PHYSICS

CHAPTER 2 – KINEMATICS IN ONE DIMENSION

Introduction

Mechanics = Study of moving objects
Kinematics and Dynamics are the two main parts of Mechanics.
Kinematics → describes **HOW** objects move
Dynamics → explains **WHY** objects move, dealing with **forces and causes of motion**

Translational motion = motion without rotation
This chapter → **1D straight-line motion**
Next chapter → **2D/3D motion** (curved paths)
Rotation → Chapter 8
Particle Model → object treated as a **point particle** with no size.
Useful when only translational motion is **significant.**
Examples: billiard ball, spacecraft to the Moon

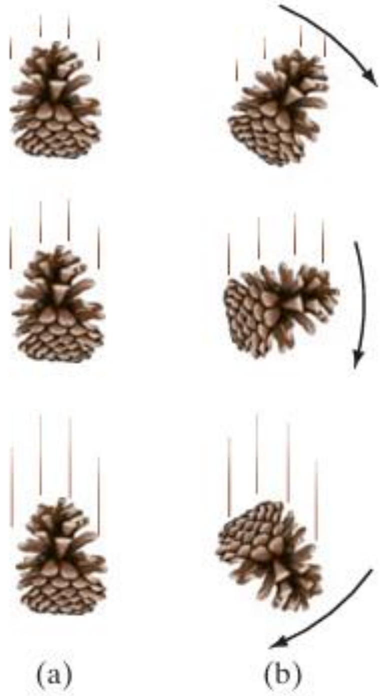


FIGURE 2–1 A falling pinecone undergoes (a) pure translation; (b) it is rotating as well as translating.

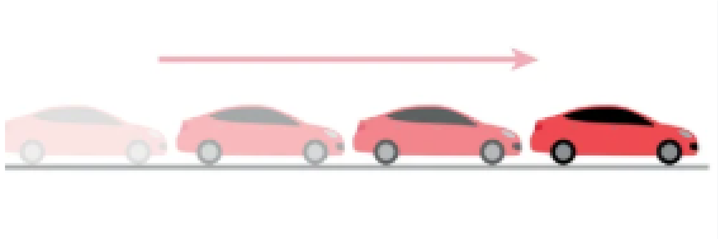
Introduction

In physics, we can categorize motion into three types: translational, rotational, and vibrational.

1. Translational Motion

Translational motion refers to the movement of an object from one point to another in a straight line

Example: A car driving on a straight road.



2. Rotational Motion

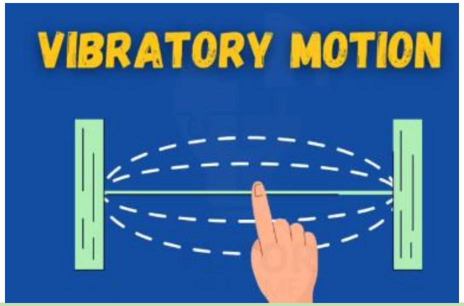
Rotation or rotational/rotary motion is the circular movement of an object around a central line, known as an axis of rotation

Example: A spinning wheel or Earth's rotation.

3. Vibrational (or Oscillatory) Motion

Vibrational motion is the repetitive back-and-forth motion of a body about a fixed point or equilibrium position.

Example: A pendulum, a mass on a spring, or guitar strings



2.1. Reference Frames and Displacement

Any measurement of **position, distance, or speed** must be made relative to a **reference frame**.
Example: On a train moving at 80 km/h, a person walks forward at 5 km/h relative to the train.

Speed relative to train: 5 km/h
Speed relative to ground: 80 + 5 = 85 km/h

FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

Always **specify the frame of reference** to avoid confusion. In everyday life, we usually mean **relative to Earth.**

Coordinate Systems in Physics

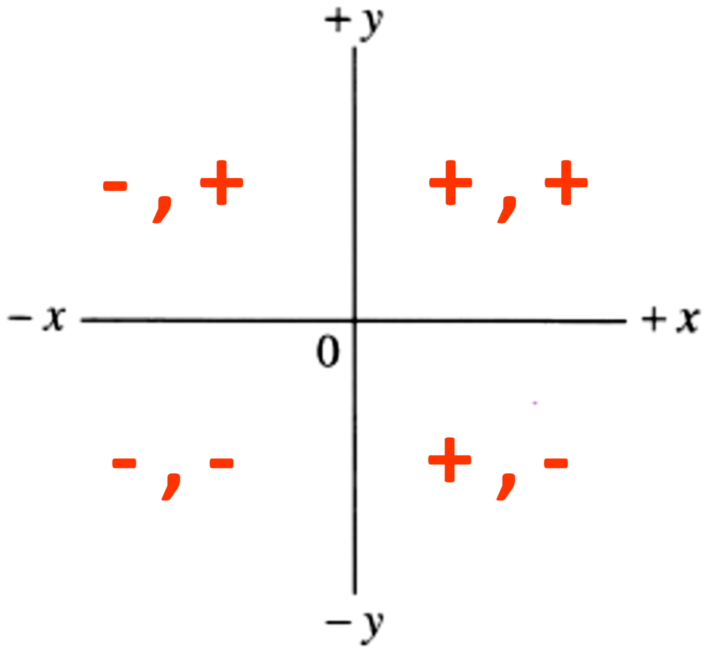
A **frame of reference** is often represented using **coordinate axes** (x, y, z).

The **origin is (x,y)=(0,0).**

Axes are **perpendicular**:

- x-axis: right = positive, left = negative
- y-axis: above origin = positive, below = negative

Any point is specified by its **x and y coordinates** (or x, y, z in 3D).



For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Then the position of an object at any moment is given by its x coordinate. If the motion is vertical, as for a dropped object, we usually use the y axis.

Distance vs. Displacement
Distance: total path length traveled by an object.
Displacement: change in position; how far the object is from its **starting point**.
Example:
A person walks **70 m east**, then **30 m west**.

- **Total distance traveled:** 100 m
- **Displacement:** 40 m (east)

Displacement has **both magnitude and direction**.
Quantities with magnitude and direction are called **vectors**.
Represented in diagrams by **arrows**:

- Arrow length = magnitude
- Arrow direction = direction of displacement

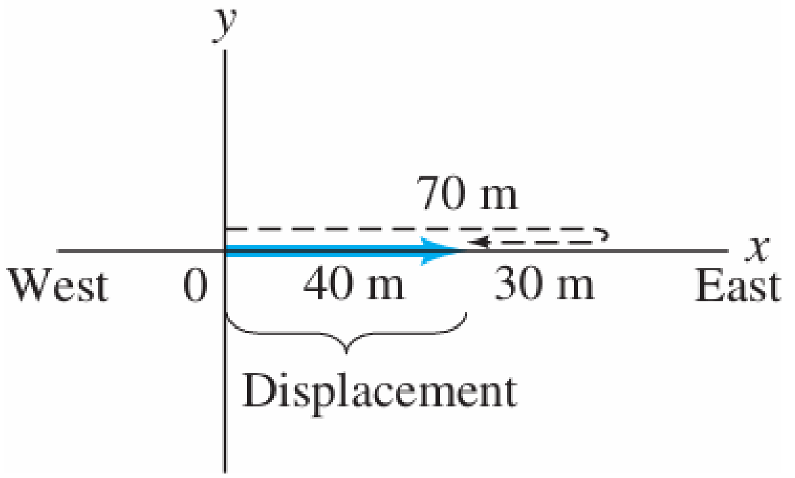
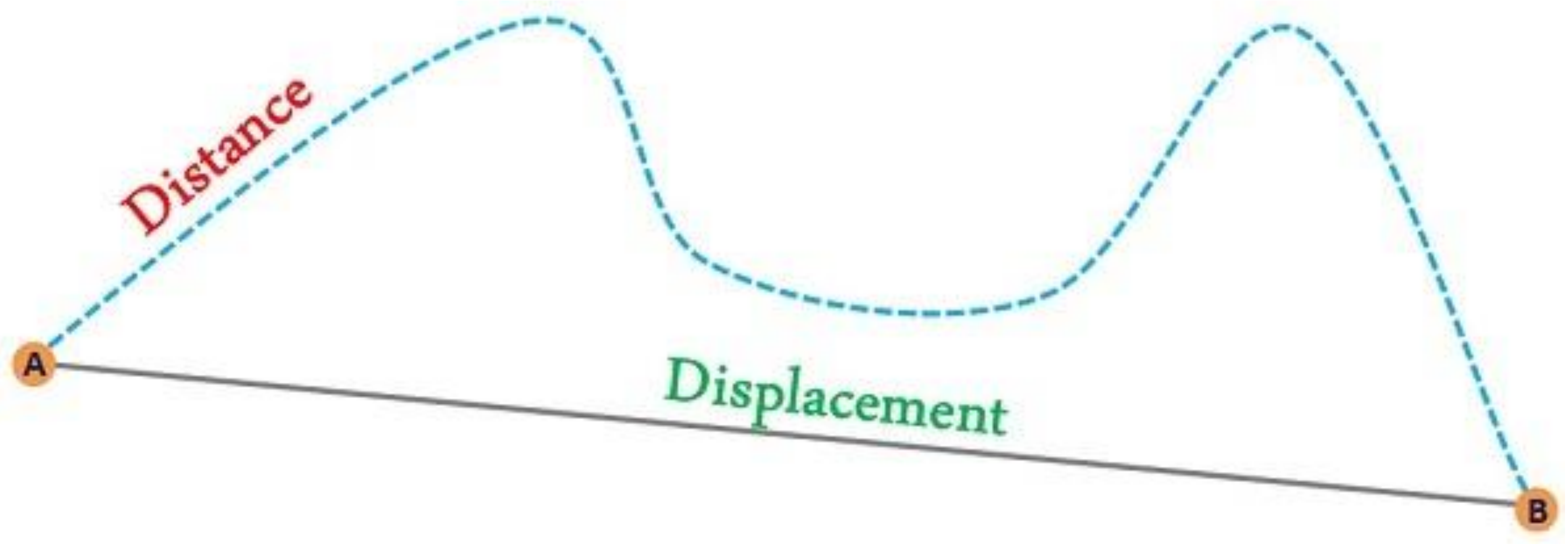


FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown dashed in black); but the displacement, shown as a solid blue arrow, is 40 m to the east.

2.1. Reference Frames and Displacement

While distance is the length of the actual path between two locations, displacement, on the other hand, is the length of the shortest path between two locations.



bird fly distance

Distance

- Distance is the length of the path travelled by a body while a moving from an initial position to a final position
- Distance is a scalar quantity.
- Distance measured is always positive.
- The total distance covered is equal to the algebraic sum of all the distances travelled in different directions.
- There is always a distance covered whenever there is a motion.
- Unit: meter (m)

Displacement

- Displacement is the shortest distance between the initial position and the final position of the body.
- Displacement is a vector quantity.
- Displacement can be positive or negative depending on the reference point.
- The net displacement is the vector sum of individual displacements in different directions.
- Displacement will be zero if the body comes back to its initial position.
- Unit: meter (m)

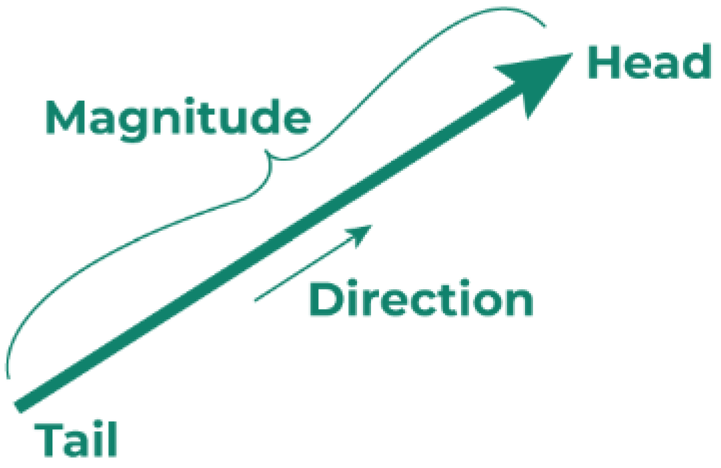
Definition of Displacement: The **displacement** Δx of a particle is defined as its change in position in some time interval. As the particle moves from an initial position x_i to a final position x_f, its displacement is given by;

$$\Delta x = x_f - x_i$$

We use the capital Greek letter delta (Δ) to denote the *change* in a quantity. From this definition, we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i.

Many quantities in physics, like displacement, have a ***magnitude and a direction***. Such quantities are called ***VECTORS***.

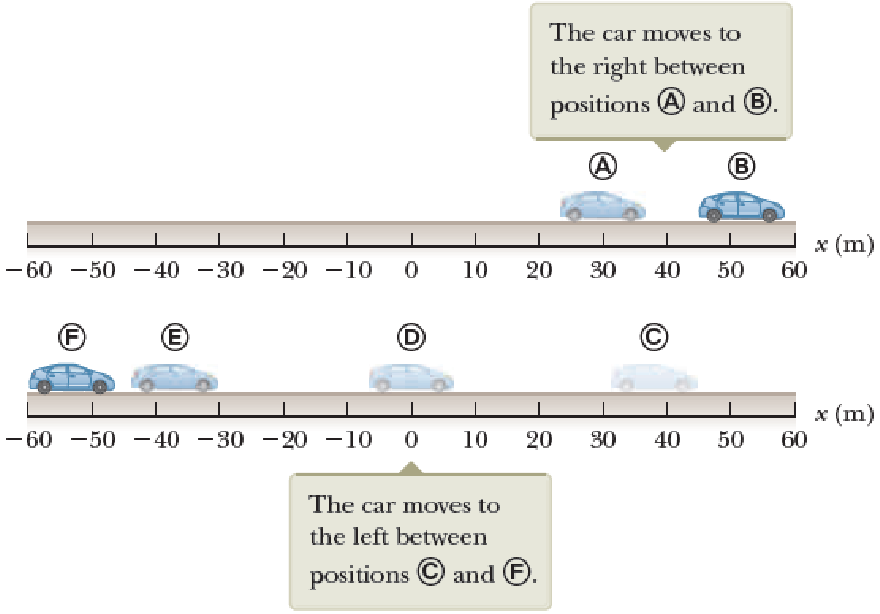
Other quantities which are vectors: velocity, acceleration, force, momentum, ...



Many quantities in physics, like distance, have a ***magnitude only***. Such quantities are called ***SCALARS***.

Other quantities which are scalars: speed, temperature, mass, volume, ...

