

Review of momentum

$$L = \text{Angular Momentum}$$

$$\uparrow$$

$$I = C \cdot I$$

Moment of inertia

$$I = T \cdot t$$

$$\uparrow$$

$$T = r \cdot F \sin \theta$$

$$I = CMR^2$$

Linear momentum

impulse

$$i = F \cdot t$$

$$F = \frac{di}{dt}$$

$$\frac{dL}{dt} = T$$

$$P = mv$$

- 2.1 Equation of Motion
- Newton-Euler Formulation (2.1)

$$\begin{cases} F^a + F^r = m\alpha \\ L^a + L^r = I\ddot{\alpha} + \omega \dot{\omega} \end{cases}$$

what is this?

\uparrow \uparrow \downarrow
 Applied Force and loads inertia & Angular acceleration
 and loads Torque

Constraint Force (we change?)
loads

(is L^a, L^r torque?)

Write EOM for a multi body

$$\begin{cases} F_i^a + F_i^r = m_i \alpha_i \\ L_i^a + L_i^r = I_i \ddot{\alpha}_i + \tilde{S}(\omega_i) I_i \omega_i \end{cases}$$

\tilde{S} : cross product operator . $\tilde{S} = S' S^\top$

Each is a function of time

q : generalized coordinate vector q (What is q ?)
(generalized velocities q')

$$\left\{ \begin{array}{l} a_i(q, q') = V_i'(q, q') = V_{iq'} \cdot q'' + V_{iq} \cdot q' = J_{t,i}(q, q') q'' + \bar{v}_i \\ \alpha_i(q, q') = \omega_i'(q, q') = \omega_{iq'} \cdot q'' + \omega_{iq} \cdot q' = J_{r,i}(q, q') q'' + \bar{\alpha}_i \end{array} \right.$$

translational Jacobian
rotational Jacobian

Recall Jacobian matrix: $J(x) = \begin{bmatrix} \nabla f_1^T \\ \nabla f_2^T \\ \vdots \\ \nabla f_n^T \end{bmatrix}$

$$J_t = \begin{bmatrix} \nabla v_0(q, q') \\ \vdots \\ \nabla v_n(q, q') \end{bmatrix} = \begin{bmatrix} \frac{\partial v_1}{\partial q'} & \frac{\partial v_1}{\partial q} \\ \vdots & \vdots \\ \frac{\partial v_n}{\partial q'} & \frac{\partial v_n}{\partial q} \end{bmatrix}$$

($0 \leq i \leq n$)

$$a_i(q, q') = J_t \cdot \begin{bmatrix} q'' \\ q' \end{bmatrix} = \begin{bmatrix} \frac{\partial v_1}{\partial q'} & \frac{\partial v_1}{\partial q} \\ \vdots & \vdots \\ \frac{\partial v_n}{\partial q'} & \frac{\partial v_n}{\partial q} \end{bmatrix} \begin{bmatrix} q'' \\ q' \end{bmatrix}$$

$$= \frac{\partial v_i}{\partial q'} \cdot q'' + \frac{\partial v_i}{\partial q} \cdot q'$$

Replace the Newton-Euler EOM

$$\left\{ \begin{array}{l} F_i^a + F_i^r = m_i a_i \\ L_i^a + L_i^r = I_i \alpha_i + \tilde{S}(\omega_i) I_i \omega_i \end{array} \right.$$

$$m_i \ddot{q}_i = m_i \cdot \left(\begin{bmatrix} \frac{\partial V_1}{\partial q'_1} & \frac{\partial V_1}{\partial q'_1} \\ \vdots & \vdots \\ \frac{\partial V_n}{\partial q'_n} & \frac{\partial V_n}{\partial q'_n} \end{bmatrix} \begin{bmatrix} q'' \\ q' \end{bmatrix} \right)$$

$$m_i \ddot{q}_i = m_i \cdot J_{t,i} \cdot q'' + m_i \cdot J_{t,i} \omega_2 \cdot q'$$

In (4), what is $J_{t,i}$ mean?

$$(5) M(q) q'' + K(q, q') = P(q, q')$$

Why $M(q)$ but not $M(q, q')$?

2.1.2 Lagrangian Formulation

$$L(q_i, \dot{q}_i) = \sum_{i=1}^N (T(\dot{q}'_i) - V(q_i))$$

kinetic energy - potential energy

Second-kind Lagrange EOM

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$$

A note about generalized coordinates

$$q(t) = (q_1(t), q_2(t), \dots, q_n(t))$$

Each is a function of time.

