

$$E \dot{x}' = F$$

$$x_i = \begin{bmatrix} \zeta \\ v_\zeta \\ \eta \\ v_\eta \\ \alpha \\ \omega \end{bmatrix} \begin{array}{l} \text{surge position} \\ \text{surge velocity} \\ \text{heave position} \\ \text{heave velocity} \\ \text{pitch position} \\ \text{pitch velocity} \end{array} \quad \text{consider 3-DOFs and respect velocity.}$$

$$\zeta' = v_\zeta$$

$$1 \cdot \zeta' = v_\zeta$$

$$\eta' = v_\eta$$

$$M_x \cdot v_\zeta' + \omega' \cdot M_d \cos \alpha = Q_\zeta + M_d \omega^2 \sin \alpha$$

$$1 \cdot \eta' = v_\eta$$

$$M_y \cdot v_\eta' + \omega' \cdot M_d \sin \alpha = Q_\eta - M_d \omega^2 \cos \alpha$$

$$1 \cdot \alpha' = \omega$$

$$M_d \cos \alpha \cdot v_\zeta' + M_d \sin \alpha \cdot v_\eta' + J_{\text{TOT}} \cdot \omega' = Q_\alpha$$

The forces consider:

Weight Q^w

Buoyancy Q^b

tie rod Q^t

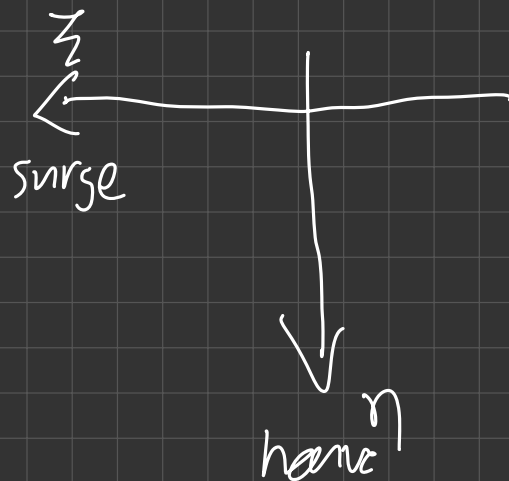
win

Simplification

- 1) Only 2D has been considered
- 2) The system is rigid
- 3) Wind thrust through 1-D approach