2.2. Average Velocity



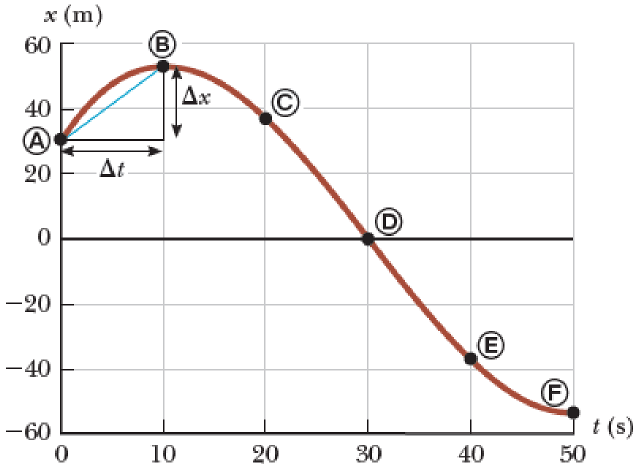


Table 2.1 Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

A particle's position x is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system. The motion of a particle is completely known if the particle's position in space is known at all times. Consider a car moving back and forth along the x axis as in the figure above. When we start collecting position data, the car is 30 m to the right of the reference point at x = 0.We start our clock and record the car's position every 10 seconds. During the first 10 seconds, the car moves to the right (positive direction) from point A to point B.After B, the position values begin to decrease, which means the car goes back from B toward F.In fact, 30 seconds after starting the measurement, the car is at point D, which is the origin (x = 0).It keeps moving to the left, and after the sixth data point, when we stop recording, the car is more than 50 m to the left of x = 0.The graphical display of this information is called **a position–time graph.**

2.2. Average Velocity

The term "speed" refers to how far an object travels in a given time interval, regardless of direction. If a car travels 240 kilometers (km) in 3 hours (h), we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the total distance traveled along its path divided by the time it takes to travel this distance*:

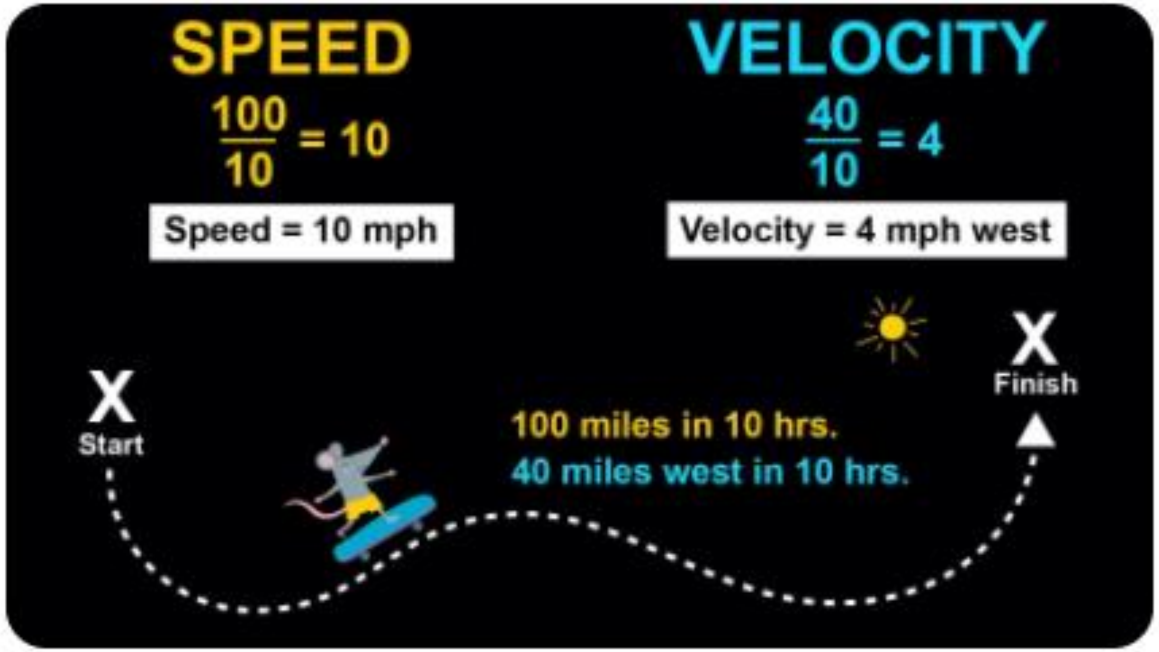
$$\text{Average Speed} = \frac{\text{Distance } travelled}{\text{Time } elapsed}$$

The terms "velocity" and "speed" are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and also the *direction* in which it is moving. Velocity is therefore a *vector*. There is a second difference between speed and velocity: namely, the average velocity is defined in terms of displacement, rather than total distance traveled:

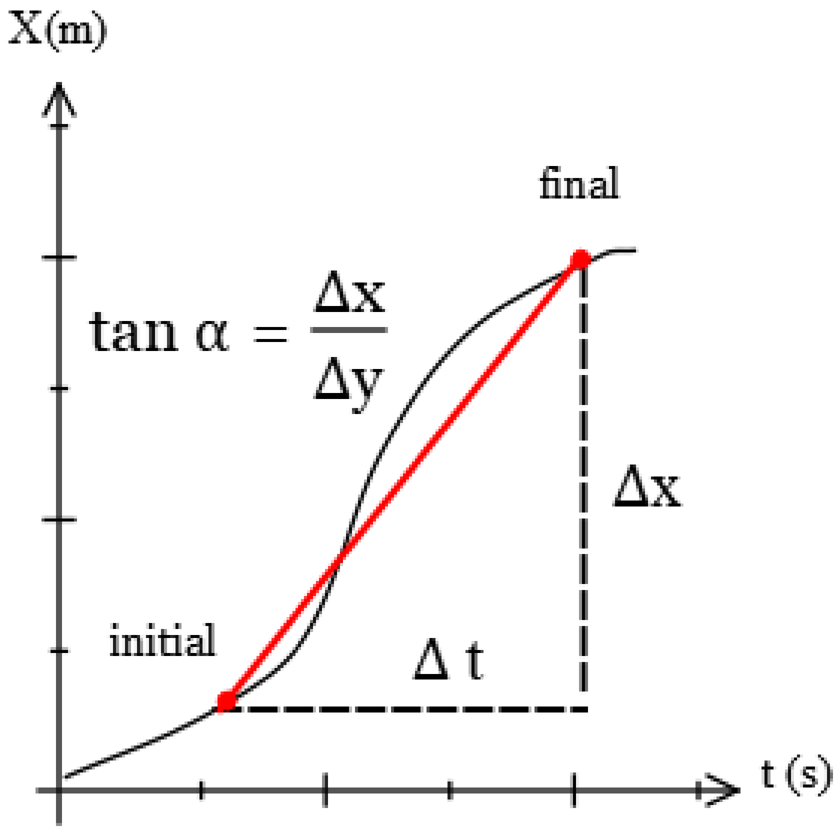
$$\text{Average Velocity} = \frac{Displacement}{\text{Time } elapsed} = \frac{\Delta x}{\Delta t}$$

Speed - the rate of distance traveled by a moving object over time

Velocity - the rate of displacement of a moving object over time

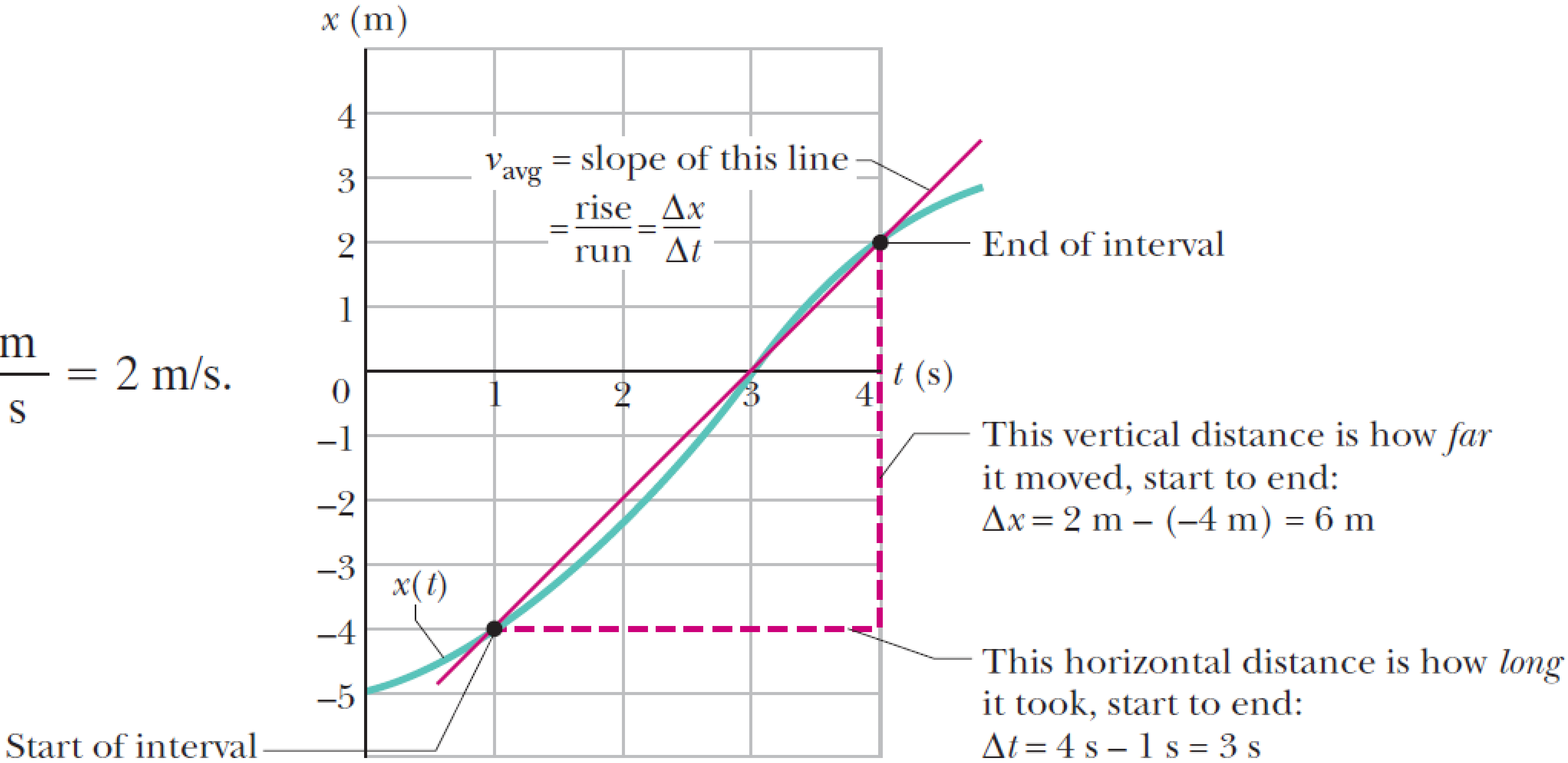


2.2. Average Velocity

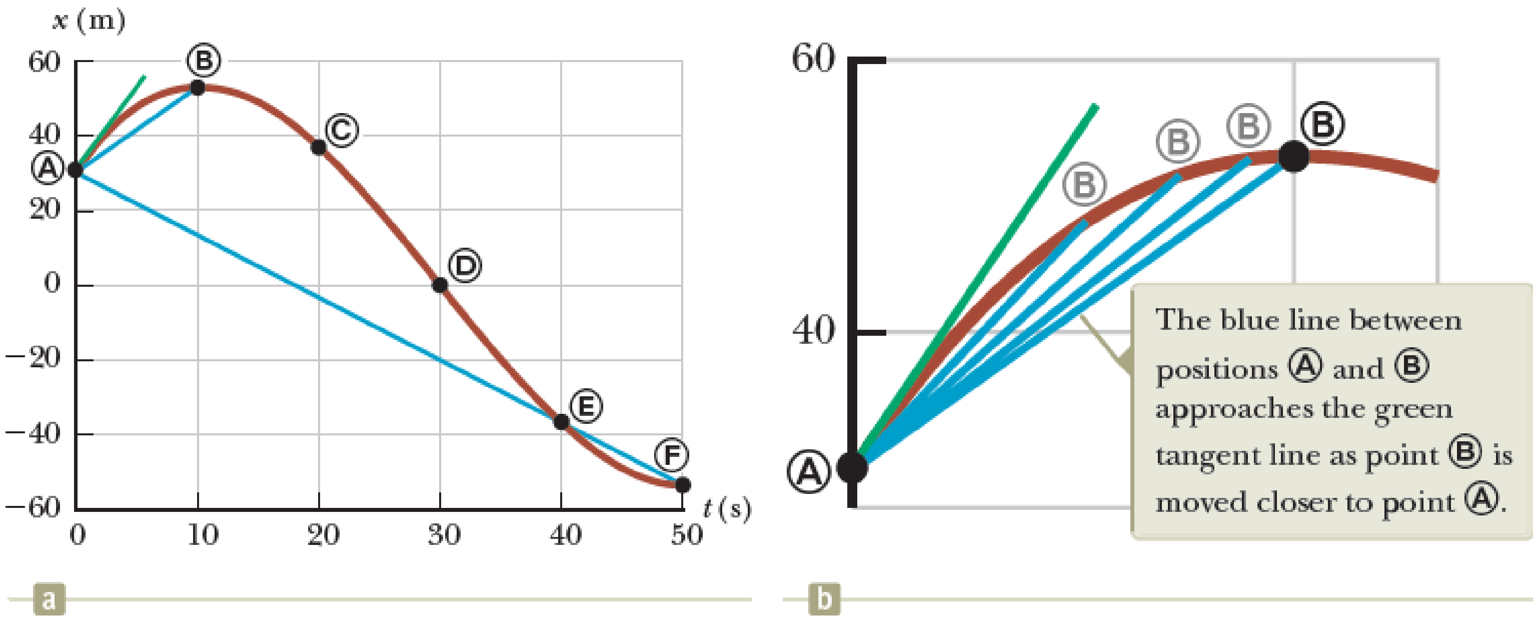


The slope of the straight-line connection
is the average velocity between these points

$$v_{avg} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}.$$



2.3. Instantaneous Velocity



Since the car begins moving to the **right (positive direction)**, the average velocity from **A → B** is more representative of the **initial velocity** than the negative value from **A → F**. Now consider moving point **B closer to A** along the curve: The line joining A and B becomes steeper. As B approaches A, this line becomes the **tangent** at A. The slope of this tangent line gives the **instantaneous velocity** at that moment. **Instantaneous velocity** is defined as the limiting value of the ratio of displacement to time as the time interval approaches zero:

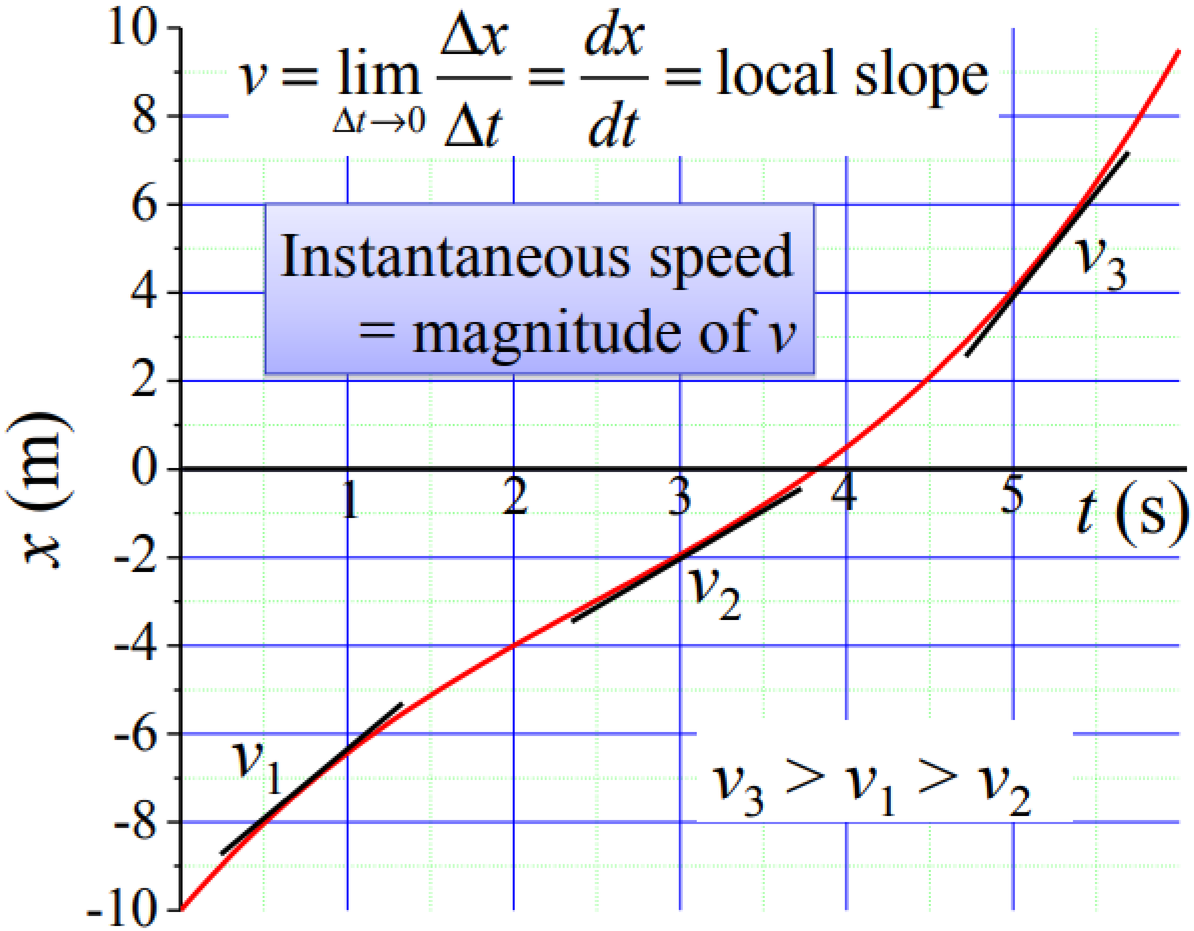
$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

In calculus notation, this limit is called the *derivative* of *x* with respect to *t*, written *dx*/*dt:*

$$\boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

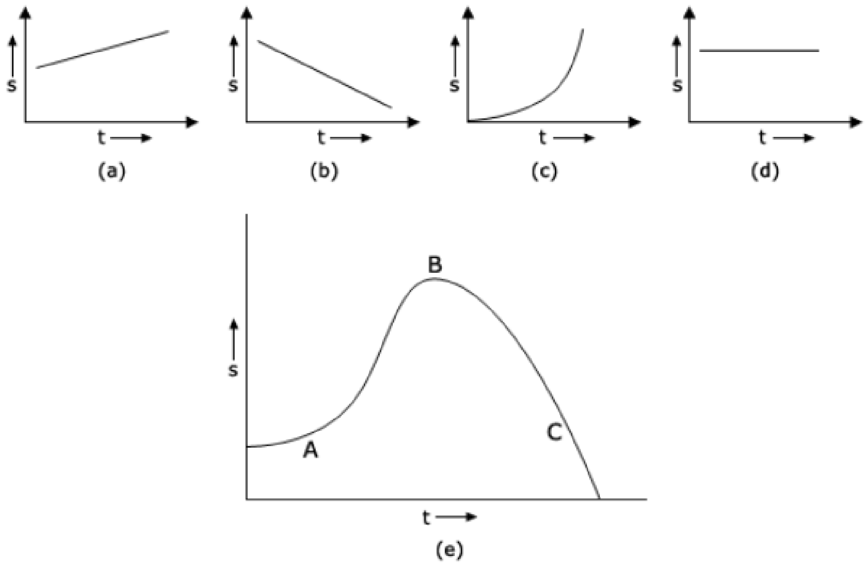
From here on, we use the word *velocity* to designate instantaneous velocity. When we are interested in *average velocity,* we shall always use the adjective *average.* The **instantaneous speed** of a particle is defined as the magnitude of its instantaneous velocity. As with average speed, instantaneous speed has no direction associated with it. For example, if one particle has an instantaneous velocity of +25 m/s along a given line and another particle has an instantaneous velocity of -25 m/s along the same line, both have a speed of 25 m/s.

Instantaneous velocity and speed



2.3. Instantaneous Velocity

From the Displacement-time graphs of a particle shown below, what conclusions can be drawn about the velocity of the particle?



Ans:

From the graph (a), the slope is constant and positive. The particle moves with the constant velocity.

From the graph (b), the slope is constant and negative. The particle moves with the negative constant velocity.

From the graph (c), the slope is variable and positive. the particle moves with variable velocity.

From the graph (d), the slope is zero. The particle is at rest.

From the graph (e), at the point A, the slope increases, the velocity increases, at B, the slope is zero, the body is at rest and at c, the slope is decreasing, the velocity is decreasing.

An object whose velocity is changing is said to be accelerating. For instance, a car whose velocity increases in magnitude from zero to 80 km/h is accelerating. Acceleration specifies how *rapidly* the velocity of an object is changing. *Average acceleration* is defined as the change in velocity divided by the time taken to make this change:

$$Average\ Acceleration = \frac{Change\ of\ Velocity}{Time\ Elapsed}$$

In symbols, the average acceleration, $\bar{a}$, over a time interval $\Delta t = t_2 - t_1$, during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as;

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

We saw that velocity is a vector (it has magnitude and direction), so acceleration is a vector too. But for one dimensional motion, we need only use a plus or minus sign to indicate acceleration direction relative to a chosen coordinate axis (Usually, right is + left is -).

The **instantaneous acceleration**, a, can be defined in analogy to instantaneous velocity as the average acceleration over an infinitesimally short time interval at a given instant:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

The acceleration equals the *second derivative* of *x* with respect to time.

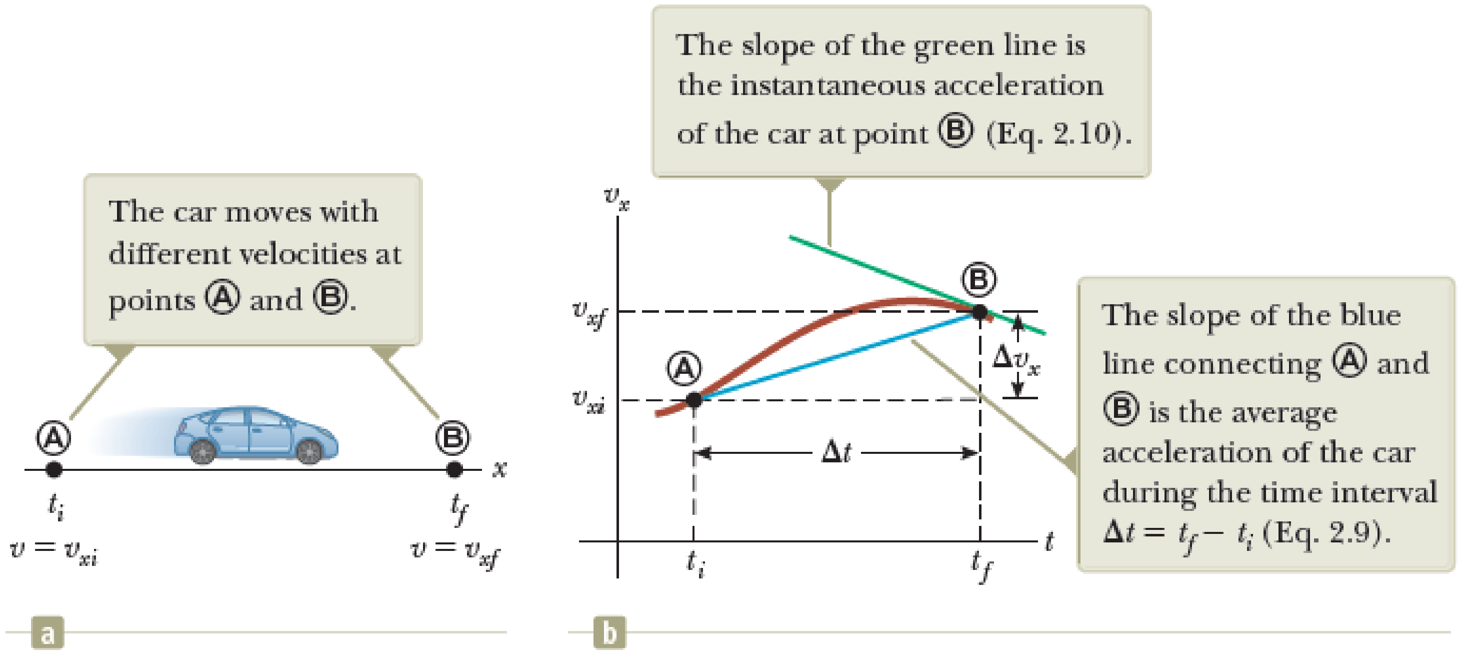


Figure: (a) A car, modeled as a particle, moving along the x axis from Ⓐ to Ⓑ, has velocity v_{xi} at $t = t_i$ and velocity v_{xf} at $t = t_f$. (b) Velocity–time graph (red-brown) for the particle moving in a straight line.

2.4 Acceleration

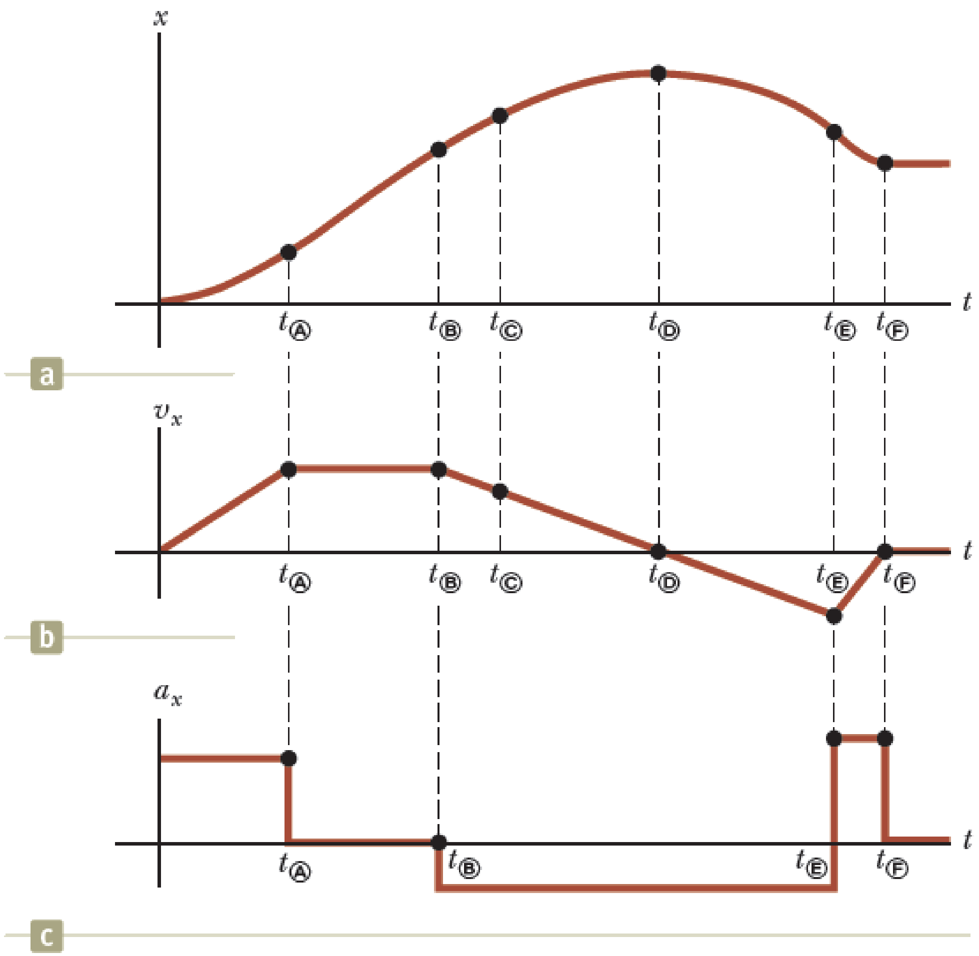


Figure 2.8 (Conceptual Example 2.5) (a) Position–time graph for an object moving along the *x* axis. (b) The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant. (c) The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

Example: The velocity of a particle moving along the *x* axis varies according to the expression

$$v_x = 40 - 5t^2$$

where v_x is in meters per second and *t* is in seconds.

a) Find the average acceleration in the time interval t=0 to t=2.0 s.
b) Determine the acceleration at t=2.0 s.

