14.1

$\alpha = \beta = 2$

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$N_1 = 80$ $\quad N_2 = 20$ $\quad \alpha_2 = \alpha_1 + m_1 = 16$

$m_1 = 14$ $\quad m_2 = 11$ $\quad \beta_2 = (N_1 - m_1) + \beta_1$

$= 66 + 2 = 68$

$\mu_1 = \frac{m + \alpha - 1}{N + \alpha + \beta - 2}$ $\qquad \mu_2 = \frac{11 + 16 - 1}{20 + 68 + 16 - 2}$

$= \frac{14 + 1}{80 + 2}$ $\qquad = \frac{26}{102}$

$= \frac{15}{82}$

$\Delta\mu = \frac{26}{102} - \frac{15}{82} = .071975$ C

14.2

$D = \{10, 5, 6, 0\}$

$\ln L(\hat{\mu}_{MLE}, \sigma^2_{UB}) = -\frac{N}{2}\ln(2\pi) - \frac{N}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum(x_i - \mu)^2$

$\hat{\mu}_{MLE} = \frac{10 + 5 + 6}{4} = 5.25$

A

$\sigma^2_{UB} = \frac{\sum_{i}^{4}(x_i - \hat{\mu})^2}{N - 1} = \frac{203}{12}$

$\ln(\hat{\mu}_{MLE}, \sigma^2_{UB}) = -10.8324$

15.1 $\quad N = 7, \ n = 2, \ K = 2$

x		y
-1	-2	0
0	0	0
1	2	0
3	-1	1
4	-1	1
4	1	1
5	1	1

Σ dijeljena i dijagonalna

$h_0(x) = \ln(x, y = 0) = \ln p(\vec{x} \mid y = 0) + \ln p(y = 0)$

$= -\frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|\hat{\Sigma}| - \frac{1}{2}(\vec{x} - \vec{\mu}_0)^T \Sigma^{-1}(\vec{x} - \vec{\mu}_0) + \ln p(y = 0)$

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$p(y=0) = \frac{3}{7} = p_0 \quad p_1 = \frac{4}{7}$

$\vec{\mu}_0 = \frac{1}{3} \cdot \begin{pmatrix} (-1 & -2) \\ + & \\ (0 & 0) \\ + & \\ (1 & 2) \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Sigma_0 = \underset{=2}{\frac{1}{N_0-1}} \left(\overset{x_1 \cdot x_1^T}{\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}} + \overset{x_2 \cdot x_2^T =}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} + \overset{x_3 \cdot x_3^T =}{\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}} \right)$

$= \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$\vec{\mu}_1 = \frac{1}{4} \cdot \begin{bmatrix} 3 & -1 \\ + 4 & -1 \\ + 4 & 1 \\ + 5 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 16 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \end{bmatrix}$

$\Sigma = \frac{1}{N_1 - 1} \cdot \sum_i (x_i - \mu)$

$\hat{\Sigma}_1 = \frac{1}{3} \cdot \left(\overset{\begin{bmatrix} -1 \\ -1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} + \overset{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} + \overset{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} + \overset{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \right)$

$= \frac{1}{3} \cdot \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2/3 & 2/3 \\ 2/3 & 4/3 \end{bmatrix}$

Dijag.

$\hat{\Sigma} = \sum p_j \Sigma_j = \frac{3}{7} \cdot \begin{bmatrix} 1 & \\ & 4 \end{bmatrix} + \frac{4}{7} \cdot \begin{bmatrix} \frac{2}{3} & \\ & 4/3 \end{bmatrix} = \begin{bmatrix} \frac{9}{21} + \frac{8}{21} & \\ & \frac{36}{21} + \frac{16}{21} \end{bmatrix}$

$= \begin{bmatrix} 17/21 & \\ & 52/21 \end{bmatrix}$

$|\hat{\Sigma}| = \frac{17}{21} \cdot \frac{52}{21} = \frac{884}{441}$

$h_0(x) = -\ln(2\pi) - \frac{1}{2} \ln\left(\frac{884}{441}\right) - \underbrace{\frac{1}{2} \begin{bmatrix} 0 & 0 \end{bmatrix} \hat{\Sigma}^{-1} \begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{=0} + \ln\left(\frac{3}{7}\right)$