The position vector r_k of particle k is a function of all the n generalized coordinates.

$$r_k = r_k(q(t))$$

The corresponding time dev of q are generalized velocities.

$$q' = \frac{dq}{dt} = (q_1'(t), q_2'(t), \dots, q_n'(t))$$

The velocity vector v_k is the total dev of r_k

$$v_k = r'_k = r_k(q_1(t), q_2(t), \dots, q_n(t))$$

Recall chain rule

$$f(x(t), y(t))$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

$$\text{then } r'_k(q_1(t), q_2(t), \dots, q_n(t))$$

$$= \frac{\partial r_k}{\partial q_1} \cdot \frac{dq_1}{dt} + \frac{\partial r_k}{\partial q_2} \cdot \frac{dq_2}{dt} + \dots + \frac{\partial r_k}{\partial q_n} \frac{dq_n}{dt}$$

$$= \sum_{j=1}^n \frac{\partial r_k}{\partial q_j} q'_j$$

2.2 Aerodynamic Models

CFD: Ability to solve the complex aerodynamic flow
with cost of expensive computation time

Not suitable choice for control development

- Details provided are not relevant

- Expensive computation time slows down the design process.

Blade Element Momentum (BEM) Theory:

Most common theory for wind turbine aerodynamic model.

Divide the total blade into several small parts

Developed for onshore wind turbines

Need appropriate number of sections

balance of accuracy and calculation time suitable for controllers.

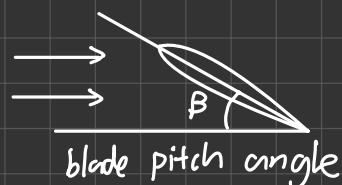
Simplification:

λ - tip-speed ratio (the linear speed of the blade's tip)

β - blade pitch angle

$C_p(\lambda, \beta)$ - power coefficient

$C_t(\lambda, \beta)$ - thrust coefficient



The axial aero force $F_A = \frac{1}{2} \rho A_{r,tip} C_t(\lambda, \beta) V_{rel}^2$

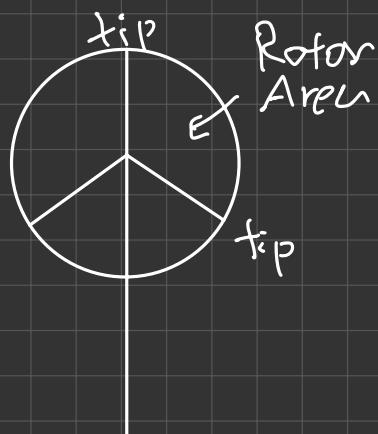
torque $M_A = \frac{1}{2} \rho A_{r,tip} \frac{C_p(\lambda, \beta)}{\omega_r} V_{rel}^3$

ρ - air density

$A_{r,tip}$ - disc rotor area

ω_r - rotor speed

V_{rel} - relative wind speed



Free Vortex Wake (FVW) Theory:

Determine vertical induction at each blade element

BEM computes average induction

Better efficiency than (F) but higher resolution time
than BEM

Able to analysis wake propagation phenomena
but could be neglected.

Impact of floating

Relative wind speed is effected by floating moving.

$$V_{rel} = V_{wind} + V_{surge} + V_{pitch}$$

$$= V_{wind} + V_{surge} + d \alpha' \cos(\lambda)$$

2.3 Hydrodynamic Models (502) 547 9365

- The linear potential flow theory case ID: ABZ 8548 396 Z

Cummin equation \leftarrow hydrostatic based on $F = Ma$
radiation
diffraction

Require external BEM software for the computation
account effects except the viscous

- Viscous Hydrodynamic Theory

Morison equation (cylinder?)

