def update weights(m, b, X, Y, learning rate): m deriv = 0.5b deriv = 0.5N = len(X)for i in range(N): # Calculate partial derivatives # -2x(y - (mx + b)) $m \ deriv += -2*X[i] * (Y[i] - (m*X[i] + b))$ # -2(y - (mx + b))b deriv += -2*(Y[i] - (m*X[i] + b))# We subtract because the derivatives point in direction of steepest ascent m -= (m deriv / float(N)) * learning rate b -= (b deriv / float(N)) * learning rate return m, b Gradient descent algorithm Initial Gradient weight Global loss minimum import numpy as np import matplotlib.pyplot as plt X=np.array([0.5,2,2.5,3,3.5,2.75,3.25,1.3,1.4,1.7,1.6])In [4]: Y=np.array([54,65,69,71,75,77,78,62,63,61,59]) i = 0m, b=0.01, 50while i<400: m, b=update weights (m, b, X, Y, 0.001) print('Iteration:{} {},{}'.format(i,m,b)) i=i+1Iteration: 0.09392004545454545, 50.03341181818182 Iteration:1 0.17679747956219005,50.06639824526033 Iteration: 2 0.2586452995402268,50.09896458681168 Iteration: 3 0.33947634057105236,50.1311160822673 Iteration: 4 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6.779853214935973,52.44740656191473 Iteration: 393 6.78072486992582,52.446997830508906 Iteration: 394 6.781588924794759,52.446586192221844 Iteration: 395 6.782445473457126,52.44617168534055 Iteration: 396 6.783294608656647,52.44575434767419 Iteration: 397 6.784136421981034,52.445334216560035 Iteration:398 6.784971003876399,52.44491132886936 Iteration: 399 6.785798443661482,52.44448572101324 Out[10]: 6.785798443661482 Out[11]: 52.44448572101324 Out[6]: array([0.5 , 2. , 2.5 , 3. , 3.5 , 2.75, 3.25, 1.3 , 1.4 , 1.7 , 1.6]) Out[7]: array([54, 65, 69, 71, 75, 77, 78, 62, 63, 61, 59]) #m,b=0.09392004545454545,50.03341181818182 Yhat=X*m+b plt.scatter(X,Y,color='r') plt.plot(X,Yhat,color='b') Out[12]: [<matplotlib.lines.Line2D at 0x212916b3a00>] 75 70 65 60 55 1.0 1.5 2.0 2.5 * HAPPY LEARNING**