MAP: for an exponential prior on the average of a gaussian distrib°:

$$x_i \sim \mathcal{N}(\mu, \sigma^2)$$

$\hookrightarrow p(\mu) = \lambda e^{-\lambda \mu}$

$$\theta^*_{MAP} = \operatorname{argmax}_\theta P(\theta | X) = \operatorname{argmax} \left(\frac{P(X|\theta) \cdot P(\theta)}{P(X)} \right)$$

$$= \operatorname{argmax} \left(\underbrace{\log P(X|\theta)}_{\text{as MLE}} + \underbrace{\log P(\theta)}_{\text{prior}} \right)$$

$$\theta^*_{MAP} = \operatorname{argmax}_{\mu,\sigma} \underbrace{\left(\log \frac{1}{\sqrt{2\pi}\sigma} - \sum_{i=1}^{N} \frac{(x_i - \mu)^2}{2\sigma^2} + \log \lambda - \lambda \mu \right)}_{J}$$

$$\mu : \frac{\partial J}{\partial \mu} = 0 \iff 0 = 0 + \frac{\cancel{2}}{\cancel{2}} \frac{\sum_{i=1}^{N} (x_i - \mu)}{\sigma^2} + 0 - \lambda$$

$$0 = \sum_{i=1}^{N} (x_i - \mu) - \lambda \sigma^2$$

$$\mu^* = \frac{1}{N} \sum_{i=1}^{N} x_i - \frac{\lambda \sigma^2}{N}$$

Remark: as always, when $N \to \infty$, the prior (λ) becomes irrelevant. Also, if $\sigma^2 \to 0$, the x_i's are narrowly distributed, the prior is irrelevant.

Also, if $\lambda \to 0$ (average of expo. dist is $\to \infty$) then we have a (kind of) flat prior (although, on the half-space $\mu > 0$ only)

Remark: it would have make more sense to use this prior on a positive-definite variable, like σ^2.