Review of momentum

$L = \omega \cdot I$ ← Moment of inertia

↑ Angular momentum

$I = T \cdot t$ ↑ Torque $T = r \cdot F \sin\theta$

$I = CMR^2$

Linear momentum

impulse

$i = F \cdot t \qquad F = \frac{di}{dt}$

$\frac{dL}{dt} = T$

$P = mV$

- 2.1 Equation of Motion

- Newton - Euler Formulation (2.1)

$$\begin{cases} F^a + F^r = ma \\ L^a + L^r = I\alpha + \omega I \omega \end{cases}$$

← what is this?

L^a: Applied Force and loads

L^r: Constraint Force loads (we change?)

$I\alpha$: inertia * Angular acceleration, Torque

(is L^a, L^r torque?)

Write EOM for a multi body

$$\begin{cases} F_i^a + F_i^r = m_i a_i \\ L_i^a + L_i^r = I_i \alpha_i + \tilde{S}(\omega_i) I_i \omega_i \end{cases}$$

$\tilde{S}$: cross product operator. $\tilde{S} = S' S^T$

Each is a function of time

q : generalized coordinate vector q (What is q?)
(generalized velocities q')

translational Jacobian

$$\begin{cases} a_i(q,q') = v_i'(q,q') = v_{iq'} \cdot q'' + v_{iq} \cdot q' = J_{t,i}(q,q')\, q'' + \bar{v}_i \\ \alpha_i(q,q') = \omega_i'(q,q') = \omega_{iq'} \cdot q'' + \omega_{iq}\, q' = J_{r,i}(q,q')\, q'' + \bar{\alpha}_i(q,q') \end{cases}$$

rotational Jacobian

Recall Jacobian matrix : $J(x) = \begin{bmatrix} \nabla f_1^T \\ \nabla f_2^T \\ \vdots \\ \nabla f_n^T \end{bmatrix}$

$$J_t = \begin{bmatrix} \nabla v_0(q,q') \\ \vdots \\ \nabla v_n(q,q') \end{bmatrix} = \begin{bmatrix} \frac{\partial v_1}{\partial q'} & \frac{\partial v_1}{\partial q} \\ \vdots & \vdots \\ \frac{\partial v_n}{\partial q'} & \frac{\partial v_n}{\partial q} \end{bmatrix}$$

$(0 \le i \le n)$

$$a(q,q') = J_t \cdot \begin{bmatrix} q'' \\ q' \end{bmatrix} = \begin{bmatrix} \frac{\partial v_1}{\partial q'} & \frac{\partial v_1}{\partial q} \\ \vdots & \vdots \\ \frac{\partial v_n}{\partial q'} & \frac{\partial v_n}{\partial q} \end{bmatrix} \begin{bmatrix} q'' \\ q' \end{bmatrix}$$

$$= \frac{\partial v_i}{\partial q'} \cdot q'' + \frac{\partial v_i}{\partial q} \cdot q'$$

Replace the Newton-Euler EOM

$$\begin{cases} F_i^a + F_i^r = m_i a_i \\ L_i^a + L_i^r = I_i \alpha_i + \tilde{S}(\omega_i) I_i \omega_i \end{cases}$$

$$m\,a = m\cdot\left(\begin{bmatrix} \frac{\partial v_1}{\partial q'} & \frac{\partial v_1}{\partial q} \\ \vdots & \vdots \\ \frac{\partial v_n}{\partial q'} & \frac{\partial v_n}{\partial q} \end{bmatrix}\begin{bmatrix} q'' \\ q' \end{bmatrix}\right)$$

$$m_i a_i = m_i \cdot J_{t;i}\, col\,1 \cdot q'' + m_i \cdot J_{t;i}\, col\,2 \cdot q'$$

In (4), what is $\dot{J}_{t,i}$ mean?

(5) $M(q)\,q'' + k(q,q') = P(q,q')$

Why $M(q)$ but not $M(q,q')$?

