The Unsteady Aerodynamics Module for FAST 8

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Executive Summary

The new modularization framework of FAST v.8 (Jonkman 2013) required a complete overhaul of the aerodynamics routines. AeroDyn is an aerodynamics module that can utilize either blade element momentum theory, dynamic blade element momentum theory, or generalized dynamic wake to calculate aerodynamic forces on blade elements. Under asymmetric conditions, such as wind shear, yawed, and tilted flow, the individual blade elements undergo variations in angle of attack that lead to unsteady aerodynamics phenomena, which can no longer be captured through the static airfoil lift and drag look-up tables. This study covers the main theory and the organization of the modularization framework of the new unsteady aerodynamics module (UAM), which includes unsteady aerodynamics under attached flow conditions and dynamic stall. The UAM can be called by either blade element momentum theory, dynamic blade element momentum theory, or (if ever implemented) generalized dynamic wake.

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List of Acronyms

2-D	two dimensional
AOA	angle of attack
AOI	angle of incidence
BE	blade element
BEMT	blade element momentum theory
DBEMT	dynamic blade element momentum theory
DS	dynamic stall
GDW	generalized dynamic wake
LBM	Leishman-Beddoes model
LE	leading edge
MT	momentum theory
TE	trailing edge
UA	unsteady aerodynamics
UAM	unsteady aerodynamics module

List of Symbols

AFI_{Params}	Airfoil static tables of C_l, C_d, C_m, and UA parameters
A_1	Constant in the expression of ϕ_α^c andϕ_q^c; experimental results (Leishman 2011) set it equal to 0.3; this value is relatively insensitive for thin airfoils, but may be different for turbine airfoils; generally speaking, it should not be tuned by the user
A_2	Constant in the expression of ϕ_α^c andϕ_q^c; experimental results (Leishman 2011) set it to 0.7; this value is relatively insensitive for thin airfoils, but may be different for turbine airfoils; generally speaking, it should not be tuned by the user
A_5	Constant in the expression of K_q''', $C_{m_q}^{nc}(s,M)$, and $k_{m,q}(M)$; experimental results (Leishman 2006) set it equal to 1
C_{LP}	Low-pass filter constant

D	Rotor diameter
$F_R(s)$	Response function to generic disturbance $\varepsilon(s)$
$F_R(t)$	Response function to generic disturbance $\varepsilon(t)$
$FirstPass$	Flag indicating first time step
$LESF$	Leading-edge separation flag
M	Mach number
$NumBlades$	Number of blades
$NumOuts$	Number of output channels
R	Rotor radius
S_1	Constant in the f curve best-fit for $\alpha_0 \leq \alpha \leq \alpha_1$; by definition it depends on the airfoil
S_2	Constant in the f curve best-fit for $\alpha > \alpha_1$; by definition it depends on the airfoil
S_3	Constant in the f curve best-fit $\alpha_2 \leq \alpha < \alpha_0$; by definition it depends on the airfoil
S_4	Constant in the f curve best-fit for $\alpha < \alpha_2$; by definition it depends on the airfoil
St_{sh}	Strouhal's shedding frequency constant, commonly taken equal to 0.19
T'_α	Mach-dependent, nondimensional time constant in the expression of ϕ_α^{nc}; it is equal to $2UT_\alpha(M)/c$
T'_q	Mach-dependent time constant in the expression of ϕ_q^{nc}
$TESF$	Trailing-edge separation flag
T_I	Time constant in the expression of $\phi_\alpha^{nc} = c/a_s$
T_V	Time constant associated with the vortex lift decay process; it is used in the expression of C_n^v. It depends on Re, M, and airfoil type
$T_\alpha(M)$	Mach-dependent time constant in the expression of ϕ_α^{nc}
T_f	Constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f''
T_p	Boundary-layer, LE pressure gradient time constant in the expression of D_p, which should be tuned based on airfoil experimental data
$T_q(M)$	Mach-dependent time constant in the expression of ϕ_q^{nc}
T_{V0}	Initial value of T_V
T_{VL}	Time constant associated with the vortex advection process; it represents the nondimensional time in semichords, needed for a vortex to travel from LE to TE; it is used in the expression of C_n^v; it depends on Re, M (weakly), and airfoil. Value's range $= [6;13]$
T_{f0}	Initial value of T_f
$T'_{m,q}$	Mach-dependent time constant in the expression of $\phi_{m,q}^{nc}$
$T_{m,q}(M)$	Mach-dependent time constant in the expression of $\phi_{m,q}^{nc}$

T_{sh}	Time constant associated with the vortex shedding; it allows multiple vortices to be shed at a Strouhal's frequency of 0.19
$UAmod$	Switch to select handling of options and possible methods in the UA treatment
U	Relative air speed
$VRTX$	Vortex advection flag
X_1	Deficiency function used in the development of $C^c_{n_\alpha}(s,M)$
X_2	Deficiency function used in the development of $C^c_{n_\alpha}(s,M)$
X_3	Deficiency function used in the development of $C^c_{n_q}(s,M)$
X_4	Deficiency function used in the development of $C^c_{n_q}(s,M)$
Δs	Incremental variation in s for the Δt time step
Δt	Time step
$\bar{\bar{x}}_{cp}$	Constant in the expression of $\hat{x}^v_{cp}$, usually equal to 0.2
$K_{\alpha LP,-1}$	Previous time-step value of low-pass- filtered K_α
$K_{qLP,-1}$	Previous time-step value of low-pass-filtered K_q
$\alpha_{,-1}$	Previous time-step value of α
$f_m{}''$	CENER's proposed version of lagged $f_m{}'$
$f_m{}'$	CENER's proposed lookup version of f'
$f''_{,-1}$	Previous time-step value of f''
$f'_{,-1}$	Previous time-step value of f'
$q_{,-1}$	Previous time-step value of q
$q_{LP,-1}$	Previous time-step value of low-pass-filtered q
$\hat{k}_1$	Constant in the C_c expression due to leading edge (LE) vortex effects
$\hat{k}_2$	Constant in the C_c expression due to LE vortex effects, taken equal to $2(C'_n - C_{n1}) + (f'' - f)$
$\hat{x}_{AC}$	Aerodynamic center distance from LE in percent chord
$\hat{x}^v_{cp}$	Center-of-pressure distance from the 1/4-chord, in percent chord, during the LE vortex advection process
$\hat{x}_{cp}$	Center-of-pressure distance from LE in percent chord
^{c}	Circulatory component of the quantity at the base
^{nc}	Noncirculatory component of the quantity at the base
$_\alpha$	Relative to a step change in α
$_n$	relative to the n-th time step
$_q$	Relative to a step change in q
$_t$	Relative to the n-th time step
a_s	Speed of sound

b_1	Constant in the expression of ϕ_α^c andϕ_q^c; experimental results (Leishman 2011) set it equal to 0.14; this value is relatively insensitive for thin airfoils, but may be different for turbine airfoils; generally speaking, it should not be tuned by the user
b_2	Constant in the expression of ϕ_α^c andϕ_q^c; experimental results (Leishman 2011) set it equalto 0.53. This value is relatively insensitive for thin airfoils, but may be different for turbine airfoils; generally speaking, it should not be tuned by the user
b_5	Constant in the expression of K_q''', $C_{m_q}^{nc}(s,M)$, and $k_{m,q}(M)$; experimental results (Leishman 2006) set it equal to 5
f_c''	Lagged version of f_c'
$f_{c,-1}''$	previous time-step value of f_c''
f_c'	f' calculated from Kirchhoff's expression containing C_c function of f
f_n'	f' calculated from Kirchhoff's expression containing C_n function of f
$f_{c,-1}'$	Previous time-step value of f_c'
$flookup$	Logical flag to indicate whether a lookup (True) or an interpolation of the airfoil data tables (False) is used to retrieve the values for f
$iBlade$	Blade index
$jBladeNode$	Blade node index
k_0	Constant in the $\hat{x}_{cp}$ curve best-fit; $= (\hat{x}_{AC} - 0.25)$
k_1	Constant in the $\hat{x}_{cp}$ curve best-fit
k_2	Constant in the $\hat{x}_{cp}$ curve best-fit
k_3	Constant in the $\hat{x}_{cp}$ curve best-fit
$k_\alpha(M)$	Mach-dependent constant in the expression of $T_\alpha(M)$
$k_q(M)$	Mach-dependent constant in the expression of $T_q(M)$
$k_{m,q}(M)$	Mach-dependent constant in the expression of $T_{m,q}(M)$ and $C_{m_q}^{nc}(s,M)$
$miscVars$	Other states that are NOT used for linearization
$nNodesPerBlade$	Number of nodes per blade
q	Nondimensional pitching rate $=\dot{\alpha}c/U$
s	Nondimensional distance
t	Time
x_d	Discrete states
$D_{\alpha f,-1}$	Previous time-step value of $D_{\alpha f}$
$D_{f,-1}$	Previous time-step value of D_f
$D_{p,-1}$	Previous time-step value of D_p
$K_{\alpha,-1}'$	Previous time-step value of K_α'
$K_{\alpha LP}$	Modified value of K_α due to filtered α and q
$K_{q,-1}'''$	Previous time-step value of K_q'''
$K_{q,-1}''$	Previous time-step value of K_q''
$K_{q,-1}'$	Previous time-step value of K_q'

