Dec. 12

Unit 3 - Derivatives

Quiz - Tue Nov. 15 Quiz - Wed Nov. 30 Pretest - Tue, Dec. 6 Test - Mon

ex. A ball rolling down a hill accelerate at $0.4 m/s^2$. It's distance travelled, d meters is given by $d = 0.2t^2$, where t is the number of seconds since the ball's release. Find its instaneous speed at t=5s.

$$d' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The derivative of a function f(x) at a number n, f'(n), is given by

$$f'(n) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{x \to n} \frac{f(x) - f(n)}{x - n}$$

ex. Find the slope of the line tangent to the curve $y = \sqrt{x}$ at each point.

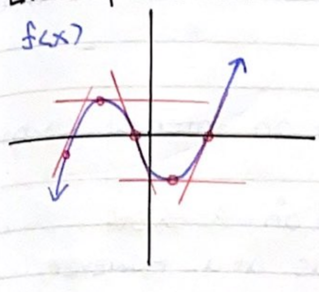
a) (1,1) $\frac{1}{2}$ b) (9,3) $\frac{1}{6}$ c) (54, 43617.31) $\to \frac{50}{1462}$

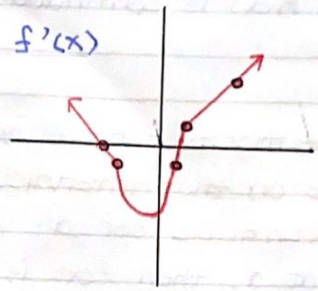
$$y' = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \to \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \left(\frac{\sqrt{x} + \Delta x + \sqrt{x}}{\sqrt{x} + \Delta x + \sqrt{x}}\right)$$

$$\hookrightarrow \lim_{\Delta x \to 0} \frac{(x+\Delta x) - x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})} \to \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})}$$

132 # 3,4,5,14,15,27,28,33

ex. Graph $f'(x)$





* take the tangent slope.

ex. Given $g(x)$ in the table, make an appox. table for $g'(x)$

x	-80	-70	-60	-50	-40	-30	-20
y	4	1	0	-3	-1	2	7

x	-70	-60	-50	-40	-30
y	-0.2	-0.2	-0.05	0.25	0.4

• For $x = -70$: $g'(-70) = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} \to \frac{g(-70+\Delta x) - g(-70)}{\Delta x}$

1. ↳ let $\Delta x = 10$. $g'(-70) \doteq \frac{g(-70+10) - g(-70)}{10} \doteq \frac{0-1}{10} \doteq -0.1$

2. ↳ let $\Delta x = -10$. $g'(-70) \doteq \frac{g(-70-10) - g(-70)}{-10} \doteq \frac{4-1}{-10} \doteq -0.3$

* Aveg. $(-0.1 + -0.3) \div 2 = -0.2$

• For $x = -60$: 1. $g'(-60) \doteq \frac{g(-60+10) - g(-60)}{10} \doteq -0.3$

use 10 b/c it's diff. btw the x in T.O.V.

2. $g'(-60) \doteq \frac{g(-60-10) - g(-60)}{-10} \doteq -0.1$

* Avg. $(-0.1 + -0.3) \div 2 = 0.2$

ex. where is the function $h(x) = \sqrt{x}$ differentiable?

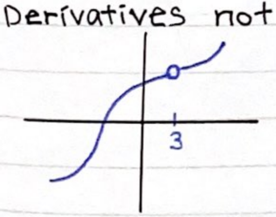
ID: $x \geq 0$ $h'(x) = \lim_{\Delta x \to 0} \frac{h(x+\Delta x) - h(x)}{\Delta x} \to \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \left(\frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}} \right)$

↳ $\frac{(x+\Delta x) - x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})} \to \frac{\Delta x}{\Delta x (\sqrt{x+\Delta x} + \sqrt{x})} \to \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}} \to \frac{1}{2\sqrt{x}}$

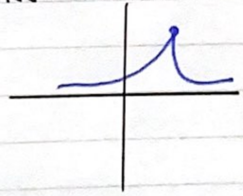
ID: $x > 0$, $x \in \mathbb{R}$ $h(x)$ is differentiable where $x > 0$