(What is added mass for both model)

i.e. Ass - infinite added mass

C_D and C_L

Two parameters for both

$k \cdot a = k_c$: the wavenumber, the incident waves
(Impact of diffraction)

a : floating platform radius

Period number: Importance of the drag

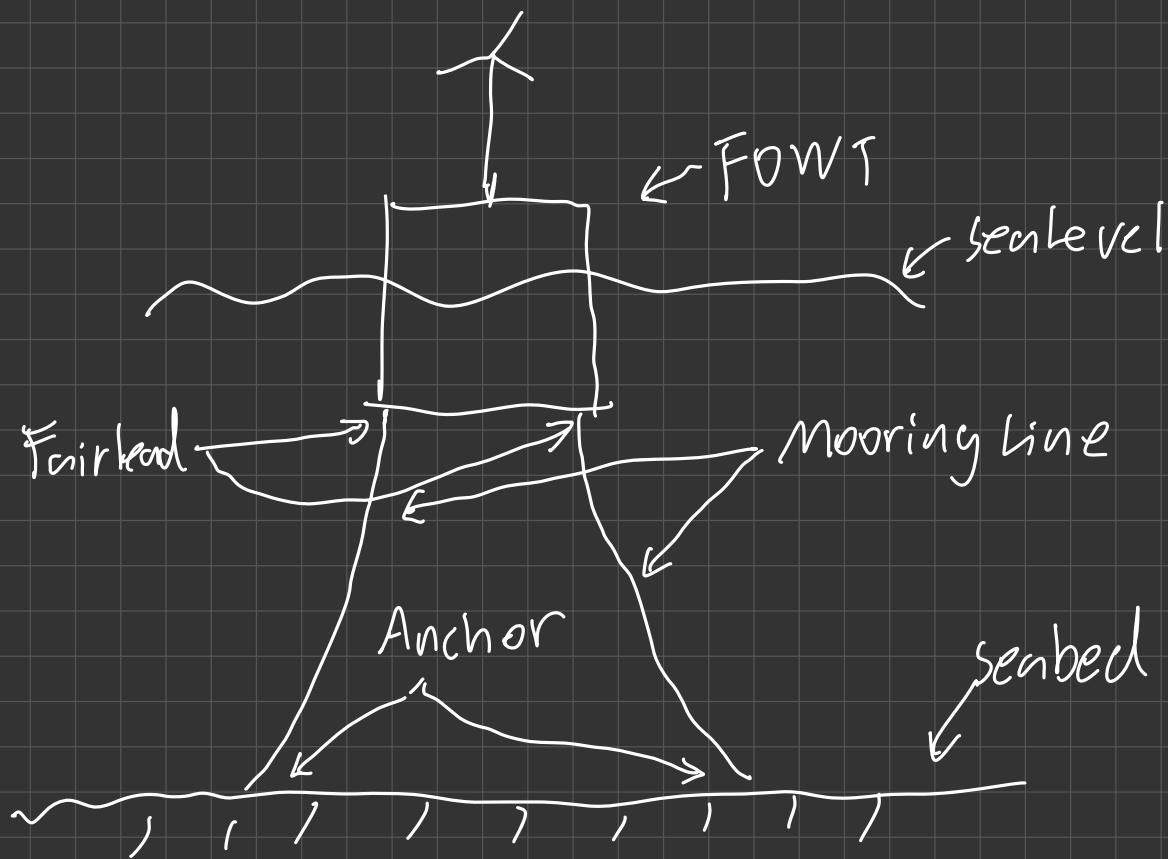
Kemleyan - Carpenter (KCC) number

The good model is in the middle of both theories

Mix two theories, consider

- added masses
 - radiation
 - diffraction
 - hydrostatic restoring effects
 - F_{ext}
- } left-hand side of the
Cummins equation
- } right-hand side, Morison equation

2.4 Mooring Line Models



Three models - Static, quasi-static, dynamic

2.4.1 Static Model:

Neglects the mooring inertia and damping

$$\bar{F}_L^{1,2,3} = F_{L,0}^{1,2,3} - C_L \quad \begin{matrix} \uparrow & \uparrow \\ \text{total static} & \text{pretensioned} \end{matrix} \quad \begin{matrix} b \times b \text{ matrix} \\ \text{total load force} \end{matrix}$$

force

C_L is obtained by
 quasi-static
 dynamic model

2.4.1 The Quasi-Static Model

refers the MAP + assume small displacement

neglects the dynamic effect, added mass, damping, inertia

Two equations

- Total length of the line is floating

- Part does not float

$$\left. \begin{array}{l} \left. \begin{array}{ll} X_F(H_F, V_F) & X_F - \text{Horizontal position} \\ Z_F(H_F, V_F) & Z_F - \text{Vertical position} \end{array} \right\} \\ \text{fairlead displacement} \\ H_F: \text{Horizontal tension force} \\ V_F: \text{Vertical tension force} \end{array} \right\} \text{at fairlead}$$