The system is divided to three components

Nacelle N
Floater and tower S
Blade P



weight

$$Q_z^{we} = 0$$

$$Q_\eta^{we} = (M_N + M_P + M_S)g$$

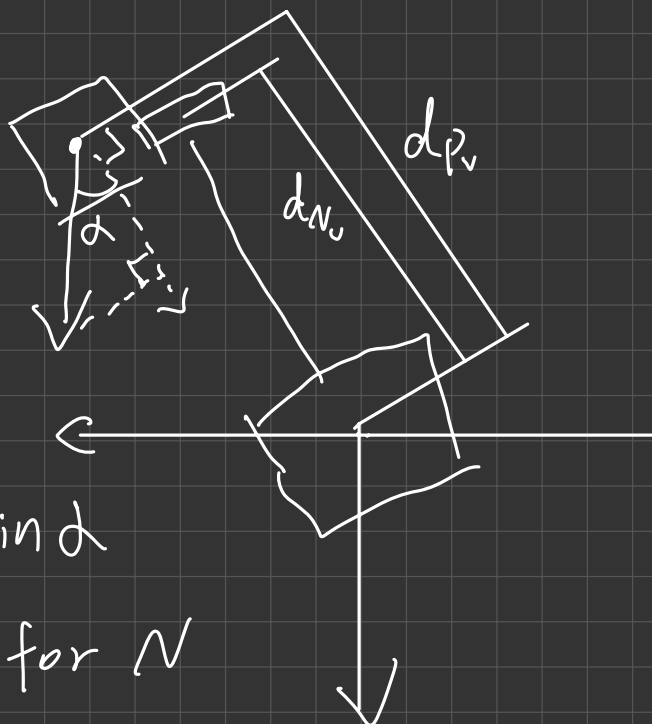
$$Q_\alpha^{we}$$

$$\sin \alpha = \frac{Q_{\alpha P}^{we}}{M_P g}$$

$$Q_{\alpha P}^{we} = M_P g \sin \alpha$$