Solution:

a) $a_{x,ave} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{20-40}{2-0} = \frac{-20}{2} = -10\ m/s^2$

b) Knowing that the initial velocity at any time t is $v_1 = 40 - 5t^2$, find the velocity at any later time $t + \Delta t$:

$$v_2 = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5\,(\Delta t)^2$$

Find the change in velocity over the time interval Δ*t:*

$$\Delta v_x = v_2 - v_1 = 40 - 5t^2 - 10t\,\Delta t - 5\,(\Delta t)^2 - 40 + 5t^2 = -10t\,\Delta t - 5\,(\Delta t)^2$$

To find the acceleration at any time t, divide this expression by Δ*t* and take the limit of the result as Δ*t* approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} \frac{-10t\,\Delta t - 5\,(\Delta t)^2}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t$$

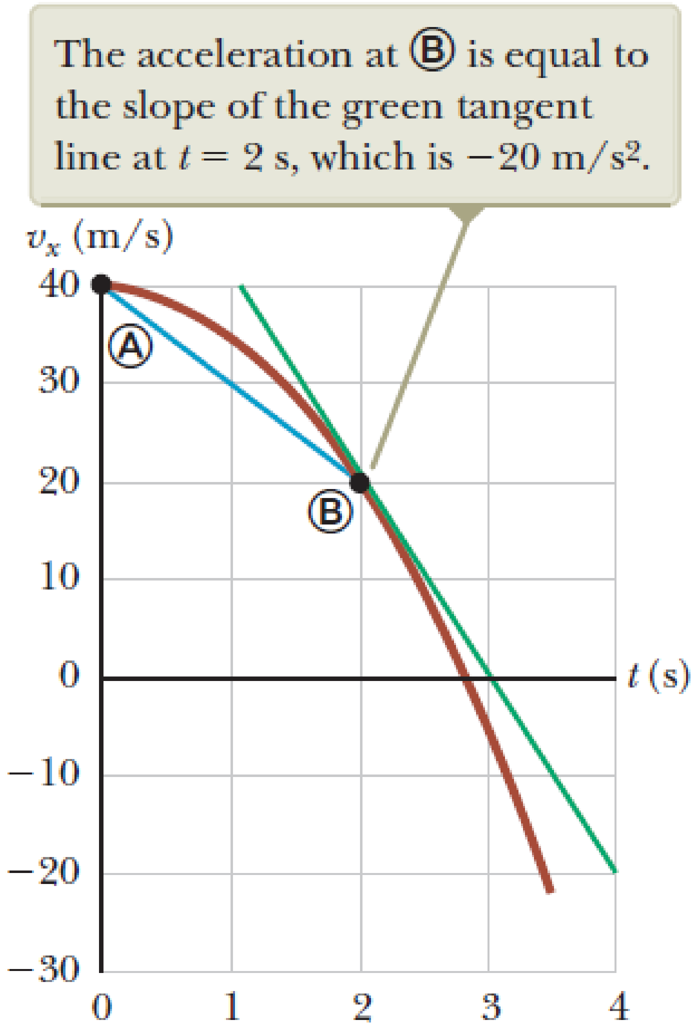
Substitute t=2, $a_x = -10 \cdot 2 = -20$

Solution: Alnernatively, we can find the instantaneous acceleration much more quickly by using the derivative:

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(40 - 5t^2) = -10t$$

For t=2 we have $a_x = -10 \cdot 2 = -20\, {}^{m}/_{s^2}$

Notice that the answers to parts (A) and (B) are different. The average acceleration in part (A) is the slope of the blue line in the figure connecting points A and B. The instantaneous acceleration in part (B) is the slope of the green line tangent to the curve at point B. Notice also that the acceleration is *not* constant in this example.



Deceleration: When an object is slowing down, we can say it is decelerating. But be careful: deceleration does not mean that the acceleration is necessarily negative. The velocity of an object moving to the right along the positive x axis is positive; if the object is slowing down (as in Fig. 2–11), the acceleration is negative. But the same car moving to the left (decreasing x), and slowing down, has positive acceleration that points to the right, as shown in Fig. 2–12. We have a deceleration whenever the magnitude of the velocity is decreasing; thus the velocity and acceleration point in opposite directions when there is deceleration.

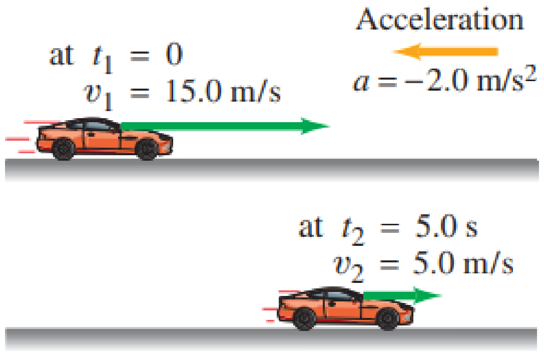


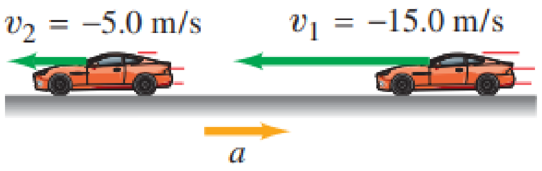
FIGURE 2–11 Example 2–6, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left because the car slows down as it moves to the right.

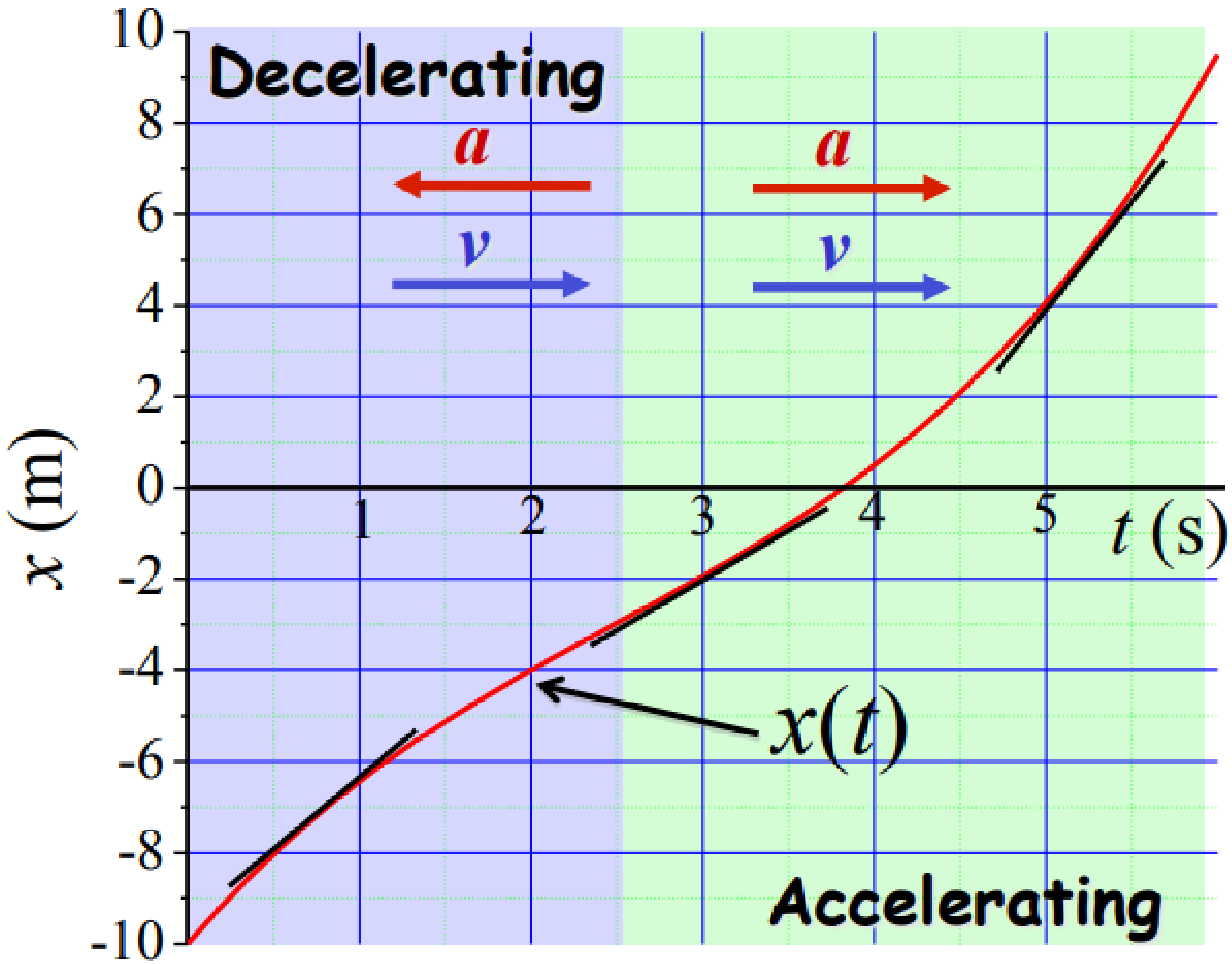
$$\bar{a} = \frac{5.0\ \mathrm{m/s} - 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = -2.0\ \mathrm{m/s^2}.$$

FIGURE 2–12 The car of Example 2–6, now moving to the *left* and decelerating. The acceleration is $a = (v_2 - v_1)/\Delta t$, or

$$a = \frac{(-5.0\ \mathrm{m/s}) - (-15.0\ \mathrm{m/s})}{5.0\ \mathrm{s}}$$

$$= \frac{-5.0\ \mathrm{m/s} + 15.0\ \mathrm{m/s}}{5.0\ \mathrm{s}} = +2.0\ \mathrm{m/s^2}.$$




Decelerating
a
v
a
v
x(t)
Accelerating
x (m)
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6
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1
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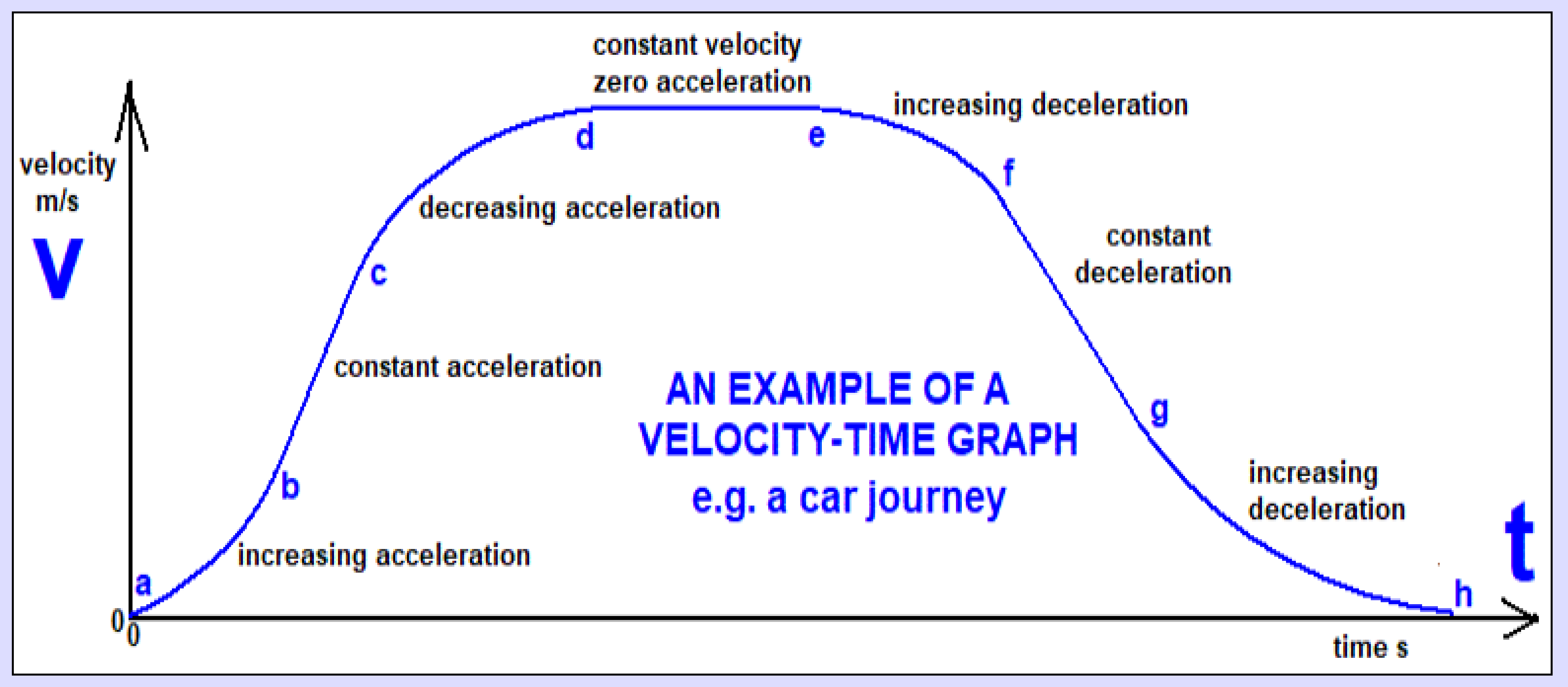

constant velocity
zero acceleration
increasing deceleration
velocity
m/s
V
d
e
f
decreasing acceleration
c
constant
deceleration
constant acceleration
AN EXAMPLE OF A
VELOCITY-TIME GRAPH
e.g. a car journey
g
b
increasing
deceleration
increasing acceleration
t
a
h
0
0
time s

We now examine motion in a straight line when the magnitude of the acceleration is constant. In this case, the instantaneous and average accelerations are equal. We use the definitions of average velocity and acceleration to derive a set of valuable equations that relate x, a, and when a is constant, allowing us to determine any one of these variables if we know the others.

The initial position: x_0
The initial velocity: v_0
The initial time: $t_0 = 0$

Average velocity ($\bar{v}$) during the time interval $t - t_0$ will be:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

The acceleration, assumed constant in time, is

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$$

Multiply both sides by t:

$$at = v - v_0 \text{ or } v - v_0 = at$$

Adding $v_0$to both sides, we obtain

$$v = v_0 + at$$

Next, let us see how to calculate the position x of an object after a time when it undergoes constant acceleration. The definition of average velocity is $\bar{v} = \frac{x-x_0}{t}$

Multiplying both sides by t, we get:

$$x = x_0 + \bar{v}t$$

Because the velocity increases at uniform rate, the average velocity will be midway between the initial and final velocities:

$$\bar{v} = \frac{v_0 + v}{2}$$

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

Remember that:

$$v = v_0 + at$$

Therefore:

$$x = x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

$$v = v_0 + at$$

$$t = \frac{v - v_0}{t}$$

$$x = x_0 + \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{t}\right)$$

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

We solve this for v^2 and obtain:

$$v^2 = v_0^2 + 2a(x - x_0)$$

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these kinematic equations for constant acceleration here in one place for future reference.

$$v = v_0 + at \qquad [a = \text{constant}] \qquad \textbf{(2–11a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \qquad \textbf{(2–11b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \qquad \textbf{(2–11c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \qquad \textbf{(2–11d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

EXAMPLE 2–7 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at $2.00\ \mathrm{m/s^2}$. (*a*) If the runway is 150 m long, can this airplane reach the required speed for takeoff? (*b*) If not, what minimum length must the runway have?

SOLUTION (*a*) Of the above four equations, Eq. 2–11c will give us v when we know v_0, a, x, and x_0:

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 0 + 2(2.00\ \mathrm{m/s^2})(150\ \mathrm{m}) = 600\ \mathrm{m^2/s^2} \\ v &= \sqrt{600\ \mathrm{m^2/s^2}} = 24.5\ \mathrm{m/s}. \end{aligned}$$

This runway length is *not* sufficient, because the minimum speed is not reached. (*b*) Now we want to find the minimum runway length, $x - x_0$, for a plane to reach $v = 27.8\ \mathrm{m/s}$, given $a = 2.00\ \mathrm{m/s^2}$. We again use Eq. 2–11c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \mathrm{m/s})^2 - 0}{2(2.00\ \mathrm{m/s^2})} = 193\ \mathrm{m}.$$

A 200-m runway is more appropriate for this plane.