$K_{q_{LP_{n-1}}}$	Low-pass-filtered value of K_q at the (n-1)-th time step
$K_{q_{LP}}$	Low-pass-filtered value of K_q
$X_{1,-1}$	Previous time-step value of X_1
$X_{2,-1}$	Previous time-step value of X_2
$X_{3,-1}$	Previous time-step value of X_3
$X_{4,-1}$	Previous time-step value of X_4
$C_n^{pot}{}_{,-1}$	Previous time-step value of C_n^{pot}
$C_{n,-1}^{v}$	Previous time-step value of C_n^v
$C_{V,-1}$	Previous time-step value of C_V
c	Chord length
C_c	2-D tangential (along chord) force coefficient
C_c^{fs}	2-D tangential (along chord) force coefficient under separated trailing edge (TE) flow separation conditions
C_c^{pot}	2-D along-chord force coefficient under attached (potential) flow conditions
C_d	2-D drag coefficient
C_{d0}	2-D drag coefficient at 0-lift
C_l	2-D lift coefficient
$C_{l\alpha}$	Slope of the 2-D lift coefficient curve
C_m	2-D pitching moment coefficient about ¼-chord; positive if nose up
C_{m0}	2-D pitching moment coefficient at 0-lift, positive if nose up
$C_{m_\alpha}(s,M)$	Pitching moment coefficient response to step change in α
$C_{m_\alpha}^{c}(s,M)$	Circulatory component of the pitching moment coefficient response to step change in α
$C_{m_\alpha}^{nc}(s,M)$	Noncirculatory component of the pitching moment coefficient response to step change in α
$C_{m_{\alpha,q}}(s,M)$	Moment coefficient response to step change in α and q
$C_{m_{\alpha,q}}^{c}(s,M)$	Circulatory component of $C_{m_{\alpha,q}}(s,M)$
$C_{m_{\alpha,q}}^{nc}(s,M)$	Noncirculatory component of $C_{m_{\alpha,q}}(s,M)$
$C_{m_q}(s,M)$	Pitching moment coefficient response to step change in q
$C_{m_q}^{c}(s,M)$	Circulatory component of the pitching moment coefficient response to step change in q
$C_{m_q}^{nc}(s,M)$	Noncirculatory component of the moment coefficient response to step change in q
$C_{m\alpha}$	Slope of the 2-D pitching moment coefficient curve
C_m^{fs}	2-D tangential ¼-chord pitching moment coefficient under separated TE flow separation conditions.
C_m^{pot}	2-D moment coefficient under attached (potential) flow conditions about ¼-chord location
$C_{mq}(s,M)$	Slope of the pitching moment coefficient versus q curve

C_m^v	Pitching moment coefficient due to the presence of LE vortex
C_n	2-D normal-to-chord force coefficient
C_{n1}	Critical value of C_n' at LE separation. It should be extracted from airfoil data at a given Mach and Reynolds number. It can be calculated from the static value of C_n at either the break in the pitching moment or the loss of chord force at the onset of stall. It is close to the condition of maximum lift of the airfoil at low Mach numbers.
C_{n2}	Critical value of C_n' at LE separation for negative AOAs; analogous to C_{n1}
$C_{n_\alpha}(s,M)$	Normal force coefficient response to step change in α
$C_{n_\alpha}^c(s,M)$	Circulatory component of the normal force coefficient response to step change in α
$C_{n_\alpha}^{nc}(s,M)$	Noncirculatory component of the normal force coefficient response to step change in α
$C_{n_{\alpha,q}}(s,M)$	Normal force coefficient response to step change in α and q
$C_{n_{\alpha,q}}^c(s,M)$	Circulatory component of $C_{n_{\alpha,q}}(s,M)$
$C_{n_{\alpha,q}}^{nc}(s,M)$	Noncirculatory component of $C_{n_{\alpha,q}}(s,M)$
$C_n^c(s,M)$	Circulatory component of $C_{n_{\alpha,q}}(s,M)$
$C_{n_q}(s,M)$	Normal force coefficient response to step change in q
$C_{n_q}^c(s,M)$	Circulatory component of the normal force coefficient response to step change in q
$C_{n_q}^{nc}(s,M)$	Noncirculatory component of the normal force coefficient response to step change in q
$C_{n\alpha}$	Slope of the 2-D normal coefficient curve, similar to $C_{l\alpha}$
$C_{n\alpha}^c(s,M)$	Slope of the circulatory normal force coefficient versus α curve
C_n^{fs}	Normal force coefficient under separated TE flow separation conditions
C_n'	Lagged component of C_n in the TE separated treatment
C_n^{pot}	2-D normal-to-chord force coefficient under attached (potential) flow conditions
$C_n^{pot,c}$	Circulatory part of 2-D normal-to-chord force coefficient under attached (potential) flow conditions
$C_n^{pot,nc}$	Noncirculatory part of 2-D normal-to-chord force coefficient under attached (potential) flow conditions
$C_{nq}(s,M)$	Slope of the normal force coefficient versus q curve
C_n^v	Normal force coefficient due to the presence of LE vortex
C_V	Contribution to the normal force coefficient due to accumulated vorticity in the LE vortex

$D_{\alpha f}$	Deficiency function for α_f
D_f	Deficiency function for f'
$D_{f_c,-1}$	Previous time-step value of D_{f_c}
D_{f_c}	Deficiency function for f'_c
D_p	Deficiency function for C'_n
f	Separation point distance from LE in percent chord
f_m	CENER's proposed version of f extracted from the C_m static tables, assuming $C_m = C_n f_m$
f'_m	Version of f_m extracted from the airfoil C_m static tables with α_f as input parameter
f''_m	Lagged version of f'_m
f'	Separation point distance from LE in percent chord under unsteady conditions
f''	Lagged version of f'accounting for unsteady boundary layer response
k	Reduced frequency
K_α	Backward finite difference of α at the n-th time step
K'_α	Deficiency function for $C^{nc}_{n_\alpha}(s,M)$
K_q	Backward finite difference of q at the n-th time step
K'_q	Deficiency function for $C^{nc}_{n_q}(s,M)$
K''_q	Deficiency function for $C^{nc}_{m_q}(s,M)$
K'''_q	Deficiency function for $C^{c}_{m_q}(s,M)$
Re	Airfoil-chord Reynolds Number
U	Air velocity magnitude relative to the airfoil
U_{inf}	Freestream air velocity magnitude

List of Greek Symbols

$\Delta_{\alpha 0}$	α-α_0
α_0	0-lift angle of attack
α_1	Angle of attack at f=0.7, (approximately the stall angle) for $\alpha \geq \alpha_0$
α_2	Angle of attack at f=0.7, for $\alpha < \alpha_0$
α_e	Effective angle of attack at ¾-chord
α_n	Value of α at the n-th time step, i.e., $t = n\Delta t$
$\alpha_{LP,-1}$	Previous time-step value of low-pass-filtered α
α_{LP_n}	Low-pass-filtered value of α
$\alpha_{LP_{n-1}}$	Low-pass-filtered value of α at the (n-1)-th time step
α_{n-1}	Value of α at the (n-1)-th time step, i.e., $t = (n-1)\Delta t$
α	Angle of attack
$\varepsilon(s)$	Generic disturbance function of s
$\varepsilon(t)$	Generic disturbance function of t

η_e	Recovery factor $\simeq [0.85-0.95]$ to account for viscous effects at limited or no separation on C_c
$\tau_{V,-1}$	Previous time-step value of τ_V
ω	Generic frequency
$\phi(s,M)$	Indicial response function
$\phi(t,M)$	Indicial response function
ϕ_α^c	Normal force coefficient, circulatory indicial response function to a step change in α
ϕ_α^{nc}	Normal force coefficient, noncirculatory indicial response function to a step change in α
ϕ_q^c	Normal force coefficient, circulatory indicial response function to a step change in q
ϕ_q^{nc}	Normal force coefficient, noncirculatory indicial response function to a step change in q
$\phi_{m,\alpha}^{nc}$	Pitching moment coefficient, noncirculatory indicial response function to a step change in α
$\phi_{m,q}^c$	Pitching moment coefficient, circulatory indicial response function to a step change in q
$\phi_{m,q}^{nc}$	Pitching moment coefficient, noncirculatory indicial response function to a step change in q
σ_1	Generic multiplier for T_f
σ_3	Generic multiplier for T_V
σ_s	Generic integrand coordinate
σ_t	Generic integrand coordinate
τ_V	Time variable that tracks the travel of the LE vortex over the airfoil suction surface. It is made dimensionless via the semichord: $\tau_V = t * 2U/c$. If less than $2T_{VL}$, it renders the logical flag $VRTX$=True; if less than T_{VL}, then the vortex is still on the airfoil
ζ_{LP}	Low-pass filter frequency cutoff ($-3\,$dB)
q_{LPn-1}	Value of q_{LP} at the (n-1)-th time step
q_{LP}	Low-pass-filtered value of q
q_{n-1}	Value of q at the (n-1)-th time step
$\alpha_{f,-1}$	Previous time-step value of α_f
α_f	Effective angle of incidence (AOI) which would give the same unsteady LE pressure gradient under static conditions; used to calculate f'
α_f'	Lagged version of α_f; used in Minnema's (1998) calculation of C_m under separated conditions
β_M	Prandtl-Glauert compressibility correction factor $\sqrt{1-M^2}$

1 Overview

Because of turbulence, wind shear, control inputs, and off-axis operations, wind turbine rotors experience unsteady aerodynamic loading. Unsteady aerodynamics is primarily caused by two physical mechanisms. First, the unsteady (indicial) variation in the two dimensional (2-D) lift, drag, and moment coefficient associated with an unsteady variation of the angle of attack; wherein the timescale is on the order of tenths of seconds, or $\sim c/\omega R$ with c representing the chord length, ω the generic frequency, and R the rotor radius. Second, induction effects driven by the midwake region with a time constant of a few seconds, or $\sim D/U_{\text{inf}}$, with D as the rotor diameter and U_{inf} as the freestream air velocity magnitude. The first mechanism pertains to the near-wake field, which affects the BE portion of the blade element momentum theory (BEMT), and is the focus of this manual. The second mechanism is affected by the dynamics of the vorticity shed in the midwake and affects the blade element momentum theory (MT) portion of the BEMT, and is described in Damiani (2016, forthcoming). In this document, UA refers to unsteady aerodynamics, or the first physical mechanism.