Differentiable = where the derivaties exist

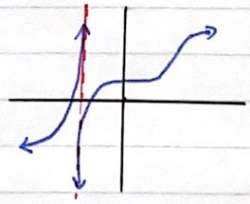
Derivatives not differentiable :



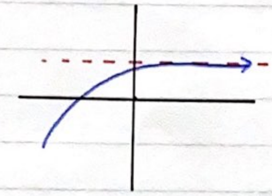
Discountinuity



A kink



vertical tangent line



horizontal tangent line is differentiable

If f(x) is diferentiable at a value a, the f(x) is countinous at a.

ex. $h(x) = \sqrt{x}$ $\lim_{x \to 0} \sqrt{x} = 0$, $h(x)$ is continuous at 0 but NOT differentiable.

Differentiability:

- A function $f(x)$ is differentiable for all values on an interval (a,b), then $f(x)$ is differentiable on that intervals.
- A function $f(x)$ is differentiable at a number a if $f'(a)$ exists
- If $f(x)$ is differentiable at a, then $f(x)$ is continous at a. However, the converse is not always ture ; a function can be continous at a but not differentiable at a.

142 # 1, 4, 7, 10, 19, 25, 29, 35, 46.

Product & Quotient Rules :

1. $\frac{d}{dx}(C) = 0$ 2. $\frac{d}{dx}(x) = 1$ 3. $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$

4. $\frac{d}{dx} x^n = nx^{n-1}$ 5. $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$

6. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$2(1x^0) = 2$

ex. Find f'(x). $f(x) = x^5 + x^4 + 2x + 1$ (4) → $5x^4 + 4x^3 + 2 + 0$

ex. Find f'(x). $f(x) = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$ (4)

↳ $8x^7 + 12(5x^4) - 4(4x^3) + 10(3x^2) - 6(1x^0) + 0 = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$

ex. f'(x) of $f(x) = \underbrace{(3x - 2x^2)}_{f}\underbrace{(5 + 4x)}_{g}$ (5) $f(x)g'(x) + g(x)f'(x)$

↳ $f'(x) = (3 - 4x)(5 + 4x) + (3x - 2x^2)(4)$

ex. f'(x) of $y = \frac{x^2 + x - 2}{x^3 + 6}$ $\frac{f}{g}$ (6) $\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

↳ $\frac{(x^3 + 6)(2x + 1) - (x^2 + x - 2)(3x^2)}{(x^3 + 6)^2}$

ex. f'(x) of $\frac{1}{x^2}$ → x^{-2} → $-2x^{-3}$ = $\frac{-2}{x^3}$

ex. f'(x) of $y = \sqrt{x} = x^{\frac{1}{2}}$ → $\frac{1}{2}x^{-\frac{1}{2}}$ = $\frac{1}{2\sqrt{x}}$

ex. Find the equation of the tangent line to the curve at (1,1) $y = \frac{2x}{x+1}$ $\frac{f}{g}$

$y' = \frac{(x+1)(2) - (2x)(1)}{(x+1)^2} = \frac{2x + 2 - 2x}{(x+1)^2} = \frac{2}{(x+1)^2} \rightarrow \frac{2}{(1+1)^2} = \frac{1}{2}$

154 # 2, 3, 5, 7, 11, 13, 17, 19,
21, 25, 35, 41, 53, 67, 72

Apply in Science: ' is speed $= \vec{v}$ in. $\vec{v}$ = in. r. of. c.

ex. The location of a particle is given by the equation $S(t) = \frac{2}{3}t^3 - 7t^2 + 20t$, where s metres is its location and t is time in seconds.

a) Find the particle's velocity at time t.

$v(t) = s'(t) = \frac{2}{3}(3t^2) - 7(2t) + 20(1) \rightarrow 2t^2 - 14t + 20 = v(t)$

b) How fast is the particle moving at time 3 sec? 10 sec?

3s: $v(\underline{3}) = 2(3)^2 - 14(3) + 20 = -4\,m/s$

10s: $v(\underline{10}) = 2(10)^2 - 14(10) + 20 = 80\,m/s$

c) when is the particle at rest?