1. Read the whole problem and make sure you understand it. Then read it again.
2. Decide on the objects under study and what the time interval is.
3. Draw a diagram and choose coordinate axes.
4. Write down the known (given) quantities, and then the unknown ones that you need to find.
5. What physics applies here? Plan an approach to a solution.

6. Which equations relate the known and unknown quantities? Are they valid in this situation? Solve algebraically for the unknown quantities, and check that your result is sensible (correct dimensions).

7. Calculate the solution and round it to the appropriate number of significant figures.

8. Look at the result—is it reasonable? Does it agree with a rough estimate?

9. Check the units again.

EXAMPLE 2–8 **Acceleration of a car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if the car accelerates from rest at a constant $2.00\ \mathrm{m/s^2}$?

APPROACH We follow the Problem Solving Strategy on the previous page, step by step.

SOLUTION

1. **Reread** the problem. Be sure you understand what it asks for (here, a time interval: "how long does it take").
2. The **object** under study is the car. We need to choose the **time interval** during which we look at the car's motion: we choose $t = 0$, the initial time, to be the moment the car starts to accelerate from rest $(v_0 = 0)$; the time t is the instant the car has traveled the full 30.0-m width of the intersection.
3. **Draw** a **diagram**: the situation is shown in Fig. 2–15, where the car is shown moving along the positive x axis. We choose $x_0 = 0$ at the front bumper of the car before it starts to move.
4. The "**knowns**" and the "wanted" information are shown in the Table in the margin. Note that "starting from rest" means $v = 0$ at $t = 0$; that is, $v_0 = 0$. The wanted time t is how long it takes the car to travel 30.0 m.
5. The **physics**: the car, starting from rest (at $t_0 = 0$), increases in speed as it covers more distance. The acceleration is constant, so we can use the kinematic equations, Eqs. 2–11.

PROBLEM SOLVING

"Starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$]

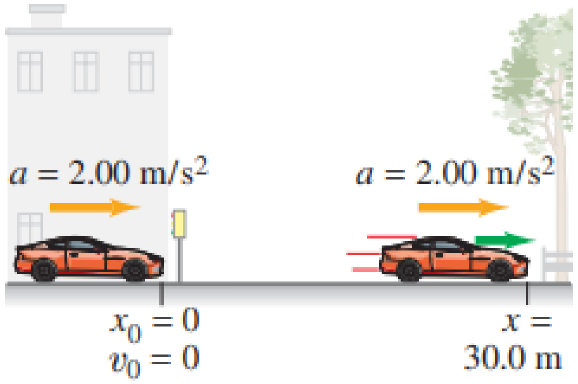


FIGURE 2–15 Example 2–8.

Known	Wanted
$x_0 = 0$	t
$x = 30.0\ \mathrm{m}$	
$a = 2.00\ \mathrm{m/s^2}$	
$v_0 = 0$	

6. **Equations**: we want to find the time, given the distance and acceleration; Eq. 2–11b is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$ in Eq. 2–11b $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$, we have

$$x = \tfrac{1}{2}at^2.$$

We solve for t by multiplying both sides by $\dfrac{2}{a}$:

$$\frac{2x}{a} = t^2.$$

Taking the square root, we get t:

$$t = \sqrt{\frac{2x}{a}}.$$

7. The **calculation**:

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\mathrm{m})}{2.00\,\mathrm{m/s^2}}} = 5.48\,\mathrm{s}.$$

This is our answer. Note that the units come out correctly.

8. We can check the **reasonableness** of the answer by doing an alternate calculation: we first find the final velocity

$$v = at = (2.00\,\mathrm{m/s^2})(5.48\,\mathrm{s}) = 10.96\,\mathrm{m/s},$$

and then find the distance traveled

$$x = x_0 + \bar{v}t = 0 + \tfrac{1}{2}(10.96\,\mathrm{m/s} + 0)(5.48\,\mathrm{s}) = 30.0\,\mathrm{m},$$

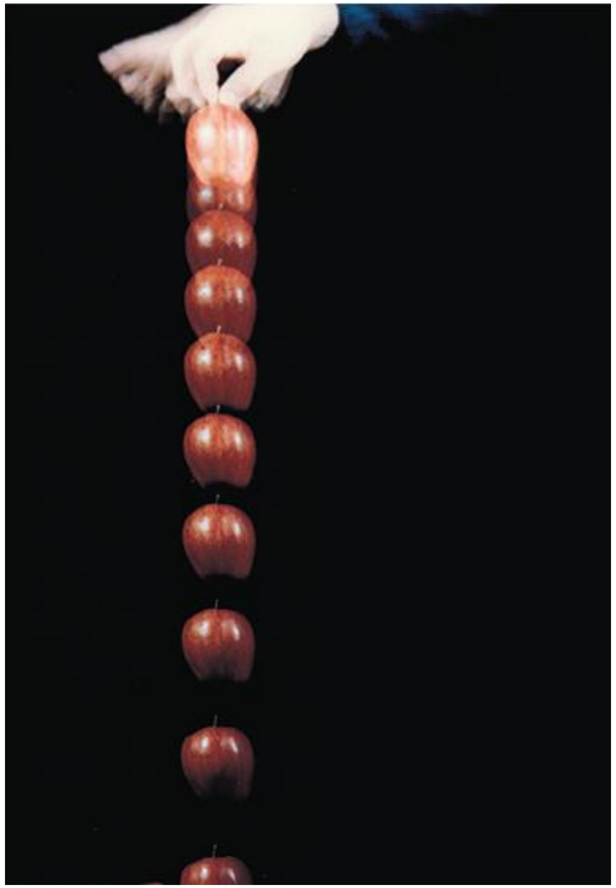
which checks with our given distance.

9. We checked the **units** in step 7, and they came out correctly (seconds).

PROBLEM SOLVING
Check your answer

NOTE In steps 6 and 7, when we took the square root, we should have written $t = \pm\sqrt{2x/a} = \pm\,5.48\,\mathrm{s}$. Mathematically there are two solutions. But the second solution, $t = -5.48\,\mathrm{s}$, is a time *before* our chosen time interval and makes no sense physically. We say it is "unphysical" and ignore it.

PROBLEM SOLVING
"Unphysical" solutions

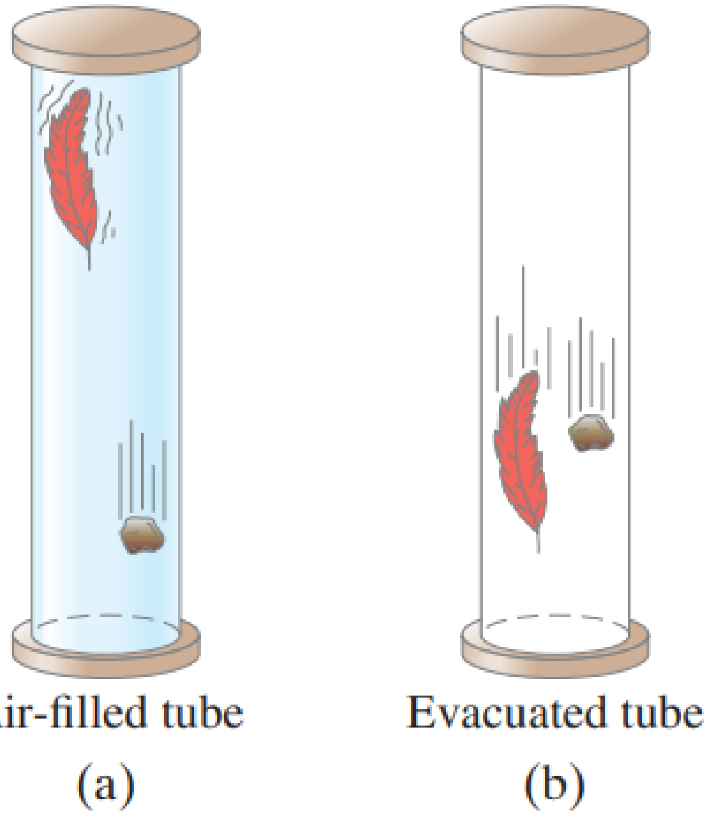


Near the surface of the Earth, all objects experience approximately the same acceleration due to gravity

This is one of the most common examples of motion with constant acceleration.

In the absence of air resistance, all objects fall with the same acceleration

FIGURE 2–21 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.



The acceleration due to gravity at the Earth's surface is approximately 9.80 m/s^2. At a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

- **Note:** Some books treat the falling objects motion slightly different than the book which we are following, but it is, of course, equivalent!
- For the free fall you may use the same equations we already have, but change notation slightly:

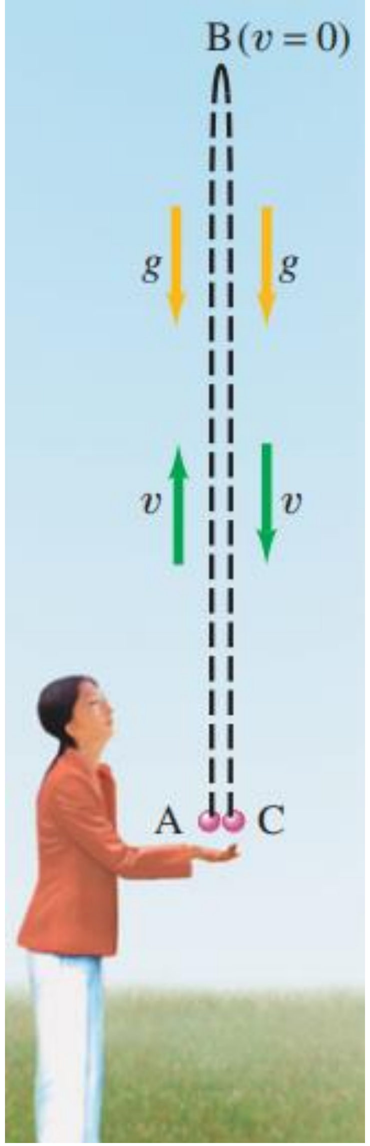
  Replace a by g = 9.8 m/s²

  But in the equations it could have a + or a – sign in front of it!
- Usually, we consider vertical motion to be in the **y** direction, so replace **x** by **y** and $\mathbf{x_0}$ by $\mathbf{y_0}$ (often $\mathbf{y_0 = 0}$)

The magnitude (**size**) of **g = 9.8 m/s²** (***Always a Positive Number!***)

- But, **acceleration is a *vector*** (1 dimen), with ***2 possible directions.*** Call these **+** and **-**.
- **However, which way is + which way is – is**

  ARBITRARY & ***UP TO US***!
- May seem "natural" for "up" to be **+ y** and "down" to be **- y**, but we could also choose (we sometimes will!) "down" to be **+ y** and "up" to be **- y**
  - So, in the equations **g** could have a **+** or a **-** sign in front of it, depending on our choice!



Directions of Velocity & Acceleration

- Objects in free fall ***ALWAYS* have *downward* acceleration.**
- Still use the same equations for objects thrown ***upward*** with some initial velocity $\mathbf{v_0}$
- An object goes up until it stops at some point & then it falls back down. Acceleration is ***always*** **g** in the ***downward*** direction. For the first half of flight, the velocity is ***UPWARD***.

  $\Rightarrow$ **For the first part of the flight, velocity & acceleration are in *opposite directions!***

Equations for Objects in Free Fall

Written **taking “up” as + y**!

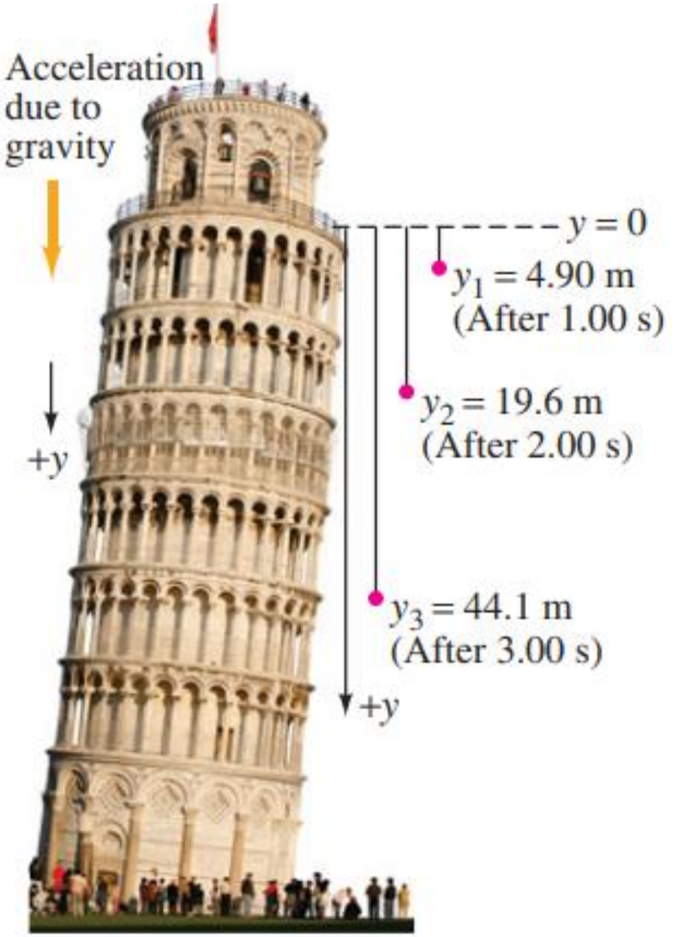
$$v = v_0 - gt \quad (1)$$
$$y = y_0 + v_0 t - (½)gt^2 \quad (2)$$
$$v^2 = (v_0)^2 - 2g(y - y_0) \quad (3)$$
$$v_{ave} = (½)(v + v_0) \quad (4)$$
$$g = 9.8\ m/s^2$$

Usually $y_0 = 0$. Sometimes $v_0 = 0$

Written **taking “down” as + y**!

$$v = v_0 + gt \quad (1)$$
$$y = y_0 + v_0 t + (½)gt^2 \quad (2)$$
$$v^2 = (v_0)^2 + 2g(y - y_0) \quad (3)$$
$$v_{ave} = (½)(v + v_0) \quad (4)$$
$$g = 9.8\ m/s^2$$

Usually $y_0 = 0$. Sometimes $v_0 = 0$



EXAMPLE 2–10 **Falling from a tower.** Suppose that a ball is dropped ($v_0 = 0$) from a tower. How far will it have fallen after a time $t_1 = 1.00\,\mathrm{s}$, $t_2 = 2.00\,\mathrm{s}$, and $t_3 = 3.00\,\mathrm{s}$? Ignore air resistance.