The main theory follows the work by Leishman and Beddoes (1986, 1989), Pierce and Hansen (1995), Pierce (1996), Leishman (2011), and Damiani (2011). UA is driven mostly by 2-D flow aspects, including that of dynamic stall. Dynamic stall (DS) is a well-known phenomenon that can affect wind turbine performance and loading especially during yawed operations, and can result in large unsteady stresses on the structures. Dynamic stall manifests as a delay in the onset of flow separation to higher angles-of-attack (AOAs) that would otherwise occur under static (steady) conditions, followed by an abrupt flow separation from the LE of the airfoil (Leishman 2011). The LE separation is the fundamental characteristic of the DS of an airfoil; in contrast, quasi-steady stall would start from the airfoil TE.

DS occurs for reduced frequencies (k) above 0.02, where:

$$k = \frac{\omega c}{2U} \tag{1.1}$$

The five stages of dynamic stall are as follows and shown in Figure 1 and Figure 2:

1. Onset of flow reversal
2. Flow separation and vorticity accumulation at the leading edge
3. Shedding of the vortex and convection along the suction surface of the airfoil (lift increases)
4. Lift stall (vortex is shed in the wake and lift, causing an abrupt drop-off)
5. Reattachment of the flow at AOAs considerably lower than static AOAs (hysteresis).

The model chosen to represent UA and DS is the Leishman-Beddoes model (LBM) because it is the most widely used, has the most support throughout the community, and has shown reasonable success when compared to experimental data. The LBM is a postdictive model, and as such it will not solve equations of motion, though the principles are fully rooted in the physics of unsteady flow.

In the LBM, the different processes are modeled as first-order subsystems with differential equations with predetermined constants to match experimental results. Therefore, *knowledge of the airfoil characteristics under unsteady aerodynamics is a prerogative* of the LBM. The LBM can also be described as an indicial response (i.e., response to a series of small disturbances) model for attached flow, extended to account for separated flow effects and vortex lift. Forces are computed as normal and tangential (to chord) and pitching moment about the 1/4-chord location. See also Figure 3 and Eq. (1.2).

$$\begin{aligned} C_l &= C_n \cos\alpha + C_c \sin\alpha \\ C_d &= C_n \sin\alpha - C_c \cos\alpha + C_{d0} \end{aligned} \tag{1.2}$$

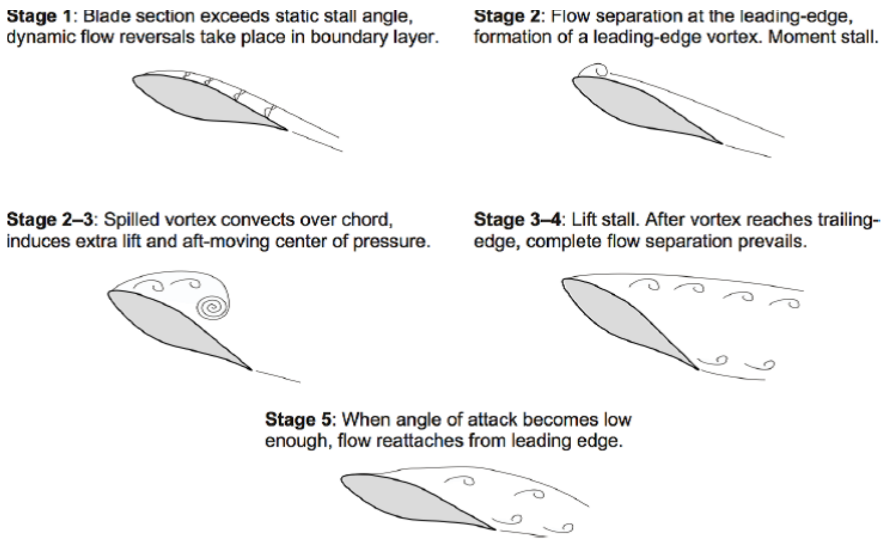

Figure 1. Conventional stages of dynamic stall (Leishman 2006)

$$\begin{aligned} C_n &= C_l \cos\alpha + (C_d - C_{d0}) \sin\alpha \\ C_c &= C_l \sin\alpha - (C_d - C_{d0}) \cos\alpha \end{aligned} \tag{1.3}$$

The original model was developed for helicopters but has been successfully applied to wind turbines (see Pierce (1996) and Gupta and Leishman (2006)). Yawed flowed conditions, Coriolis, and centrifugal forces that lead to three-dimensional effects were not included in the original model.

The LBM considers a number of unsteady aerodynamics conditions including attached flow conditions and TE separation before stall, delays associated with the unsteady onset of dynamic stall and accompanying boundary layer development, advection of the LE vortex, shedding in the wake, and suppression of TE separation in favor of LE separation. The LBM can be divided into three main submodules:

1. Unsteady, attached flow solution via indicial treatment (potential flow)
2. TE flow separation
3. DS vorticity advection.

1.1 Unsteady Attached Flow and Its Indicial Treatment

The advantage of the indicial treatment is that a response to an arbitrary forcing can be obtained by superpositioning response-functions to a step variation in AOA, pitch rate, or heave (plunging) motion. The superposition is carried out via the so-called Duhamel Integral (Leishman 2006), which for the generic response $F_R(t)$ to a generic disturbance $\varepsilon(t)$ can be written as:

$$F_R(t) = \varepsilon(0)\phi(t,M) + \int_0^t \frac{\mathrm{d}\varepsilon}{\mathrm{d}\sigma_t}(\sigma_t)\,\phi(t-\sigma_t,M)\mathrm{d}\sigma_t$$

or

$$F_R(s) = \varepsilon(0)\phi(s,M) + \int_0^s \frac{\mathrm{d}\varepsilon}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi(s-\sigma_s,M)\mathrm{d}\sigma_s \tag{1.4}$$

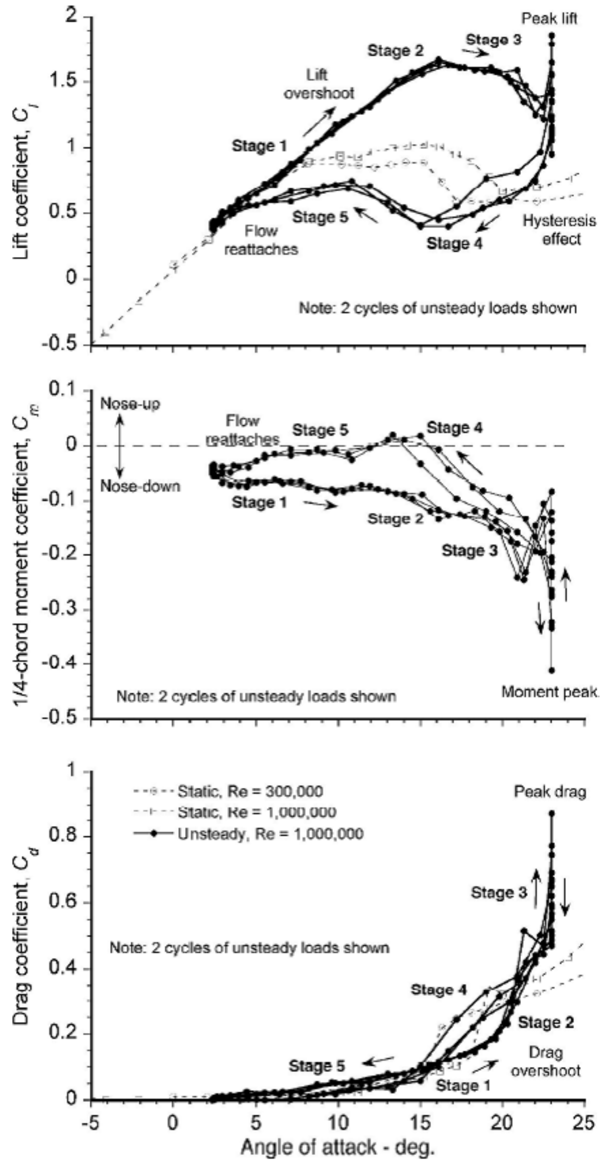

Figure 2. Conventional stages of dynamic stall and associated C_l, C_d, and C_m as functions of AOA from Leishman (2006)

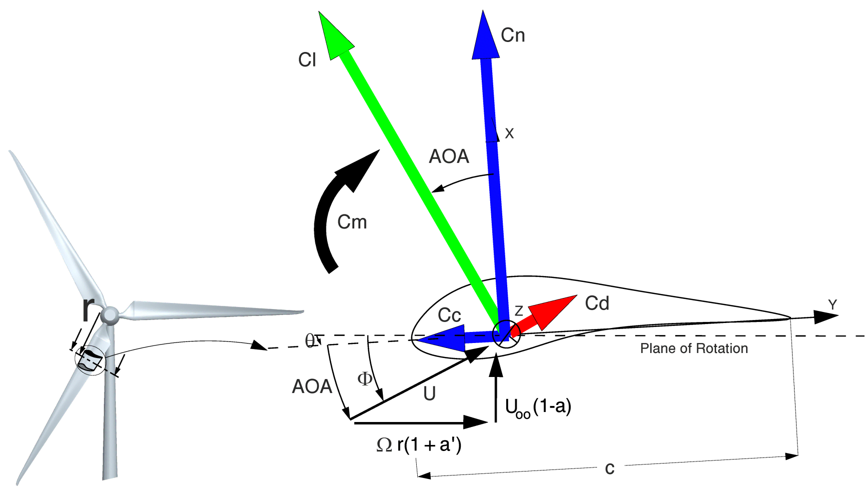

Figure 3. Main definitions of BE forces (denoted via their normalized coefficients) for the unsteady aerodynamics treatment (Damiani 2011)

where t is time, M is the Mach number, σ_t and σ_s are generic integrand coordinates, and s is the nondimensional distance, defined as:

$$s = \frac{2}{c}\int_0^t U(t)dt \tag{1.5a}$$

$$\Delta s = \frac{2}{c}U(t)\Delta t \tag{1.5b}$$

where the airfoil half chord ($c/2$) was taken as the nondimensionalizing factor, and U is the air velocity magnitude relative to the airfoil.