2.1.2 Lagrangian Formulation

$$L(q_i, q_i') = \sum_{i=1}^{N} \left(T(q_i') - V(q_i)\right)$$

kinetic energy − potential energy

second-kind Lagrange EOM

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i'}\right) - \frac{\partial L}{\partial q_i} = 0$$

<u>A note about generalized coordinates</u>

$q(t) \equiv (q_1(t), q_2(t), \ldots, q_n(t))$

Each is a function of time.

The position vector r_k of particle k is a function of all the n generalized coordinates.

$r_k = r_k(q(t))$

The corresponding time dev of q are generalized velocities.

$$q' = \frac{dq}{dt} = (q_1'(t), q_2'(t), \ldots, q_n'(t))$$

The velocity vector v_k is the total dev of r_k

$$v_k = r_k' = r_k'(q_1(t), q_2(t), \ldots, q_n(t))$$

Recall chain rule

$$f(x(t), y(t))$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

then $r_k'(q_1(t), q_2(t), \ldots, q_n(t))$

$$= \frac{\partial r_k}{\partial q_1} \cdot \frac{dq_1}{dt} + \frac{\partial r_k}{\partial q_2} \cdot \frac{dq_2}{dt} + \cdots + \frac{\partial r_k}{\partial q_n}\frac{dq_n}{dt}$$

$$= \sum_{j=1}^{n} \frac{\partial r_k}{\partial q_j} q_j'$$

2.2 Aerodynamic Models

CFD: Ability to solve the complex aerodynamic flow with cost of expensive computation time

Not suitable choice for control development

- Details provided are not relevant
- Expensive computation time slows down the design process.

Blade Element Momentum (BEM) Theory:

Most common theory for wind turbine aerodynamic model.

Divide the total blade into several small parts

Developed for onshore wind turbines

Need appropriate number of sections

balance of accuracy and calculation time suitable for controllers.

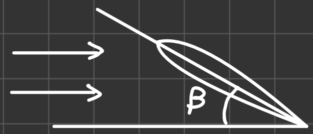
Simplification:

λ - tip-speed ratio (The linear speed of the blade's tip)

β - blade pitch angle

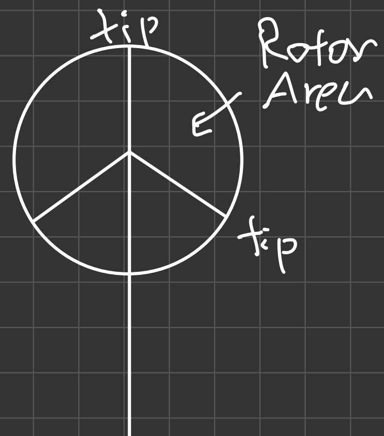
$C_p(\lambda, \beta)$ - power coefficient

$C_t(\lambda, \beta)$ - thrust coefficient



The axial aero force $F_A = \frac{1}{2} \rho A_r C_t(\lambda, \beta) V_{rel}^2$

torque $M_A = \frac{1}{2} \rho A_r \frac{C_p(\lambda, \beta)}{\omega_r} V_{rel}^3$



ρ - air density

A_r - disc rotor area

ω_r - rotor speed

V_{rel} - relative wind speed

Free Vortex Wake (FVW) Theory:

Determine vortical induction at each blade element

BEM computes average induction

Better efficiency than (FD) but higher resolution time than BEM

Able to analysis wake propagation phenomena but could be neglected.

Implact of Floating

Relative wind speed is effacted by floating moving.

$$V_{rel} = V_{wind} + V_{surge} + V_{pitch}$$

$$= V_{wind} + V_{surge} + d\,\alpha' \cos(\alpha)$$

2.3 Hydrodynamic Models

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- The linear potential flow theory

$$\text{Cummin equation} \leftarrow \begin{cases} \text{radiation} \\ \text{hydrostatic} \\ \text{diffraction} \end{cases} \text{based on } F = Ma$$

Require external BEM software for the computation

account effects expect the viscous

- Viscous Hydrodynamic Theory

  Morison equation (cylinder ?)

