

[DL] A Survey of FPGA Based Neural Network Accelerator

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Recent researches on neural network have shown great improvement in computer vision over traditional algorithms based on handcrafted features and models. Neural network is now greatly adopted in regions like image, speech and video recognition. But the great computation and storage complexity of neural network based algorithms poses great difficulty on its application. CPU platforms are hard to offer enough computation capacity. While GPU platforms are highly parallelized, the energy efficiency is low. The high energy cost of GPU causes problems for a wide application of neural network.

To address the above problems, various FPGA based hardware accelerators for neural networks have been proposed. Specialized hardware are designed to achieve high speed and low power neural network process. In this paper, we give an overview of previous work on neural network accelerators based on FPGA and summarize the main techniques used. Investigation from software to hardware, from circuit level to system level is carried out to complete analysis of FPGA based neural network accelerator design and serves as a guide to future work.

Additional Key Words and Phrases: FPGA, Neural Network

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1 INTRODUCTION

Recent research on Neural Network (NN) is showing great improvement over traditional algorithms in computer vision. Various network models, like convolutional neural network (CNN), recurrent neural network (RNN), have been proposed for image, video, and speech process. CNN [21] improves the top-5 image classification accuracy on ImageNet [38] dataset from 73.8% to 84.7% and further helps improve object detection [8] with its outstanding ability in feature extraction. RNN [15] achieves state-of-the-art word error rate on speech recognition. In general, NN features a high fitting ability to a wide range of pattern recognition problems. This makes NN a promising candidate to many artificial intelligence applications.

But the computation and storage complexity of NN models are high. The research on NN is also increasing the size of NN models. The largest neural network model for an 224×224 image classification requires upto 39 billion floating point operations (FLOP) and more than 500MB model parameters [42]. As the computation complexity is proportional to the input image size, processing images with higher resolutions may need more than 100 billion operations.

Traditional hardware platforms are not suitable for neural network process. A common CPU can perform 10-100G FLOP per second, and the power efficiency is usually below 1GOPS/W. So CPUs neither meet the high performance requirements in cloud applications nor the low power

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requirements in mobile applications. In contrast, GPUs offer upto 10TOP/s peak performance and is a good choice for high performance neural network applications. Development frameworks like Caffe [19] and Tensorflow [3] also offers easy-to-use interfaces which makes GPU the first choice of neural network acceleration. But GPUs are power consuming and thus not suitable for mobile applications.

On the other hand, FPGA is becoming a candidate to implement energy efficient neural network accelerator. With a specific hardware design, FPGAs are able to implement high parallelism and make use of the properties of neural network computation to remove unnecessary logic. Therefore FPGAs are possible to achieve higher energy efficiency compared with CPU and GPU.

But FPGA based accelerator designs are still faced with two problems:

- Current FPGAs usually support working frequency at 100-300MHz, which is much less than CPU and GPU. The FPGA's logic overhead for reconfigurability also reduces the overall system performance. Straight forward design on FPGA is hard to achieve high performance and high energy efficiency.
- Implementation of neural networks on FPGAs is much harder than that on CPUs or GPUs. Development framework like Caffe and Tensorflow for CPU and GPU is needed for FPGA.

Many researches on the above two problems have been carried out for energy efficient and flexible FPGA based neural network accelerator. In this paper, we summarize the techniques proposed in these work. Specifically, we will introduce the techniques from the following aspects:

- We investigate current techniques for high performance and energy efficient neural network accelerator designs. Techniques in both software level and hardware level are evaluated.
- We investigate state-of-the-art automatic design methods of FPGA based neural network accelerators.

The rest part of this paper is organized as follows:

2 PRELIMINARY ON NEURAL NETWORK

In this section, we introduce the basic operations included in neural network algorithms. Neural network is a bio-inspired model, which usually includes several layers. Each layer receives input from a set of neurons and output a set of neurons. The synapses connecting input and output neurons are modeled as parameters, which is referred to as weights in this paper. In the rest part of this section, we introduce different types of layers in neural network models.

Fully connected (FC) layer implements a connection between every input neuron and output neuron with a weight. This type of layer is adopted in both CNN and RNN. The input and output neurons of an FC layer are two vectors \mathbf{x} and \mathbf{y} . The weights of this layer can be modeled as a matrix W . A bias vector \mathbf{b} is added to each of the output neuron. The computation of this layer is described as equation 1.

$$\mathbf{x} = W\mathbf{y} + \mathbf{b} \quad (1)$$

Convolution (CONV) layer is used for 2-d neuron process. This is commonly adopted in CNN for image process. The input and output neurons of this layer can be described as sets of 2-d feature maps, F_{in} and F_{out} . Each feature map is denoted as a channel. A CONV layer implements a 2-d convolution kernel K_{ij} for each input and output channel pair and a bias scalar b_i for each output channel. The computation of a CONV layer with M input channels and N output channels can be described as equation 2.

$$F_{out}(j) = \sum_{i=0}^{M-1} \text{conv2d}(F_{in}(i), K_{ij}) + b_j, j = 0, 1, \dots, N-1 \quad (2)$$

There are varieties of 2-d convolutions in CONV layer. Usually standard convolution with padding is used when the kernel size is 3×3 . For larger kernels like 5×5 and 7×7 , a stride larger than 1 is usually used to reduce the number of operation. Recent work is also using 1×1 convolution kernels [16, 18].

