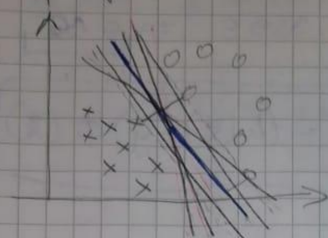
VALENTINA IVANIĆ, 0036538614

VO8

1.1

a) model SVM-a : $h(\vec{w}; \vec{w}) = \vec{w}^T \vec{x}$

pretpostavka: linearno odvojivi podatci



Neke od mogućih granica su prikazane crnom bojom, njih ima beskonačno mnogo ako govorimo o $x \in \mathbb{R}^2$.

Pravac koji će najbolje generalizirati biti će onaj u sredini (prikazan plavo).

Mi stoga želimo granicu koja maksimizira marginu gdje je margina udaljenost granice do najbližeg primjera.

vrijedi: $h(\vec{x}; \vec{w}, w_0) = \vec{w}^T \vec{x} + w_0$

primjeri lin. odvojivi $\Rightarrow \forall (\vec{x}^{(i)}, y^{(i)}) \in D,\ y^i h(x^i) \geq 0$

udaljenost hiperravnine od primjera:

$$d = \frac{y^i (\vec{w}^T \vec{x}^i + w_0)}{\|\vec{w}\|}$$

$\Rightarrow$ marg = udalj. do najbližeg primjera

$$\frac{1}{\|\vec{w}\|} \min_i \left(y^i (\vec{w}^T \vec{x}^i + w_0) \right)$$

$\Rightarrow$ želimo maksimizirati tu udaljenost:

$$\Rightarrow \underset{\vec{w}, w_0}{\operatorname{argmax}} \left\{ \frac{1}{\|\vec{w}\|} \min_i \left(y^i (\vec{w}^T \vec{x}^i + w_0) \right) \right\}$$

$\Rightarrow$ uz $yh(x) = 1$ za primjere najbliže hiperravnini

$$\Rightarrow \underset{\vec{w}, w_0}{\operatorname{argmax}} \frac{1}{\|\vec{w}\|}$$

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b) Problem kvad. programiranja

minimizirati $f(x)$

uz ograničenja $g_i(x) \leq 0 \;, i = 1, \dots, m$

$h(x) = 0 \;, i = 1, \dots, p$

Lagrangeova funkcija:

$$L(\vec{x}, \vec{\alpha}, \vec{\beta}) = f(x) + \sum_{i=1}^{m} \alpha_i g_i(\vec{x}) + \sum_{i=1}^{p} \beta_i h_i(\vec{x})$$

KKT uvjeti u stacionarnoj točki:

početna ograničenja minimizacije $\vec{x}$:

$$\left.\begin{aligned} g_i(\vec{x}) &\leq 0 \\ h_i(\vec{x}) &= 0 \end{aligned}\right\} \quad \begin{aligned} i &= 1, \dots, m \\ i &= 1, \dots, p \end{aligned}$$

$\alpha_i \geq 0 \qquad i = 1, \dots, m$ – stac. točka i li neaktivno ogran.

$\alpha_i g_i(\vec{x}) = 0$

$\hookrightarrow$ komplementarna labavost

Dualna:

$$L(\vec{\alpha}, \vec{\beta}) = \min_{\vec{x}} \mathcal{L}(\vec{x}, \vec{\alpha}, \vec{\beta})$$

c) imamo $\underset{\vec{w}, w_0}{\operatorname{argmin}} \frac{1}{2}\|\vec{w}\|^2$, uz uvjete $y^i(\vec{w}^T x^i + w_0) \geq 1 \;, i = 1, \dots, N$

prim. Lagrange f

$$L(\vec{w}, w_0, \vec{\alpha}) = \frac{1}{2}\|\vec{w}\|^2 - \sum_{i=1}^{N} \alpha_i \{y^i(\vec{w}^T \vec{x}^i + w_0) - 1\}$$

$$\Rightarrow \frac{\partial L(\vec{w}, w_0, \vec{\alpha})}{\partial \vec{w}} = 0 \Rightarrow \vec{w} = \sum_{i=1}^{N} \alpha_i y^i \vec{x}^i$$

$$\Rightarrow \frac{\partial L(\vec{w}, w_0, \vec{\alpha})}{\partial w_0} = 0 \Rightarrow \sum_{i=1}^{N} \alpha_i y^i = 0$$

$\Rightarrow$ Dualni opt. problem SVM-a

$$\text{maks.:} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y^i y^j (\vec{x}^i)^T \vec{x}^j$$

uz ograničenja: $\alpha_i \geq 0 \;, i = 1, \dots, N$

$$\sum_{i=1}^{N} \alpha_i y^i = 0$$

u točki rješenja vrijede uvjeti KKT:

$$\left.\begin{aligned} y^i(\vec{w}^T \vec{x}^i + w_0) &\geq 1 \\ \alpha_i &\geq 0 \\ \alpha_i (y^i h(\vec{x}^i) - 1) &= 0 \end{aligned}\right\} i = 1, \dots, N$$

VALENTINA IVANIĆ 0036538614

d) Dualni kvadratni problem može se riješiti algoritmom slijedne minimalne optimizacije (SMO).