  (What is added mass for both model)

  i.e. A_∞ - infinite added mass

  C_a and C_d

Two parameters for both

$k \cdot a$ – k: the wavenumber, the incident waves

(Impact of diffraction)

a: floating platform radius

period number: Importance of the drag

Keulegan-Carpenter (KC) number

The good model is in the middle of both theories

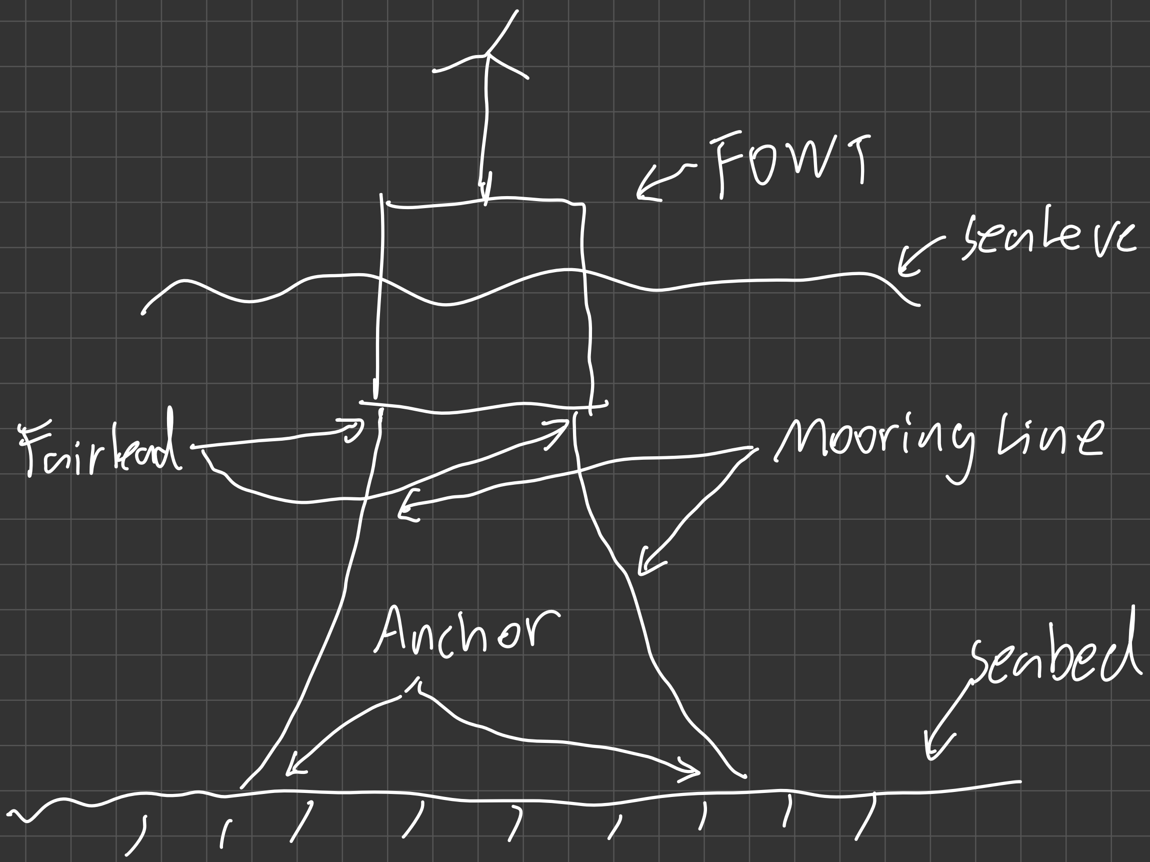
Mix two theories, consider

- added masses
- radiation
- diffraction
- hydrostatic restoring effects

} left-hand side of the Cummins equation

- F_{ext} } right-hand side, Morison equation

2.4 Mooring Line Models



Three models - Static, quasi-static, dynamic

2.4.1 Static Model:

Neglects the mooring inertia and damping

$$F_L^{1,2,3} = F_{L,0}^{1,2,3} - C_{Lq}$$

$F_L^{1,2,3}$: total static force

$F_{L,0}^{1,2,3}$: pretensioned force

C_L: 6x6 matrix, total load force

C_L is obtained by
- quasi-static
- dynamic model

2.4.1 The Quasi-Static Model

refers the MAP++ assume small displacement neglects the dynamic effect, added mass, damping, inertia

