1. KORELACIJSKA MATRICA

a) $M = b_1 \cdot a_1^T + b_2 \cdot a_2^T + b_3 \cdot a_3^T$

$$= \begin{bmatrix} 6 \\ 2 \end{bmatrix} \begin{bmatrix} -2 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 12 & 6 \\ -4 & 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 2 \\ 2 & 4 & -4 \end{bmatrix} + \begin{bmatrix} -8 & -4 & -8 \\ 4 & 2 & 4 \end{bmatrix} = \underline{\begin{bmatrix} -21 & 6 & 0 \\ 2 & 10 & 2 \end{bmatrix}}$$

b) $\|a_1\| = \sqrt{4+4+1} = \sqrt{9} = 3 \neq 1$

$a_1 \cdot a_2 = -2 \cdot 1 + 2 \cdot 2 - 1 \cdot 2 = 0$ ✓

$\|a_2\| = \sqrt{1+4+4} = \sqrt{9} = 3 \neq 1$

$a_1 \cdot a_3 = -2 \cdot 2 + 2 \cdot 1 + 1 \cdot 2 = 0$ ✓

$\|a_3\| = \sqrt{4+1+4} = \sqrt{9} = 3 \neq 1$

$a_2 \cdot a_3 = 1 \cdot 2 + 2 \cdot 1 - 2 \cdot 2 = 0$ ✓

b) $\|a_1\| = \sqrt{4+4+1} = \sqrt{9} = 3 \neq 1$

$\|a_2\| = \sqrt{1+4+4} = \sqrt{9} = 3 \neq 1$

$\|a_3\| = \sqrt{4+1+4} = \sqrt{9} = 3 \neq 1$

$a_1 \cdot a_2 = -2\cdot 1 + 2\cdot 2 - 1\cdot 2$

$a_1 \cdot a_3 = -2\cdot 2 + 2\cdot 1 + 1\cdot 2 =$

$a_2 \cdot a_3 = 1\cdot 2 + 2\cdot 1 - 2\cdot 2 =$

c) $a_1 = \dfrac{a_1}{\|a_1\|} = \dfrac{1}{3}\begin{bmatrix} -2 & 2 & 1 \end{bmatrix}^T$

$a_2 = \dfrac{a_2}{\|a_2\|} = \dfrac{1}{3}\begin{bmatrix} 1 & 2 & -2 \end{bmatrix}^T$

$a_3 = \dfrac{a_3}{\|a_3\|} = \dfrac{1}{3}\begin{bmatrix} 2 & 1 & 2 \end{bmatrix}^T$