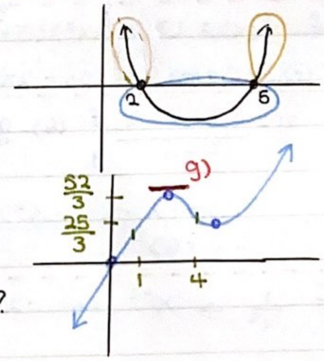
$0 = 2t^2 - 14t + 20 \rightarrow 2(t^2 - 7t + 10) \rightarrow 2(t-5)(t-2) = 5\text{ sec}, 2\text{ sec}$

d) when is it moving forward? backward?

$2(t-5)(t-2) > 0$

Forward: $t < 2$ & $t > 5$

Backward: $2 < t < 5$



e) Sketch a graph of the particle's motion.

$S(t) = \frac{2}{3}t^3 - 7t^2 + 20t \rightarrow t(\frac{2}{3}t^2 - 7t + 20)$

f) How far does it travel in the first 6 seconds?

where $0 \leq t \leq 2$: $\frac{52}{3}$

$S(2) = \frac{2}{3}(2^3) - 7(2)^2 + 20(2) = \frac{52}{3}$

where $2 < t \leq 5$: $\frac{52}{3} - \frac{25}{3} = 9$

$S(5) = \frac{2}{3}(5)^3 - 7(5)^2 + 20(5) = \frac{25}{3}$

where $5 < t \leq 6$: 12

$S(6) = \frac{2}{3}(6)^3 - 7(6)^2 + 20(6) = 12$

total: $\frac{52}{3} + 21 = \frac{115}{3}\,m$

g) what is the greatest $\vec{d}$ btw $t = 1$ & $t = 4$

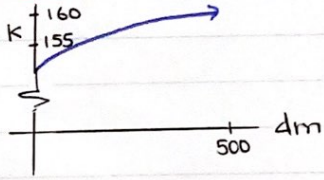
ex. In a chemical reaction, a liquid in a full 50-cm deep beaker is heated unevenly. Suppose the kelvin temperature of the liquid at depth d mm is given by $K(d) = 150 + \sqrt[3]{d}$, where $0 \leq d \leq 500$.

a) Find the kelvin temp. at 6mm & 123mm.

$K(\underline{6}) = 150 + \sqrt[3]{6} = 151.817K$

$K(\underline{123}) = 150 + \sqrt[3]{123} = 154.973K$

b)



Q: Sketch a graph showing the temp. at depth d mm.

(use T.O.V.)

c) Find the rate of temp. change per mm of depth, as a stir stick lowered into the beaker.

$K'(d) = 0 + \frac{1}{3}d^{\frac{-2}{3}} \rightarrow \frac{1}{3\sqrt[3]{d^2}}$

d) what's the rate of temperature increase per mm when the tip of the stick is 47 mm deep?

e) At what day th will the temperature be changing by 0.05 K/mm

$0.05 = \frac{1}{3\sqrt[3]{d^2}} \rightarrow 0.05(3\sqrt[3]{d^2}) = 1 \rightarrow \sqrt[3]{d^2} = \frac{1}{0.15} \rightarrow d^2 = (\frac{1}{0.15})^3$

$\hookrightarrow d = \pm\sqrt{(\frac{1}{0.15})^3} = \pm 17.213 \rightarrow 17^{th}$ day

ex. A beaver population increases until its food supply begins to run out, then it decreases. Suppose the population can be approximatly model by $P(t) = -3t^2 + 150t + 902$, where t is the number of days since Jan. 1, 2022.

a) what's the rate of population change per day at time t days?

$P'(t) = -6t + 150$

b) when does the population hit its maximum? what is the max. population?

$0 = -6t + 150 \rightarrow t = \frac{-150}{-6} = 25$

$P(25) = -3(25)^2 + 150(25) + 902 = 2777$

c) what's the rate of population increase / decrease on Feb. 9th?

$P'(31+9) = -6(40) + 150 = -90$

d) what is the population increasing by 19 beavers a day?