$$\left. \begin{array}{l} H_A \\ V_A \end{array} \right\} \text{at anchor}$$

2.4.3. Dynamic Model

Consider inertia, added mass, damping

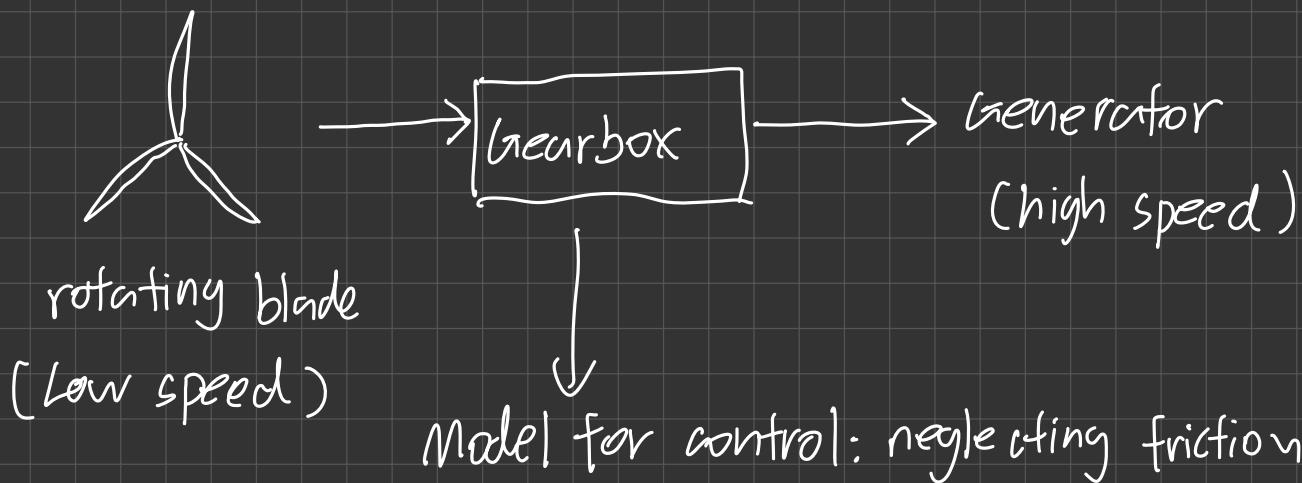
(Recall static model does not consider those)

Developed by Matthew Hall

Summary

The quasi-static model is suitable
spring equation

2.5 Drivetrain Models



One shaft and two shaft

Two shaft express low-speed shaft but make control complex.

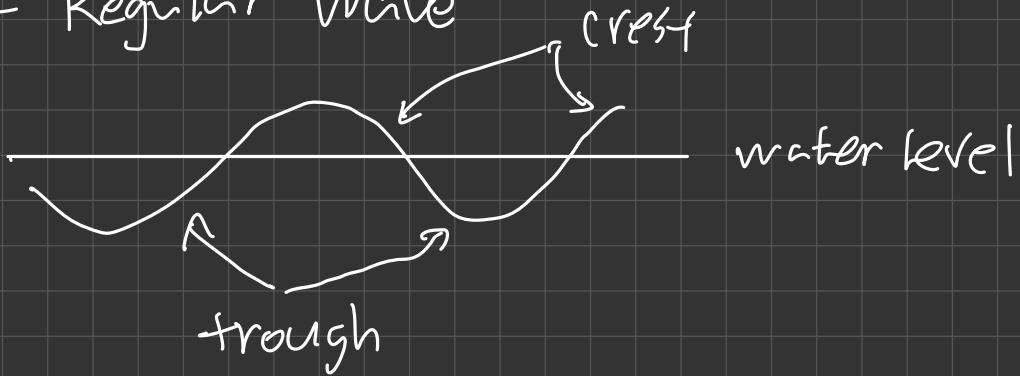
2.6 Wind Profile

The Kaimal spectrum

The Von Karman Spectrum

2.7 Wave Profile

- Regular wave



Very limited model

- Stokes waves

- Irregular waves:

Most realistic

Written as a superposition of multiple regular wave

Summary of section 2.