Non-linear layer applies a non-linear function on each of the input neurons. Sigmoid function and tanh function are commonly adopted in early models and are still used in RNN for acoustic or speech recognition. Rectified linear unit (ReLU) [21] is the adopted in many state-of-the-art models. This function maintains the positive neurons and filters negative neurons as zero. Varieties of ReLU are also used, such as PReLU and Leaky ReLU [50].

Pooling layer is also used for 2-d neuron process like CONV layer. A pooling layer downsamples each of the input channel respectively, which helps reduce feature dimension. There are two kinds of down sampling method: average pooling and max pooling. Average pooling splits a feature map into small windows, i.e. 2×2 windows, and finds the average value of each window. Max pooling method finds the maximum value in each window. Common window size includes 2×2 , stride=2 and 3×3 , stride=2.

Element-wise layer is usually used in RNN and is introduced in ResNet [16]. This layer receives two neuron vectors of the same size and applies element-wise operations on corresponding neurons of the two vectors. In ResNet, this layer is element-wise addition. For RNN, this layer can be element-wise subtraction or multiplication.

Among these types of layers, FC layer and CONV layer contributes to most of the computation and storage in neural networks. In the following sections, both software level and hardware level designs focus on these two types of layers.

3 DESIGN METHODOLOGY

Before going into the details of the techniques used for fast and energy efficient neural network accelerator, we first give an overview of the design methodology. In general, the design target of a neural network processing system includes the following three aspects: high model accuracy, high throughput and high energy efficiency. For certain applications, high flexibility should also be considered.

In general, a larger neural network model usually results in a higher model accuracy. Different network structures like the ones in [16, 21, 42] surely affect model accuracy, but is out of the discussion of this paper. With a same model, applying model compression methods can achieve the trade-off between throughput and model accuracy. Some of the model compression methods even achieves acceleration without model accuracy loss.

The throughput of a neural network processing system can be expressed by equation 3. With model compression methods, we can reduce the workload. With a certain FPGA chip, the on-chip resource is limited. Increasing the peak performance means to reduce the size of each computation unit and increase the working frequency. Reducing the size of computation units usually means to simplify the basic operations in neural network model, which is a hardware-software co-design problem. Increasing working frequency, on the other hand is pure hardware design work. A high utilization ratio is kept by reasonable parallelism implementation and efficient memory system. Most of this part is affected by hardware design. But model compression can also reduce the storage requirement of a neural network model and benefits the memory system.

$$throughput = \frac{peak_performance \times utilization}{workload} \quad (3)$$

Energy efficiency is evaluated by the number of operations (multiplication or addition in this case) executed with unit energy cost. Given a certain network model, the energy efficiency of a

neural network processing system is inversely proportional to the energy cost, which is expressed in equation 4. The energy cost comes from 2 parts: computation and memory access.

$$E_{total} = N_{effect_op} \times E_{unit_op} + N_{mem_access} \times E_{unit_mem_access} \quad (4)$$

The first item in equation 4 is the energy cost for computation. This part is greatly affected by model compression. Model compression methods can reduce the actual number of operations carried out on hardware, N_{effect_op} and simplify the operations to reduce the unit energy cost of a single operation E_{unit_op} . Given an FPGA chip, E_{unit_op} is also affected by its hardware implementation. The second item in equation 4 is the energy cost for memory access. The number of memory access N_{mem_access} is affected by the memory system and scheduling method. The energy for each memory access can be reduced by model compression methods by reducing the bit-width of data.

From the analysis of throughput and energy, we see that neural network accelerator involves software-hardware co-design. In the following sections, we will introduce previous work in both software and hardware level.

4 SOFTWARE DESIGN: MODEL COMPRESSION

As introduced in section 3, the design of energy efficient and high performance neural network accelerator involves software and hardware co-design. In this section, we investigate the software level network model compression methods. Many researches on this topic have been proposed to reduce the number of weights or reduce the number of bitwidth for the neurons and weights, which helps reduce the computation and storage complexity. But these methods can also sacrifice the model accuracy. The trade-off between model compression and model accuracy loss is discussed in this section.

4.1 Data Quantization

One of the most commonly used method for model compression is the quantization on the weights and neurons. The neurons and weights of a neural network is usually represented by floating point data in common developing frameworks. Recent work try to replace this representation with low-bit fixed-point data or even a small set of trained values. On one hand, using less bits for each neuron or weight helps reduce the bandwidth and storage requirement of the neural network processing system. On the other hand, using a simplified representation reduce the hardware cost for each operation. The benefit on hardware will be discussed in detail in section 5. Two kinds of quantization methods are discussed in this section: linear quantization and non-linear quantization.

4.1.1 Linear Quantization. Linear quantization finds the nearest fixed-point representation of each weight and neuron. The problem of this method is that the dynamic range of floating point data greatly exceeds that for fixed point data. Most of the weights and neurons will suffer from overflow or underflow. Qiu, et al. [37] finds that the dynamic range of the weights and neurons in a single layer is much more limited and differs across different layers. Therefore they assign different fractional bit-widths to the weights and neurons in different layers. To decide the fractional bit-width of a set of data, i.e. the neurons or weights of a layer, the data distribution is first analyzed. A set of possible fractional bit-widths are chosen as candidate solutions. Then the solution with the best model performance on training data set is chosen. In [37], the optimized solution of a network is chosen layer by layer to avoid an exponential design space exploration. Guo, et al. [12] further improves this method by fine tuning the model after the fraction bit-width of all the layers are fixed.

