$L \in NP$

$$\exists T, k,\ \forall x \in \{0,1\}^* \left[x \in L \iff \exists y : |y|^k \land T(x,y) = q_a \land dTIME(T,x,y) \leq |x|^k \right]$$

RICERCA ESAUSTIVA

FOR $(y \in \{0,1\}^* : |y| \leq |x|^k)$ DO $\to O(2^{|x|^k})$

  $q \leftarrow T(x,y)$,

  IF $(q = q_a)$ THEN RETURN q

END

RETURN q_R

Γ:

I_Γ

$S_\Gamma(x) \quad \forall x \in I_\Gamma$

$\pi_\Gamma(x, S_\Gamma(x)) = \exists y \in S_\Gamma(x) : \eta_\Gamma(x,y)$

FOR $(y \in S_\Gamma(x))$ DO BEGIN $\to O(|S_\Gamma(x)|)$

  IF $(\eta_\Gamma(x,y))$ THEN RETURN q_a $\to O(\text{VERIFICO } \eta_\Gamma(x,y))$

END

RETURN q_R

SE $|S_\Gamma(x)| \leq |x|^h$

$\hookrightarrow$ LA RICERCA ESAUSTIVA DIVENTA EFFICIENTE QUANDO $S_\Gamma(x)$ È POLINOMIALE

K-3 SAT → ASSEGNO VERO AD AL PIÙ K VARIABILI

$$\mathcal{I}_{K\text{-}3SAT} = \{<X, f> : f \text{ è in 3CNF su } X\}$$

K È COSTANTE   OK ALL'ALGORITMO IN P

$$S_{K\text{-}3SAT}(X, f) = \{a : X \to \{\text{VERO}, \text{FALSO}\} \wedge |\{x \in X : a(x) = \text{VERO}\}| \leq K\} \to |S_\Pi| \in O\left(\binom{|X|}{K}\right) \subseteq O(|X|^K)$$

$$\pi_{K3\text{-}SAT}(X, f, S_{K\text{-}3SAT}(X, f)) = \exists a \in S_{K\text{-}3SAT}(X, f) : f(a(X)) = \text{VERO}$$

MIN 3-SAT

$$\mathcal{I}_{MIN\text{-}3SAT} = \{<X, f> : f \text{ è in 3CNF su } X \wedge K \in \mathbb{N}\}$$

$$S_{MIN\text{-}3SAT}(X, f) = \{a : X \to \{\text{VERO}, \text{FALSO}\}\}$$

$$\pi_{MIN3\text{-}SAT}(X, f, S_{K\text{-}3SAT}(X, f)) = \exists a \in S_{K\text{-}3SAT}(X, f) : f(a(X)) = \text{VERO} \wedge |\{x \in X : a(x) = \text{VERO}\}| \leq K$$

$$3SAT \preceq MIN\ 3\text{-}SAT$$

$$<X, f> \to <\underset{X}{\underset{\downarrow}{X'}}, \underset{f}{\underset{\downarrow}{f'}}, \underset{|X|}{\underset{\downarrow}{K}}>$$

MIN 2-SAT

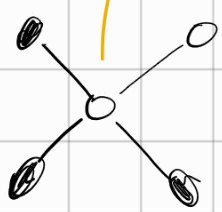
$$\mathcal{I}_{MIN\text{-}2SAT} = \{\langle X, f \rangle : f \text{ è } 3CNF \text{ su } X \wedge k \in \mathbb{N}\}$$

$$S_{MIN\text{-}2SAT}(X, f) = \{a : X \to \{VERO, FALSO\}\}$$

$$\pi_{MIN\text{-}2SAT}(X, f, S_{k\text{-}3SAT}(X, f)) = \exists a \in S_{k\text{-}3SAT}(X, f) : f(a(X)) = VERO \wedge |\{x \in X : a(x) = VERO\}$$