$19 = -6t + 150 \rightarrow t = \frac{19-150}{-6} = 22$

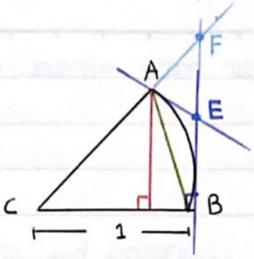
e) $0.05 = \frac{1}{3\sqrt[3]{d^2}} = \pm 17.213$

d) 17.213

47

Trig:

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$



$\widehat{AB} = \theta$

$\sin\theta = \frac{|AD|}{1} \rightarrow \sin\theta = |AD|$ $\qquad \frac{\sin\theta}{\theta} < 1$ $\uparrow$

$|AD| < |AB| < \widehat{AB} \rightarrow \sin\theta < |AB| < \theta$ # $\sin\theta < \theta$

$\widehat{AB} < |EB| + |AE| \rightarrow \theta < |EB| + |AE|$, $\theta < |EB| + |FE|$, $\theta < |BF|$. # $\theta < \tan\theta$

$\hookrightarrow \theta < \frac{\sin\theta}{\cos\theta})$

$\cos\theta < \frac{\sin\theta}{\theta}$ $\qquad$ ☆: $\cos\theta < \frac{\sin\theta}{\theta} < 1$

$\lim_{\theta \to 0} \cos\theta = 1$ $\qquad \lim_{\theta \to 0} 1 = 1 \rightarrow \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (squeeze theorem)

- $y = \frac{\sin\theta}{\theta}$ is even function

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ $\qquad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$

$y = \sin x \rightarrow y' = \cos x$ $\qquad y = \cos x \rightarrow y' = -\sin x$ $\qquad y = \tan x \rightarrow y' = \sec^2 x$

$y = \csc x \rightarrow y' = -\csc x \cot x$ $\qquad y = \sec x \rightarrow y' = \sec x \tan x$ $\qquad y = \cot x \rightarrow y' = -\csc^2 x$

ex. Find equation of line tangent to $y = \cos^2 x$ at $(\frac{\pi}{8}, 0.854)$

$y = (\cos x)(\cos x) \rightarrow y' = (\cos x)(-\sin x) + (-\sin x)(\cos x)$

$\hookrightarrow y' = -2\sin x \cos x = -\sin 2x$

$m = -\sin 2x$ $\qquad y = mx + b \rightarrow 0.854 = \frac{-\sqrt{2}}{2}\left(\frac{\pi}{8}\right) + b \rightarrow b = 1.132$

$\downarrow -\sin\left(2\left(\frac{\pi}{8}\right)\right) = \frac{-\sqrt{2}}{2}$

$\rightarrow y = \frac{-\sqrt{2}}{2} x + 1.132$

Chain Rule:

$y = f(g(x)) \rightarrow y' = [f'(g(x))][g'(x)]$

(8) # 6, 11, 13, 19, 23, 33, 39, 40, 57, 65, 72

ex. Differentiate each function:

a) $y = \sqrt{\cos x}$ $\quad f(x) = \sqrt{x}$ $\quad g(x) = \cos x$ $\quad y = f(g(x))$ $\quad g'(x) = -\sin x$

$y = (\cos x)^{\frac{1}{2}}$ $\quad y' = \frac{1}{2} \cos x^{-\frac{1}{2}} (-\sin x) \rightarrow \frac{-\sin x}{2\sqrt{\cos x}}$

b) $y = (x^3 - 2x^2 + 5)^{78}$ $\quad f(x) = x^{78}$ $\quad g(x) = x^3 - 2x^2 + 5 \rightarrow g'(x) = 3x^2 - 2(2x) + 0$

$y' = 78(x^3 - 2x^2 + 5)^{77}(3x^2 - 4x) \rightarrow 78(3x^2 - 4x)(x^3 - 2x^2 + 5)^{77}$

c) $y = \sqrt[3]{\frac{x^2 - 5}{2\sqrt{x}}}$ $\quad f(x) = \sqrt[3]{x}$ $\quad g(x) = \frac{x^2 - 5}{2\sqrt{x}}$ $\quad 2x - 0$ $\quad \rightarrow g'(x) = \frac{(2x)(2\sqrt{x}) - (x^{-\frac{1}{2}})(x^2 - 5)}{(2x^{\frac{1}{2}})^2}$

$2x^{\frac{1}{2}} \rightarrow 2\sqrt{x} \leftarrow \cancel{2}(\frac{1}{\cancel{2}}x^{-\frac{1}{2}})$