The method of choosing certain fractional bit-width equals to scale the data with a scaling factor of 2^k . Li, et al. [22] scales the weights with trained parameter W^l for each layer and quantize the

weights with 2-bit data, representing W^l , 0 and $-W^l$. The neurons in this work is not quantized. So the the network still implements 32-bit floating point operations. Zhou, et al. [58] further quantize the weights of a layer with only 1 bit to $\pm s$, where $s = E(|w^l|)$ is the expectation of the absolute value of the weights of this layer. Linear quantization is also applied to the neurons in this work.

4.1.2 Non-linear Quantization. Compared with linear quantization, non-linear quantization independently assigns values to different binary code. The translation from a non-linear quantized code to its corresponding value is thus a look-up table. This kind of methods helps further reduce the bit-width used for each neuron or weight. Chen, et al. [6] assign each of the weight to an item in the look-up table by a pre-defined hash function and train the values in look-up table. Han et al. [14] assigns the values in look-up table to the weights by clustering the weights of a trained model. Each look-up table value is set as the cluster center and further fine-tuned with training data set. This method is able to compress the weights of state-of-the-art CNN models to 4-bit without accuracy loss. Zhu, et al. [59] propose the ternary quantized network where all the weights of a layer are quantized to three values: W^n , 0, and W^p . Both the quantized value and the correspondance between weights and look-up table are trained. This method sacrifices less than 2% accuracy loss on ImageNet data set on state-of-the-art network models. The weight bit-width is reduced from 32-bit to 2-bit, which means about 16 \times model size compression.

4.1.3 Comparison. We compare different quantization methods in Figure 1. The labels describe the experiments as $BW_{weight} \times BW_{neurons}$ and network name. The "(FT)" denotes that the network is fine-tuned after a linear quantization. Comparing different methods on different models is a little bit unfair. But it still gives some insights. For linear quantization, 8-bit is a clear bound to ensure negligible accuracy loss. With 6 or less bits, using fine-tune or even training each weight from the beginning, will cause obvious accuracy degradation.

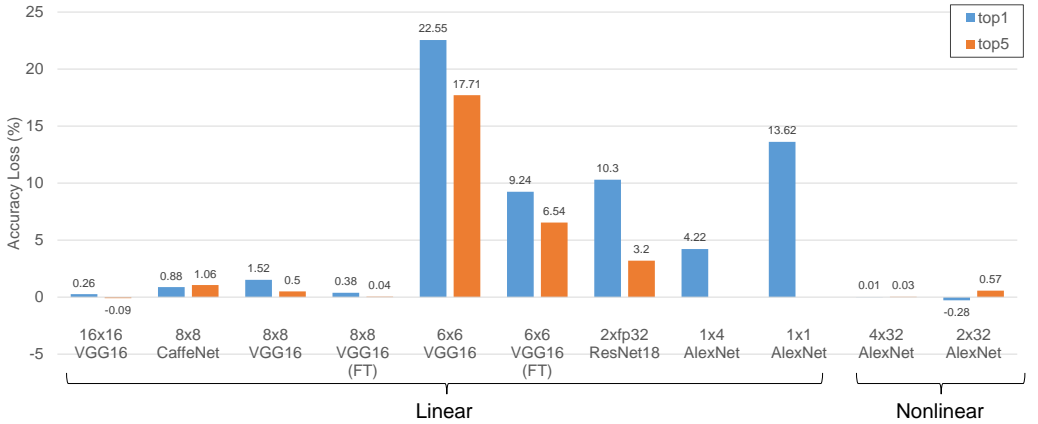


Fig. 1. Comparison between different quantization methods from [12, 14, 22, 37, 58, 59].

4.2 Weight Reduction

Besides narrowing the bit-width of neurons and weights, another method for model compression is to reduce the number of weights. One kind of method is to approximate the weight matrix with a low rank representation. Qiu, et al. [37] compress the weight matrix W of an FC layer with singular value decomposition. An $m \times n$ weight matrix W is replaced by the multiplication of two

matrices $A_{m \times p} B_{p \times n}$. For a sufficiently small p , the total number of weights is reduced. This work compress the largest FC layer of VGG network to 36% of its original size with 0.04% classification accuracy degradation. Zhang, et al. [56] use similar method for convolution layers and takes the effect of the following non-linear layer into the decomposition optimization process. The proposed method achieves 4 \times speed up on state-of-the-art CNN model targeting at ImageNet, with only 0.9% accuracy loss.

Pruning is another kind of method to reduce the number of weights. This kind of method directly remove the zeros in weights or remove those with small absolute values. The challenge in pruning is how to make more weights zero while keeping the model accuracy. One solution is the application of lasso object function during training. Liu, et al. [25] apply the sparse group-lasso object function on the AlexNet [21] model. 90% weights are removed after training with less than 1% accuracy loss. Another solution is to prune the zero weights during training. Han, et al. [14] directly removes the values in network with zero or small absolute value. The left weights are then fine-tuned are training set to recover accuracy. Experimental result on AlexNet show that 89% weights can be removed while keeping the model accuracy.

5 HARDWARE DESIGN: EFFICIENT ARCHITECTURE

In this section, we investigate the hardware level techniques used in state-of-the-art FPGA based neural network accelerator design to achieve high performance and high energy efficiency. We classify the techniques into 3 levels: computation unit level, loop unrolling level, and system level.