Two equations

- Total length of the line is floating
- part does not float

$$\begin{cases} X_F(H_F, V_F) \\ Z_F(H_F, V_F) \end{cases}$$

$\left.\begin{array}{l} X_F \text{ - Horizontal position} \\ Z_F \text{ - Vertical position} \\ \underline{\text{fairlead}} \text{ displacement} \\ H_F\text{: Horizontal tension force} \\ V_F\text{: Vertical tension force} \end{array}\right\}$ at fairlead

$$\begin{cases} H_A \\ V_A \end{cases}$$ at anchor

2.4.3. Dynamic Model

Consider inertia, added mass, damping

(Recall static model does not consider those)

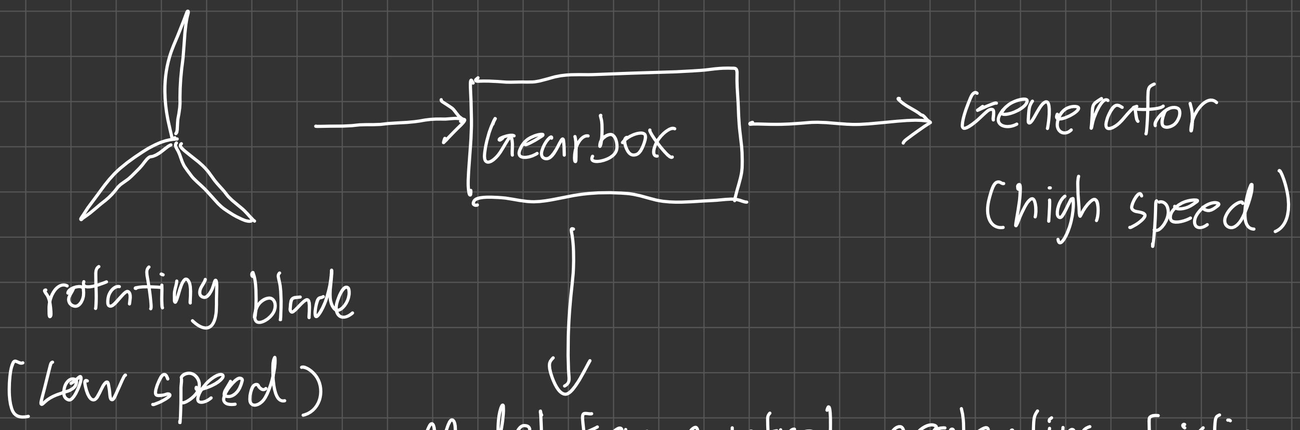
Developed by Matthew Hall

Summary

The quasi-static model is suitable

spring equation

2.5 Drivetrain Models



Model for control: neglecting friction

one shaft and two shaft

Two shaft express low-speed shaft but make control complex.

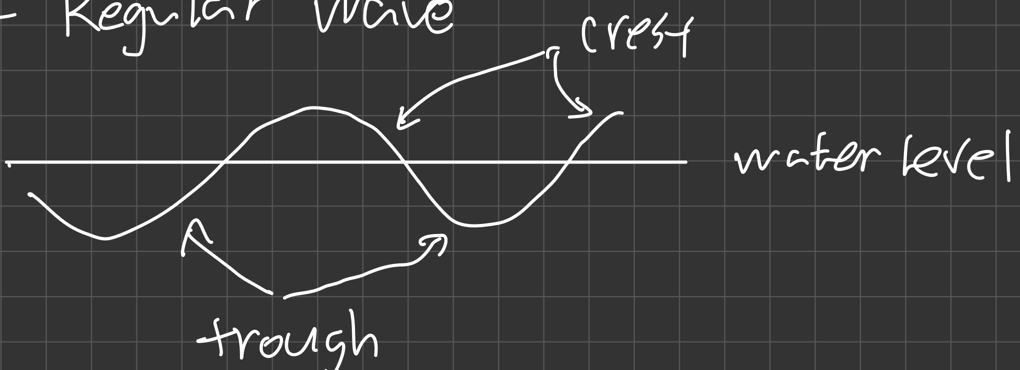
2.6 Wind Profile

The Kaimal spectrum

The Von karman Spectrum

2.7 Wave Profile

- Regular wave



Very limited model

- Stokes waves

- Irregular waves:

  Most realistic

  Written as a superposition of multiple regular wave

Summary of section 2.

