucket, calling UP the positive direction.

$$\sum F_1 = F_{T1} - mg = 0 \quad \rightarrow$$

$$F_{T1} = mg = (3.2\text{ kg})(9.80\text{ m/s}^2) = \boxed{31\text{ N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = 0 \quad \rightarrow$$

$$F_{T2} = F_{T1} + mg = 2\,mg = 2(3.2\text{ kg})(9.80\text{ m/s}^2) = \boxed{63\text{ N}}$$

(b)

Now repeat the analysis, but with a nonzero acceleration. The free-body diagrams are unchanged.

$$\sum F_1 = F_{T1} - mg = ma \quad \rightarrow$$

$$F_{T1} = mg + ma = (3.2\text{ kg})(9.80\text{ m/s}^2 + 1.25\text{ m/s}^2) = 35.36\text{ N} \approx \boxed{35\text{ N}}$$

$$\sum F_2 = F_{T2} - F_{T1} - mg = ma \quad \rightarrow \quad F_{T2} = F_{T1} + mg + ma = 2F_{T1} = \boxed{71\text{ N}}$$

EXERCISE 4

Suppose the pulley in Figure 4 is suspended by a cord C. Determine the tension in this cord after the masses are released and before one hits the ground. Ignore the mass of the pulley and cords.

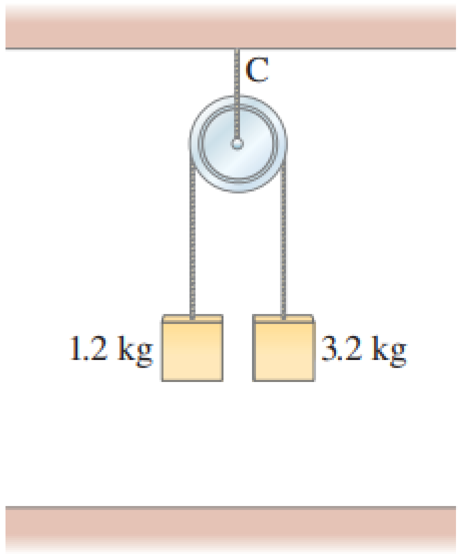
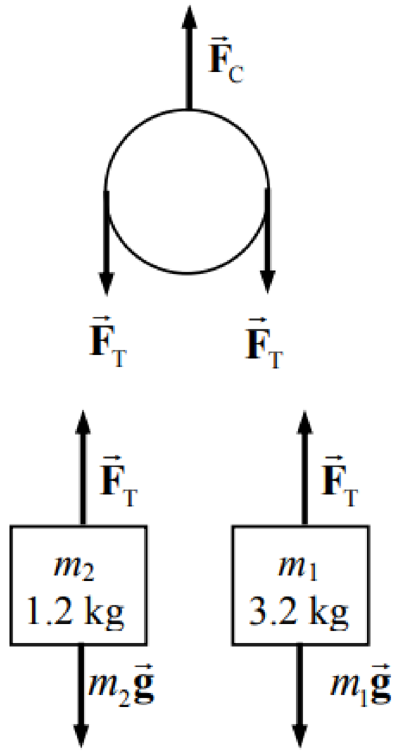


Figure 4



SOLUTION

We draw a free-body diagram for each mass. We choose up to be the positive direction. The tension force in the cord is found from analyzing the two hanging masses. Notice that the same tension force is applied to each mass. Write Newton’s second law for each of the masses.

$$F_T - m_1 g = m_1 a_1 \qquad F_T - m_2 g = m_2 a_2$$

Since the masses are joined together by the cord, their accelerations will have the same magnitude but opposite directions. Thus $a_1 = -a_2$. Substitute this into the force expressions and solve for the tension force.

$$F_T - m_1 g = -m_1 a_2 \;\rightarrow\; F_T = m_1 g - m_1 a_2 \;\rightarrow\; a_2 = \frac{m_1 g - F_T}{m_1}$$

$$F_T - m_2 g = m_2 a_2 = m_2\left(\frac{m_1 g - F_T}{m_1}\right) \;\rightarrow\; F_T = \frac{2 m_1 m_2 g}{m_1 + m_2}$$