5.1 Computation Unit Designs

Computation unit level design affect the peak performance of the neural network accelerator. With a certain FPGA chip, the available resource is limited. A smaller computation unit design means more computation units and higher peak performance. A carefully designed computation unit array can also increase the working frequency of the system and thus improve peak performance.

5.1.1 Low Bit-width Unit. Reduce the number of bit-width for computation is a direct way to reduce the size of computation units. The feasibility of using less bits comes from the quantization methods as introduced in section 4.1. Most of the state-of-the-art FPGA designs replace the 32-bit floating point units with fixed point units. Podili, et al. [35] implements 32-bit fixed point units for the proposed system. 16-bit fixed point units are widely adopted in [9, 23, 37, 49, 51]. ESE [13] adopts 12-bit fixed-point weight and 16-bit fixed point neurons design. Guo, et al. [12] use 8-bit units for their design on embedded FPGA. Recent work is also focusing on extremely narrow bit-width design. Prost-Boucle, et al. [36] implements 2-bit multiplication with 1 LUT for ternary network. Experiments in [34] shows that FPGA implementation of Binarized Neural Network (BNN) outperforms that on CPU and GPU. Though BNN suffers from accuracy loss, many designs explore the benefit of using 1 bit for computation [20, 24, 29, 31, 32, 44, 57].

The designs mentioned above are focused on computation unit for linear quantization. For non-linear quantization, translating the data back to full precision for computation is still of high cost. Samragh, et al. [39] proposes the factorized coefficients based dot product implementation. As the possible values of weights are quite limited for non-linear quantization, the proposed computation unit accumulates the multiplier for each possible value and calculate the result as the weighted sum of the values in look-up table. In this way, the multiplication needed for one output neuron is a constant as the number of values in look-up table. The original multiplications are replaced by random addressed accumulations.

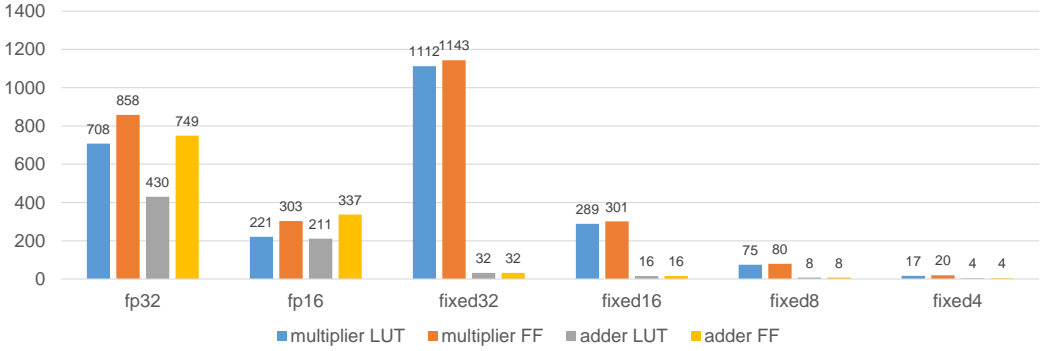


Fig. 2. FPGA resource consumption comparison for multiplier and adder with different types of data.

Most of the designs use a same bit-width through the process of a neural network. Qiu, et al. [37] finds that neurons and weights in FC layers can use less bits compared with CONV layers while the accuracy is maintained. Heterogeneous computation units are used in the designs of [11, 57].

The size of computation units of different bit-width is compared in Figure 2. The resource consumption is the synthesis result by Vivado 2017.2. All the IPs are required to not use DSP resources. Though we tend to use DSPs in real implementations, this result shows the actual hardware cost. It is also common to implement the computation units in a hybrid way like [37], where some of the operators are implemented by DSP and others by logic.

In neural network, the number of multiplication and addition is approximately the same. So the sum of resource for multiplier and adder shows the overall cost. Operations with 32-bit fixed point data consumes similar resource as 32-bit floating point operations. For 16-bit operations, using fixed-point format saves about 30% resource. As introduced in section 4.1, 8-bit fixed point is the bound for linear quantization. Compared with the 32-bit version, this allows 14× more hardware operators within the same area of logic. If further research can utilize 4-bit operations, this advantage becomes 54×.

5.1.2 Fast Convolution Unit. For CONV layers, the convolution operations can be accelerated by special algorithms. Discrete Fourier Transformation (DFT) based fast convolution is widely adopted in digital signal processing. Zhang, et al. [53] propose a 2D DFT based hardware design for efficient CONV layer execution. For an $F \times F$ filter convolved with $K \times K$ filter, DFT converts the $(F - K + 1)^2 K^2$ multiplications in space domain to F^2 complex multiplications in frequency domain. For a CONV layer with M input channel and N output channel, MN times of frequency domain multiplications are needed while only $(M + N)$ times DFT/IDFT are needed. The conversion of convolution kernels is once for all. So the domain conversion process are of low cost for CONV layers. This technique does not work for CONV layers with stride > 1 or 1×1 convolution. Ding, et al. [7] suggests that a block-wise circular constraint can be applied on the weight matrix. In this way, the matrix vector multiplication in FC layers are converted to a set of 1D convolutions and can further accelerated in frequency domain. This method can also be applied to CONV layers by treating the $K \times K$ convolution kernels as $K \times K$ matrices and is not limited by K or stride.