The indicial functions ($\phi(s,M)$) are surmised into two components: the first is related to the noncirculatory (superscript 'nc') loading (piston theory and acoustic wave theory), and the second (superscript 'c') originates from the development of circulation about the airfoil. The noncirculatory part depends not only on the instantaneous airfoil motion, but on the time history of the prior motion. The circulatory response can be calculated via the 'lumped approach, wherein the effects of step changes in AOA (α), pitch rate, heave motion, and so on are combined into an effective AOA at the 3/4-chord station.

1.1.1 Normal Force

The normal force coefficient response to a step change in nondimensional pitch rate q and a step change in AOA can be written as a function of the indicial functions as shown in Eq. (1.6):

$$\begin{aligned} C_{n_{\alpha,q}}(s,M) &= C_{n_\alpha}(s,M) + C_{n_q}(s,M) = C_{n\alpha}\alpha + C_{nq}(s,M)q \\ C_{n_\alpha}(s,M) &= \frac{4}{M}\phi_\alpha^{nc} + \frac{C_{n\alpha}}{\beta_M}\phi_\alpha^c \\ C_{n_q}(s,M) &= \frac{1}{M}\phi_q^{nc} + \frac{C_{n\alpha}}{2\beta_M}\phi_q^c \end{aligned} \tag{1.6}$$

where β_M is the Prandtl-Glauert compressibility correction factor $\sqrt{1-M^2}$, and $C_{n\alpha}$ is the slope of the 2-D normal coefficient curve, similar to $C_{l\alpha}$.

The nondimensional pitch rate q is given by:

$$\begin{aligned} q &= \frac{\dot{\alpha}c}{U} \simeq \frac{K_{\alpha n}c}{U} \\ \text{with : } K_{\alpha n} &= \frac{\alpha_n - \alpha_{n-1}}{\Delta t} \end{aligned} \tag{1.7}$$

where the subscript 'n' denotes the n-th time step.

For small Δt's, the finite difference K_α can be subjected to significant numerical noise. To smooth out the terms associated with time derivatives, a low-pass filter is introduced. The filter is applied to α, q, and its derivative K_q (which is used later) as defined in Eq. (1.8).

$$\begin{aligned} \alpha_{LP_n} &= C_{LP}\alpha_{n-1} + (1 - C_{LP})\,\alpha_n \\ q_n &= \frac{\left(\alpha_{LP_n} - \alpha_{LP_{n-1}}\right)c}{U\Delta t} \\ q_{LPn} &= C_{LP}q_{LPn-1} + (1 - C_{LP})\,q_n \\ K_{\alpha LPn} &= \frac{q_{LPn}U_n}{c} \\ K_{q_n} &= \frac{q_n - q_{n-1}}{\Delta t} \\ K_{q_{LPn}} &= C_{LP}K_{q_{LP_{n-1}}} + (1 - C_{LP})\,K_{q_n} \\ &\text{with :} \\ C_{LP} &= e^{-2\pi\Delta t\zeta_{LP}} \end{aligned} \tag{1.8}$$

and where α_{LP_n} is the low-pass-filtered value of α, q_{LP} is the low-pass-filtered value of q, $K_{\alpha LP}$ is the modified value of K_α due to filtered α and q, K_q is the backward finite difference of q at the n-th time step, $K_{q_{LP}}$ is the low-pass-filtered value of K_q, C_{LP} is the low-pass filter constant, and ζ_{LP} is the low-pass filter frequency cutoff ($-3\,$dB).

From here on, the 'LP' subscript is dropped with the understanding that quantities such as α, K_α, q, and K_q denote the respective filtered quantities α_{LP_n}, $K_{\alpha LP}$, q_{LP}, and $K_{q_{LP}}$ as defined in Eq. (1.8).

The indicial responses can then be approximated as in Eq. (1.9) (Leishman and Beddoes 1989; Johansen 1999):

$$\begin{aligned} \phi_\alpha^c &= \phi_q^c = 1 - A_1 \exp\left(-b_1\beta_M^2 s\right) - A_2 \exp\left(-b_2\beta_M^2 s\right) \\ \phi_\alpha^{nc} &= \exp\left(-\tfrac{s}{T'_\alpha}\right) \\ \phi_q^{nc} &= \exp\left(-\tfrac{s}{T'_q}\right) \end{aligned} \tag{1.9}$$

where A_1, A_2, b_1, and b_2 are constants that were tuned from experimental results on oscillating airfoils in the wind tunnel, and that are relatively insensitive to the airfoil shapes, at least for thin airfoils such as those used in rotorcraft (see Section); the time constants T'_α and T'_q are defined below.

By making use of exact results for short times $0 \leq s \leq 2M/(M+1)$ (Lomax et al. 1952), Leishman (2011) shows that:

$$\begin{aligned} T_\alpha(M) &= 0.75\tfrac{c}{2U}T'_\alpha = 0.75\frac{c}{2Ma_s}T'_\alpha = 0.75k_\alpha(M)T_I \\ T_q(M) &= 0.75\tfrac{c}{2U}T'_q = 0.75\frac{c}{2Ma_s}T'_q = 0.75k_q(M)T_I \end{aligned} \tag{1.10}$$

where:

$$k_\alpha(M) = \left[(1-M) + \frac{C_{n\alpha}}{2} M^2 \beta_M (A_1 b_1 + A_2 b_2)\right]^{-1} = \left[(1-M) + \frac{C_{n\alpha}}{2} M^2 \beta_M 0.413\right]^{-1} \tag{1.11a}$$

$$k_q(M) = \left[(1-M) + C_{n\alpha} M^2 \beta_M (A_1 b_1 + A_2 b_2)\right]^{-1} = \left[(1-M) + C_{n\alpha} M^2 \beta_M 0.413\right]^{-1} \tag{1.11b}$$

$$T_I = \frac{c}{a_s} \tag{1.11c}$$

where a_s is the speed of sound.

Note that Leishman (2011) recommends the use of the factor 0.75 for $T_\alpha(M)$ and $T_q(M)$ to account for three-dimensional effects not included in piston theory.

For the circulatory component of the aerodynamic force response, the lumped approach can lead to a direct solution of $C^c_{n_{\alpha,q}}(s,M)$. Considering the circulatory part $C^c_{n_\alpha}(s,M)$ of Eq. (1.6) for the response to the step in α, it can be written:

$$\begin{aligned} C^c_{n_\alpha}(s,M) &= \textstyle\int_{s_0}^{s} \frac{C_{n\alpha}}{\beta_M} \phi^c_\alpha \alpha(s) \mathrm{d}s \simeq C^c_{n\alpha}(s,M)\Delta\alpha \\ \text{where} \quad C^c_{n\alpha}(s,M) &= \tfrac{C_{n\alpha}}{\beta_M} \end{aligned} \tag{1.12}$$

By using Eq. (1.4) with $\phi(s,M)$ replaced by ϕ^c_α and $\varepsilon(s)$ by α, Eq. (1.12) rewrites:

$$C^c_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\left[\alpha(s_0)\phi^c_\alpha(s) + \int_{s_0}^{s} \frac{\mathrm{d}\alpha}{\mathrm{d}\sigma_s}(\sigma_s)\,\phi^c_\alpha(s-\sigma_s,M)\mathrm{d}\sigma_s\right] = C^c_{n\alpha}(s,M)\alpha_e \tag{1.13}$$

where α_e is an effective angle of attack at ¾-chord accounting for a step variation in α, pitching rate, heave, and velocity (lumped approach). By applying the first of Eq. (1.9), and setting $s_0 = 0$, Eq. (1.13) can be simplified to arrive at an expression for α_e at the n-th time step, (i.e., α_{e_n}):

$$\alpha_{e_n}(s,M) = (\alpha_n - \alpha_0) - X_{1_n}(\Delta s) - X_{2_n}(\Delta s) \tag{1.14}$$

where the $\int_{s_0}^{s}[...]\mathrm{d}\sigma_s$ was divided into two steps considering a distance interval Δs, (i.e.,$\int_0^s[...]\mathrm{d}\sigma_s$ and $\int_s^{s+\Delta s}[...]\mathrm{d}\sigma_s$), by carrying out the algebra a recursive expression for X_1 and X_2 can be found:

$$\begin{aligned} X_{1n} &= X_{1n-1}\exp\left(-b_1\beta_M^2\Delta s\right) + A_1\exp\left(-b_1\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \\ X_{2n} &= X_{2n-1}\exp\left(-b_2\beta_M^2\Delta s\right) + A_2\exp\left(-b_2\beta_M^2\tfrac{\Delta s}{2}\right)\Delta\alpha_n \end{aligned} \tag{1.15}$$

Note that α_0 was introduced into Eq. (1.14), because α_e is an effective AOI and not AOA.

Similar to the above development, the circulatory contribution to $C^c_{n_{\alpha,q}}(s,M)$ from a step change in q can be derived as:

$$C^c_{n_q}(s,M) = \frac{C^c_{n\alpha}(s,M)}{2} q - X_3(\Delta s) - X_4(\Delta s) \tag{1.16}$$

with

$$\begin{aligned} X_{3n} &= X_{3n-1}\exp\left(-b_1\beta_M^2\Delta s\right) + A_1\exp\left(-b_1\beta_M^2\tfrac{\Delta s}{2}\right)\Delta q \\ X_{4n} &= X_{4n-1}\exp\left(-b_2\beta_M^2\Delta s\right) + A_2\exp\left(-b_2\beta_M^2\tfrac{\Delta s}{2}\right)\Delta q \end{aligned} \tag{1.16a}$$

Eqs. (1.16)-(1.16a) are used by González (2014).

However, following the original LBM method, the lumped approach can account for any effect to α, including step changes in q, so Eq. (1.16) is not necessary, and is virtually included via Eq. (1.13) and (1.14).