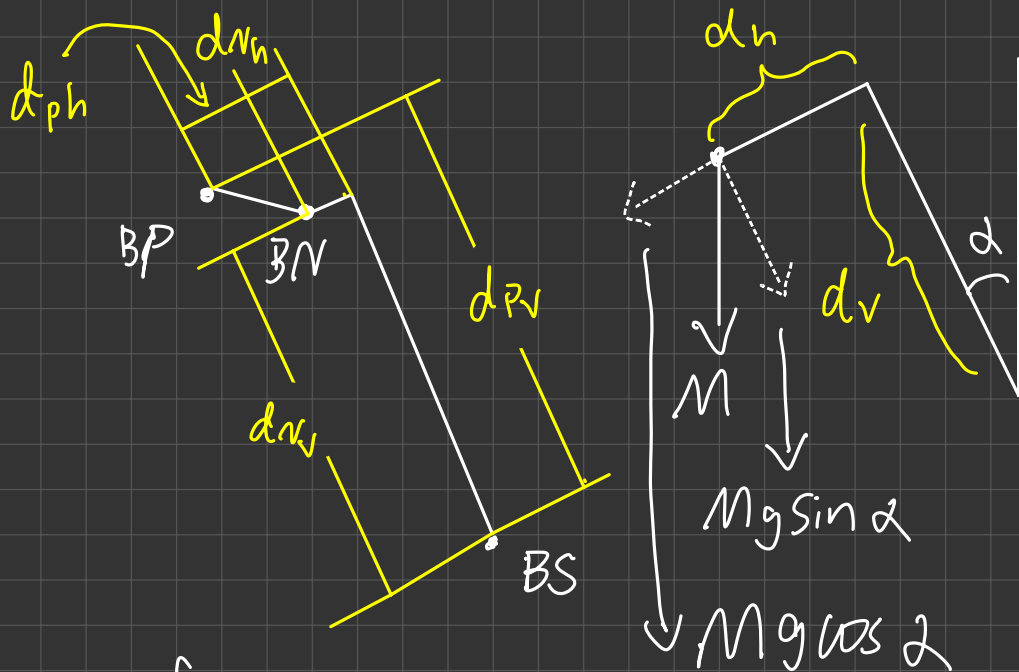
Same analysis for N

$$Q_{\alpha N}^{we} = M_N g \sin \alpha$$

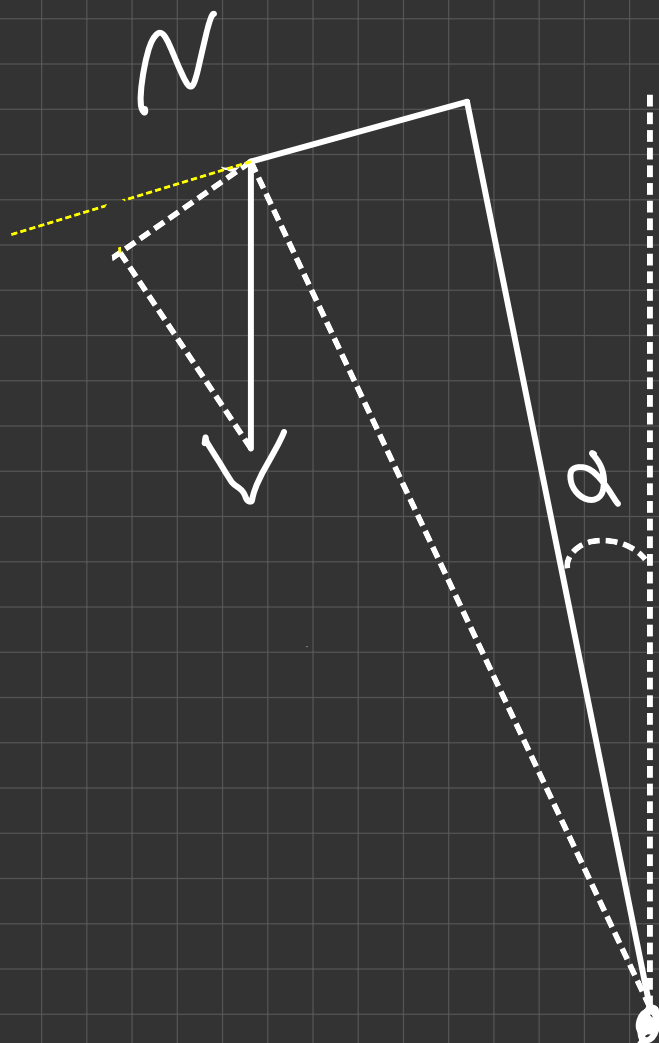


$$\bar{Q}_{\alpha_{NP}}^{we} = (M_N + M_P) \sin \alpha g$$

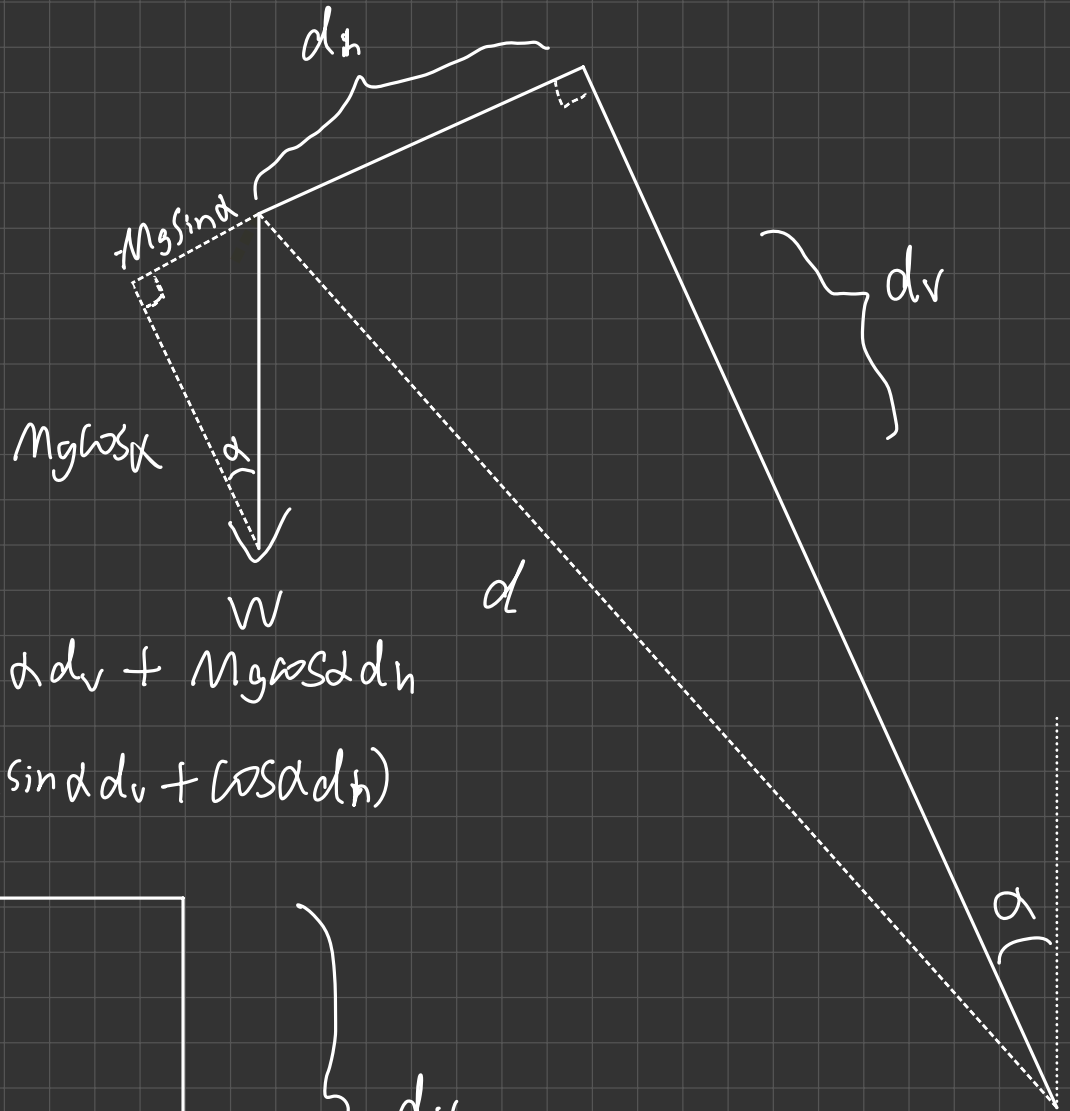
$$Q_{\alpha}^{we} = M_N d_{N_v} \sin \alpha g + M_P d_{P_v} \sin \alpha g$$