Wind Profile

- The Kolmogorov spectrum
- The Von Karman spectrum

Wave Profile

- regular wave
- Stokes wave
- Irregular wave

Aerodynamic model

- CFD (Not good for control)
- BEM (Best) good compromise between accuracy and computational time
- FVW (more detailed) than BEM

Accuracy:

$$CFD > FVW > BEM$$

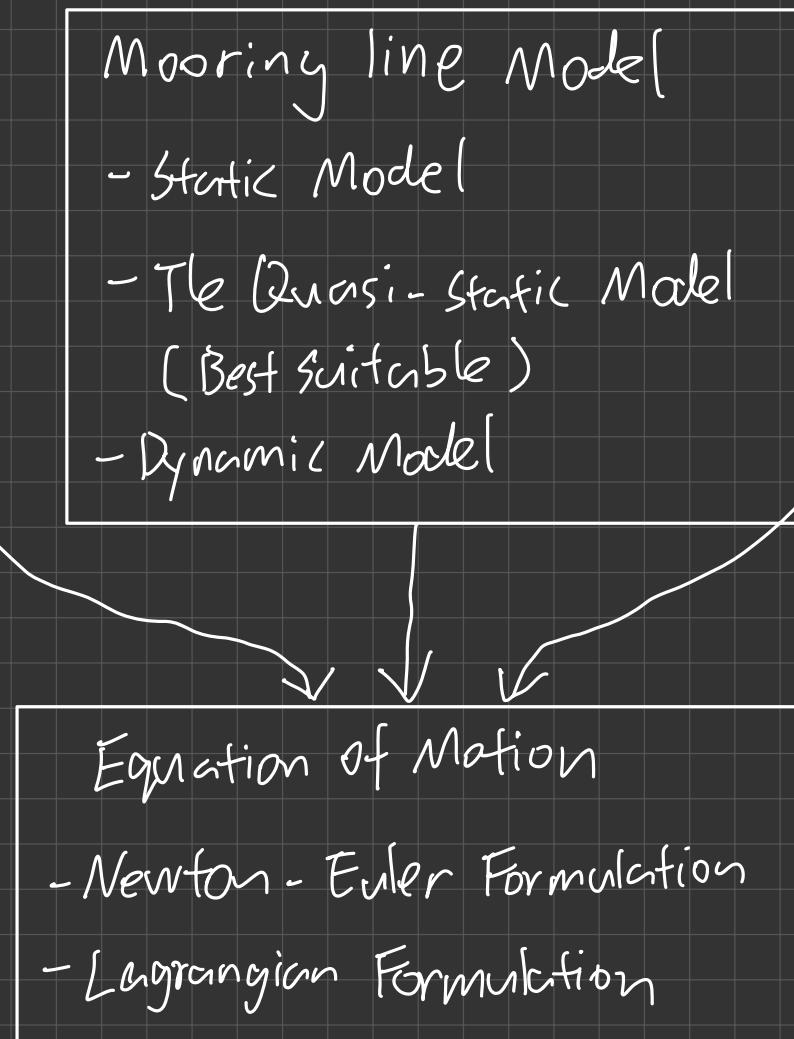
Time complexity

$$CFD > FVW > BEM$$

Hydrodynamic model

- The Linear Potential Flow
 - The viscous effect theory
- Good model mix both

$$V_{rel} = V_{wind} + V_{surge} + V_{pitch}$$



3. COMs of FOWTs

Focused on Non-linear time-domain Control-oriented Models (COM)