Isplativo posebice ako $N << n$, gdje SMO složenost je $O(N^2)$.

e) primarno: dualno:

$$h(\vec{x}) = \vec{\omega}^T \vec{x} + w_0 = \sum_{i=1}^{N} \alpha_i y^i \vec{x}^T (\vec{x}^i) + w_0$$

f) Potporni su vektori primjeri iz ulaznog skupa primjera za koje vrijedi:

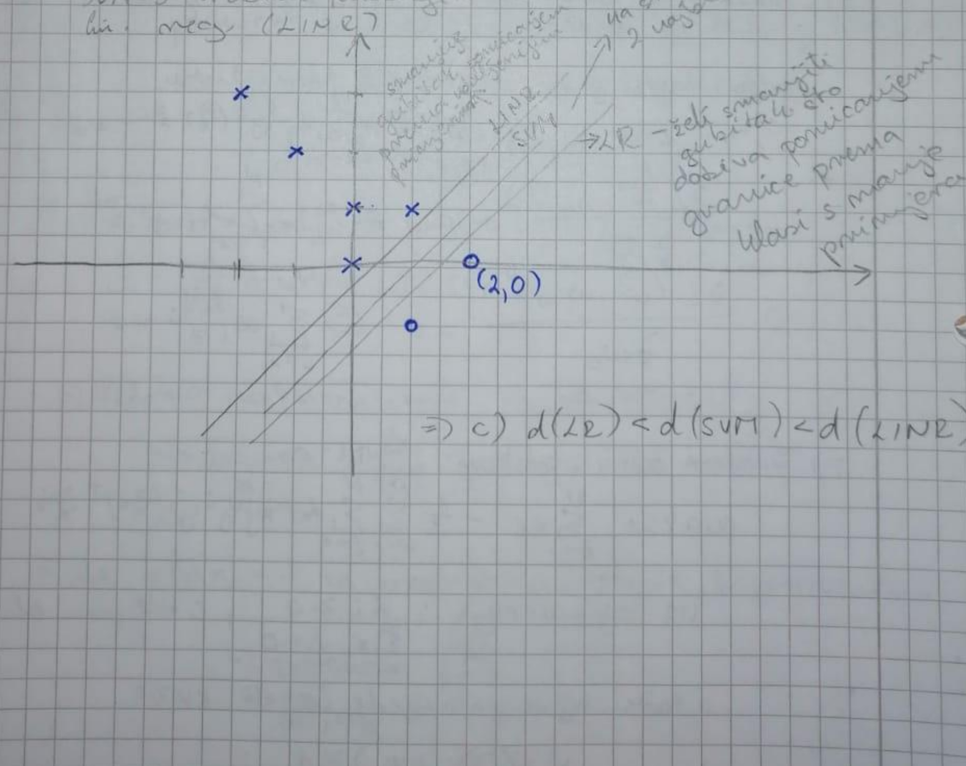
$$y^i h(\vec{x}^i) = 1$$

Ti primjeri su oni koji se nalaze na margini jer za najbliže primjere imamo definirano upravo taj izraz.

g) Skaliranjem dobivamo upravo gornji izraz što uvelike pojednostavljuje algoritam.

2.4

Skup $\mathcal{D}$
logistička reg. (LR)
SVM s tvrdom marginom
lin. reg. (LINR)



$\Rightarrow$ c) $d(LR) < d(SVM) < d(LINR)$

$\boxed{V09}$ VALENTINA NANIĆ, 0036538614

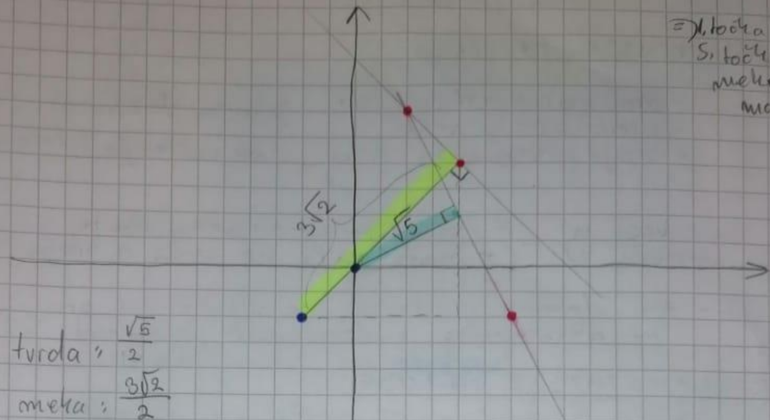
(2.1)