Q_{α}^{we} is the sum of torque due to pitch

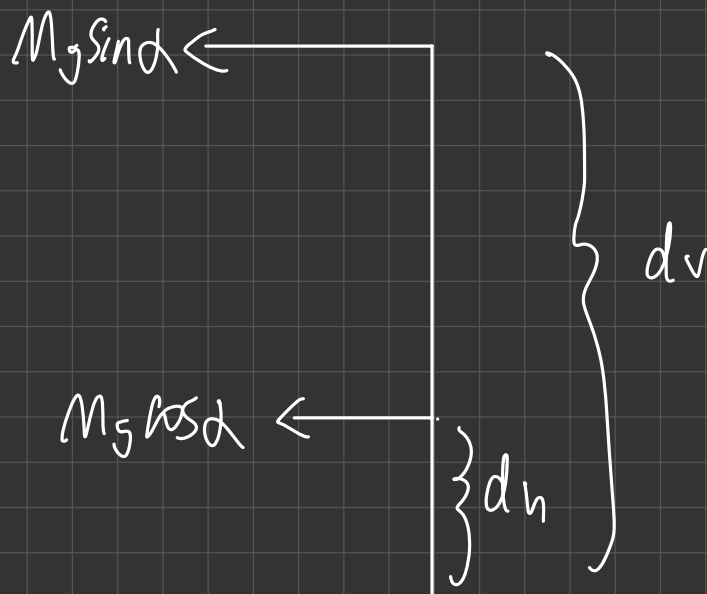


Weight torque

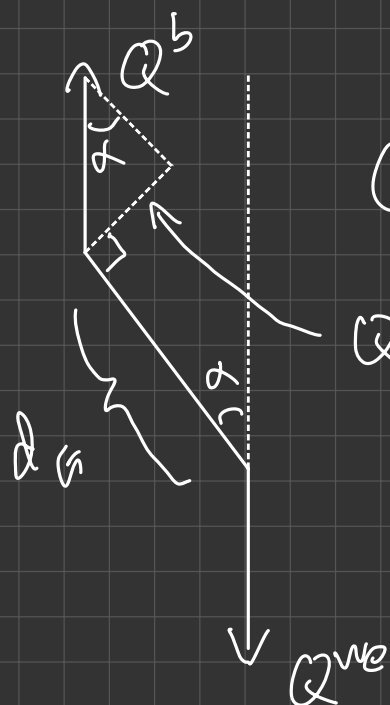
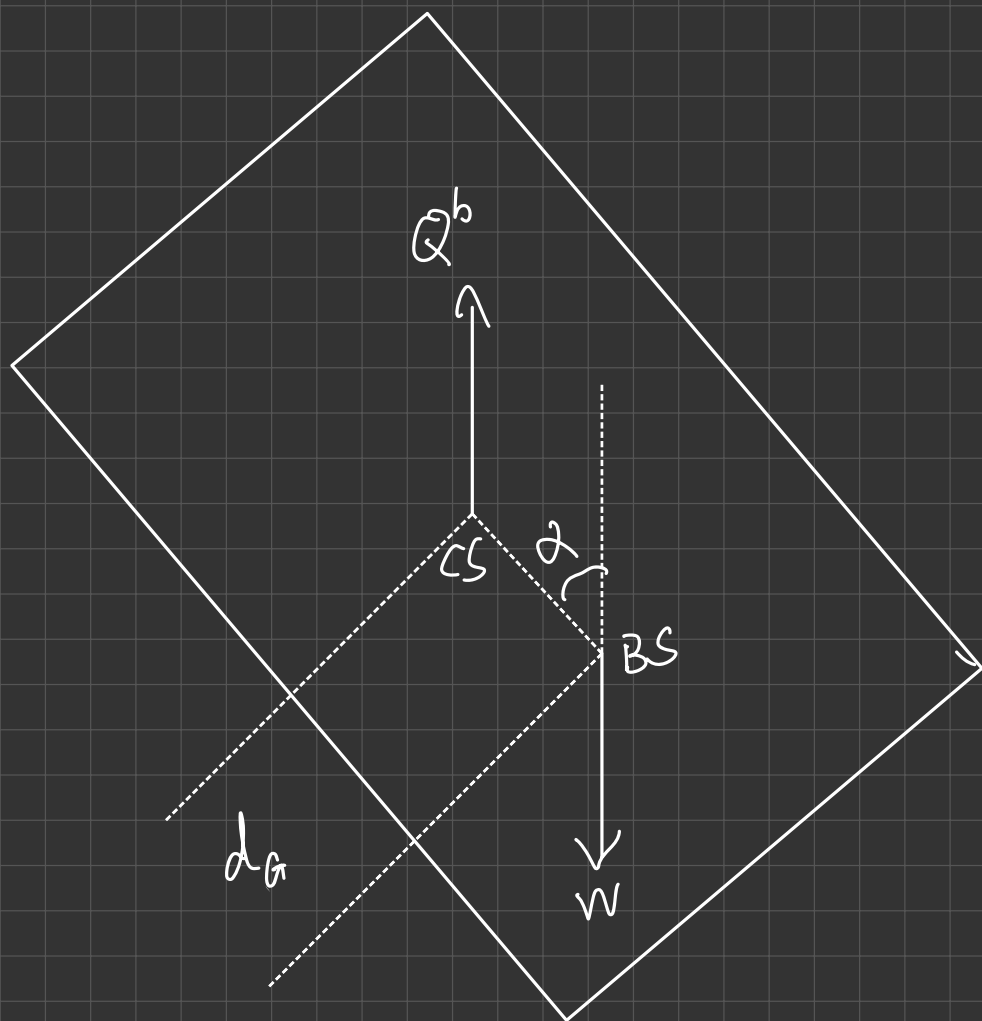