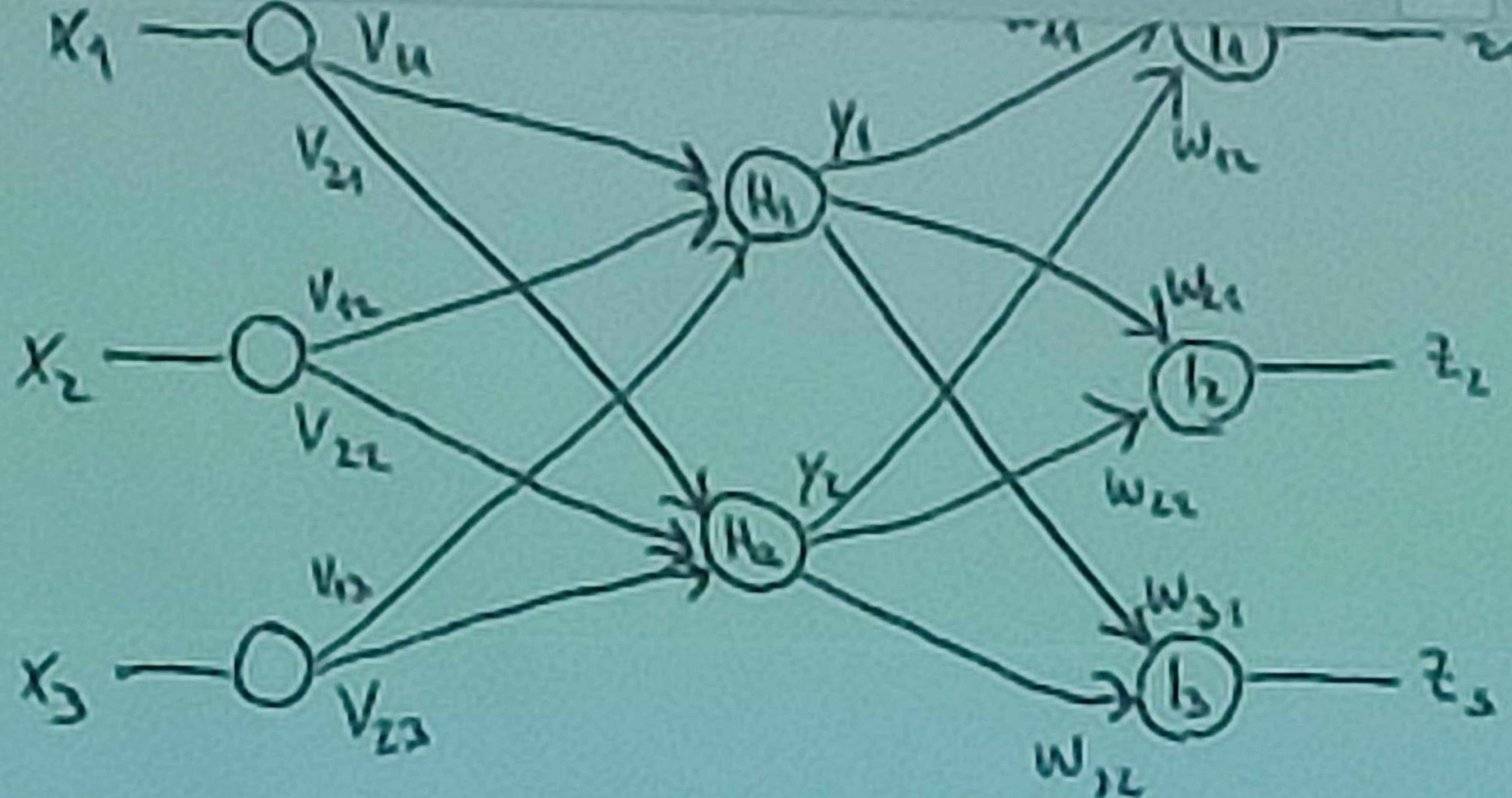
d)

$$\|a_4\| = \sqrt{\frac{100}{225} + \frac{121}{225} + \frac{4}{225}} = \sqrt{\frac{225}{225}} = 1 \quad \checkmark$$

$$a_1 \cdot a_4 = \frac{1}{3} \cdot \frac{1}{15}(-2 \cdot 10 + 2 \cdot 11 + 1 \cdot 2) = \frac{1}{45} \cdot 4 \quad \times$$

$$a_2 \cdot a_4 = \frac{1}{3} \cdot \frac{1}{15}(1 \cdot 10 + 2 \cdot 11 - 2 \cdot 2) = \frac{1}{45} \cdot 28 \quad \times$$

$$a_3 \cdot a_4 = \frac{1}{3} \cdot \frac{1}{15}(2 \cdot 10 + 1 \cdot 11 + 2 \cdot 2) = \frac{1}{45} \cdot 35 \quad \times$$



$V_1 = \begin{bmatrix} \\ 2 \\ -2 \end{bmatrix} \quad V_2 = \begin{bmatrix} \\ 1 \\ -1 \end{bmatrix}$

$w_1 = \begin{bmatrix} -1 \\ -3,5 \end{bmatrix} \quad w_2 = \begin{bmatrix} 0,5 \\ -1,2 \end{bmatrix} \quad w_3 = \begin{bmatrix} -0,3 \\ 0,6 \end{bmatrix}$

- SIGMOIDA, BEZ BIASA, ULAZ $u = [1 \;\; 1 \;\; 0]^T$

$$f(x) = \frac{1}{1+e^{-x}}$$

$$y_1 = f(H_1) = f(v_1^T \cdot u) = f\left([-2 \;\; 2 \;\; -2]\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = f(0) = \frac{1}{1+e^{-0}} = \boxed{\frac{1}{2}}$$

$$y_2 = f(H_2) = f(v_2^T \cdot u) = f\left([1 \;\; 1 \;\; -1]\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = f(2) = \frac{1}{1+e^{-2}} = \boxed{0,88}$$

$$z_2 = f(I_2) = f(w_2^T \cdot H) = f\left([0,5 \;\; -1,2]\begin{bmatrix} 0,5 \\ 0,88 \end{bmatrix}\right) = f(-0,806) = \frac{1}{1+e^{0,806}} = \boxed{0,31}$$