Frequency domain methods require complex number multiplication. Another kind of fast convolution involves only real number multiplication [47]. The convolution of a 2D feature map F_{in} with a kernel K using Winograd algorithm is expressed by equation 5.

$$F_{out} = A^T [(GF_{in}G^T) \odot (BF_{in}B^T)]A \quad (5)$$

G , B and A are transformation matrix which only related to the sizes of kernel and feature map. \odot denotes an element-wise multiplication of two matrices. For a 4×4 feature map convolved with 3×3 kernel, the transformation matrices are described as follows:

$$G = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & -1 \\ 0 & -1 \end{bmatrix}$$

Winograd based methods are also limited by the kernel size and stride as DFT based methods. The most commonly used Winograd transformation is for 3×3 convolution in [27, 49].

5.1.3 DSP Optimization. Recent FPGAs implement hardened DSP units together with the re-configurable logic to offer a high computation capacity. The basic function of a DSP unit is a multiplication accumulation (MAC). The bit-width for multiplication and addition is fixed. When the bit-width used in neural network does not match that of the DSP units, the FPGA is not fully utilized. The latest DSP units in Altera's FPGA implements $2 \times 18 \times 19$ multipliers and can be configured into a 27×27 multiplier or a 32-bit floating point multiplier [1]. That of Xilinx's FPGA implements one 27×18 multiplier [2]. As mentioned in section 5.1.1, many designs adopt multiplication with less or equal than 16 bits, which can cause great DSP under utilization.

Nguyen, et al. [33] propose the design to implement two narrow bit-width fixed-point multiplication with a single wide bit-width fixed-point multiplier. In this design, AB and AC is executed with one multiplication $A(B \ll k + C)$. If k is sufficiently large, the bits for AB and AC does not overlap in the multiplication result and can be directly separated. The design in [33] implements two 8-bit multiplications with one 25×18 multiplier, where k is 9. Similar method can be applied on other bit-width and DSPs.

5.1.4 Frequency Optimization Methods. All the above techniques introduced targets at increasing the number of computation units within a certain FPGA. Increasing the working frequency of the computation units also improves the peak performance.

To implement high parallelism, neural network accelerators usually implements matrix-vector multiplication or matrix-matrix multiplications rather than vector inner product as the basic operation. Different computation units share operators. Simply broadcast data to different computation units leads to large fan-out and high routing cost and thus reduce the working frequency. Wei, et al. [46] use the systolic array structure in their design. The shared data are transferred from one computation unit to the next in a chain mode. So the data is not broadcasted and only local connections between different computation units are needed. The drawback is the increase in latency. As the process of a neural network model is determined and the systolic structure is fully pipelined, the latency overhead can be fully covered.

Latest FPGAs support 700-900MHz DSP theoretical peak working frequency. But existing designs usually work at 100-300MHz [12, 28, 37, 51]. As claimed in [48], the working frequency is limited by the routing between on-chip SRAM and DSP units. The design in [48] use different working frequencies for DSP units and surrounding logic. Neighbour slices to each DSP unit are used as local RAMs to separate the clock domain. The prototype design in [48] achieves the peak DSP working frequency at 741MHz and 891MHz on FPGA chips of different speed grades. Yet this method is not adopted by a complete neural network accelerator design.

5.2 Loop Unrolling Strategies

CONV layers and FC layers contribute to most of the computations and storage requirement of a neural network. As introduced in section 2. We express the CONV layer function in equation 2 as

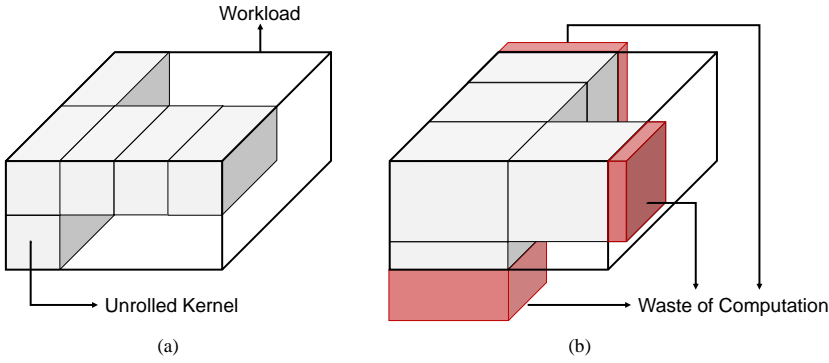


Fig. 3. Comparison between appropriate and inappropriate loop unroll parameters. (a) Appropriate parameters. (b) Inappropriate parameters.

nested loops in Algorithm 1. To make the code clear to read, we merge the loops along x and y directions for feature maps and 2-D convolution kernels respectively. An FC layer can be treated as a CONV layer with feature map and kernel both of size 1×1 . Besides the loops in Algorithm 1, we also parallelize the process of multiple inputs as a batch. This forms the batch loop.