$VC \propto MIN\text{-}2SAT$

NON È UN VERTEX COVER



$\langle G = (V, E), k \rangle \longrightarrow \langle X, f, k \rangle$

$X = \{x_v : v \in V\}$

$f = \{C_{uv} = (x_u \vee x_v) : (u, v) \in E\}$

$k' = k$

$$\forall (u, v) \in E \; [u \in V' \vee v \in V']$$

$$\Downarrow$$

$$\forall (u, v) \in E \; [(x_u \vee x_v)]$$

DATO G E UN $K \in \mathbb{N}$ DECIDERE SE OGNI SOTTOINSIEME DI K NODI CONTIENE ALMENO UN ARCO

M

$$J_M = \{ <G=(V,E), K> : K \in \mathbb{N} \}$$

$$S_M(G,K) = \{ V' \subseteq V \}$$

$$\pi_M(G,K,S(G,K)) = \forall V' \in S_M(G,K) : |V'| \geq K \left[\exists u,v \in V' : (u,v) \in E \right]$$

$$\neg\pi_M(G,K,S(G,K)) = \exists V' \in S_M(G,K) : |V'| \geq K \left[\forall u,v \in V : (u,v) \notin E \right]$$

↳ INDIPENDENT SET $\propto M^C$

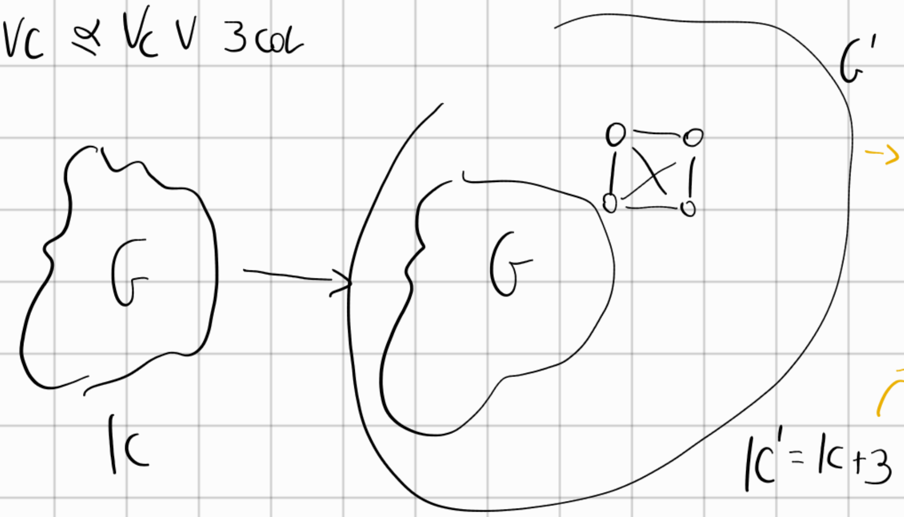
VC V 3COL

$$J_{VCV3COL} = \{ <G=(V,E), K> : K \in \mathbb{N} \}$$

$$S_{VCV3COL}(G,K) = \{ <V', c> : V' \subseteq V \wedge c : V \to \{1,2,3\} \}$$

$$\pi_{VCV3COL}(G,K,S(G,K)) = \exists <V', c> \in S_M(G,K) : \left[\forall (u,v) \in E \left[u \in V' \vee v \in V' \right] \right] \vee \left[\forall (u,v) \in E \left[c(u) \neq c(v) \right] \right]$$