Apply Newton’s second law to the stationary pulley.

$$F_C - 2F_T = 0 \;\rightarrow\; F_C = 2F_T = \frac{4 m_1 m_2 g}{m_1 + m_2} = \frac{4(3.2\text{ kg})(1.2\text{ kg})(9.80\text{ m/s}^2)}{4.4\text{ kg}} = \boxed{34\text{ N}}$$

EXERCISE 5

In Figure 5 the coefficient of static friction between mass m_A and the table is 0.40, whereas the coefficient of kinetic friction is 0.20. **(a)** What minimum value m_A of will keep the system from starting to move? **(b)** What value(s) m_A of will keep the system moving at constant speed? [Ignore masses of the cord and the (frictionless) pulley.]

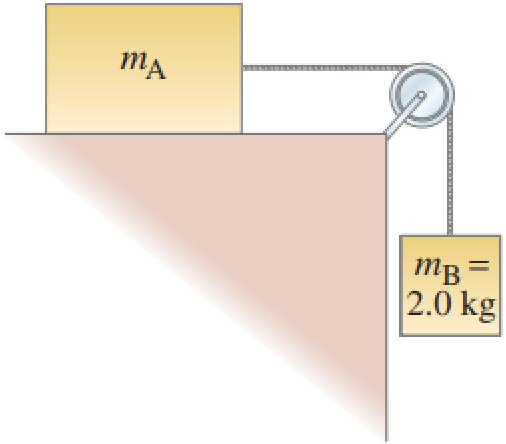


Figure 5

SOLUTION

(a) For m_B not to move, the tension must be equal to $m_B g$, so $m_B g = F_T$. For m_A not to move, the tension must be equal to the force of static friction, so $F_{fr} = F_T$. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

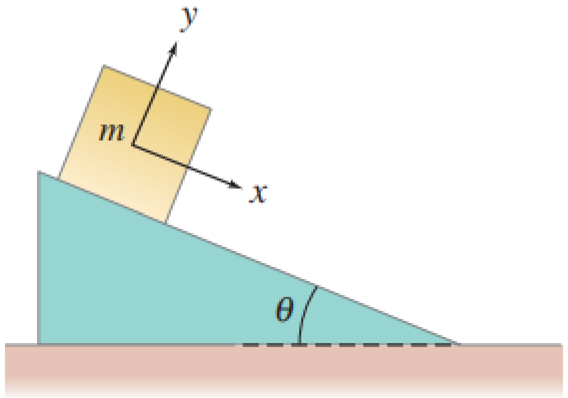
$$m_B g = F_T = F_{fr} \le \mu_s m_A g \quad \rightarrow \quad m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\text{ kg}}{0.40} = 5.0\text{ kg} \quad \rightarrow \quad m_A \ge \boxed{5.0\text{ kg}}$$

(b) For m_B to move with constant velocity, the tension must be equal to $m_B g$. For m_A to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_{fr} = \mu_k m_A g \quad \rightarrow \quad m_A = \frac{m_B}{\mu_k} = \frac{2.0\text{ kg}}{0.20} = \boxed{10\text{ kg}}$$

EXERCISE 6

The block shown in Figure 6 has mass m = 7.0 kg and lies on a fixed smooth frictionless plane tilted at an angle θ = 22.0 to the horizontal. **(a)** Determine the acceleration of the block as it slides down the plane. **(b)** If the block starts from rest 12.0 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline?