APPROACH Let us take y as positive downward, so the acceleration is $a = g = +9.80\,\mathrm{m/s^2}$. We set $v_0 = 0$ and $y_0 = 0$. We want to find the position y of the ball after three different time intervals. Equation 2–11b, with x replaced by y, relates the given quantities (t, a, and v_0) to the unknown y.

SOLUTION We set $t = t_1 = 1.00\,\mathrm{s}$ in Eq. 2–11b:

$$\begin{aligned} y_1 &= v_0 t_1 + \tfrac{1}{2} a t_1^2 \\ &= 0 + \tfrac{1}{2} a t_1^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.00\,\mathrm{s})^2 = 4.90\,\mathrm{m}. \end{aligned}$$

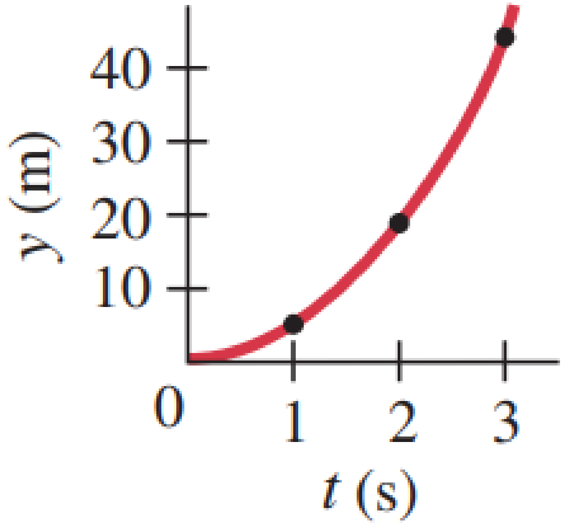
The ball has fallen a distance of 4.90 m during the time interval $t = 0$ to $t_1 = 1.00\,\mathrm{s}$. Similarly, after 2.00 s ($= t_2$), the ball's position is

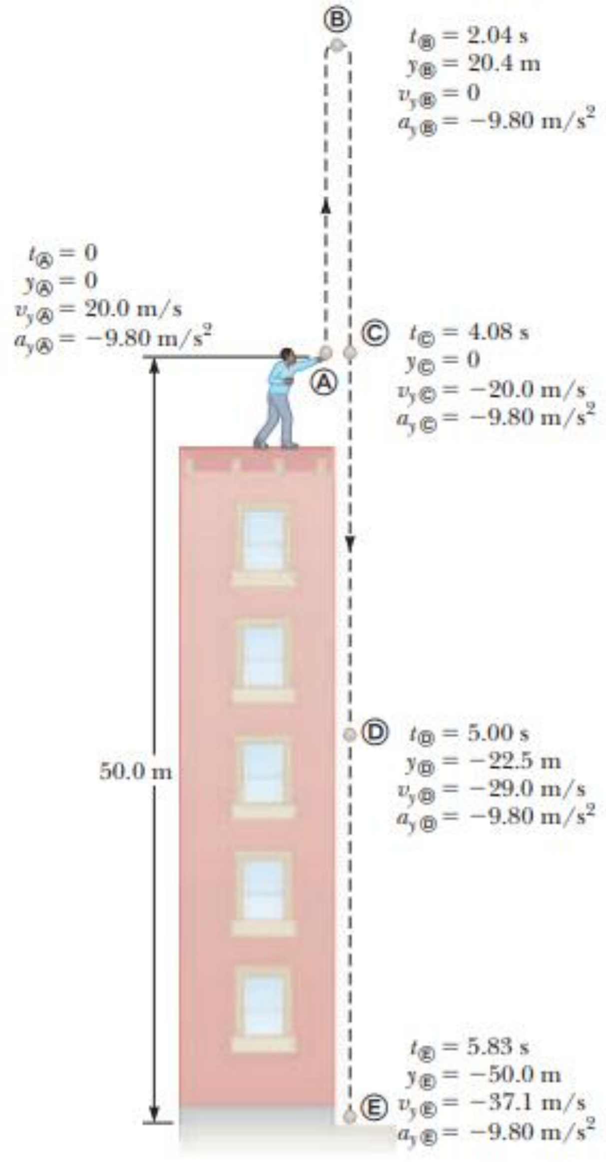
$$y_2 = \tfrac{1}{2} a t_2^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(2.00\,\mathrm{s})^2 = 19.6\,\mathrm{m}.$$

Finally, after 3.00 s ($= t_3$), the ball's position is (see Fig. 2–22)

$$y_3 = \tfrac{1}{2} a t_3^2 = \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(3.00\,\mathrm{s})^2 = 44.1\,\mathrm{m}.$$

NOTE Whenever we say "dropped," it means $v_0 = 0$. Note also the graph of y vs. t (Fig. 2–22b): the curve is not straight but bends upward because y is proportional to t^2.



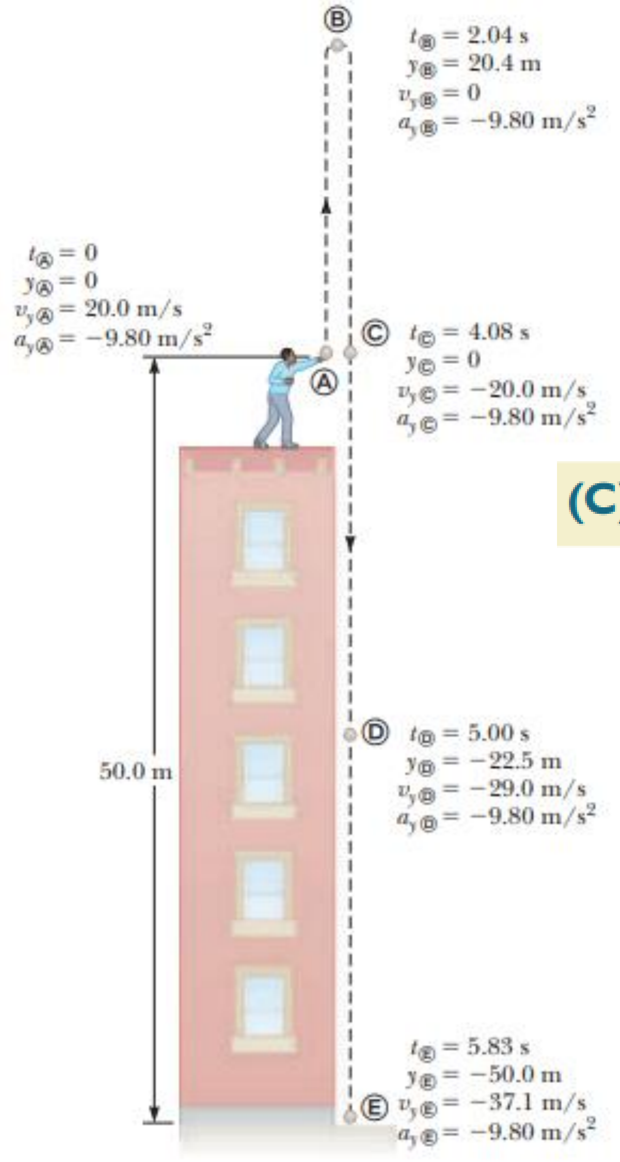


A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_{Ⓐ} = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

$$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad t = \frac{v_{yf} - v_{yi}}{a_y}$$

$$t = t_{Ⓑ} = \frac{0 - 20.0\ \text{m/s}}{-9.80\ \text{m/s}^2} = \boxed{2.04\ \text{s}}$$



(B) Find the maximum height of the stone.

$$y_{max} = y_Ⓑ = y_Ⓐ + v_{xⒶ} t + \tfrac{1}{2}a_y t^2$$

$$y_Ⓑ = 0 + (20.0\text{ m/s})(2.04\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(2.04\text{ s})^2 = \boxed{20.4\text{ m}}$$

(C) Determine the velocity of the stone when it returns to the height from which it was thrown.

$$v_{yⒸ}{}^2 = v_{yⒶ}{}^2 + 2a_y(y_Ⓒ - y_Ⓐ)$$

$$v_{yⒸ}{}^2 = (20.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(0 - 0) = 400\text{ m}^2/\text{s}^2$$

$$v_{yⒸ} = \boxed{-20.0\text{ m/s}}$$

(D) Find the velocity and position of the stone at $t = 5.00$ s.

$$v_{y\text{Ⓓ}} = v_{y\text{Ⓐ}} + a_y t = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(5.00\ \text{s}) = \boxed{-29.0\ \text{m/s}}$$

$$\begin{aligned} y_{\text{Ⓓ}} &= y_{\text{Ⓐ}} + v_{y\text{Ⓐ}} t + \tfrac{1}{2} a_y t^2 \\ &= 0 + (20.0\ \text{m/s})(5.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(5.00\ \text{s})^2 \\ &= \boxed{-22.5\ \text{m}} \end{aligned}$$

WHAT IF? What if the throw were from 30.0 m above the ground instead of 50.0 m? Which answers in parts (A) to (D) would change?

Answer None of the answers would change. All the motion takes place in the air during the first 5.00 s. (Notice that even for a throw from 30.0 m, the stone is above the ground at $t = 5.00$ s.) Therefore, the height of the throw is not an issue. Mathematically, if we look back over our calculations, we see that we never entered the height of the throw into any equation.

Velocity as slope

When an object moves with **constant velocity**, its position x increases **linearly** with time t. The equation of motion is

$$x = vt.$$

The **graph of position versus time** is therefore a **straight line** passing through the origin.

The **slope** of this line represents the **velocity** of the object:

$$v = \frac{\Delta x}{\Delta t}.$$

In the example, during a 1.0-s interval, the object's position changes by 11 m, giving

$$v = \frac{11\,\mathrm{m}}{1.0\,\mathrm{s}} = 11\,\mathrm{m/s}.$$

Thus, the slope of the x–t graph equals the object's velocity, and a steeper line indicates a higher speed.

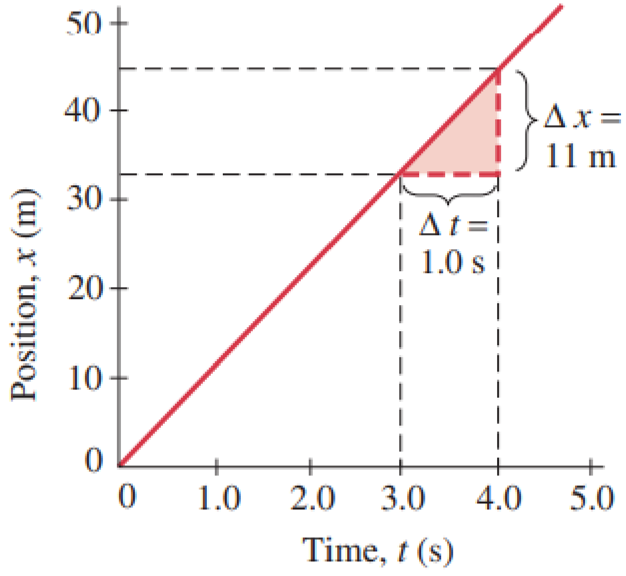


FIGURE 2–26 Graph of position vs. time for an object moving at a constant velocity of 11 m/s.

Average Velocity as the Slope of the x–t Graph

When an object's velocity changes with time, the **position–time graph (x vs. t)** is **curved** rather than a straight line.

If the object is at position x_1 at time t_1 and at position x_2 at time t_2, a straight line connecting these two points (P_1 and P_2) can be drawn.

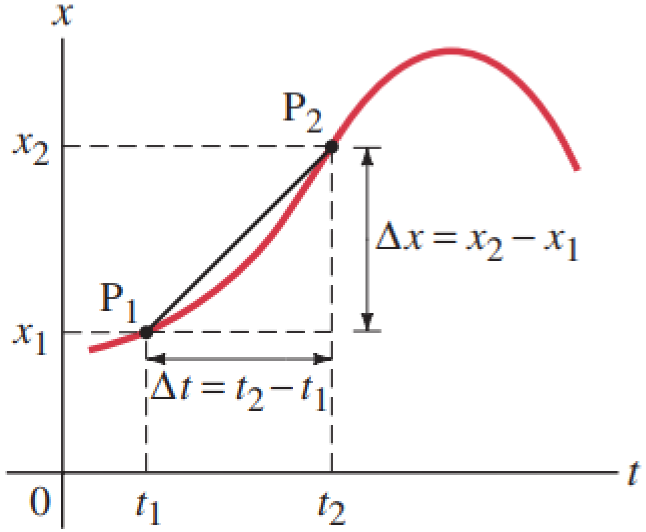
The **slope of this line** is

$$\frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1},$$

and it represents the **average velocity** of the object during that time interval.

Thus, **the average velocity of an object over any time interval equals the slope of the straight line (or chord)** joining the two points on its x-versus-t graph.

FIGURE 2–27 Graph of an object's position x vs. time t. The slope of the straight line P_1P_2 represents the average velocity of the object during the time interval $\Delta t = t_2 - t_1$.



2.8 Graphical Analysis of Linear Motion

Instantaneous Velocity as the Slope of the Tangent

If we choose a point P_3 on the **position–time graph** between P_1 and P_2, the **slope** of the line P_1P_3 represents the **average velocity** over that smaller interval.

As P_3 moves closer to P_1, the time interval $\Delta t = t_3 - t_1$ becomes smaller, and the line P_1P_3 approaches a **tangent** to the curve at P_1.

In the limit as $\Delta t \to 0$, the average velocity approaches the **instantaneous velocity** at that point.

Therefore, **the instantaneous velocity at any point on an x-versus-t graph equals the slope of the tangent line** to the curve at that point.

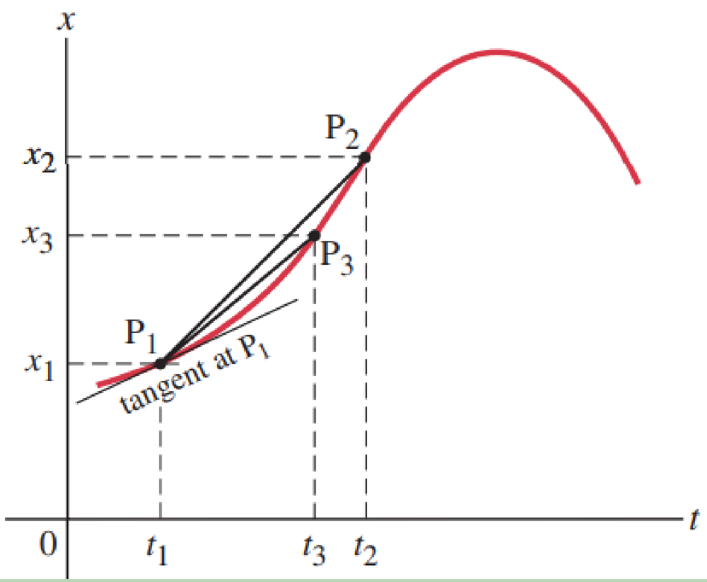


FIGURE 2–28 Same position vs. time curve as in Fig. 2–27. Note that the average velocity over the time interval $t_3 - t_1$ (which is the slope of P_1P_3) is less than the average velocity over the time interval $t_2 - t_1$. The slope of the line tangent to the curve at point P_1 equals the *instantaneous* velocity at time t_1.