$\hookrightarrow \frac{4x\sqrt{x} - \frac{x^2}{\sqrt{x}} + \frac{5}{\sqrt{x}}}{4x}$ $\quad f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$y' = \frac{1}{3}\left(\frac{x^2 - 5}{2\sqrt{x}}\right)^{-\frac{2}{3}} \left(4x\sqrt{x} - \frac{x^2}{\sqrt{x}} + \frac{5}{\sqrt{x}}\right)\left(\frac{1}{4x}\right) = \left(\frac{2\sqrt{x}}{x^2 - 5}\right)^{\frac{2}{3}} \left(\frac{3x^2 + 5}{12x\sqrt{x}}\right)$

$\frac{4x^2 - x^2 + 5}{\sqrt{x}} \div 4x$

ex. $y = \cos((\sin x)^5)$ $\quad y = f(g(h(x)))$

$f(x) = \cos x$ $\quad g(x) = x^5$ $\quad h(x) = \sin x$

$y' = -\sin((\sin x)^5) \cdot 5(\sin x)^4 \cdot \cos x$

Implicit : 188 #1, 4, 9, 13, 19, 26, 29, 36, 39

$y = f(x)$ $\quad x = y^2 \to y_1 = \sqrt{x} \to \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\to y_2 = -\sqrt{x} \to -\frac{1}{2}x^{-\frac{1}{2}} = \frac{-1}{2\sqrt{x}}$

or $x = y^2 \to 1 = 2y\frac{dy}{dx} \to \frac{1}{2y} = \frac{dy}{dx} \to \frac{dy}{dx} = \boxed{\frac{1}{2\sqrt{x}}}$

$\to \frac{dy}{dx} = \frac{1}{2(-\sqrt{x})} = \boxed{\frac{-1}{2\sqrt{x}}}$

ex. differentiate $x^2+y^2=16$

$2x + 2y\frac{dy}{dx} = 0 \to \frac{dy}{dx} = \frac{-2x}{2y} \to \frac{dy}{dx} = \boxed{-\frac{x}{y}}$

ex. Find the tangent to the hyperbola. $\frac{(x-3)^2}{16} - \frac{(y+5)^2}{9} = -1$ at point $(3+4\sqrt{3}, 1)$.

$\frac{1}{16}(x-3)^2 - \frac{1}{9}(y+5)^2 = -1 \to \frac{2}{16}(x-3)(1) - \frac{2}{9}(y+5)\left(\frac{dy}{dx}\right) = 0$

$\hookrightarrow \frac{1}{8}(x-3) - \frac{2}{9}(y+5)\frac{dy}{dx} = 0 \to 72\left(\frac{1}{8}(x-3)\right) - \frac{2}{9}(y+5)\frac{dy}{dx}) = 72(0)$

$\hookrightarrow 9(x-3) - 16(y+5)\frac{dy}{dx} = 0 \to -16(y+5)\frac{dy}{dx} = -9(x-3)$

$\hookrightarrow \frac{dy}{dx} = \frac{9(x-3)}{16(y+5)}$ $\quad m = \frac{9(3+4\sqrt{3}-3)}{16(1+5)} = \frac{3\sqrt{3}}{8}$

ex. Find y' if $\sin(2x+y^2) = y^2\cos x$.

$\cos(2x+y^2)(2+2y\frac{dy}{dx}) = y^2(-\sin x) + \cos x(2y\frac{dy}{dx})$

$2\cos(2x+y^2) + 2y\cos(2x+y^2)\frac{dy}{dx} = -y^2\sin x + 2y\cos x\frac{dy}{dx}$

$2\cos(2x+y^2) + 2y\cos(2x+y^2)\frac{dy}{dx} - 2y\cos x\frac{dy}{dx} = -y\sin x$

$\frac{dy}{dx} = \frac{-y^2\sin x - 2\cos(2x+y^2)}{2y((\cos(2x+y^2)-\cos x)}$

Second Derivative:

$y = 7x^6 - 3x^3 + 2x^2$

$y' = 42x^5 - 6x^2 + 4x$

$y'' = 210x^4 - 18x + 4$

$y''' = 840x^3 - 18$

$y^{(4)} = 2520x^2$

$y^{(5)} = 5040x$

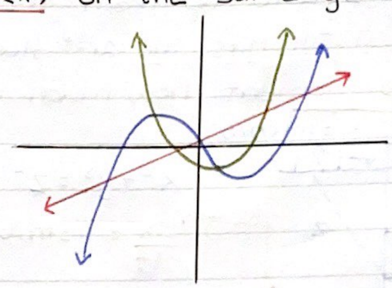
$y^{(6)} = 5040$

$y^{(7)} = 0$

ex. $f(x) = x^3 - 3x^2 - 9x$. Graph $f(x)$, $f'(x)$ and $f''(x)$ on the same grid.