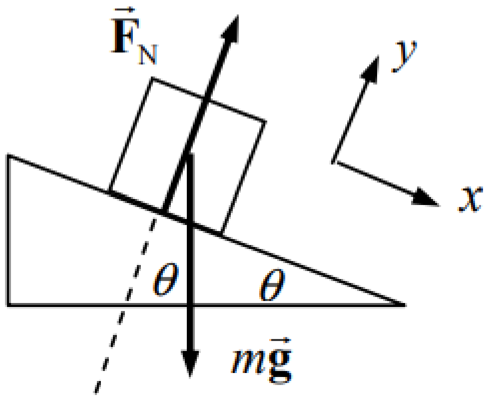


Figure 6

SOLUTION

(a)

Consider the free-body diagram for the block on the frictionless surface. There is no acceleration in the y direction. Use Newton's second law for the x direction to find the acceleration.

$$\sum F_x = mg \sin\theta = ma \quad \rightarrow$$

$$a = g \sin\theta = (9.80\ \text{m/s}^2) \sin 22.0^\circ = \boxed{3.67\ \text{m/s}^2}$$

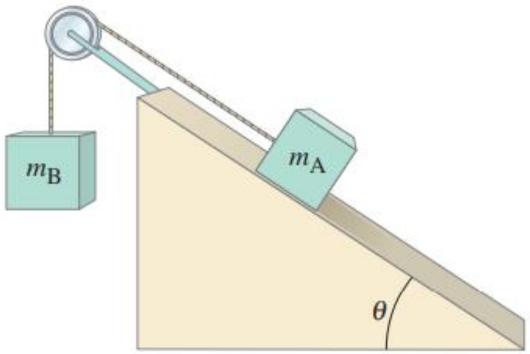
(b)

Use Eq. 2–11c with $v_0 = 0$ to find the final speed.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x - x_0)} = \sqrt{2(3.67\ \text{m/s}^2)(12.0\ \text{m})} = \boxed{9.39\ \text{m/s}}$$

EXERCISE 7

(a) Suppose the coefficient of kinetic friction between m_A and the plane in Figure 7 is μ_k=0.15, and that m_A=m_B=2.7kg. As m_B moves down, determine the magnitude of the acceleration of m_A and m_B given θ=34° **(b)** What smallest value of μ_k will keep the system from accelerating? [Ignore masses of the (frictionless) pulley and the cord.]



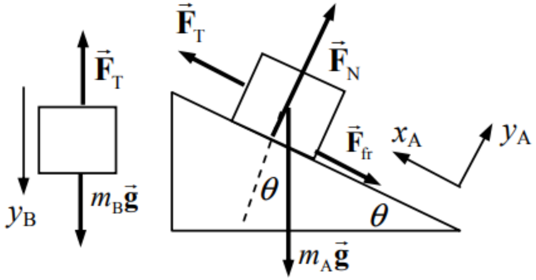


Figure 7

SOLUTION

(a)

Given that m_B is moving down, m_A must be moving up the incline, so the force of kinetic friction on m_A will be directed down the incline. Since the blocks are tied together, they will both have the same acceleration, so $a_{yB} = a_{xA} = a$. Write Newton’s second law for each mass.

$$\sum F_{yB} = m_B g - F_T = m_B a \quad \rightarrow \quad F_T = m_B g - m_B a$$
$$\sum F_{xA} = F_T - m_A g \sin\theta - F_{fr} = m_A a$$
$$\sum F_{yA} = F_N - m_A g \cos\theta = 0 \quad \rightarrow \quad F_N = m_A g \cos\theta$$

Take the information from the two y equations and substitute into the x equation to solve for the acceleration.

$$m_B g - m_B a - m_A g \sin\theta - \mu_k m_A g \cos\theta = m_A a \quad \rightarrow$$
$$a = \frac{m_B g - m_A g \sin\theta - m_A g \mu_k \cos\theta}{(m_A + m_B)} = \tfrac{1}{2} g(1 - \sin\theta - \mu_k \cos\theta)$$
$$= \tfrac{1}{2}(9.80\ \text{m/s}^2)(1 - \sin 34° - 0.15 \cos 34°) = \boxed{1.6\ \text{m/s}^2}$$

(b)