$$Mg \sin \alpha dv + Mg \cos \alpha dh$$

$$= Mg (\sin \alpha dv + \cos \alpha dh)$$



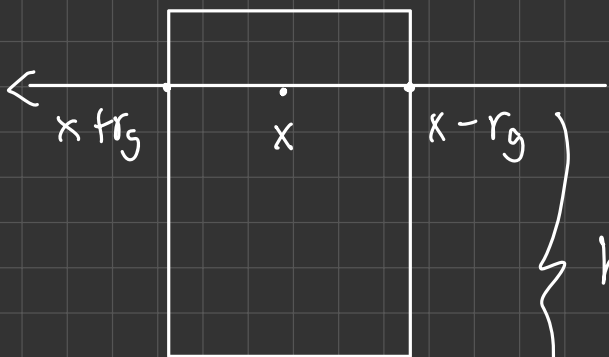
Buoyancy torque



$$Q_{\alpha}^b = Q_{\eta}^b \cdot \sin \alpha \, da$$

$$Q^b \sin \alpha$$

Approximate V_g



Water surface

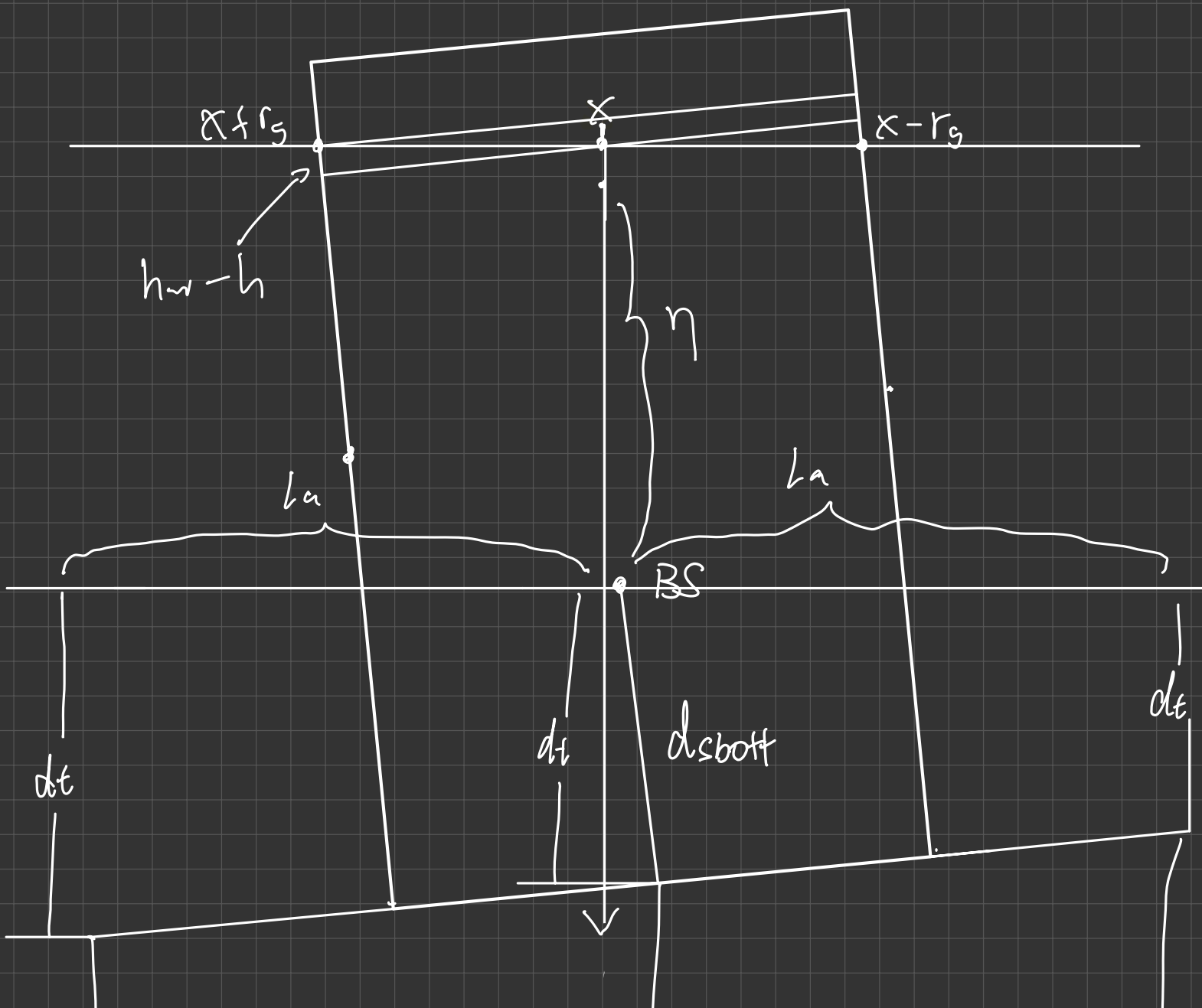
(Assume flat and horizontal)