We can obtain the velocity of an object at any instant from its graph of x vs. t. For example, in Fig. 2–29 (which shows the same graph as in Figs. 2–27 and 2–28), as our object moves from x_1 to x_2, the slope continually increases, so the velocity is increasing. For times after t_2, the slope begins to decrease and reaches zero $(v = 0)$ where x has its maximum value, at point P_4 in Fig. 2–29. Beyond point P_4, the slope is negative, as for point P_5. The velocity is therefore negative, which makes sense since x is now decreasing—the particle is moving toward decreasing values of x, to the left on a standard xy plot.

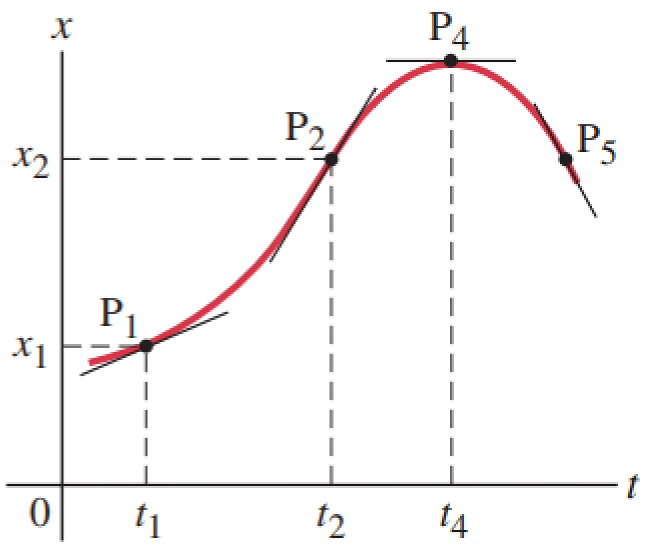


FIGURE 2–29 Same x vs. t curve as in Figs. 2–27 and 2–28, but here showing the slope at four different points: At P_4, the slope is zero, so $v = 0$. At P_5 the slope is negative, so $v < 0$.

Slope and Acceleration

We can also draw a graph of the *velocity*, v, vs. time, t, as shown in Fig. 2–30. Then the average acceleration over a time interval $\Delta t = t_2 - t_1$ is represented by the slope of the straight line connecting the two points P_1 and P_2 as shown. [Compare this to the position vs. time graph of Fig. 2–27 for which the slope of the straight line represents the average velocity.] The instantaneous acceleration at any time, say t_1, is the slope of the tangent to the v vs. t curve at that time, which is also shown in Fig. 2–30. Using this fact for the situation graphed in Fig. 2–30, as we go from time t_1 to time t_2 the velocity continually increases, but the acceleration (the rate at which the velocity changes) is decreasing since the slope of the curve is decreasing.

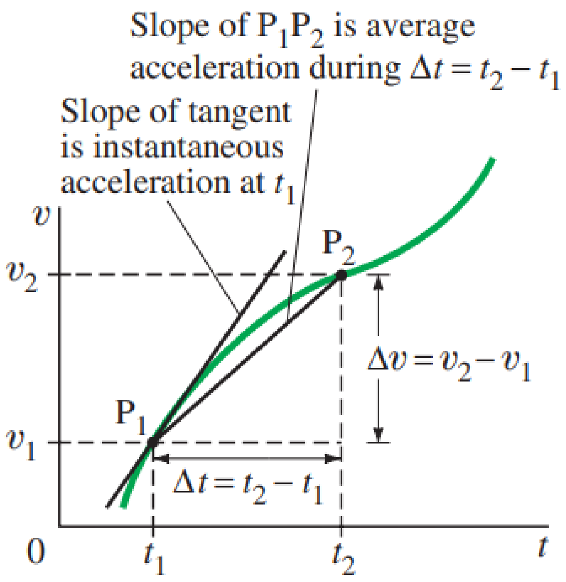
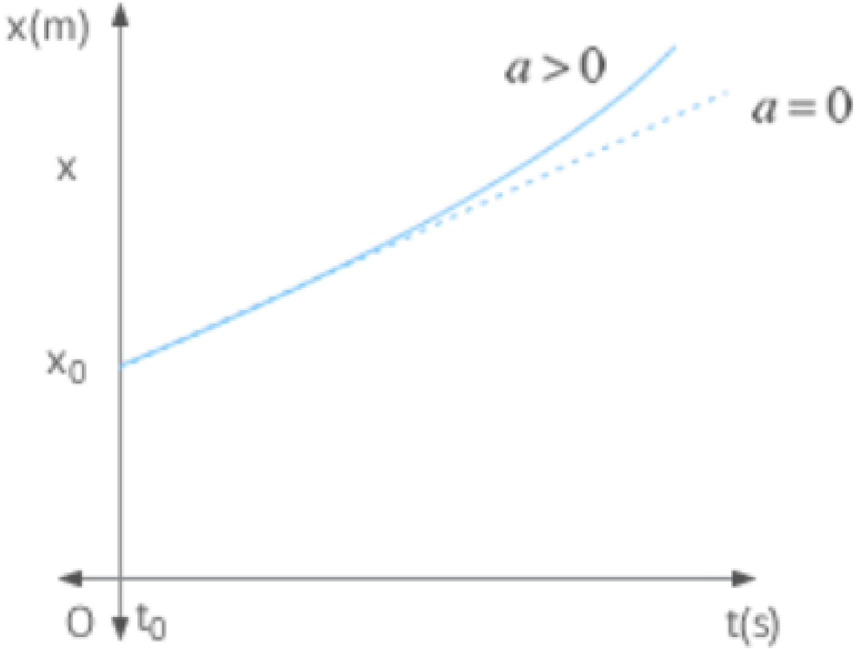
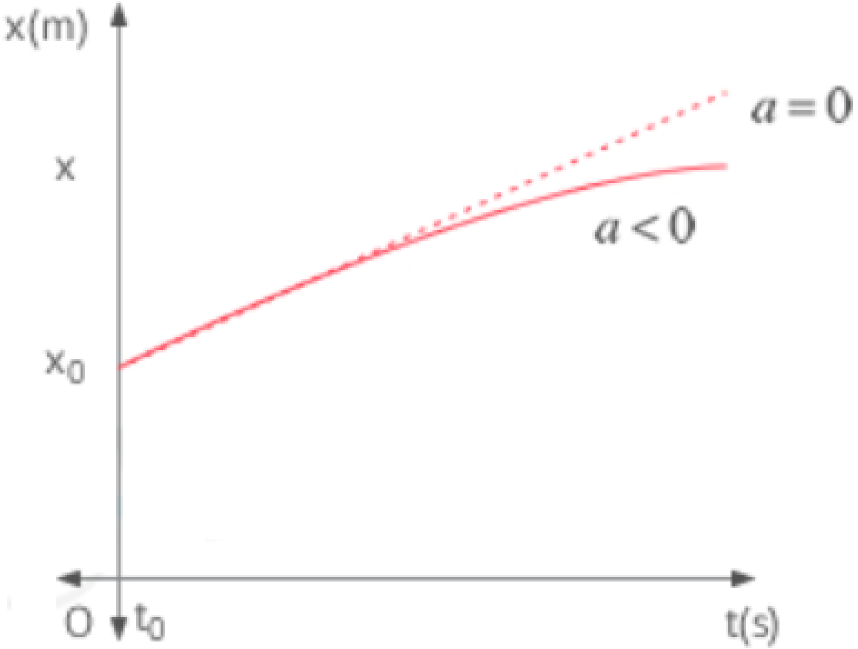


FIGURE 2–30 A graph of velocity v vs. time t. The average acceleration over a time interval $\Delta t = t_2 - t_1$ is the slope of the straight line P_1P_2: $\bar{a} = \Delta v/\Delta t$. The instantaneous acceleration at time t_1 is the slope of the v vs. t curve at that instant.

x-t graph in constant acceleration motion

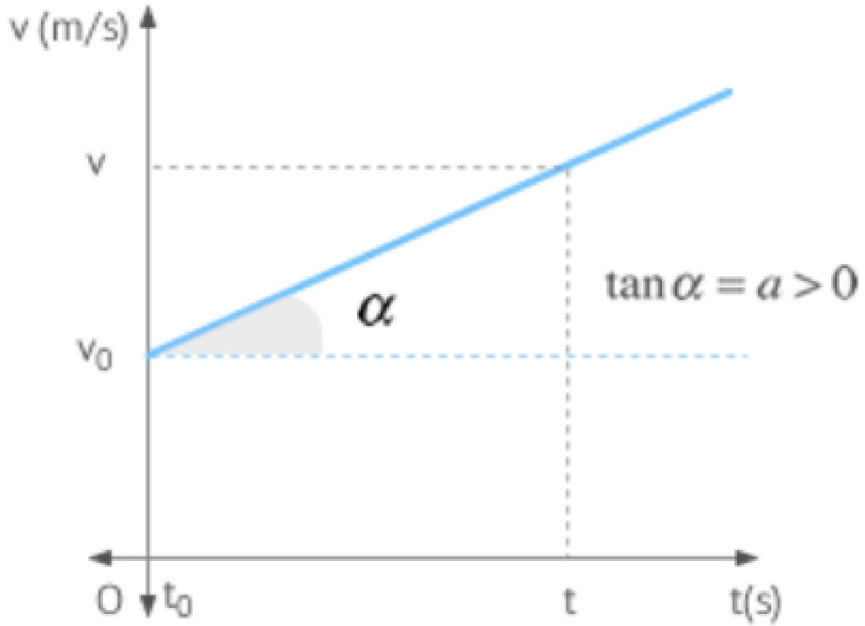


positive acceleration

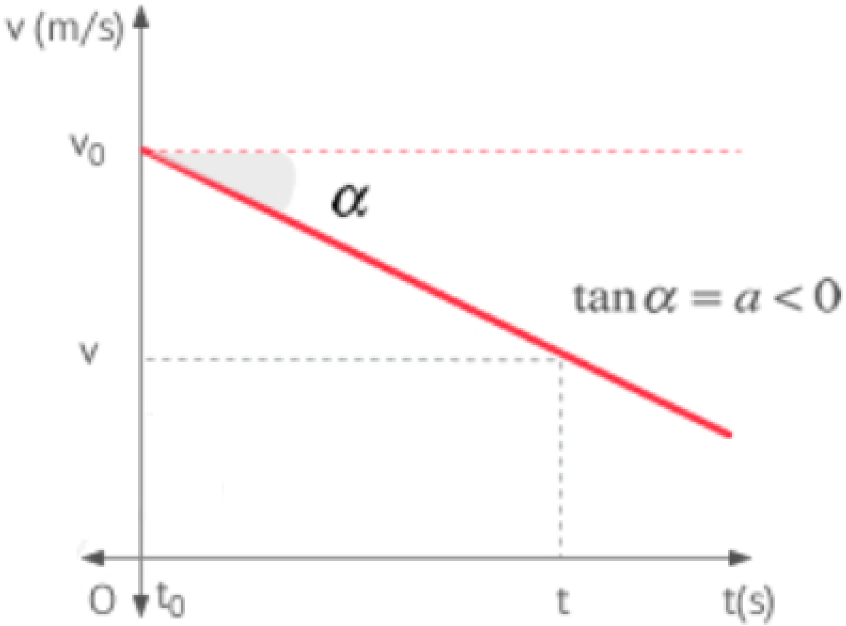


negative acceleration

v-t graph in constant acceleration motion



positive acceleration



negative acceleration

Kinematic Equations (for constant acceleration) Derived from Calculus

1. Start from the definition

Let $x(t)$ be position, $v(t) = \frac{dx}{dt}$ velocity, $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ acceleration.

For **constant acceleration** $a(t) = a$ where a is a constant.

2. Integrate acceleration → velocity

$$\frac{dv}{dt} = a \quad \Rightarrow \quad dv = a\,dt.$$

Integrate from initial time t_0 (often take $t_0 = 0$) to t:

$$\int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a\,dt \quad \Rightarrow \quad v(t) - v(t_0) = a(t - t_0).$$

Set $v_0 = v(t_0)$. Often with $t_0 = 0$:

$$\boxed{v(t) = v_0 + at.}$$

Kinematic Equations (for constant acceleration) Derived from Calculus

3. Integrate velocity → position

$$\frac{dx}{dt} = v(t) = v_0 + at.$$

Integrate from t_0 to t:

$$x(t) - x(t_0) = \int_{t_0}^{t} (v_0 + at)\,dt = v_0(t - t_0) + \tfrac{1}{2}a(t - t_0)^2.$$

With $x_0 = x(t_0)$ and usually $t_0 = 0$:

$$\boxed{x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2.}$$

Kinematic Equations (for constant acceleration) Derived from Calculus

4. Other useful kinematic relation (no explicit time)

Start from $a = \frac{dv}{dt}$. Use chain rule $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$. So for constant a:

$$v\frac{dv}{dx} = a \quad \Rightarrow \quad v\,dv = a\,dx.$$

Integrate from x_0 to x:

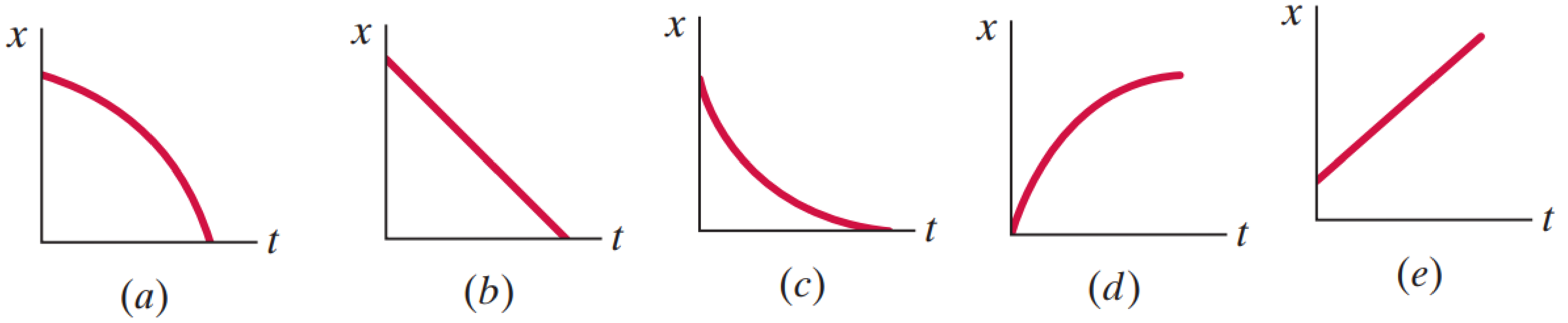
$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a\,dx \quad \Rightarrow \quad \tfrac{1}{2}(v^2 - v_0^2) = a(x - x_0).$$

So

$$\boxed{v^2 = v_0^2 + 2a(x - x_0).}$$

Exercise 1.