skup D

SVM tvrda $\wedge$ meka, $C = 1$

$\alpha_1 = 1,\ \alpha_2 > 0,\ \alpha_3 > 0,\ \alpha_4 > 0,\ \alpha_5 = 1$

$\Rightarrow$ 1. točka $\wedge$ 5. točka unutar meke margine



tvrda : $\frac{\sqrt{5}}{2}$

meka : $\frac{3\sqrt{2}}{2}$

omjer $\frac{m}{t} = \frac{3}{5}\sqrt{10}$ $\boxed{B}$

$\boxed{V10}$

(1,3)

a) $k(\vec{x}, \vec{z}) = (\vec{x}^T \vec{z} + 1)^2$ $(u = 2)$

$= (x_1 z_1 + x_2 z_2 + 1)^2$

$= x_1^2 z_1^2 + x_2^2 z_2^2 + 1 + 2x_1 x_2 z_1 z_2 + 2x_1 z_1 + 2x_2 z_2$

$$\Rightarrow k(\vec{x}, \vec{z}) = \begin{bmatrix} 1 \\ x_1^2 \\ x_2^2 \\ \sqrt{2}\, x_1 x_2 \\ \sqrt{2}\, x_1 \\ \sqrt{2}\, x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & z_1^2 & z_2^2 & \sqrt{2}\, z_1 z_2 & \sqrt{2}\, z_1 & \sqrt{2}\, z_2 \end{bmatrix}$$

$= \phi(\vec{x})^T \cdot \phi(\vec{z})$ $\Rightarrow$ Mercerova jezgra

Bitno jer nam omogućuje uporabu jezgrenog trika.

b) $x = (2, 3)$

$\Rightarrow \phi(\vec{x}) = [1 \quad 4 \quad 9 \quad 6\sqrt{2} \quad 2\sqrt{2} \quad 3\sqrt{2}]$

c) Kada $N > u$, gdje $u = 5$ jer 5dimenzijski prostor, N broj primjera.

d)

x	y
0 0	0
0 1	1
1 0	1
1 1	0

$\Longrightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 4 \end{bmatrix}$ $\Rightarrow$ prvi stupac sve 0 droga leže svi u ortog 1 ravnini

$\Rightarrow$ zanemarimo prvi stupac i preostala 3 ucrtamo 3D

$\Rightarrow$ linearno odvojiv

za $k(\vec{x}, \vec{z}) = (\vec{x}^T \vec{z})^2$ vrijedi isto

$\phi(x) = (x_1^2, x_2^2, \sqrt{2}\, x_1 x_2) \Rightarrow \phi(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

VA1

VALENTINA [illegible]
0036538614

(1,2)

a)

$\vec{x}$	y	$(4,2,1)$ za x_1		$(0,7,7)$ za x_2	
[4 4 0]	1	$\sqrt{5}$	1/6	≈ 5.1	1/27
4 3 1	1	1	1/2	$2\sqrt{5} \approx 4.472$	1/21
6 0 2	1	3	1/10	≈ 6.782	1/47
5 2 2	0	$\sqrt{2}$	1/3	$3\sqrt{3} \approx 5.196$	1/28
5 1 1	0	$\sqrt{2}$	1/3	≈ 5.75	1/34
7 2 0	0	$\sqrt{10}$	1/11	≈ 7.681	1/60

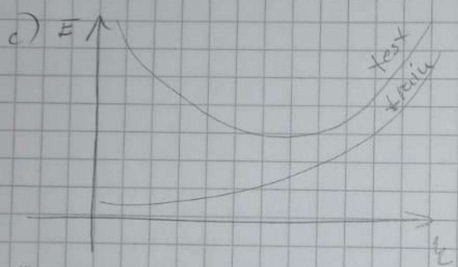
4 najbliža za: x_1 : x^1, x^2, x^4, x^5
x_2 : — 11 —

broj klasa za x_1 : $(2, 2)$ $\Rightarrow$ nemamo odluke ni za x_1, ni x_2
x_2 : $(2, 2)$

b) Inverzna kvadratna jezgra
→ udaljenosti gore plavom

težinski broj klasa za $x_1 = \left(\frac{1}{6} + \frac{1}{2}, \frac{1}{3} + \frac{1}{2}\right) \Rightarrow$ nema odluke
$x_2 = \left(\frac{1}{27} + \frac{1}{21}, \frac{1}{28} + \frac{1}{34}\right) \Rightarrow$ klasa 1

c)



d)