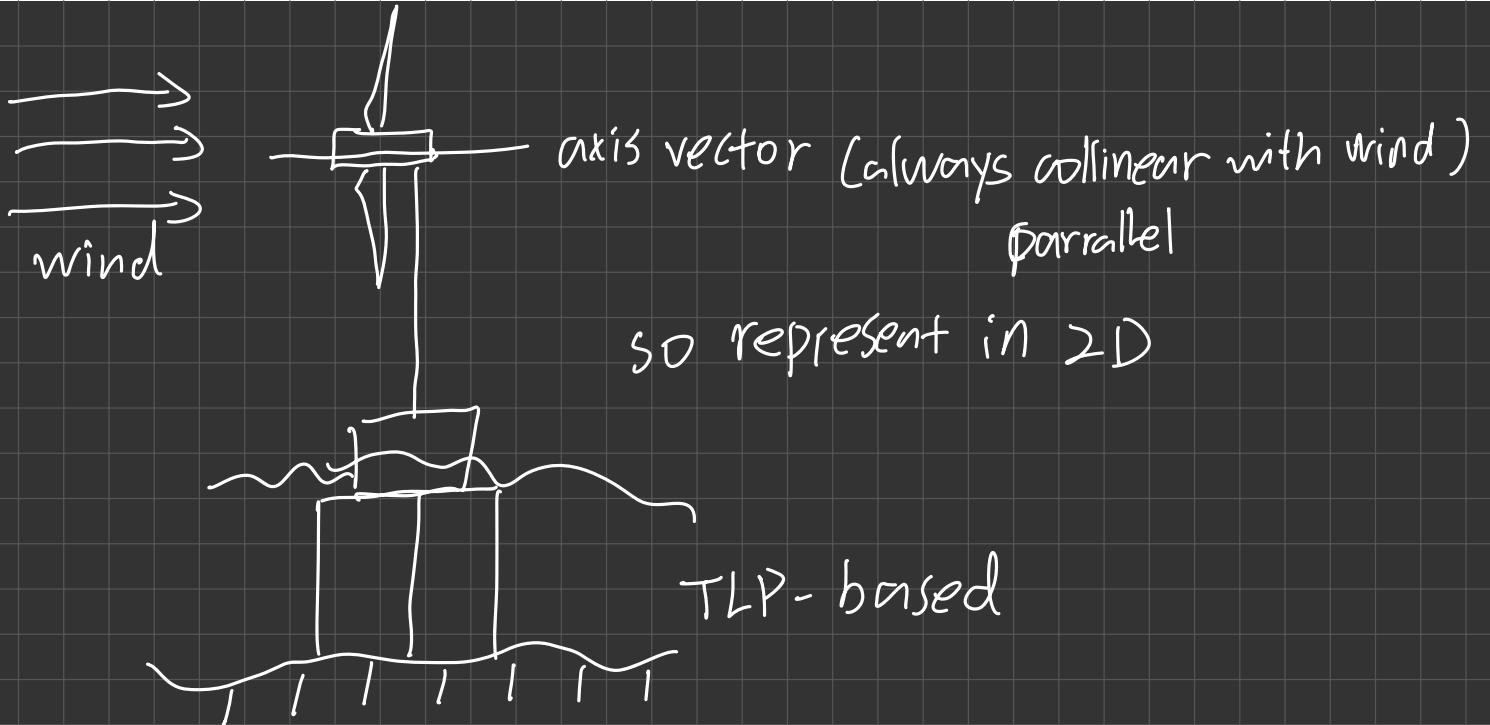
Three models: Betti model, Lemmer model, Homer model.

3.1. Betti Model

(What is heave?)

Seven states: Surge, heave, pitch (respect velocities), rotor speed.