The noncirculatory part cannot be handled via the superposition (lumped approach), therefore, the contribution from step changes in α and q need to be kept separate:

$$C^{nc}_{n_{\alpha,q}}(s,M) = C^{nc}_{n\alpha}(s,M) + C^{nc}_{n_q}(s,M) \tag{1.17}$$

Now, using Duhamel's integral (1.4) on the noncirculatory component $C^{nc}_{n\alpha}(s,M)$ [see Eq. (1.17)] with the ϕ^{nc}_{α} from Eq. (1.9), the following can be arrived at:

$$\begin{aligned} C^{nc}_{n\alpha}(s,M) &= \frac{4T_\alpha(M)}{M}\left(K_\alpha - K'_\alpha\right) \\ K'_{\alpha n} &= K'_{\alpha n-1}\exp\left(-\frac{\Delta t}{T_\alpha(M)}\right) + \left(K_{\alpha n} - K_{\alpha n-1}\right)\exp\left(-\frac{\Delta t}{2T_\alpha(M)}\right) \end{aligned} \tag{1.18}$$

Note that in Eq. (1.18), K'_α is the deficiency function for $C^{nc}_{n\alpha}(s,M)$.

For $C^{nc}_{n_q}(s,M)$, an analogous procedure leads to:

$$\begin{aligned} C^{nc}_{n_q}(s,M) &= -\frac{T_q(M)}{M}\left(K_{q_n} - K'_{q_n}\right) \\ K'_{q_n} &= K'_{q_{n-1}}\exp\left(-\frac{\Delta t}{T_q(M)}\right) + \left(K_{q_n} - K_{q_{n-1}}\right)\exp\left(-\frac{\Delta t}{2T_q(M)}\right) \end{aligned} \tag{1.19}$$

So finally, the expression for the total normal force under attached conditions C^{pot}_n can be expressed as:

$$\begin{aligned} & C^{pot}_n = C^{pot,c}_n + C^{pot,nc}_n \\ & C^{pot}_n = C^c_{n_{\alpha,q}}(s,M) + C^{nc}_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\alpha_e + \frac{4T_\alpha(M)}{M}\left(K_{\alpha n} - K'_{\alpha n}\right) + \frac{T_q(M)}{M}\left(K_q - K'_q\right) \\ & \text{with} \\ & C^{pot,c}_n = C^c_{n_{\alpha,q}}(s,M) = C^c_{n\alpha}(s,M)\alpha_e \\ & C^{pot,nc}_n = C^{nc}_{n_{\alpha,q}}(s,M) = \frac{4T_\alpha(M)}{M}\left(K_\alpha - K'_\alpha\right) + \frac{T_q(M)}{M}\left(K_q - K'_q\right) \end{aligned} \tag{1.20}$$

1.1.2 Chordwise Force

The chordwise force can be written as in Eq. (1.21) from Leishman (2011):

$$C^{pot}_c = C^{pot,c}_n \tan\left(\alpha_e + \alpha_0\right) \tag{1.21}$$

In potential flow, D'Alambert's paradox leads to the absence of drag; therefore, from Eq. (1.3), $C_l \cos\alpha = C_n$ and $C_c = C_l \sin\alpha$, which bring forth Eq. (1.21). Because α_e is a virtual angle of incidence at ¾-chord, we needed to add the α_0. Because this drag treatment has roots only in the circulatory lift derivation, the noncirculatory part is dropped as shown in Eq. (1.21).

1.1.3 Pitching Moment

Analogous to the normal force treatment, the pitching moment coefficient about the 1/4-chord can be derived via indicial response as shown in Eq. (1.22):

$$
\begin{aligned}
C_{m_{\alpha,q}}(s,M) &= C_{m_\alpha}(s,M) + C_{m_q}(s,M) = C_{m\alpha}\alpha + C_{mq}(s,M)q \\
C_{m_\alpha}(s,M) &= -\frac{1}{M}\phi^{nc}_{m,\alpha} - \frac{C_{n\alpha}}{\beta_M}\phi^{c}_{\alpha}\left(\hat{x}_{AC} - 0.25\right) + C_{m0} \\
C_{m_q}(s,M) &= -\frac{7}{12M}\phi^{nc}_{m,q} - \frac{C_{n\alpha}}{16\beta_M}\phi^{c}_{m,q}
\end{aligned}
\tag{1.22}
$$

where $\hat{x}_{AC}$ is the aerodynamic center distance from LE in percent chord, C_{m0} (the 2-D pitching moment coefficient at 0-lift, positive if nose up) is positive if it causes a pitch up of the airfoil, as seen in Figure 3. Also note that the circulatory component of the pitching moment response to a step change in α is a function of the $C^c_{n_\alpha}(s,M)$.

The indicial response can be approximated [see also Eq. (1.9)] as:

$$
\phi^{nc}_{m,q} = \exp\left(-\frac{s}{T'_{m,q}}\right) \tag{1.23}
$$

Analogous expressions can be found for $\phi^{c}_{m,q}$ and $\phi^{nc}_{m,\alpha}$, but they are not shown here because further simplified expressions will be derived below. In Eq. (1.23), $T'_{m,q}$ is the Mach-dependent time constant in the expression of $\phi^{nc}_{m,q}$, whose expression can be derived in a similar fashion to those of the constants in Eq. (1.10):

$$
T_{m,q}(M) = \frac{c}{2U}T'_{m,q} = \frac{c}{2Ma_s}T'_{m,q} = k_{m,q}(M)T_I \tag{1.24}
$$

Following Johansen (1999), the circulatory component $C^c_{m_q}(s,M)$ can be written as:

$$
C^c_{m_q}(s,M) = -\frac{C_{n\alpha}}{16\beta_M}\left(q - K'''_q\right)\frac{c}{U} \tag{1.25}
$$

where:

$$
K'''_{q\,n} = K'''_{q\,n-1}\exp\left(-b_5\beta_M{}^2\Delta s\right) + A_5\Delta q_n \exp\left(-b_5\beta_M{}^2\frac{\Delta s}{2}\right) \tag{1.26}
$$

with A_5 and b_5 constants set to 1 and 5, respectively, from experimental results (Leishman 2006).

The noncirculatory component of the pitching moment response to the step change in α, $C^{nc}_{m_\alpha}(s,M)$, writes (Leishman and Beddoes 1986; Johansen 1999):

$$
C^{nc}_{m_\alpha}(s,M) = -\frac{1}{M}\phi^{nc}_{m,\alpha} = -\frac{C^{nc}_{n\alpha}(s,M)}{4} \tag{1.27}
$$

which implies

$$
\phi^{nc}_{m,\alpha} = \phi^{nc}_{\alpha} \tag{1.28}
$$

The other noncirculatory component, $C^{nc}_{m_q}(s,M)$, writes (Leishman 2006):

$$
\begin{aligned}
C^{nc}_{m_q}(s,M) &= -\tfrac{7}{12M}\phi^{nc}_{m,q} = -\tfrac{7k_{m,q}(M)^2T_I}{12M}\left(K_q - K''_q\right) \\
&\text{with:} \\
k_{m,q}(M) &= \tfrac{7}{15(1-M)+1.5C_{n\alpha}A_5b_5\beta_M M^2} \\
K''_{q\,n} &= K''_{q\,n-1}\exp\left(-\tfrac{\Delta t}{k_{m,q}(M)^2T_I}\right) + \left(K_{q_n} - K_{q_{n-1}}\right)\exp\left(-\tfrac{\Delta t}{2k_{m,q}(M)^2T_I}\right)
\end{aligned}
\tag{1.29}
$$

where the same procedure was used to arrive at a deficiency function using Duhamel's integral [Eq. (1.4)] and equating the expressions of the $C_{m_{\alpha,q}}(s,M)$ at $s \to 0$ (see Leishman (2006)).

Finally, the expression for the total pitching moment at 1/4-chord under attached conditions can be expressed as:

$$\begin{aligned}
&C_m^{pot} = C^c_{m_{\alpha,q}}(s,M) + C^{nc}_{m_{\alpha,q}}(s,M) = \\
&C_{m0} - \tfrac{C_{n\alpha}}{\beta_M}\phi^c_\alpha\left(\hat{x}_{AC} - 0.25\right) + \\
&-\tfrac{C_{n\alpha}}{16\beta_M}\left(q - K_q'''\right)\tfrac{c}{U} + \\
&-\tfrac{T_\alpha(M)}{M}\left(K_\alpha - K_\alpha'\right) + \\
&-\tfrac{7k_{m,q}(M)^2 T_I}{12M}\left(K_q - K_q''\right)
\end{aligned} \tag{1.30}$$

Note that the pitching moment treatment is slightly different from what is in AeroDyn v13 and Damiani (2011), and it is more in line with Leishman (2006) and Johansen (1999). If Minnema's (1998) method is used, then the $C^{nc}_{m_q}(s,M)$ [last term in Eq. (1.30)] is to be replaced by:

$$C^{nc}_{m_q}(s,M) = -\frac{7}{12M}\phi^{nc}_{m,q} = -\frac{C^{nc}_{n_q}(s,M)}{4} - \frac{k_\alpha(M)^2 T_I}{3M}\left(K_q - K_q''\right) \tag{1.31}$$

1.2 TE Flow Separation

The basis of this dynamic system is Kirchhoff's theory, which can be expressed as follows (Leishman 2006):

$$\begin{aligned}
C_n(\alpha,f,s,M) &= C^c_{n\alpha}(s,M)(\alpha-\alpha_0)\left(\tfrac{1+\sqrt{f}}{2}\right)^2 \\
C_c(\alpha,f,s,M) &= \eta_e C^c_{n\alpha}(s,M)(\alpha-\alpha_0)\sqrt{f}\tan(\alpha)
\end{aligned} \tag{1.32}$$

where f is the separation point distance from LE in percent chord and η_e the recovery factor $\simeq [0.85-0.95]$ to account for viscous effects at limited or no separation on C_c.