VC $\propto$ VC V 3COL



→ AGGIUNGO UN GRAFO CON COLORAZIONE

COPRIAMO ANCHE IL PEZZO AGGIUNTO

$VC \wedge 3COL$

$$I_{VC \wedge 3COL} = \{\langle G=(V,E), K \rangle : K \in \mathbb{N}\}$$

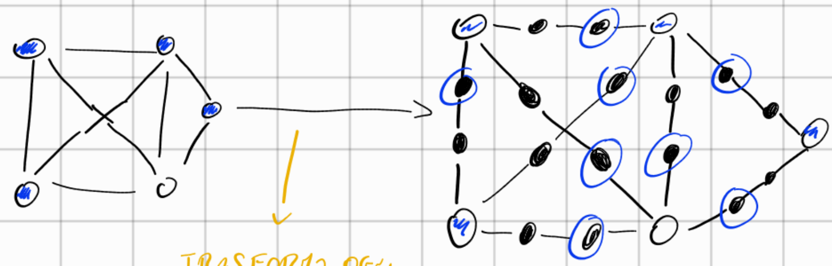
$$S_{VC \wedge 3COL}(G,K) = \{\langle V', c \rangle : V' \subseteq V \wedge c : V \to \{1,2,3\}\}$$

$$\pi_{VC \wedge 3COL}(G,K,S(G,K)) = \exists \langle V', c \rangle \in S_\pi(G,K) : \left[\forall (u,v) \in E \left[u \in V' \vee v \in V'\right]\right] \wedge \left[\forall (u,v) \in E \left[c(u) \neq c(v)\right]\right]$$

$VC \propto VC \wedge 3COL$

SICURAMENTE 3 COLORABILE

G



K

TRASFORMO OGNI ARCO IN 3 ARCHI MINORI CON 2 NODI AGGIUNTIVI

$K' = K + |E|$

PRENDO IL NODO MINORE PIÙ VICINO AL NODO ORIGINALE SE IL NODO $\notin VC$

$VC \wedge \neg 3COL$

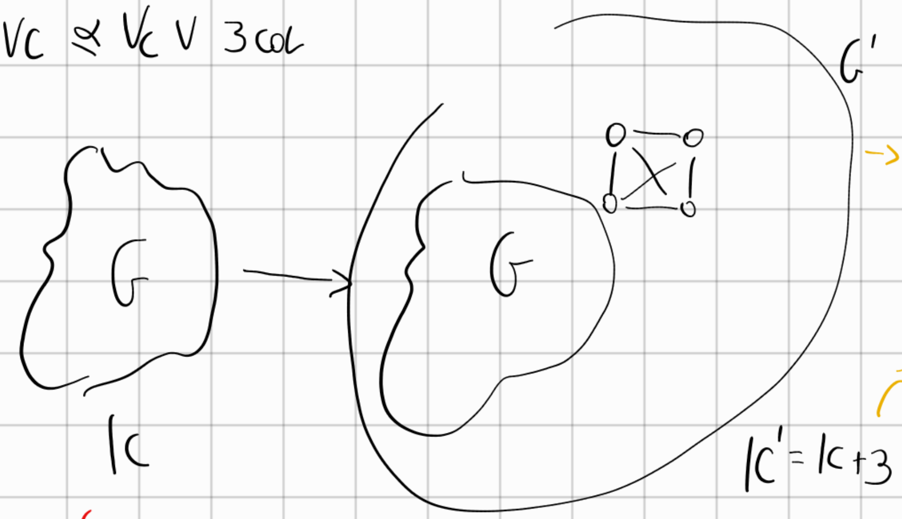
$$I_{VC \wedge \neg 3COL} = \{\langle G=(V,E), K \rangle : K \in \mathbb{N}\}$$

$$S_{VC \wedge \neg 3COL}(G,K) = \{[V' \subseteq V] \cup \{c : V \to \{1,2,3\}\}\}$$

$$\pi_{VC \wedge \neg 3COL}(G,K,S(G,K)) = \exists\, V' \subseteq V : |V'| \leq K \wedge \forall (u,v) \in E \left[u \in V' \vee v \in V'\right]\}$$

$$\wedge \{\forall c : V \to \{1,2,3\} \left[\exists (u,v) \in E : c(u) = c(v)\right]$$

VC $\propto$ VC V 3COL



→ AGGIUNGO UN GRAFO CON COLORAZIONE

COPRIAMO ANCHE IL PEZZO AGGIUNTO

NON SAPPIAMO VERIFICARE IL CERTIFICATO IN TEMPO POLINOMIALE

NP-HARD