Two control inputs: Generator torque, blade pitch angle



EOM: Lagrangian approach

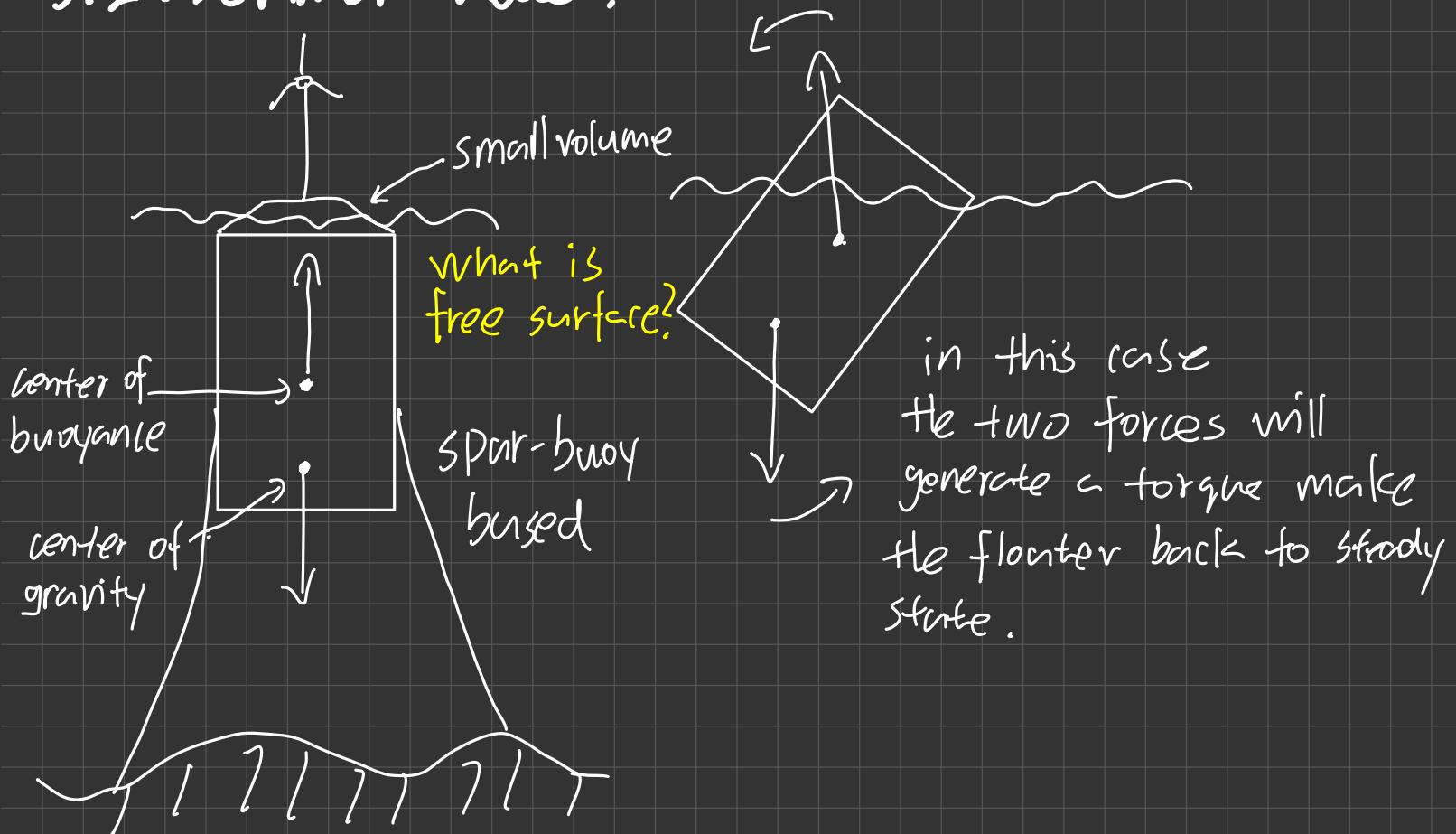
Drive train : one-mass shaft

Aerodynamic : BEM, Bernoulli equation

Hydrodynamic: Viscous theory

Mooring line : spring equation (quasi-static)

3.2. Lemmer Model.



Six states: surge, pitch (displacement and velocity)

nacelle (displacement and velocity)

Note: consider tower flexibility

Control inputs: generator torque, blade pitch angle

The wind turbine is modeled in 2D (same as BEM)

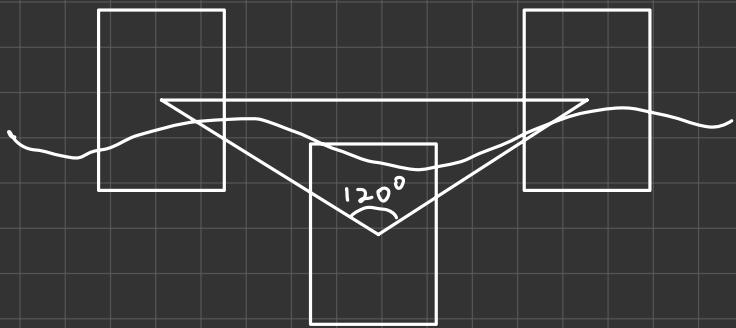
EOM: Newton - Euler

neglect constrain force

Aero: BEM

Hydro: neglect viscous theory, consider no incident waves

3.3. Homer Model



Large volume is in contact with the waves
sensitive to water flow

16 states: 6 DOFs of the platform

$x, y, z, \omega_x, \omega_y, \omega_z$

$x', y', z' \quad \omega'_x, \omega'_y, \omega'_z$

rotor, generator (angle, velocity)

3D model of the FOWT

Control input: generator torque, blade pitch angle, nacelle yaw angle
inertia tensor is constant (what is inertia tensor)
Neglect hydrostatic effect

Mooring line: quasi-static

3.4 Comparison

Betti model is validated in still-water

In the hydro model, only buoyancy force are assessed, which is not enough

Lemmer Model has large error for pitch angle, surge, tower top fore-aft without effect of kinetic wave.

Smallest RMSE, STD means highest accuracy

Betti is the most inaccurate.

Betti validated on a non-turbulent wind profile without wave profile

Lemmer model - Fast control design development.

but large error (only model consider the flexibility)

Hornet most realistic, consider turbulent multi-directional wind profiles wave kinematics
but more complex