If the airfoil's C_l, C_d, and C_m characteristics are known, then Eq. 1.32 may be solved for f. Leishman (2011) suggests the use of best-fit curves obtained from static measurements on airfoils, of the type:

$$f = \begin{cases}
1 - 0.3\exp\left(\frac{\alpha-\alpha_1}{S_1}\right), \text{if } \alpha_0 \le \alpha \le \alpha_1 \\
1 - 0.3\exp\left(\frac{\alpha_2-\alpha}{S_3}\right), \text{if } \alpha_2 \le \alpha < \alpha_0 \\
0.04 + 0.66\exp\left(\frac{\alpha_1-\alpha}{S_2}\right), \text{if } \alpha > \alpha_1 \\
0.04 + 0.66\exp\left(\frac{\alpha-\alpha_2}{S_4}\right), \text{if } \alpha < \alpha_2
\end{cases} \tag{1.33}$$

S_1-S_2 (and the analogous S_3-S_4 for $\alpha < \alpha_0$) are best-fit constants that define the abruptness of the static stall. α_1 is the angle of attack at f=0.7, (approximately the stall angle) for $\alpha \ge \alpha_0$, whereas α_2 is the angle of attack at f=0.7, for $\alpha < \alpha_0$.

Accounting for unsteady conditions, the TE separation point gets modified because of temporal effects on airfoil pressure distribution and boundary layer response. LE separation occurs when a critical pressure at the leading edge, corresponding to a critical value of the normal force C_{n1}, is reached.

1.2.1 Normal Force

The circulatory normal force needs to be modified to account for the lagged boundary layer response. To arrive at a new expression for C_n, we start by accounting for the separation point location under unsteady conditions, which can be calculated starting from an effective AOI, α_f:

$$\alpha_f = \frac{C'_n}{C^c_{n\alpha}(s,M)} + \alpha_0 \tag{1.34}$$

where an effective C'_n is used and calculated as in Eq. (1.35):

$$\begin{aligned} C'_n &= C_n^{pot} - D_p \\ D_{p_n} &= D_{p_{n-1}} \exp\left(-\tfrac{\Delta s}{T_p}\right) + \left(C_n^{pot}{}_n - C_n^{pot}{}_{n-1}\right) \exp\left(-\tfrac{\Delta s}{2T_p}\right) \end{aligned} \tag{1.35}$$

Note T_p is the boundary-layer, LE pressure gradient time constant in the expression of D_p, which should be tuned based on airfoil experimental data. Johansen (1999) employs two time constants $T_{p\alpha}$ and T_{pq} as the C'_n is separated into two contributions, one from α and from q.

Given the new C'_n, a new formulation can be obtained for f (i.e.,f''), which accounts for delays in the boundary layer and will be used via Kirchhoff's treatment to arrive at the new C_n:

$$\begin{aligned} f'' &= f' - D_f \\ D_{f_n} &= D_{f_{n-1}} \exp\left(-\tfrac{\Delta s}{T_f}\right) + \left(f'_n - f'_{n-1}\right) \exp\left(-\tfrac{\Delta s}{2T_f}\right) \end{aligned} \tag{1.36}$$

where f' is the separation point distance from LE in percent chord under unsteady conditions that can be obtained from the best-fit in Eq. (1.33) replacing α with α_f.

Alternatively, f' can be derived from a direct lookup table of static airfoil data reversing Eq. (1.32). In fact, two values of f' could be calculated: one for C_n (f'_n) and one for C_c (f'_c).

Also note that T_f is a constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f'', and it is associated with the motion of the separation point along the suction surface of the airfoil. T_f gets modified via multipliers (σ_1) that depend on the phase of the separation or reattachment as discussed later; here it suffices noting that T_f can be written as a modified version of the initial value T_{f0}:

$$T_f = T_{f0}/\sigma_1 \tag{1.37}$$

Finally, the normal force coefficient C_n^{fs}, after accounting for separated flow from the TE becomes:

$$C_n^{fs} = C^{nc}_{n_{\alpha,q}}(s,M) + C^c_{n_{\alpha,q}}(s,M)\left(\frac{1+\sqrt{f''}}{2}\right)^2 = C^{nc}_{n_{\alpha,q}}(s,M) + C^c_{n\alpha}(s,M)\alpha_e\left(\frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.38}$$

Note that González (2014) and Sheng, Galbraith, and Coton (2007) propose the corrective factor to be:

$$C_n^{fs} = C^{nc}_{n_{\alpha,q}}(s,M) + C^c_{n\alpha}(s,M)\alpha_e\left(\frac{1+2\sqrt{f''}}{3}\right)^2 + C^c_{n_q}(s,M) \tag{1.39}$$

The last term in Eq. (1.39) is calculated via Eq. (1.16). The corrective factor $\left(\frac{1+2\sqrt{f''}}{3}\right)^2$ was proposed to account for lower values of C_n when f=0 that were seen from experimental data.

1.2.2 Chordwise Force

The along-chord force coefficient analogously becomes:

$$C_c^{fs} = C_c^{pot}\eta_e\left(\sqrt{f''} - 0.2\right) \tag{1.40}$$

where the 0.2 factor is used by González (2014) to modify $\sqrt{f''}$ to account for negative values seen at f=0.

1.2.3 Pitching Moment

Now turning to the pitching moment, the contribution caused by unsteady separated flow is on the circulatory component alone. Leishman (2011) suggests using this formulation that modified C_m^{pot} for the $C_{m\alpha}^c(s,M)$ component:

$$C_m^{fs} = C_{m0} - C_{n_{\alpha,q}}^c(s,M)(\hat{x}_{cp} - 0.25) + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.41}$$

where $\hat{x}_{cp}$ is the center-of-pressure distance from LE in percent chord and can be approximated by (Leishman 2011):

$$\hat{x}_{cp} = k_0 + k_1\left(1 - f''\right) + k_2 \sin\left(\pi f''^{k_3}\right) \tag{1.42}$$

where k_0=0.25-$\hat{x}_{AC}$, and the k_1-k_3 constants are calculated via best fits of experimental data. Other expressions could be used to perform the best fit of $\hat{x}_{cp}$ versus f from static C_m airfoil data.

Minnema (1998) suggests a different approach where an effective lagged AOI is calculated as follows:

$$\begin{aligned} \alpha'_f &= \alpha_f - D_{\alpha f} \\ D_{\alpha f_n} &= D_{\alpha f_{n-1}} \exp\left(-\tfrac{\Delta s}{0.1T_f}\right) + \left(\alpha_{f_n} - \alpha_{f_{n-1}}\right)\exp\left(-\tfrac{\Delta s}{2*0.1T_f}\right) \end{aligned} \tag{1.43}$$

Note that T_f is factored by 0.1 following Minnema (1998) validation results. Then the new AOI is used to derive the contribution to the circulatory component of the pitching moment, which is extracted from a lookup table of static coefficients C_m versus α.

$$C_m^{fs} = C_m\left(\alpha'_f\right) + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.44}$$

The method proposed by González (2014) [see Eq. (1.45)] uses a value of f' (=f'_m) extracted from a static data table where it is assumed that $C_m - C_{m0} = f_m C_n$ (loosely correlating f to $\hat{x}_{cp}$). The angle used for interpolation of the lookup table is α_f. To resolve the contribution to C_m from the unsteady TE separation, this method should then use the full C_n^{fs} from Eq. (1.39) and the lagged f''_m calculated from f'_m and Eq. (1.36), replacing f' with f'_m. However, for the current implementation of the UAM, there is no further lagging of the f'_m quantity. This may change in future versions of UAM.

$$C_m^{fs} = C_{m0} + C_n^{fs} f'_m + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) \tag{1.45}$$

In this case, the two treatments (Minnema 1998; González 2014) seem somewhat equivalent, with the exception that González (2014) proposes 21 (7 for each f'' related to C_n, C_c, and C_m) different multipliers for T_f depending on the state of the airfoil aerodynamics (e.g., increasing AOA and above a critical C_{n1}, increasing AOA and below a critical C_{n1}).

1.3 Dynamic Stall

1.3.1 Normal Force

During dynamic stall, there is shear layer roll up at the leading edge with associated vortex formation, and vortex travel over the upper surface of the airfoil that will be subsequently shed in the wake. The main condition to be met for the shear layer roll up is:

$$\begin{array}{lll} C'_n > C_{n1} & \text{for} & \alpha \geq \alpha_0 \\ C'_n < C_{n2} & \text{for} & \alpha < \alpha_0 \end{array} \tag{1.46}$$

The normal force coefficient contribution from the additional lift associated with the low pressure LE vortex can be written as (Leishman (2011)):

$$C^v_{n,n} = C^v_{n,n-1} \exp\left(-\frac{\Delta s}{T_V}\right) + (C_{V\,n} - C_{V\,n-1}) \exp\left(-\frac{\Delta s}{2T_V}\right) \tag{1.47}$$

T_V is the time constant associated with the vortex lift decay process, and it depends on Re, M, and airfoil type. Note that the C^v_n contribution is not allowed to have a sign opposite to that of C^{fs}_n.

T_V gets modified via a multiplier σ_3 to account for various stages of the process as discussed later, but here it suffices to say that:

$$T_V = T_{V0}/\sigma_3 \tag{1.48}$$

C_V represents the contribution to the normal force coefficient due to accumulated vorticity in the LE vortex. C_V is modeled proportionally to the difference between the attached and separated circulatory contributions to C_n:

$$C_V = C^c_{n_{\alpha,q}}(s,M) - C^c_{n_{\alpha,q}}(s,M)\left(\frac{1+\sqrt{f''}}{2}\right)^2 = C^c_{n\alpha}(s,M)\alpha_e\left(1-\frac{1+\sqrt{f''}}{2}\right)^2 \tag{1.49}$$

If the method proposed by González (2014) is used, then C_V can be written as:

$$C_V = C^c_{n\alpha}(s,M)\alpha_e\left(1-\frac{1+2\sqrt{f''}}{3}\right)^2 \tag{1.50}$$

The position of the LE vortex along the chordwise direction is tracked via a nondimensional time variable, τ_V, defined in Eq. (1.51):

$$\tau_V = t\frac{2U}{c} \tag{1.51}$$

If τ_V=0, the vortex is at the LE; if τ_V=T_{VL}, the vortex is at the TE.