Wind Profile
- The Kaimal spectrum
- The Von Karman spectrum

↓

Aerodynamic model
- CFD (Not good for control)
- BEM (Best) good compromise between accuracy and computational time
- FVW (more detailed than BEM)

Accuracy:

CFD > FVW > BEM

Time complexity

CFD > FVW > BEM

Wave Profile
- regular wave
- Stokes wave
- Irregular wave

↓

Hydrodynamic model
- The Linear Potential Flow
- The viscous effect theory

Good model mix both

$V_{rel} = V_{wind} + V_{surge} + V_{pitch}$

Mooring line Model

- Static Model
- The Quasi-Static Model (Best suitable)
- Dynamic Model

Equation of Motion

- Newton-Euler Formulation
- Lagrangian Formulation

3. COMs of FOWTs

Focused on Non-linear time-domain Control-oriented Models (COM)

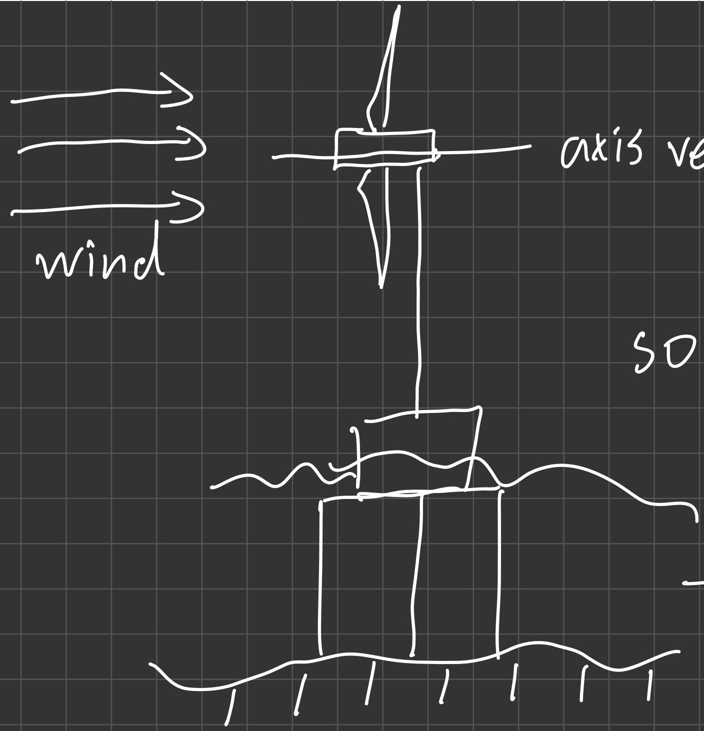
Three models: Betti model, Lemmer model, Homer model.

3.1. Betti Model

(what is heave?)

Seven states: surge, heave, pitch (respect velocities), rotor rotor speed.