Algorithm 1 Convolution Layer

Require: feature map F_{in} of size $M \times Y \times X$; convolution kernel Ker of size $N \times M \times K \times K$; bias vector b of size N

Ensure: feature map F_{out}

```

1: function CONV_LAYER( $F_{in}, Ker$ )
2:   Let  $F_{out} \leftarrow$  zero array of size  $N \times (Y - K + 1) \times (X - K + 1)$ 
3:   for  $n = 1; n < N; n++$  do                                ▶ Output channel loop
4:     for  $m = 1; m < M; m++$  do                                ▶ Input channel loop
5:       for each  $(y, x)$  within  $(Y - K + 1, X - K + 1)$  do    ▶ Feature map loop
6:         for each  $(ky, kx)$  within  $(K, K)$  do                ▶ Kernel loop
7:            $F_{out}[n][y][x] += F_{in}[m][y - ky + 1][x - kx + 1] * K[n][m][ky][kx]$ 
8:        $F_{out}[n] += b[n]$ 
9:   return  $F_{out}$ 

```

To parallelize the execution of the loops, we unroll a certain part of the loops and map every operation in this part as a hardware computation unit. An inappropriate set of loop unroll parameter may lead to serious hardware underutilization. We take an example of three nested loops as shown in Figure 3. The big cube denotes all the operations within the loops. The length of each edge denotes the trip count of a loop. The small cube denotes the unrolled kernel, whose edges denote the unrolling parameter. A complete execution of the workload means to fulfill the big cube with small cubes. Figure 3(a) shows an appropriate set of unroll parameters. But for Figure 3(b), the red part of some of the small cubes are out of the big cube, which means the hardware is wasted.

It is obvious from Figure 3 that if the trip count of a loop is too small, the unroll parameter for this loop is limited. For a CNN model, the loop dimension varies greatly among different layers. For a common network used on ImageNet classification like ResNet [16], the channel numbers vary from 3 to 2048, the feature map sizes vary from 224×224 to 7×7 , the convolution kernel

sizes vary from 7×7 to 1×1 . Besides the under utilization problem, loop unrolling also affect the datapath and on-chip memory design. Thus loop unrolling strategy is a key feature for a neural network accelerator design.

Various work are proposed focusing on how the unroll parameter should be chosen. Zhang, et al. [52] propose the idea of unrolling the input channel and output channel loops and choose the optimized unroll parameter by design space exploration. Along these two loops, there is no input data cross dependency between neighbouring iterations. So no multiplexer is needed to route data from on-chip buffer to computation units. But the parallelism is limited as $7 \times 64 = 448$ multipliers. For larger parallelism, this solution is easy to suffer from the under utilization problem. Ma, et al. [28] further extends the design space by allowing parallelism on the feature map loop. The parallelism reaches $1 \times 16 \times 14 \times 14 = 3136$ multipliers. A shift register structure is used to route feature map pixels to the computation units.

The kernel loop is not chosen in the above work because kernel sizes vary greatly. Motamedi, et al [30] analysis the kernel unrolling on AlexNet. Even with 3×3 unroll for the 11×11 and 5×5 kenrels, the overall system performance still reaches 97.4% of its peak performance for the convolution layers. For certain networks like VGG [42], only 3×3 convolution kernels are used. Qiu, et al. [37] use the line-buffer structure to achieve 3×3 sliding window function and fully parallelize the kernel loop. Another reason to unroll kernel loop is to achieve acceleration with fast convolution algorithms. Design in [53] implements fully parallelized frequency domain multiplication on 4×4 feature map and 3×3 kernel. Lu, et al. [27] implement Winograd algorithm on FPGA with a dedicated pipeline for equation 5. The convolution of 3×3 kernel on 6×6 kernel is fully parallelized.

The above solutions are only for a single layer. But there is hardly a one-size-fits-all solution for a whole network, especially when we need high parallelism. Designs in [23, 26] propose fully pipelined structure with each layer a pipe stage. As each layer is executed with an independent part of hardware and each part is small, loop unrolling method can be easily chosen. But this method is memory consuming because ping-pong buffers are needed between neighbouring layers for the feature maps. Design in [54] is similar but implemented on FPGA clusters to resolve the scalability problem. Shen, et al. [40] group the layers of a CNN by the loops' trip count and map each group onto one hardware module. Actually these solutions can be treated as unrolling the batch loop, because different inputs are processed in parallel on different layer pipe stage. The design in [27] implements parallelized batch both within a layer and among different layers. The drawback of batch parallel method is that the latency is higher compared with a *batch* = 1 design of a same parallelism.

Most of the current designs follow one of the above methods for loop unrolling. A special kind of design is for sparse neural network. Han, et al. [13] propose the ESE architecture for sparse LSTM network acceleration. Unlike processing a dense network, all the computation units will not work synchronously. This causes difficulty in sharing data between different computation units. ESE implements only the output channel (the output neurons of the FC layers in LSTM) loop unroll within a layer to simplify hardware design and parallelize batch process.

5.3 System Design

A typical FPGA based neural network accelerator system is shown in Figure 4. The logic part of the whole system are denoted with the blue boxes. The host CPU issues workload or commands to the FPGA logic part and monitors its working status. On the FPGA logic part, a controller is usually implemented to communicate with host and generates control signals to all the other modules on FPGA. The controller can be an FSM or an instrction decoder. The on the fly logic part is implemented for certain designs if the data loaded from external memory needs preprocess. This