To have an acceleration of zero, the expression for the acceleration must be zero.

$$a = \tfrac{1}{2} g(1 - \sin\theta - \mu_k \cos\theta) = 0 \quad \rightarrow \quad 1 - \sin\theta - \mu_k \cos\theta = 0 \quad \rightarrow$$
$$\mu_k = \frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin 34°}{\cos 34°} = \boxed{0.53}$$

EXERCISE 8

Three climbers are tied together and climbing a 31° ice slope. The last climber slips and pulls the middle climber. The first climber holds both of them. Each climber weighs 75 kg. Find the tension in the two ropes. Ignore friction with the ice (second and third climbers).

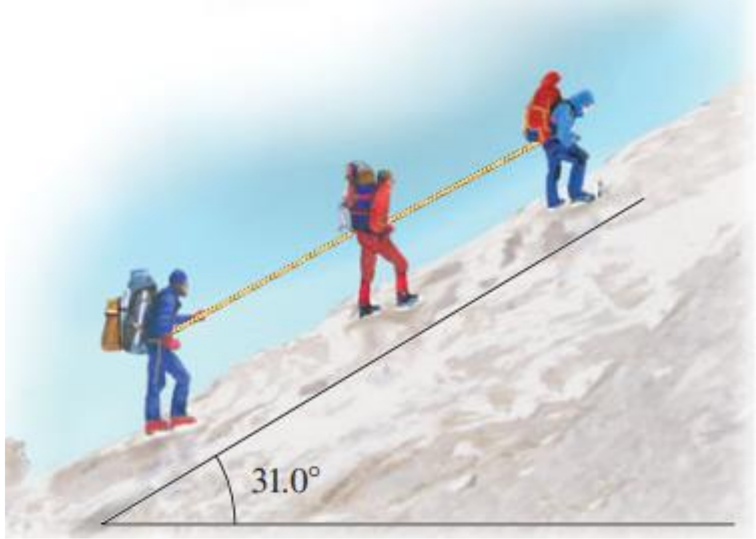
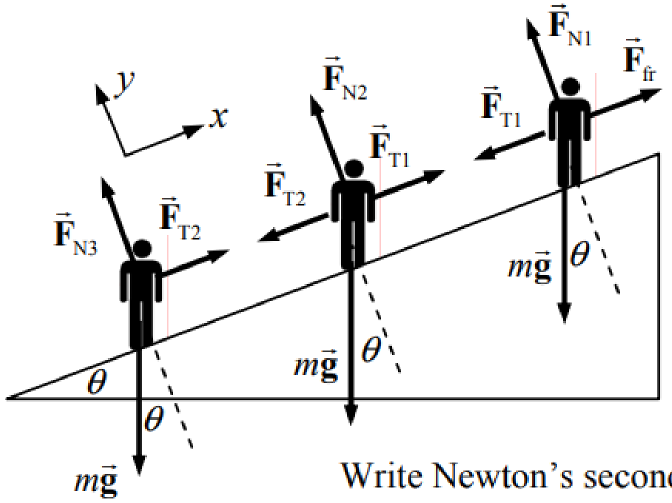


Figure 8

SOLUTION

Since the climbers are on ice, the frictional force for the lower two climbers is negligible. Consider the free-body diagram as shown. Note that all the masses are the same. Write Newton's second law in the *x* direction for the lowest climber, assuming he is at rest.



$$\sum F_x = F_{T2} - mg \sin\theta = 0$$

$$F_{T2} = mg \sin\theta = (75\text{ kg})(9.80\text{ m/s}^2)\sin 31.0°$$

$$= \boxed{380\text{ N}}$$

Write Newton's second law in the *x* direction for the middle climber, assuming he is at rest.

$$\sum F_x = F_{T1} - F_{T2} - mg \sin\theta = 0 \quad \rightarrow \quad F_{T1} = F_{T2} + mg\sin\theta = 2F_{T2}g\sin\theta = \boxed{760\text{ N}}$$