A car travels along the x axis with increasing speed. We don't know if to the left or the right. Which of the graphs in Fig. 2–34 most closely represents the motion of the car?

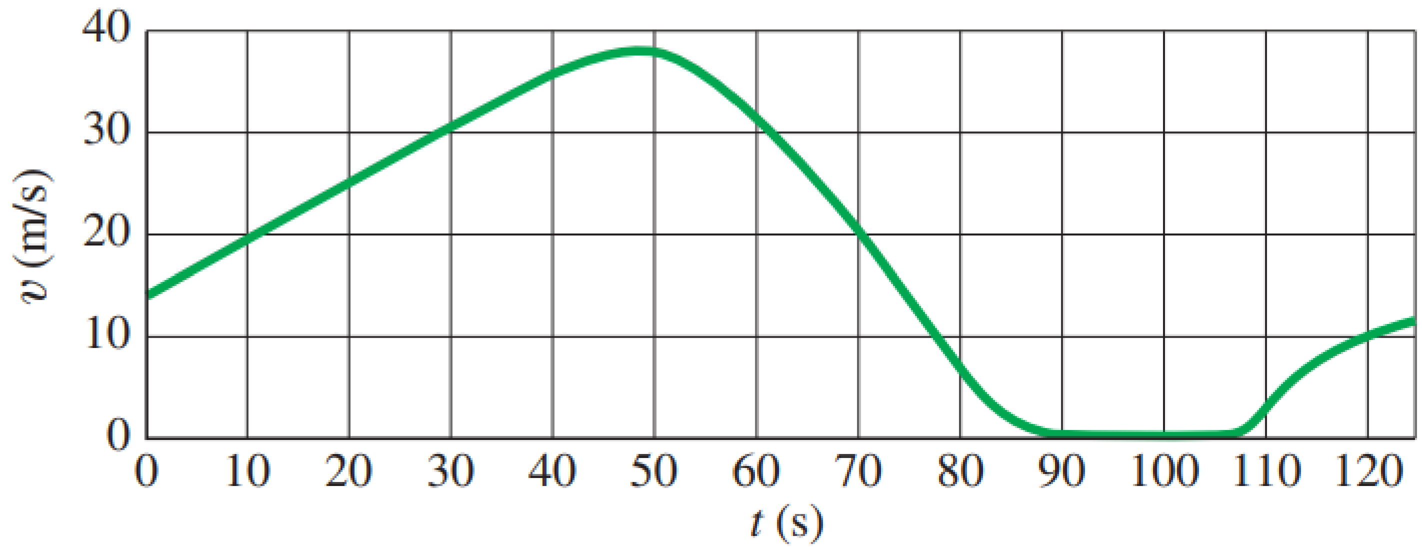


Answer

Increasing speed means that the slope must be getting steeper over time. In graphs (b) and (e), the slope remains constant, so these are cars moving at constant speed. In graph (c), as time increases x decreases. However, the rate at which it decreases is also decreasing. This is a car slowing down. In graph (d), the car is moving away from the origin, but again it is slowing down. The only graph in which the slope is increasing with time is graph (a).

Exercise 2.

Figure 2–42 shows the velocity of a train as a function of time. (a) At what time was its velocity greatest? (b) During what periods, if any, was the velocity constant? (c) During what periods, if any, was the acceleration constant? (d) When was the magnitude of the acceleration greatest?



Answer

(*a*) The greatest velocity is found at the highest point on the graph, which is at $\boxed{t \approx 48\text{ s}}$.

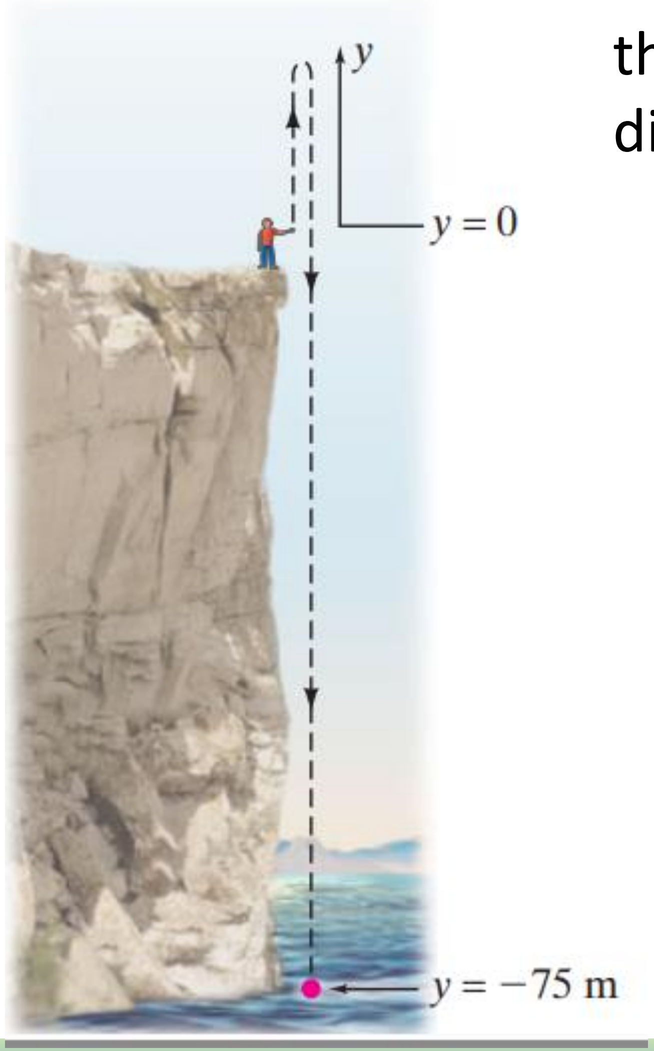
(*b*) The indication of a constant velocity on a velocity vs. time graph is a slope of 0, which occurs from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*c*) The indication of a constant acceleration on a velocity vs. time graph is a constant slope, which occurs from $\boxed{t = 0\text{ s to } t \approx 42\text{ s}}$, again from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$, and again from $\boxed{t = 90\text{ s to } t \approx 108\text{ s}}$.

(*d*) The magnitude of the acceleration is greatest when the magnitude of the slope is greatest, which occurs from $\boxed{t \approx 65\text{ s to } t \approx 83\text{ s}}$.

Exercise 3.

A stone is thrown vertically upward with a speed of from the edge of a cliff 75.0 m high. (a) How much later does it reach the bottom of the cliff? (b) What is its speed just before hitting? (c) What total distance did it travel?



Answer

Choose upward to be the positive direction and $y_0 = 0$ to be at the throwing location of the stone. The initial velocity is $v_0 = 15.5\ \text{m/s}$, the acceleration is $a = -9.80\ \text{m/s}^2$, and the final location is $y = -75\ \text{m}$.

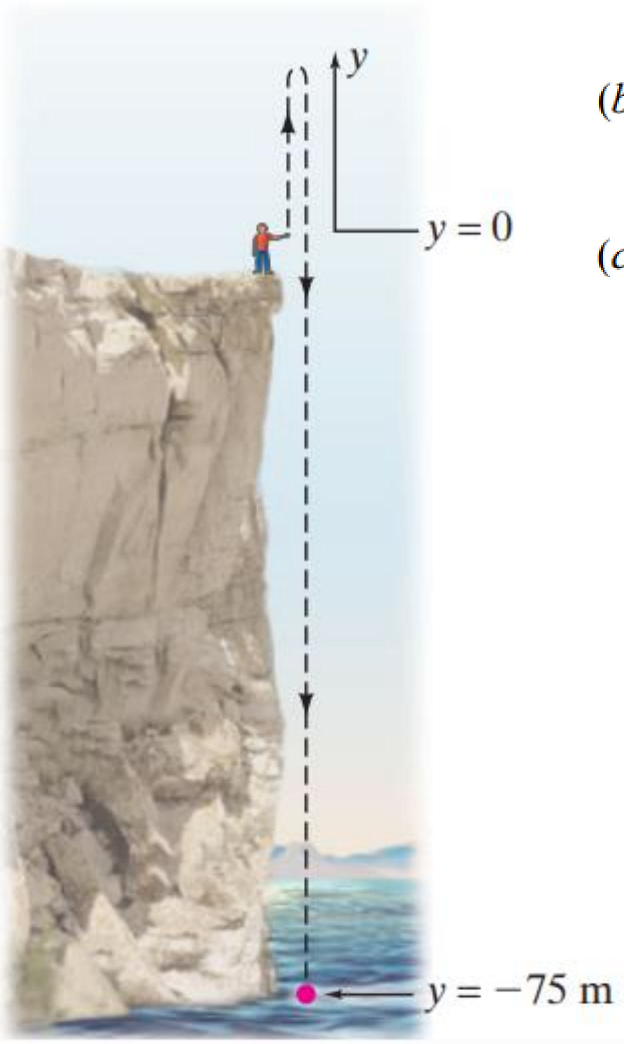
(*a*) Using Eq. 2–11b and substituting *y* for *x*, we have the following:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2 \quad \rightarrow \quad (4.9\ \text{m/s}^2)t^2 - (15.5\ \text{m/s})t - 75\ \text{m} = 0 \quad \rightarrow$$

$$t = \frac{15.5 \pm \sqrt{(15.5)^2 - 4(4.9)(-75)}}{2(4.9)} = 5.802\ \text{s}, \quad -2.638\ \text{s}$$

The positive answer is the physical answer: $\boxed{t = 5.80\ \text{s}}$.

Answer



(*b*) Use Eq. 2–11a to find the velocity just before hitting.

$$v = v_0 + at = 15.5\text{ m/s} + (-9.80\text{ m/s}^2)(5.802\text{ s}) = -41.4\text{ m/s} \quad \rightarrow \quad |v| = \boxed{41.4\text{ m/s}}$$

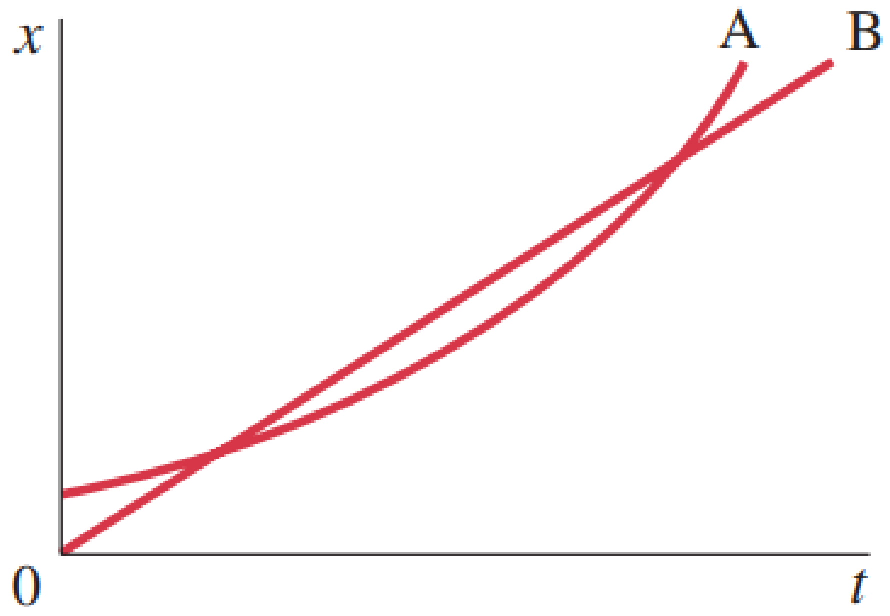
(*c*) The total distance traveled will be the distance up plus the distance down. The distance down will be 75 m more than the distance up. To find the distance up, use the fact that the speed at the top of the path will be 0. Then using Eq. 2–11c we have the following:

$$v^2 = v_0^2 + 2a(y - y_0) \quad \rightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 0 + \frac{0 - (15.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 12.26\text{ m}$$

Thus the distance up is 12.26 m, the distance down is 87.26 m, and the total distance traveled is $\boxed{99.5\text{ m}}$.

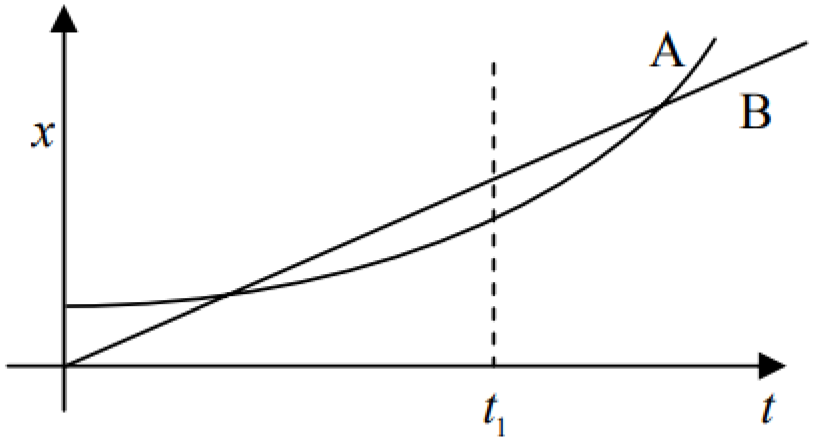
Exercise 4.

Figure below shows the position vs. time graph for two bicycles, A and B. (a) Identify any instant at which the two bicycles have the same velocity. (b) Which bicycle has the larger acceleration? (c) At which instant(s) are the bicycles passing each other? Which bicycle is passing the other? (d) Which bicycle has the larger instantaneous velocity? (e) Which bicycle has the larger average velocity?



Answer

(*a*) The two bicycles will have the same velocity at any time when the instantaneous slopes of their x vs. t graphs are the same. That occurs near the time t_1 as marked on the graph.



(*b*) Bicycle A has the larger acceleration, because its graph is concave upward, indicating a positive acceleration. Bicycle B has no acceleration because its graph has a constant slope.

(*c*) The bicycles are passing each other at the times when the two graphs cross, because they both have the same position at that time. The graph with the steepest slope is the faster bicycle, so it is the one that is passing at that instant. So at the first crossing, bicycle B is passing bicycle A. At the second crossing, bicycle A is passing bicycle B.

Answer

(*d*) Bicycle B has the highest instantaneous velocity at all times until the time t_1, where both graphs have the same slope. For all times after t_1, bicycle A has the highest instantaneous velocity. The largest instantaneous velocity is for bicycle A at the latest time shown on the graph.

(*e*) The bicycles appear to have the same average velocity. If the starting point of the graph for a particular bicycle is connected to the ending point with a straight line, the slope of that line is the average velocity. Both appear to have the same slope for that "average" line.