If $\tau_V > T_{VL}$ and if α_f is not moving away from stall (i.e.,$[(\alpha_f - \alpha_0) * (\alpha_{f\,n} - \alpha_{f\,n-1})] > 0)$, then the vorticity is no longer allowed to accumulate, in which case Eq. (1.47) can be rewritten as:

$$\begin{array}{rll} & C^v_{n,n} & = C^v_{n,n-1}\exp\left(-\frac{\Delta s}{T_{V0}/\sigma_3}\right) \\ \text{with} & \sigma_3 & = 2 \end{array} \tag{1.52}$$

where the decay of the normal force (due to vorticity at the LE) is accelerated at twice the original rate and no further accretion of vorticity is allowed. Eq. (1.52) should also be used when conditions in Eq. (1.46) are not met. Note that T_{VL} represents the time constant associated with the vortex advection process; it represents the nondimensional time in semichords, needed for a vortex to travel from LE to TE; it is used in the expression of C^v_n; it depends on Re, M (weakly), and airfoil. Value's range $= [6;13]$

Finally, the total normal force can be written as:

$$C_n = C^{fs}_n + C^v_n = C^c_{n\alpha}(s,M)\alpha_e\left(\frac{1+\sqrt{f''}}{2}\right)^2 + C^{nc}_{n_{\alpha,q}}(s,M) + C^v_n \tag{1.53}$$

Again, if González's (2014) method is used, then the correction factor for the separated flow treatment is slightly modified as in Eq. (1.50), for example:

$$C_n = C_n^{fs} + C_n^v = C_{n\alpha}^c(s,M)\alpha_e \left(\frac{1+2\sqrt{f''}}{3}\right)^2 + C_{n_{\alpha,q}}^{nc}(s,M) + C_n^v \tag{1.53b}$$

Note that multiple vortices can be shed at a given shedding frequency corresponding to:

$$T_{sh} = 2\frac{1-f''}{St_{sh}} \tag{1.54}$$

Therefore, τ_V is reset to 0 if $\tau_V = T_{VL} + T_{sh}$.

1.3.2 Chordwise Force

The along-chord force coefficient gets modified by the presence of the LE vortex as in Pierce (1996):

$$C_c = C_c^{fs} + C_n^v \tan(\alpha_e)\left(1 - \frac{\tau_V}{T_{VL}}\right) \tag{1.55}$$

Note that in the current release of UA, the $\tan(\alpha_e) \simeq \alpha_e$ approximation is made. This may be changed after testing in future releases.

González (2014) does not contain the vortex contribution to C_c based on experimental validation:

$$C_c = C_c^{fs} \tag{1.55b}$$

The original Leishman and Beddoes (1989) model had C_c written as:

$$C_c = \begin{cases} \eta_e C_c^{pot}\sqrt{f''}\sin(\alpha_e + \alpha_0) & , \quad C_n' \le C_{n1} \\ \hat{k}_1 + C_c^{pot}\sqrt{f''}f''^{\hat{k}_2}\sin(\alpha_e + \alpha_0) & , \quad C_n' > C_{n1} \end{cases} \tag{1.56a}$$

$$\text{with:} \quad \hat{k}_2 = 2(C_n' - C_{n1}) + f'' - f \tag{1.56b}$$

where $\hat{k}_1$ is a constant required to fit the C_c curve under static conditions for 2-D airfoils.

1.3.3 Pitching Moment

Leishman (2011) offers a form for the $\hat{x}_{cp}^v$, which is the center-of-pressure distance from the ¼-chord, in percent chord, during the LE vortex advection process:

$$\begin{aligned} C_m^v &= -\hat{x}_{cp}^v C_n^v \\ \hat{x}_{cp}^v(\tau_V) &= \bar{\bar{x}}_{cp}\left(1 - \cos\left(\frac{\pi\tau_V}{T_{VL}}\right)\right) \end{aligned} \tag{1.57}$$

where $\bar{\bar{x}}_{cp}$ is a constant in the expression of $\hat{x}_{cp}^v$, usually equal to 0.2.

Finally, the expression for the total pitching moment can be written as:

$$C_m = C_{m0} - C_{n_{\alpha,q}}^c(s,M)(\hat{x}_{cp} - 0.25) + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.58}$$

If Minnema's (1998) approach is used then Eq. (1.58) can be rewritten as:

$$C_m = C_m\left(\alpha_f'\right) + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.59}$$

and if González's (2014) treatment is used, the total moment becomes:

$$C_m = C_n f_m'' + C_{m_q}^c(s,M) + C_{m\alpha}^{nc}(s,M) + C_{m_q}^{nc}(s,M) + C_m^v \tag{1.60}$$

2 Inputs, Outputs, Parameters, States, and Implementation of UA

The UA implementation loosely follows the FAST modularization framework. This includes key interface routines for module initialization (UA_Init), updating the module's states (UA_UpdateStates), and generating module outputs (UA_CalcOutput). Because the UAM module does not communicate directly with the FAST glue code, we have taken some liberties with regards to the arguments to these interface routines. Additionally, the typical framework data structures for inputs (u) and outputs (y) have been modified to allow for a more intuitive integration into either a BEMT/dynamic blade element momentum theory (DBEMT) algorithm or the generalized dynamic wake (GDW) algorithm. In the following sections, the inputs/states/parameters are listed, then each interface routine will be discussed, and any deviations from the FAST framework will be highlighted.

To provide the context for the UAM, the flow diagram from AeroDyn to UA is shown in Figure 4.

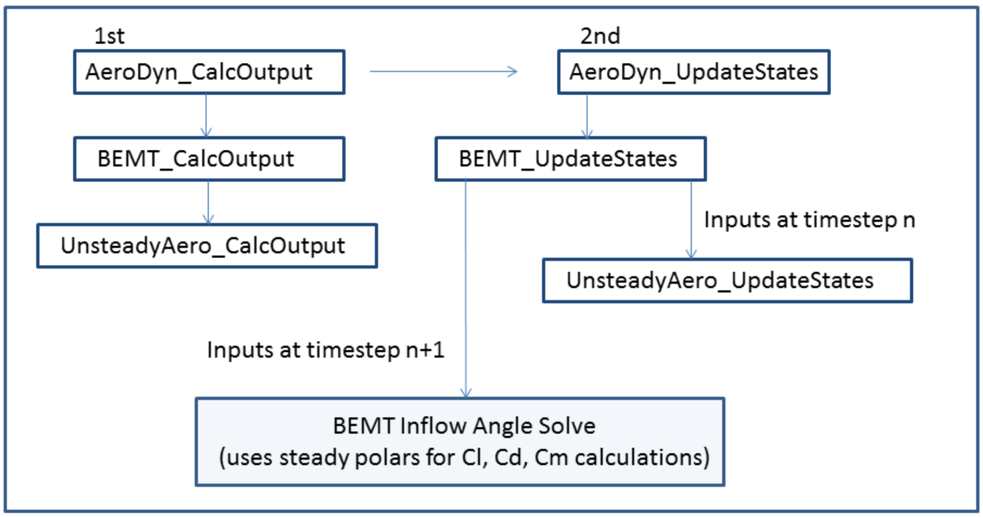

Figure 4. Block diagram showing the order of the calls to the subroutines (AeroDyn_CalcOutput happens first, AeroDyn_UpdateStates happens second) and the overall organization from the parent module AeroDyn to UA

2.1 Init_Inputs

Besides the standard variables common to all modules (e.g.,$NumOuts$ (number of output channels), the Init_Inputs to the UA are:

- Δt - time step
- a_s - speed of sound
- $UAmod$ - switch to select handling of options and possible methods in the UA treatment
- $flookup$ - logical flag to indicate whether a lookup (True) or an interpolation of the airfoil data tables (False) is used to retrieve the values for f
- $nNodesPerBlade$ - number of nodes per blade (used for array allocation)
- $NumBlades$ - number of blades (used for array allocation)
- c - airfoil chord at each blade station for each blade.

2.2 Inputs u

The inputs to the UA are:

- α
- U
- Re.

Note: these are for a given node (within a given blade).

2.3 Outputs y

The outputs from the UA are:

- C_n
- C_c
- C_m
- C_l
- C_d.

Note: these are for a given node (within a given blade).

2.4 States x_d

The states for the UA are:

Discrete states:

- $\alpha_{,-1}$ - previous time-step value of α
- $\alpha_{LP,-1}$ - previous time-step value of low-pass-filtered α
- $\alpha_{f,-1}$ - previous time-step value of α_f
- $q_{,-1}$ - previous time-step value of q
- $q_{LP,-1}$ - previous time-step value of low-pass-filtered q
- $K_{\alpha LP,-1}$ - previous time-step value of low-pass- filtered K_α
- $K_{qLP,-1}$ - previous time-step value of low-pass-filtered K_q
- $X_{1,-1}$ - previous time-step value of X_1
- $X_{2,-1}$ - previous time-step value of X_2
- $X_{3,-1}$ - previous time-step value of X_3
- $X_{4,-1}$ - previous time-step value of X_4
- $K'_{\alpha,-1}$ - previous time-step value of K'_α
- $K'_{q,-1}$ - previous time-step value of K'_q
- $K''_{q,-1}$ - previous time-step value of K''_q
- $K'''_{q,-1}$ - previous time-step value of K'''_q

- $D_{p,-1}$ - previous time-step value of D_p
- $D_{f,-1}$ - previous time-step value of D_f
- $D_{f_c,-1}$ - previous time-step value of D_{f_c} (D_{f_c} is the deficiency function for f'_c analgous to D_f)
- $C_n^{pot}{}_{,-1}$ - previous time-step value of C_n^{pot}
- T_f - constant dependent on Mach, Re, and airfoil shape; it is used in the expression of D_f and f''
- $f'_{,-1}$ - previous time-step value of f'
- $f'_{c,-1}$ - previous time-step value of f'_c
- $f''_{,-1}$ - previous time-step value of f''
- $f''_{c,-1}$ - previous time-step value of f''_c (f''_c is the lagged version of f'_c)
- τ_V - time variable that tracks the travel of the LE vortex over the airfoil suction surface. It is made dimensionless via the semichord: $\tau_V = t * 2U/c$. If less than $2T_{VL}$, it renders the logical flag $VRTX$=True; if less than T_{VL}, then the vortex is still on the airfoil
- $\tau_{V,-1}$ - previous time-step value of τ_V
- $C^v_{n,-1}$ - previous time-step value of C^v_n
- $C_{V,-1}$ - previous time-step value of C_V
- $D_{\alpha f,-1}$ - previous time-step value of $D_{\alpha f}$

The FAST 8 framework does not allow logicals or discontinuous variables within states. For this reason, the following are declared as either *Other States* or *miscVars* (other states that are NOT used for linearization).