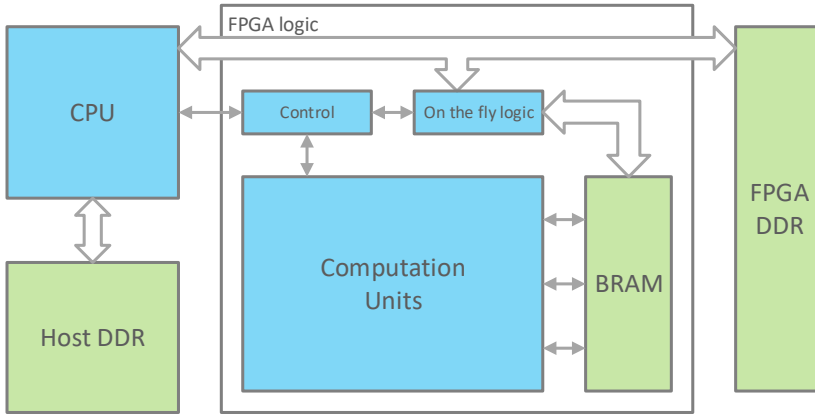


Fig. 4. Block graph of a typical FPGA based neural network accelerator system

module can be data arrangement module, data shifter [37], FFT module [53], etc. The computation units are as discussed in section 5.1 and section 5.2.

The memory hierarchy of the system mainly contains three parts, denoted as the green boxes in Figure 4: Host DDR, FPGA DDR and on-chip block RAM. For state-of-the-art network, the number of weights can reach up to 100M. Using even 8-bit or less bit-width quantization will result in tens of MB storage requirement. Most of the current FPGAs implements less than 10MB on-chip memory and 1-8GB external DDR integrated on board. So for common designs, a two level memory hierarchy is used with DDR and on-chip memory.

From system level, the performance of a neural network accelerator is limited by two factor: the on-chip computation resource and the off-chip memory bandwidth. Various researches have been proposed to achieve the best performance within a certain off-chip memory bandwidth. Zhang, et al. [52] introduce the roofline model in their work to analyze whether a design is memory bounded or computation bounded. An example of a roofline model is shown in Figure 7.

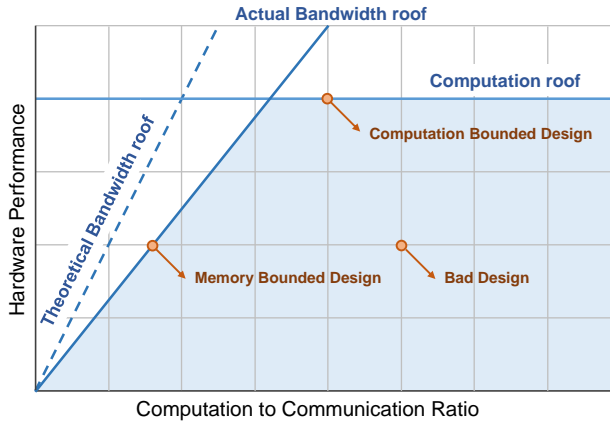


Fig. 5. An example of the roofline model. The shaded part denotes the valid design space given bandwidth and resource limitation.

The figure use the computation to communication (CTC) ratio as x -axis and hardware performance as y -axis. CTC is the number of operations that can be executed with a unit size of memory access. Each hardware design can be treated as a point in the figure. So y/x equals to the bandwidth requirement of the design. Given a certain platform, the available bandwidth is limited and can be described as the theoretical bandwidth roof in Figure 7. But the actual bandwidth roof is below the theoretical roof because for DDR access, the achievable bandwidth depends on the data access pattern. Sequential DDR access achieves much higher bandwidth than random access. The other roof is the computation roof, which is limited by the available resource on FPGA.

So a higher CTC ratio means the hardware is more likely to achieve the computation bound. Increasing the CTC ratio also reduce DDR access, which greatly reduce the energy cost according to [17]. In section 5.2, we have discussed the loop unrolling strategies to increase the parallelism while reducing the waste of computation for a certain network. When the loop unrolling strategy is decided, the scheduling of the rest part of the loops decides how the hardware can reuse data with on-chip buffer. This involves loop tiling and loop interchange strategy.

Loop tiling is a higher level of loop unrolling. All the input data of a loop tile will be stored on-chip and the loop unrolling hardware kernel works on these data. A larger loop tile size means that each tile will be loaded from external memory to on-chip memory less times. Loop interchange strategy decides the processing order of the loop tiles. External memory access happens when the hardware is moving from one tile to the next. Neighbouring tile may share a part of data. For example in a CONV layer, neighbouring tile can share input feature map or the weights. This is decided by the execution order of the loops.

In [28, 52], design space exploration is done on all the possible loop tiling sizes and loop orders. Many designs also explores the design space though some of the loop unrolling, tiling and loop order is already decided [30, 37]. Shen, et al. [41] also discuss the affect of batch parallelism over the CTC for different layers. This is a loop dimension not focused on in previous work.

All the above work give one optimized loop unrolling strategy and loop order for a whole network. Guo, et al. [12] implements flexible unrolling and loop order configuration for different layers with an instruction interface. The data arrangement in on-chip buffers are controlled through instructions to fit with different feature map sizes. This means the hardware can always fully utilize the on-chip buffer to use the largest tiling size according to on-chip buffer size. This work also propose the "back and forth" loop execution order to avoid total on-chip data refresh when a innermost loop finishes.

Alwani, et al. [4] address the external memory access problem by fusing two neighbouring layers together to avoid the intermediate result transfer between the two layers. This strategy helps reduce 95% off-chip data transfer with extra 20% on-chip memory cost. Even software program gains 2 \times speedup with this scheduling strategy.