Two control inputs: Generator torque, blade pitch angle



axis vector (always collinear with wind)
parrallel

so represent in 2D

TLP-based

EOM: Lagrangian approach

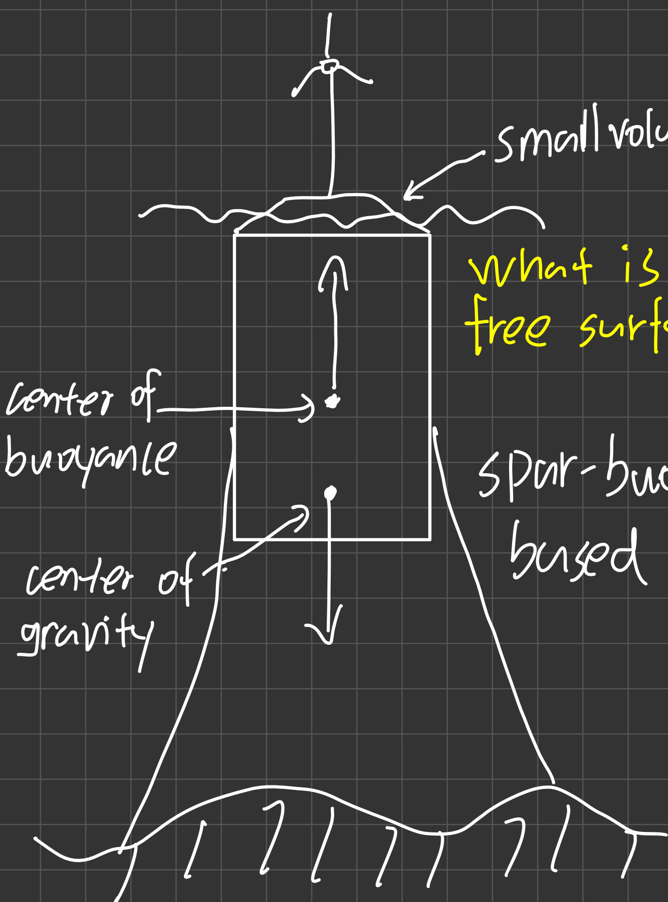
Drive train : one-mass shaft

Aerodynamic : BEM, Bernouilli equation

Hydrodynamic: viscous theory

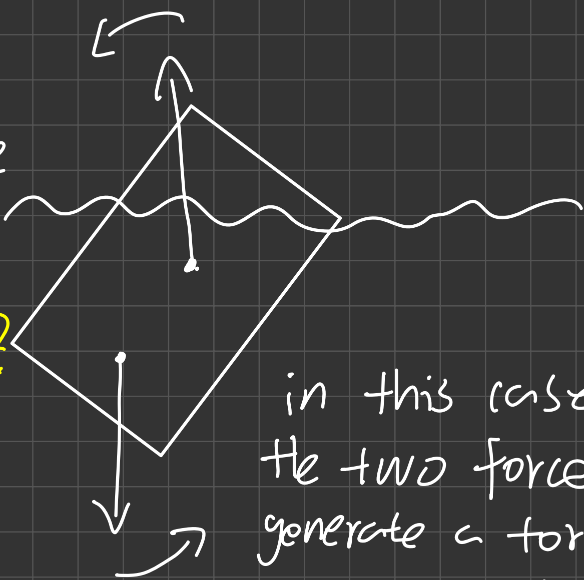
Mooring line : spring equation (quasi-static)

3.2. Lemmer Model.



what is free surface?

spar-buoy based



in this case the two forces will generate a torque make the floater back to steady state.

Six states: surge, pitch (displacement and velocity)
nacelle (displacement and velocity)
Note: consider tower flexibility

Control inputs: Generator torque, blade pitch angle

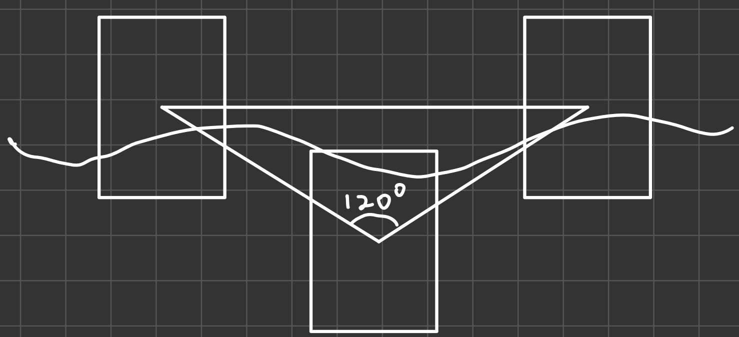
the wind turbine is modeled in 2D (same as Betti)

EOM: Newton - Euler
neglect constrain force

Aero: BEM

Hydro: neglect viscous theory, consider no incident waves

3.3. Homer Model



Large volume is in contact with the waves
sensitive to water flow

16 states: 6 DOFs of the plateform
$x, y, z, \omega_x, \omega_y, \omega_z$
x', y', z' $\omega_x', \omega_y', \omega_z'$
rotor, generator (angle, velocity)

3D model of the FOWT

Control input: generator torque, blade pitch angle, nacelle yaw angle

inertia tensor is constant (what is inertia tensor)

Neglect hydrostatic effect

Mooring line: quasi-static

3.4 Comparison

Betti model is valideted in still-water

In the hydro model, only buoyancy force are assessed, which is not enough

Lemmer Model has large error for pitch angle, surge, tower top fore-aft without effect of kinetic wave.

