

How I have solved the task

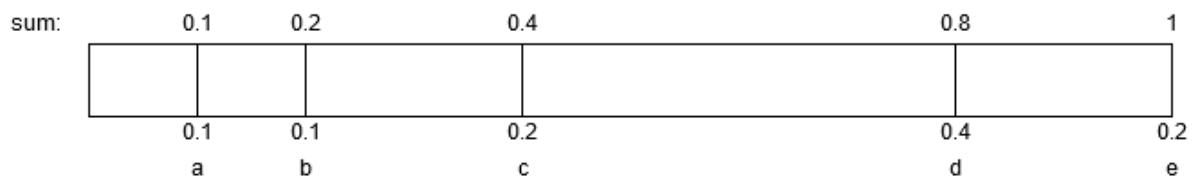
Analysing the zeroth order source statistics

- Create an array the same size as the number of characters in our alphabet (also keep a reference to which character is represented by which slot in the array).
- Go through the text and store the number of times each character (only those in our alphabet) occurs.
- Then divide the stored numbers by the total of characters analysed to get the probability of each.

Generate text based on the zeroth order statistics

- Uniformly generate a number between 0 and 1 (exclusive)
- Sum the probabilities until the sum is strictly greater than the randomly generated number and append the character whose probability was last added to the sum.
- Example (using a smaller alphabet for simplicity) :

Probabilities:		Random generated nr:	Result:
a	0.1	0.33	c
b	0.1	0.1	b
c	0.2	0.0	a
d	0.4	0.02	a
e	0.2	0.99	e



Analysing orders > zeroth

- Create a transition matrix with initially zero rows
 - o It can store unique prefixes as rows, with columns of possible suffixes and how many times they have occurred.
 - o For each prefix, also store the total of transitions recorded
- Start by looking at the character at position n , where n is equal to the order. If it is not in the alphabet, continue to the next one.
- Then, check if the n previous characters are all in the alphabet. Continue to next position if any are not in the alphabet.
- Check if the transition matrix has a row for the current prefix.
 - o If it does not, add a new row with the prefix, and the suffix character as a possible transition
 - o If it exists, add an occurrence of the suffix character.

Generate text based on order > zeroth

- Analyse the source statistics from zeroth to the desired order
- Generate the first character based on the zeroth order statistics, second character based on 1st order, third character based on 2nd order etc, up to desired order -1.

- Generate the rest of the text based on the statistics of the given order.
- Characters are generated in a similar manner as described for zeroth order;
 - o A random number between 0 and the total occurrences of suffixes is generated.
 - o Sum the number of occurrences for each suffix until the sum is greater than the random number.
 - o Append the last suffix of which occurrence number was added.

Observation

- Zeroth order
 - o The text does not make any sense, but the frequency of spaces seems to be about correct for Norwegian text.
- 1st order
 - o Some parts of the text are actual words, but mostly, it's not readable
- 2nd order
 - o About half of the "words" are actual words
- 3rd order
 - o Almost all the text is made up of readable words.

Are they unifilar?

At order $n > 0$, they are unifilar because all states produce distinct symbols, which determine the next state. I do not know how the unifilar property is defined for a source which has no state, as with zeroth order.

Entropy

Zeroth order

Entropy calculated by

$$H(s) = \sum_{i=1}^a P_i * \log_2 \frac{1}{P_i}$$

where a is the alphabet size.

Result: 3.999233313493003

1st Order

$$H_{\infty}(U) = - \sum_{k=1}^r W_k \sum_{i=1}^{n_k} P_{kk_i} \log P_{kk_i}$$

Result: 3.024102190660886