Besides increasing CTC, increasing the actual bandwidth roof helps improve the attainable performance with a certain CTC ratio. This is achieved by regularize the DDR access pattern. Common data format in external memory includes *NCHW* or *CHWN*, where *N* means the batch dimension, *C* means the channel dimension, *H* and *W* means the feature map y and x dimension. Using any of these format, a feature map tile may be cut into small data blocks stored in discontinuous addresses. Guan [9] suggest that a channel major storage format should be used for their design. This format avoids data duplication while long DDR access burst is ensured. Qiu et al. [37] propose a feature map storage format that arranges the $H \times W$ feature map into (HW/rc) tile blocks of size $r \times c$. So the write burst size can be increased from $c/2$ to $rc/2$.

Table 1. Performance and resource utilization of state-of-the-art neural network accelerator designs

	Data Format	Perf. (GOP/s)	Power (W)	Efficiency (GOP/J)	Resource(%)			FPGA chip
					DSP	logic	BRAM	
[5]	FP16	1382	45	30.7	97	58	92	Arria 10 GX1150
[13]	INT16/12	2520	41	61.5	54.4	88.6	87.7	XCKU060
[45]	INT16	12.73	1.75	7.27	94.54	66.64	6.07	XC7Z020
[53]	FP32	123.5	13.18	9.37	87.5	85.4	64	Stratix V
[55]	INT16	1790	37.46	47.8	91	43	53	GX1150
	FP32	866	41.73	20.75	87	-	46	
[28]	INT16-8	645.25	21.2	30.43	100	38	70	GX1150
[37]	INT16	136.97	9.63	14.22	89.2	83.5	86.7	XC7Z045
[43]	INT16-8	117.8	19.1	6.2	12.5	22	65.2	5SGSD8
[52]	FP32	61.62	18.61	3.3	80	61.3	50	XC7VX485T
[9]	INT16	364.4	25	14.6	65	25	46	5SGSMD5
[27]	INT16	2940.7	23.6	124.6	-	-	-	ZCU102
[35]	INT32	229	8.04	28.5	100	83.7	17.6	Stratix V
[31]	1bit	329.47	2.3	143.2	0.5	34.4	11.4	Zynq XC7Z020
[20]	2bit	410.22	2.26	181.51	40.5	82.7	37.7	Zynq XC7Z020
[29]	1bit	40770	48	849.38	-	-	-	GX1155
[23]	INT16	565.94	30.2	22.15	59.56	63.21	65.07	XC7VX690T
[26]	INT16-8	222.1	24.8	8.96	39.9	26.6	39.7	XC7VX690T
[54]	INT16	1280.3	160	8	-	-	-	XC7Z020+ XC7VX690T×6
[12]	INT8	84.3	3.5	24.1	87	84	89	XC7Z045
[49]	INT16	229.5	9.4	24.42	91.9	71	83.2	XC7Z045
[10]	FP32	7.26	19.63	0.37	42	65.31	52.04	XC7VX485T
[51]	INT16	354	26	13.6	78	81	42	XC7VX690T

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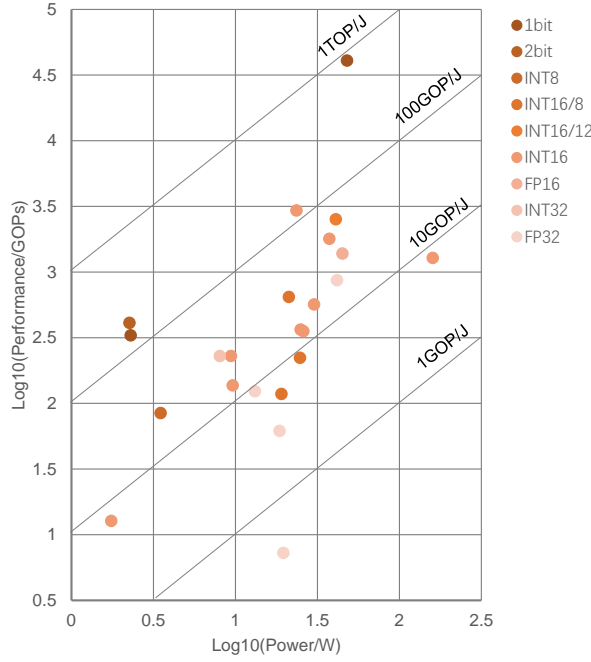


Fig. 6. A comparison between different designs on a logarithm coordinate of power and performance.

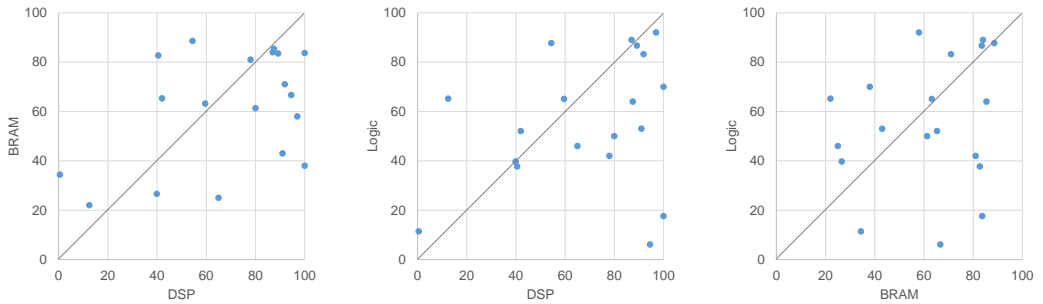


Fig. 7. Resource utilization ratio of different accelerator designs.

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