Other states:

- σ_1 - generic multiplier for T_f
- σ_3 - generic multiplier for T_V

miscVars:

- $iBlade$ - blade index
- $jBladeNode$ - blade node index
- $TESF$ - trailing-edge separation flag
- $LESF$ - leading-edge separation flag
- $VRTX$ - vortex advection flag
- $FirstPass$ - flag indicating first time step

Note that, in contrast to inputs and outputs, the states must be tracked by the UA module, therefore they are a 2-D array (per blade, per node).

2.5 Parameters p

The parameters for the UA are:

- Δt - time step
- c - chord length

- $UAmod$ - switch to select handling of options and possible methods in the UA treatment
- a_s - speed of sound
- $flookup$ - logical flag to indicate whether a lookup (True) or an interpolation of the airfoil data tables (False) is used to retrieve the values for f
- ζ_{LP} - low-pass filter frequency cutoff (-3 dB)

An airfoil data structure (AFI_{Params}) is passed directly to the UAM framework routines, and is indexed to the airfoil of interest. AFI_{Params} contains airfoil-specific quantities, i.e., parameters and constants for the UA, alhtough it is not formally a parameter of the FAST8 framework:

- α_0, α_1, α_2, $C_{n\alpha}$, C_{n1}, C_{n2}, η_e, C_{d0}, C_{m0}, $\bar{\bar{x}}_{cp}$, St_{sh}
- A_1,b_1,A_2,b_2,A_5,b_5
- S_1, S_2, S_3, S_4
- T_p (fairly independent of airfoil type)
- T_{f0}, T_{V0}, T_{VL}
- k_0, k_1, k_2, k_3
- $\hat{k}_1$.

These parameters were introduced in this manual, and their meanings are provided in the list of symbols at the beginning of the document.

2.6 UA Implementation

2.6.1 UA_Init Routine

This routine allocates the module's data structures, initializes the module's states, and sets the nontime-varying parameters (copies them from the initialization input data section).

2.6.2 UA_UpdateStates Routine

The typical list of arguments to UA_UpdateStates gets augmented to pass indices to the blade and blade node of interest and the structure AFI_{Params}, which contains the airfoil data.

The model is of the parsimonious, open loop, Kelvin-chain kind. Outputs of each subsystem serve as inputs to the next subsystem. There are no differential equations to solve. There is no solver per se, for this reason states are discrete states only (see Section 2.4 for other states).

2.6.2.1 UAmod Logical Flags

The options implemented in the code are selected via the $UAmod$ switch and $flookup$ flag:

- $UAmod$=1: closest model to the original Leishman-Beddoes formulation
- $UAmod$=2: modifications to the original model and simplifications following González (2014)
- $UAmod$=3: modifications to the original model and simplifications following Pierce (1996) and Minnema (1998)
- $flookup$=True: Eq. (1.33) gets replaced by lookup values for f'_n f'_c. Note that if $UAmod$=2 or 3, the flag is automatically set to True.

In what follows, the modifications to the algorithm for *UAmod*=2 or 3 are given with respect to the sequence of equations used for *UAmod*=1.

If *UAmod*=2, then:
Replace Eq. (1.58) with Eq. (1.60)
Replace Eq. (1.56) with Eq. (1.55b)
Replace Eq. (1.53) with Eq. (1.53b)
Replace Eq. (1.49) with Eq. (1.50)
Replace Eq. (1.41) with Eq. (1.45)
Replace Eq. (1.38) with Eq. (1.39)
Add Eq. (1.16) to $C^c_{n_{\alpha,q}}(s,M)$ Eq. (1.13).

If *UAmod*=3, then:
Replace Eq. (1.56) with Eq. (1.55)
Replace Eq. (1.58) with Eq. (1.59)
Replace Eq. (1.41) with Eq. (1.43)-(1.44)
Replace Eq. (1.30) with Eq. (1.31).

If CCLBMswitch, then:
Replace Eq. (1.55) with Eq. (1.56)

2.6.2.2 Update Discrete States

For a given set of inputs, (u), and at the current step in time, (t), the Kelvin chain is performed through the following equations:

- Eq. (1.11c)
- Eq. (1.5b)
- Eq. (1.7)-(1.8)
- Eq. (1.11a)
- Eq. (1.11b)
- Eq. (1.10)
- Eq. (1.37)
- Eq. (1.48)
- Eq. (1.18)
- Eq. (1.19)
- Eq. (1.17)
- Eq. (1.15)
- Eq. (1.14)
- Eq. (1.13)
- Eq. (1.26)
- Eq. (1.25)
- Eq. (1.20)
- Eq. (1.29)

- Eq. (1.21)
- Eq. (1.35)
- Eq. (1.34)
- Eq. (1.33)
- Eq. (1.36)
- Eq. (1.38) [or Eq. (1.39)]
- Eq. (1.49) [or Eq. (1.50)]
- Eq. (1.47) [or Eq. (1.52)].

2.6.2.3 Update Other States

- If $C_n' > C_{n1}$ ($C_n' < C_{n2}$ for $\alpha < \alpha_0$), then:
  $LESF$=True: this means LE separation can occur
  ELSE $LESF$=False: this means reattachment can occur
- If $f''_t < f''_{t-1}$, then:
  $TESF$=True: this means TE separation is in progress
  ELSE $TESF$=False: this means TE reattachment is in progress
- If $0 < \tau_V \leq 2T_{VL}$, then:
  $VRTX$=True: this means vortex advection is in progress
  else $VRTX$=False: this means vortex is in wake
- If $\tau_V \geq 1 + \dfrac{T_{sh}}{T_{VL}}$ and $LESF$=True, then:
  τ_V is reset to 0.

2.6.2.3.1 T_f modifications

The following conditional statements operate on a multiplier σ_1 that affects T_f, i.e., the actual T_f is given by Eq. (1.37).

$$T_f = T_{f0}/\sigma_1 \qquad \text{(1.37 revisited)}$$

where T_{f0} is the initial value of T_f
σ_1 = 1 (initialization default value)
$\Delta_{\alpha 0}$ = α-α_0
If $TESF$=True, then: (separation)
If $K_\alpha \Delta_{\alpha 0} < 0$, then: σ_1 = 2 (accelerate separation point movement)
else if $LESF$=False, then: σ_1 = 1 (default value, LE separation can occur)
else if $f''_{n-1} \leq 0.7$ then: σ_1 = 2 (accelerate separation point movement if separation is occurring)
else σ_1 =1.75 (accelerate separation point movement)
Else: (reattachment, this means $TESF$=False)
IF $LESF$ = False, then: σ_1 = 0.5 (default: slow down reattachment)
IF $VRTX$=True and $0 \leq \tau_V \leq T_{VL}$, then:
    σ_1 = 0.25 - No flow reattachment if vortex shedding is in progress
If $K_\alpha \Delta_{\alpha 0} > 0$, then: σ_1 = 0.75.
Note the last three conditional statements are separate "ifs."

Although this logic was tested and proved to be effective, the current version of UA uses a simpler version.

2.6.2.3.2 T_V modifications

For T_V, an analogous set of conditions is used to set the proper value of the time constant depending on subsystem stages:

σ_3=1 (initialization default value)
If $T_{VL} \leq \tau_V \leq 2T_{VL}$, then
  σ_3=3 (postshedding)
  If $TESF$=False, then:
    σ_3=4 (accelerate vortex lift decay)
    If $VRTX$=True and $0 \leq \tau_V \leq T_{VL}$, then:
      If $K_\alpha \Delta_{\alpha 0} < 0$, then σ_3=2 (accelerate vortex lift decay)
      else σ_3=1 (default)
Else if $K_\alpha \Delta_{\alpha 0} < 0$, then: σ_3=4 (vortex lift must decay fast)
If $TESF$=False and $K_q \Delta_{\alpha 0} < 0$, then: σ_3=1 (default).

2.6.2.3.3 Update 'previous time step' states

After the states are updated to the next time step (t+1) values, the current values at time step, t, are stored into the (t-1) states.

2.6.3 UA_CalcOutput

This routine determines the outputs, C_n, C_c (and the transformed versions, C_l and C_d), and C_m given the inputs of U, α, and Re (currently unused), for a given blade element. Because the routine only generates outputs for a specific blade element, the FAST framework arguments are augmented to include indices to the blade and blade node of interest. The routine uses the same Kelvin chain as in UA_UpdateStates, however, the state variables themselves are not updated during these calculations.

For the first time step, outputs are determined by static lookup tables.

The equations implemented in this routine are the same as in Section 2.6.2.2, plus the following:

- Eq. (1.53)
- Eq. (1.55)
- Eq. (1.2)
- Eq. (1.58).

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