$z = \begin{bmatrix} 0{,}83 \\ 0{,}26 \\ 0{,}56 \end{bmatrix}$

$w_{12} = -3{,}5$

$w_{22} = -1{,}2$

$w_{32} = 0{,}6$

$y_2 = 0{,}6$

$t = \begin{bmatrix} 0{,}68 & 0{,}26 & 0{,}56 \end{bmatrix}^T$

$\eta = 0{,}03$

$\Delta w_{12} = \eta \cdot \delta_1 \cdot y_2$

$\delta_1 = e_1 \cdot f'(l_1) = (t_1 - z_1) \cdot z_1(1 - z_1)$

$\delta_1 = (-0{,}15) \cdot 0{,}83 \cdot 0{,}17 = -0{,}02117$

[redacted]

$\Delta w_{12} = 0{,}03 \cdot (-0{,}02117) \cdot 0{,}6 = -0{,}00038$

- $w_{12}' = w_{12} + \Delta w_{12} = \boxed{-3{,}50038}$

$\Delta w_{22} = \eta \cdot \delta_2 \cdot y_2$

$\delta_2 = e_2 \cdot f'(l_2) = (t_2 - z_2) \cdot z_2(1 - z_2) = 0$

$\Delta w_{22} = 0$

$\boxed{w_{22}' = -1{,}2}$

$\Delta w_{32} = \eta \cdot \delta_3 \cdot y_2$

$\delta_3 = e_3 \cdot f'(l_3) = (t_3 - z_3) \cdot z_3(1 - z_3) = 0$

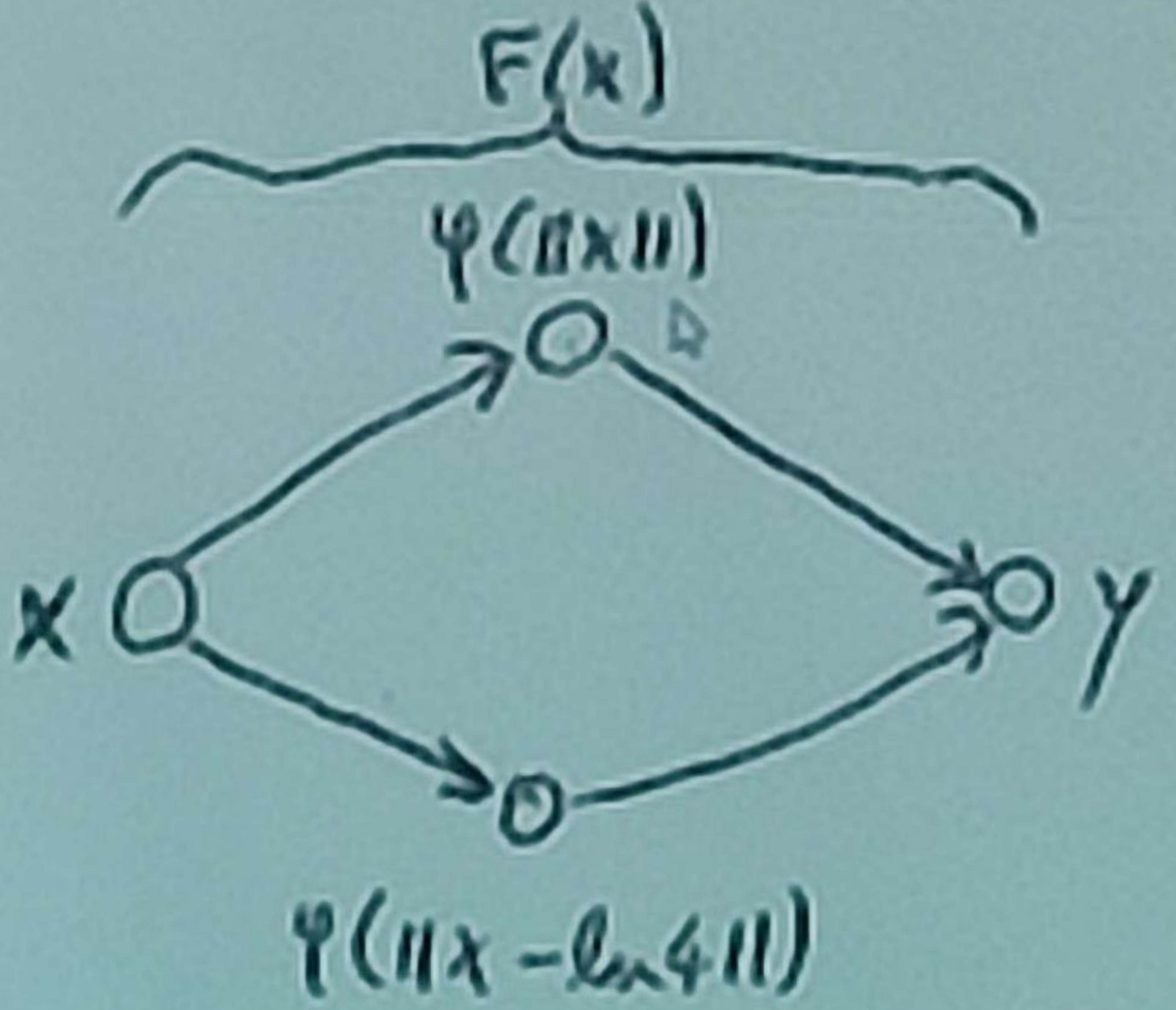
$\Delta w_{32} = 0$

$\boxed{w_{32}' = 0{,}6}$

3. RADIJALNE MREŽE

$F(0) = 3$

$F(\ln(4)) = -4 \qquad \varphi(r) = \dfrac{1}{e^{r/2}}$



$$F(x) = \sum_{k=1}^{2} w_k \varphi(\|x - x_k\|)$$

$$F(x) = w_1 \varphi(\|x\|) + w_2 \varphi(\|x - \ln 4\|)$$

$$F(0) = w_1 \varphi(0) + w_2 \varphi(\ln 4) \qquad \varphi(0) = \frac{1}{e^0} = 1$$