Smallest RMSE, STD means highest accuracy

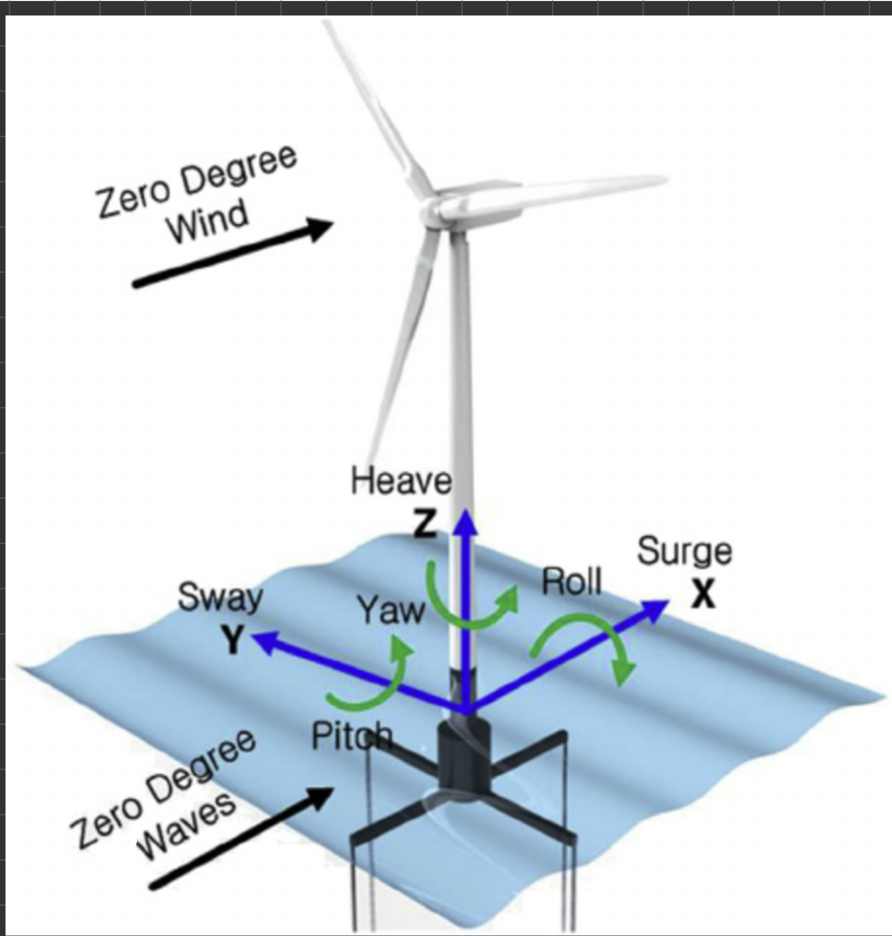
Betti is the most accurate.

Betti validated on a non-turbulent wind profile
without wave profile

Lemmer model - Fast control design development.
but large error (only model consider the flexibility)

Homer most realistic, consider turbulent
multi-directional wind profiles
wave kinematics

but more complex



4. Application Example

Lagrange equation

$$E\dot{x}_i = F, \quad x_1 = \begin{bmatrix} \text{surge position} \\ \text{surge velocity} \\ \text{heave position} \\ \text{heave velocity} \\ \text{surge displacement} \\ \text{surge velocity} \end{bmatrix}$$

Should be rotor but not surge?

E is the coefficient matrix
(Don't know the physics behind E)

In 2.2.3 $M_A = \frac{1}{2} \rho A_r \frac{C_p(\lambda, \beta)}{\omega_r} V_{rel}^3$

which M_A is torque, is T_A in here (42)

$T_A = \frac{P_A}{\omega_r}$

$T_A - T_E$ is torque

Recall physics $T = I \cdot \alpha$, $\alpha = \frac{T}{I}$ make sense!