$= -3.033 \quad$ **B**

15.2

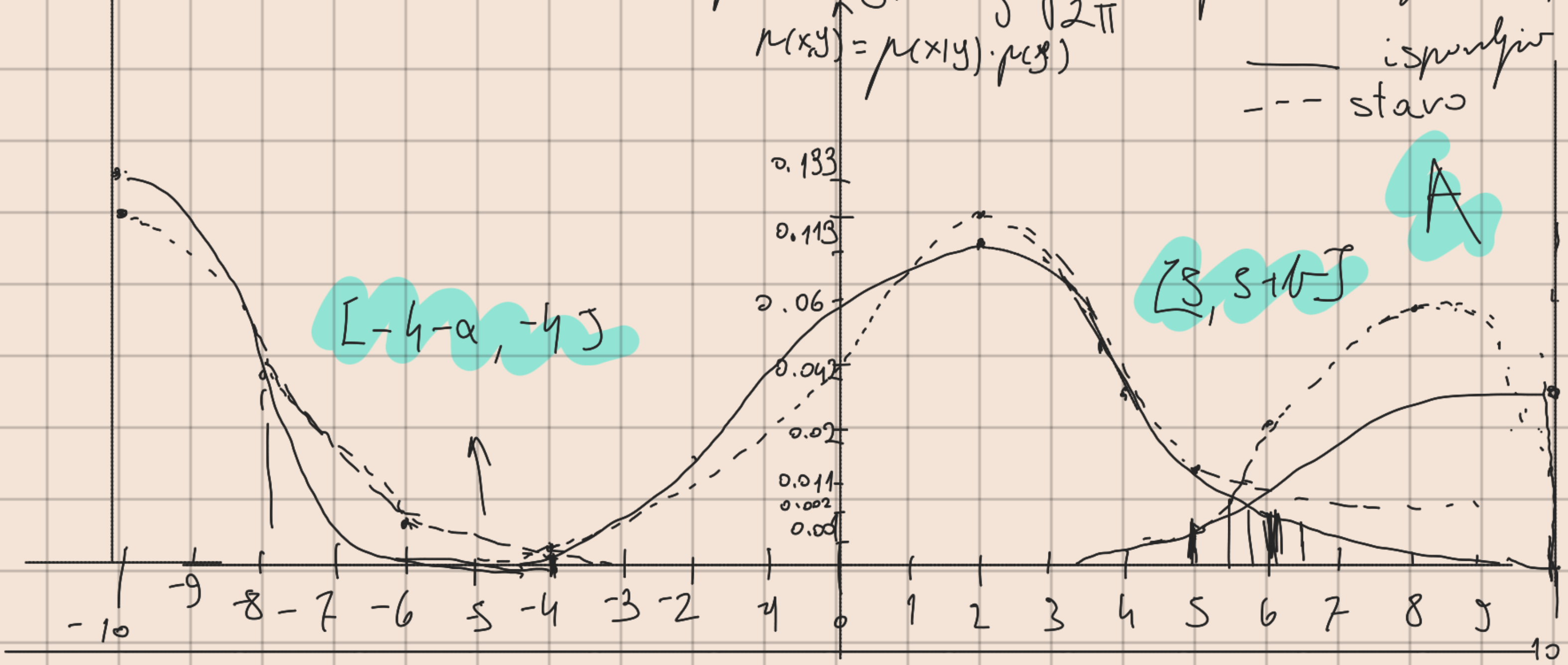
$$p(x|y=1) = \mathcal{N}(\overset{\mu}{-10}, \overset{\sigma^2}{2}) \to (-10, 1)$$

$$p(x|y=2) = \mathcal{N}(2, 2) \qquad K = 3$$

$$p(x|y=3) = \mathcal{N}(8, 2) \qquad n = 1 \qquad a, b > 0$$

x	$P(y=x)$	$P'(y=x)$
1	2/5	1/3
2	2/5	1/3
3	1/5	1/3
	h_1	h_2

$$p(x|y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



16.1

$$I_{x_1 x_2} = \sum P(x_1, x_2) \ln \frac{P(x_1, x_2)}{P(x_1) \cdot P(x_2)}$$

$I > 0.01$

$n = 3$

$I_{12, 13, 23} = ?$

1 \ 2	0	1	
0	0.1	0.1	0.2
1	0.2	0.6	0.8
	0.3	0.7	

1 \ 3	0	1	
0	0.1	0.1	0.2
1	0.5	0.3	0.8
	0.6	0.4	

2 \ 3	0	1	
0	0.2	0.1	0.3
1	0.4	0.3	0.7
	0.6	0.4	

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$$I_{1,2} = 0.1 \ln\frac{0.1}{.2 \cdot .3} + .1 \ln\frac{.1}{.2 \cdot .7} + \ln\frac{.2}{.3 \cdot .2} \cdot .2 + \ln\frac{.6}{.8 \cdot .7} \cdot .6$$

$$= 0.2996 \approx 0.3 > 0.01$$ 1 i 2 zadržijemo

$$I_{1,3} = 5,13 \cdot 10^{-3} = 0,00513 < 0.01$$

$$I_{1,3} = 4,022 \cdot 10^{-3} \approx 0.004 < 0.01$$

$$P(y)\, P(x_1, x_2 \mid y) \cdot P(x_3 \mid y) \quad B$$

15.2

$n = 200 \qquad K = 2$ – binarna klasi.

LR $\longrightarrow$ param $= \underset{\text{težine}}{n} + \underset{\text{bias}}{1} = 201$

GBC

$\downarrow$

$$h = \underbrace{-\frac{n}{2} \ln 2\pi}_{0} - \frac{1}{2} \ln|\Sigma| - \frac{1}{2}(x-\mu_i)^T \Sigma^{-1} (x-\mu_i) + \ln p(y=j)$$

$$\text{param} = \underbrace{\frac{200}{2} \cdot 201}_{\text{dijag. } \Sigma} + \underbrace{2 \cdot 200}_{\substack{k \cdot n \\ \mu_i \text{ po klasama}}} + \underbrace{2-1}_{\substack{p(y) \\ K-1}} = 20100 + 400 + 1 = 20501$$

$$20501 - 201 = 20300 \quad \underline{B}$$