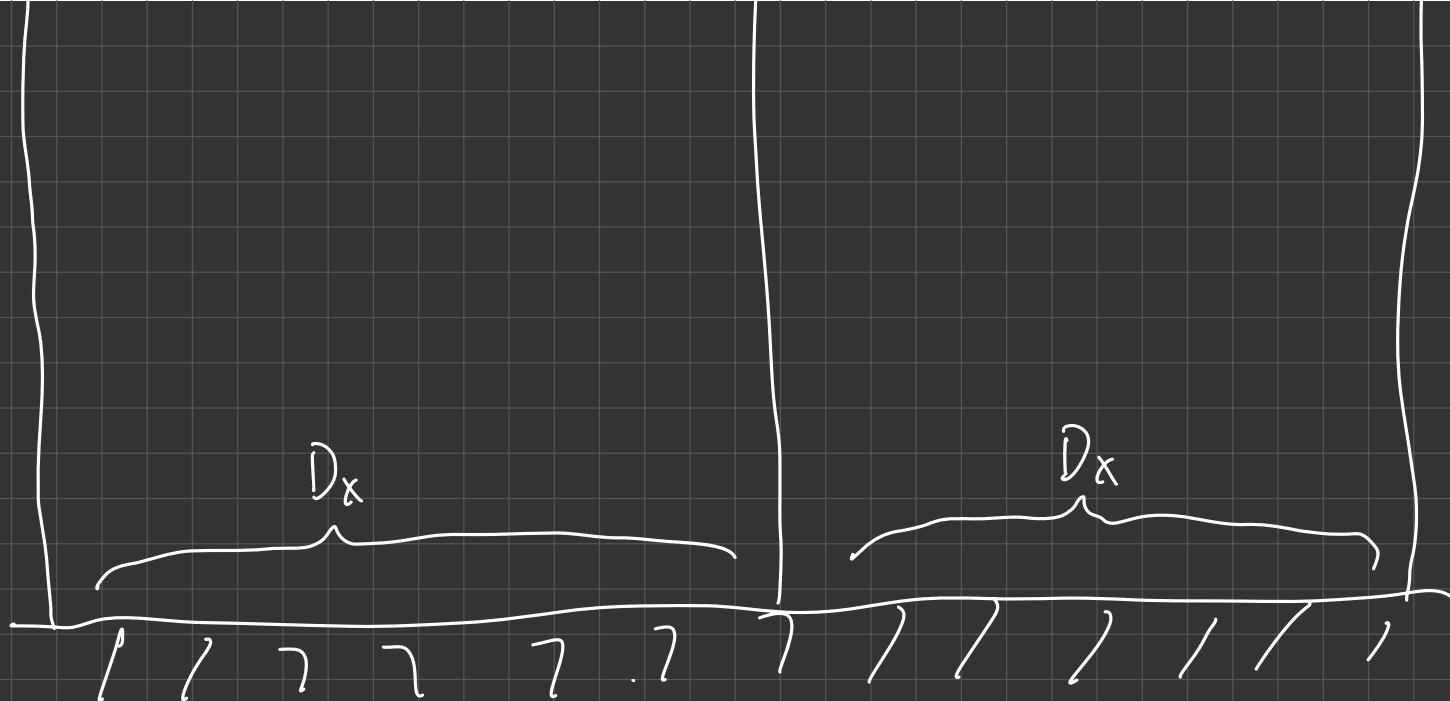
hw

Average of $x+r_5, x, x-r_5$

$$\cos(\alpha) \approx 1 \quad (\text{small pitch angle})$$

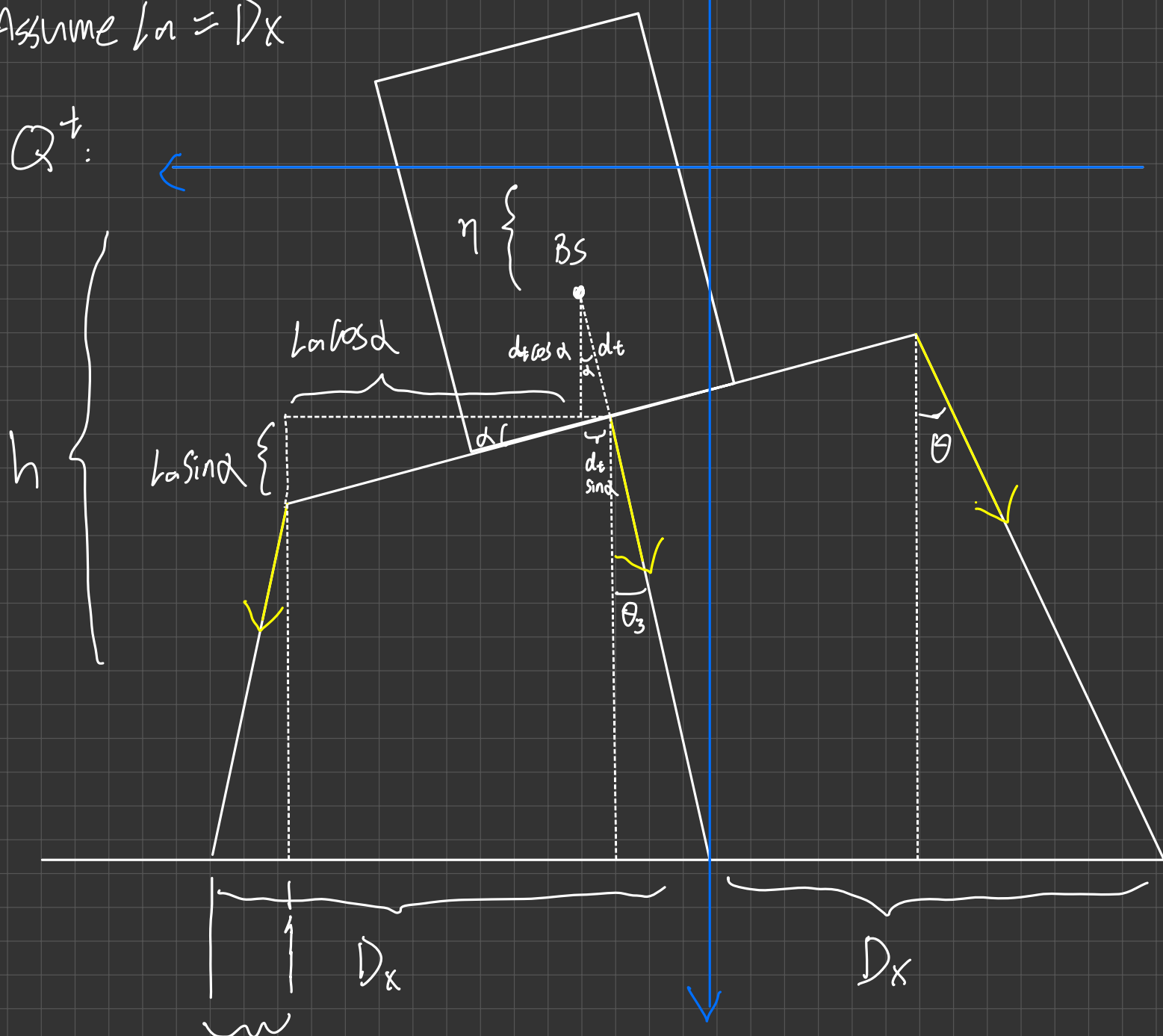
in seabed



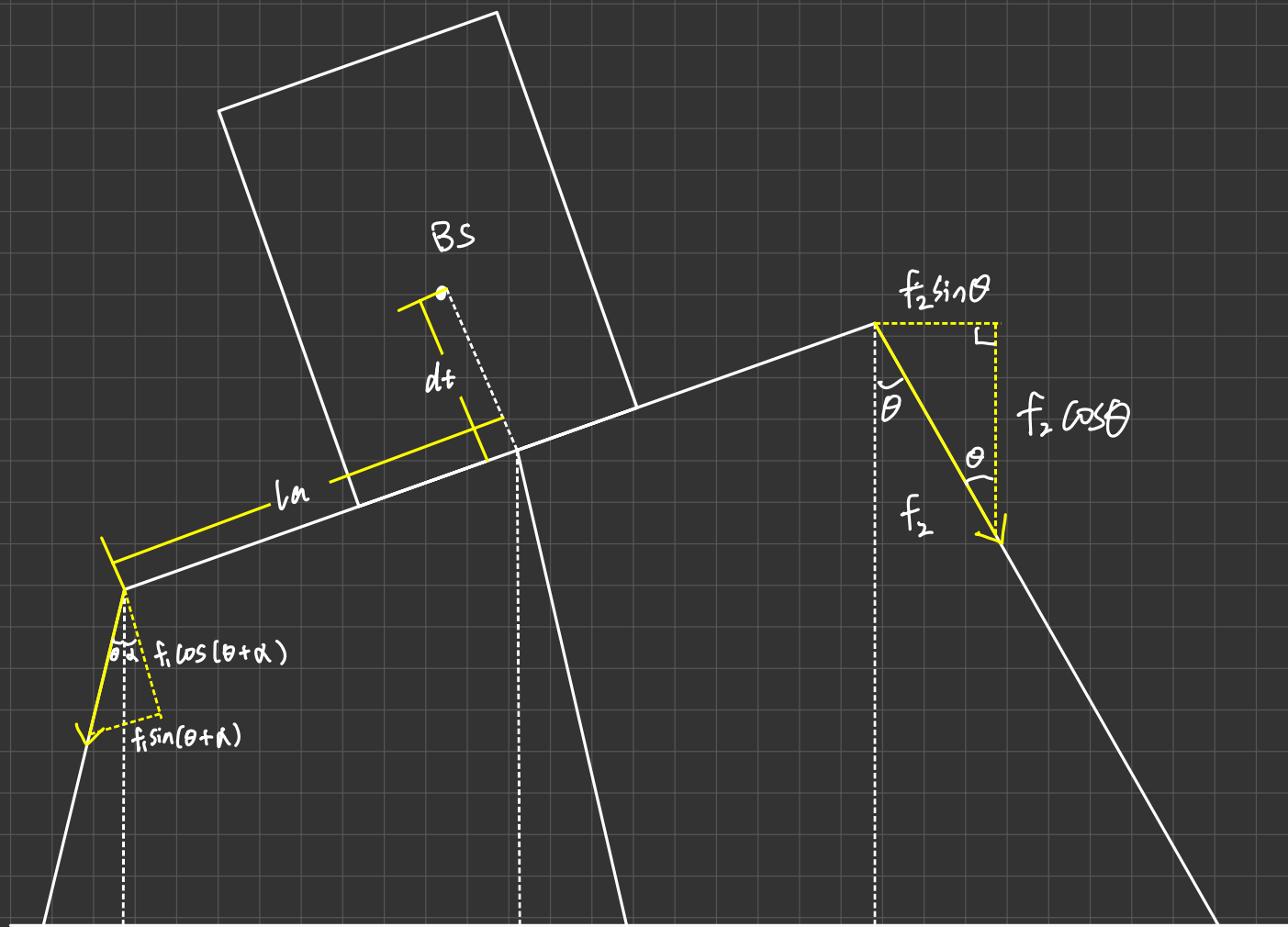


Assume $l_a = D_x$

Q^+ :



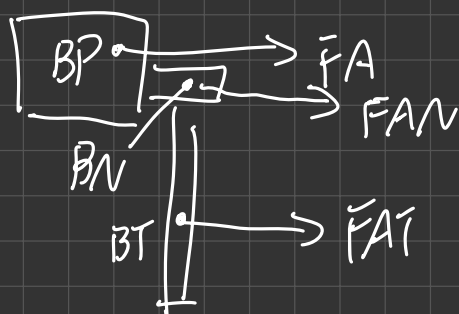
$$D_x = \frac{1}{2} l_a \cos \alpha + d_t \sin \alpha$$



Torque of f_1 :
$$Q_{\alpha f_1}^t = f_1 \cos(\theta + \alpha) \cdot l_a - f_1 \sin(\theta + \alpha) dt$$

↑
because these two
torques are opposite
direction

Wind Force



work done

$$1 \cdot \dot{\zeta}' = V_{\zeta}$$

$$M_x \cdot \dot{\zeta}' + \omega' \cdot M_d \cos \alpha = Q_{\zeta} + M_d \omega'^2 \sin \alpha$$

$$1 \cdot \dot{\eta}' = V_{\eta}$$

$$M_y \cdot \dot{\eta}' + \omega' \cdot M_d \sin \alpha = Q_{\eta} - M_d \omega'^2 \cos \alpha$$

$$1 \cdot \dot{\alpha}' = \omega$$

$$M_d \cos \alpha \cdot \dot{\zeta}' + M_d \sin \alpha \cdot \dot{\eta}' + J_{\text{TOT}} \cdot \dot{\alpha}' = Q_{\alpha}$$

$$\dot{\zeta}' = V$$

$$\dot{\zeta}' - V_{\zeta} = 0$$

$$\dot{\zeta}' -$$

$$\begin{bmatrix} \zeta \\ \eta \\ \alpha \end{bmatrix}$$

$$M_x \cdot \ddot{\zeta}'' + 0 \cdot \ddot{\eta}'' + M_d \cos \alpha \cdot \ddot{\alpha}'' = Q_{\zeta} + M_d \omega^2 \sin \alpha$$