$\dot{\omega}_r$

$\tilde{J}_r$ = rotor inertia

$$E x_1' = F$$
$$x_1' = E^{-1} F$$

$$\omega_r' = \frac{1}{\tilde{J}_r}\Big(\underbrace{\frac{\frac{1}{2}\rho A C_P(\lambda,\beta) v_{rel}^3}{\omega_r}}_{\text{Aero torque}} - \underset{\substack{\uparrow \\ \text{generator torque}}}{T_E} \Big)$$

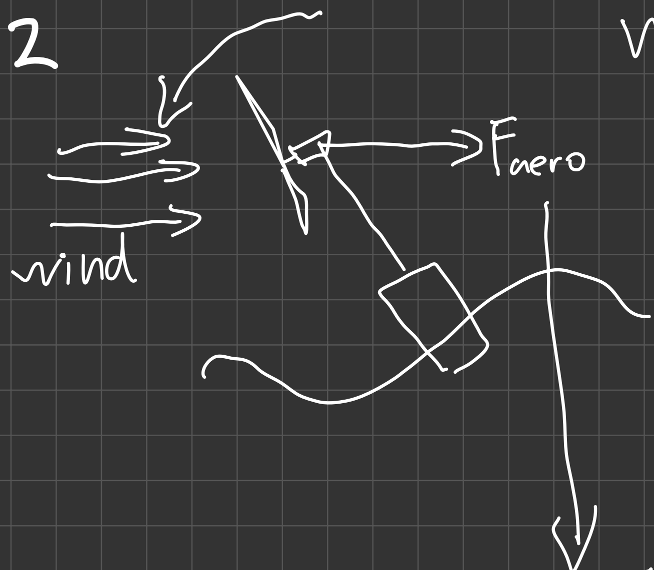
$$= \frac{1}{2\tilde{J}_r}\rho A C_p(\lambda,\beta) v_{rel}^3 - \frac{T_E}{\tilde{J}_r}$$

What is (43)?

$x_3' = \beta' = -\frac{\beta}{\tau} + \frac{\beta^*}{\tau}$ $\quad\beta$ = blade pitch angle, an input of Betti model.

$$x' = a(x) + b(x)u$$
$$y = \sigma(x,t)$$

4.2



Wave cause pitch

$V_{rel}\uparrow \rightarrow F_{aero}\uparrow \rightarrow \omega_r \uparrow$

General control:

$\uparrow\beta \rightarrow \omega_r \downarrow$

$\searrow F_{aero}\downarrow$ also

This Faero decrease cause FOWT piches forward further

This is negative damping effect

main challenge!

Control goal: $e_1 = \omega_r - \omega_{r0}$ rotor speed tracking error

$e_2 = \alpha' - \alpha_0'$ platform pitch velocity

Let e_1, e_2 converge to 0 in finite time

Note the output power tracking error:

$$e_3 = P - P_0 = \eta_G T_E e_1$$

convergence $e_1 \to$ conv e_3

4.3. Sliding Variable

$\sigma = \omega_r - \omega_r^*$

used to counteract the negative damping by accepting small oscillations on ω_r.

$\omega_r^* = \omega_{r0}(1 + k(\underbrace{\alpha' - \alpha_0'}_{e_2}))$ pitch velocity error

$\sigma = \omega_r - \omega_{r0} - \omega_{r0} k(\alpha' - \alpha_0')$

$= e_1 - k\omega_{r0} e_2$

$\sigma'' = e_1'' + ce_2'' = \underbrace{f(\cdot) + g(\cdot)}_{\text{known function}} u + \underbrace{d(\cdot)}_{\text{unknown disturbances}}$

Assume $f(\cdot), g(\cdot)$ are bdd, $\exists\, c$ s.t $|f(\cdot)| \le C$

$0 < K_m < |g(\cdot)| < K_M$

4.4 Results

ω_r not regulated at nominal rotor speed, quite smaller than ω_{r0}