$$F(\ln 4) = w_1 \varphi(\ln 4) + w_2 \varphi(0) \qquad \varphi(\ln 4) = \frac{1}{e^{\frac{\ln 4}{2}}} = \frac{1}{e^{\ln 4^{\frac{1}{2}}}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}$$

$$\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\left.\begin{aligned} w_1 + \tfrac{1}{2} w_2 &= 3 \quad \mid \cdot 2 \\ \tfrac{1}{2} w_1 + w_2 &= -4 \end{aligned}\right\} -$$

$$\frac{3}{2} w_1 = 10$$

$$\boxed{w_1 = \frac{20}{3}}$$

$$\frac{20}{3} + \frac{1}{2} w_2 = 3$$

$$\frac{1}{2} w_2 = -\frac{11}{3}$$

4. STROJ S POTPORNIM VEKTORIMA

a) $w_0^T x + b_0 = 0$

$$\begin{bmatrix} w_{01} & w_{02} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + b_0 = 0$$

$$5w_{01} + 6w_{02} + b_0 = 1$$

$$\left.\begin{aligned} 5w_{01} + 3w_{02} + b_0 &= -1 \\ 7w_{01} + 5w_{02} + b_0 &= -1 \end{aligned}\right\} -$$

$$-2w_{01} - 2w_{02} = 0$$

$$w_{01} = -w_{02}$$

$$w_{02} + b_0 = 1$$

$$-7w_{02} + 5w_{02} + 1 - w_{02} = -1$$

$$-3w_{02} = -2$$

$$-2w_{01} - 2w_{02} = 0$$

$$w_{01} = -w_{02}$$

$$w_{02} + b_0 = 1$$

$$b_0 = 1 - w_{02}$$

$$-7w_{02} + 5w_{02} + 1 - w_{02} = -1$$

$$-3w_{02} = -2$$

$$\boxed{w_{02} = \frac{2}{3} \quad w_{01} = -\frac{2}{3} \quad b_0 = \frac{1}{3}}$$

b) $\rho = 2r = \frac{2}{\|w_0\|} = \frac{2}{\sqrt{\frac{4}{9} + \frac{4}{9}}} = \frac{\cancel{2}}{\frac{\cancel{2}\sqrt{2}}{3}} = \frac{3}{\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{2}}$