$$0 \cdot \ddot{\zeta}'' + M_y \cdot \ddot{\eta}'' + M_d \sin \alpha \cdot \ddot{\alpha}'' = Q_{\eta} - M_d \omega^2 \cos \alpha$$

$$M_d \cos \alpha \cdot \ddot{\zeta}'' + M_d \sin \alpha \cdot \ddot{\eta}'' + J_{\text{TOT}} \cdot \ddot{\alpha}'' = Q_{\alpha}$$

$$\begin{bmatrix} M_x & 0 & M_d \cos \alpha \\ 0 & M_y & M_d \sin \alpha \\ M_d \cos \alpha & M_d \sin \alpha & J_{\text{TOT}} \end{bmatrix} \begin{bmatrix} \ddot{\zeta}'' \\ \ddot{\eta}'' \\ \ddot{\alpha}'' \end{bmatrix} = \begin{bmatrix} Q_{\zeta} + M_d \omega^2 \sin \alpha \\ Q_{\eta} - M_d \omega^2 \cos \alpha \\ Q_{\alpha} \end{bmatrix}$$

$$J_R \ddot{\alpha} R'' = T_A - T_E$$

work done
by weight
↓

work done
by buoyancy
↓

work done
by tie rods
↓

$$Q_z^w \cdot z + Q_z^b \cdot z + Q_z^t \cdot z$$

$$Q_\eta^w \cdot \eta + Q_\eta^b \cdot \eta + Q_\eta^t \cdot \eta$$

$$Q_\alpha^w \cdot \alpha + Q_\alpha^b \cdot \alpha + Q_\alpha^t \cdot \alpha$$

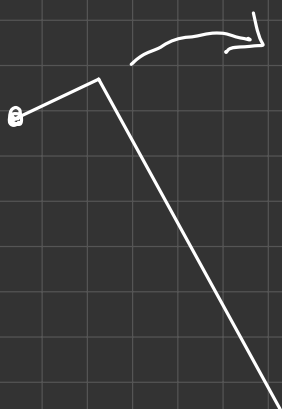
} work done by
weight, buoyancy, tie rods

we have Q_z

Q_η

Q_α

$$E_k = \frac{1}{2} (M_s + m_x) (\dot{z}')^2 + \frac{1}{2} (M_s + m_y) (\dot{\eta}')^2 \\ + \frac{1}{2} M_N \dot{V}_N^2 + \frac{1}{2} M_P \dot{V}_P^2 + \frac{1}{2} (J_s + J_N + J_P) (\dot{\alpha}')^2$$



Forces considered in R_{int}

weight, buoyancy, tension in the mooring lines

$$Q_{\xi}^w = 0$$

$$Q_{\eta}^w = (M_N + M_P + M_S)g$$

} does not depend on any state

$$Q_{\alpha}^w =$$

States : $D = \begin{bmatrix} \xi \\ \eta \\ \alpha \\ \alpha_R \end{bmatrix}$ surge position
heave position
pitch position
rotor position

$$R_{int} = \begin{bmatrix} Q_{\xi}^w + Q_{\xi}^b + Q_{\xi}^t \\ Q_{\eta}^w + Q_{\eta}^b + Q_{\eta}^t \\ Q_{\alpha}^w + Q_{\alpha}^b + Q_{\alpha}^t \\ 0 \end{bmatrix}$$

$$V_{in} = V_w + V_{\xi} + d_P \omega \cos \alpha$$

$$\tilde{J}_R \alpha_R'' = \frac{P_A}{\alpha_R'} - \tilde{T}_E$$

$$\bar{F}_A = \frac{1}{2} \rho A (V_{in}^2 - V_{out}^2) + \Delta F_A$$

$$F_{AN} = \frac{1}{2} \rho C_{dN} A_N \cos(\alpha)$$

$$M_x = M_s + M_N + M_P + m_x$$

$$M_y = M_s + M_N + M_P + m_y$$

$$J_{Tot} = J_s + J_N + J_P + M_N d_N^2 + M_P d_P^2$$

$$M_x \quad \xi''$$

$$M_y \quad \eta''$$

$$J_{Tot} \quad \alpha''$$

$$\begin{bmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & J_{Tot} \end{bmatrix} \begin{bmatrix} \xi'' \\ \eta'' \\ \alpha'' \end{bmatrix} + \begin{bmatrix} Q_{\xi}^w + Q_{\xi}^b + Q_{\xi}^t \\ Q_{\eta}^w + Q_{\eta}^b + Q_{\eta}^t \\ Q_{\alpha}^w + Q_{\alpha}^b + Q_{\alpha}^t \end{bmatrix}$$

$$= \begin{bmatrix} Q_{\xi}^{wi} + Q_{\xi}^h + Q_{\xi}^{wn} \\ Q_{\eta}^{wi} + Q_{\eta}^h + Q_{\eta}^{wn} \\ Q_{\alpha}^{wi} + Q_{\alpha}^h + Q_{\alpha}^{wn} \end{bmatrix}$$

$$\tilde{J}_R \omega_R' = \frac{1}{2\omega_R} \rho A V_{in}^3 \mathcal{L}_P(\lambda, \beta) - \tilde{T}_E$$

$$V_{in} = V_w + \xi' + d_p \alpha' \cos \alpha$$