$f(x) = x(x^2 - 3x - 9)$

x	-4	-2	0	2	4	6
y	-76	-2	0	-22	-20	54



$f'(x) = 3x^2 - 6x - 9 \Rightarrow 3(x^2 - 2x + 1) + 3(-1) - 9$

$= 3(x-1)^2 - 12$

$f''(x) = 6x - 6$

ex. A particle's position is given by $f(t) = t^3 - 7t^2 + 14t - 5$, where t is the time in second.

a) Find the $\vec{v}$ at time t.

$f'(t) = 3t^2 - 14t + 14$

b) How fast is the particle moving at time t = 3 sec?

$f'(3) = 27 - 42 + 14 = -1\, m/s$

c) Find $\vec{a}$ at time t.

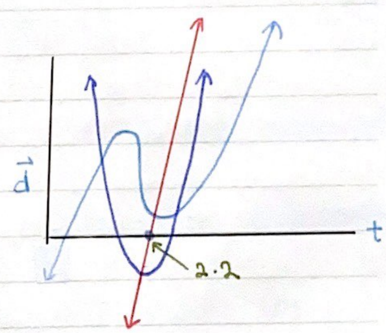
$f''(t) = 6t - 14$

d) What is $\vec{a}$ after 2 seconds?

$f''(2) = 6(2) - 14 = -2\, m/s^2$

e) Graph the position, $\vec{v}$, & $\vec{a}$ curves for $0 \le t \le 5$

t	0	1	2	3	4	5
f(t)	-5	3	3	1	3	15
f'(t)	14	3	-2	-1	6	19
f''(t)	-14	-8	-2	4	10	16



f) When is the position speeding up? slowing down?

t > 2.2 sec. t < 2.2 sec.

ex. $f(x) = \sin x$. Find $f^{(100)}x$.

$f'(x) = \cos x$ $f''(x) = -\sin x$ $f'''(x) = -\cos x$ $f^{(4)}(x) = \sin x$ $f^{(5)}(x) = \cos x$ ...

$\hookrightarrow f^{(100)}(x) = \sin x$

195 # 5, 8, 10, 13, 17, 20, 23, 29, 32, 43, 53, 54, 55

3.5 - 3.7 (trig d') (chain rule) (implicit diff.)

Relate Rates:

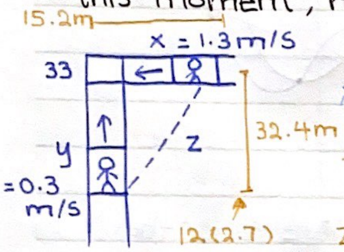
ex. A circle's radius is increasing at 4cm/s. How fast is the circle's area increasing when the diameter is 7.8 cm?

$\frac{7.8}{2} = 3.9 = r$

We seek $\frac{dA}{dt}$, know $\frac{dr}{dt} = 4\ cm/s$.

$A = \pi r^2 \rightarrow \frac{dA}{dt} = \pi(2r)\left(\frac{dr}{dt}\right) = \pi(2)(3.9)(4) = 31.2\pi$ sq. cm/sec

ex. At the Brull building, each floor is 2.7m high. Jack is on the 33rd floor, walking toward the elevator at 1.3 m/s. Jill is riding the elevator up from the lobby toward the 33rd floor, at 0.3 m/s. When Jill hits the 21st floor, Jack is 15.2m from the elevator. At this moment, how much is the distance btw Jack & Jill changing?



$x^2 + y^2 = z^2 \rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dx}{dt} = 1.3\ m/s \qquad \frac{dy}{dt} = 0.3\ m/s \qquad \frac{dz}{dt} = ?$

$z = \sqrt{32.4^2 + 15.2^2} = 35.78\cdots$

$$\frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{dz}{dt} \rightarrow \frac{15.2(-1.3) + 32.4(-0.3)}{35.78\cdots} = -0.823\ m/s = \frac{dz}{dt}$$

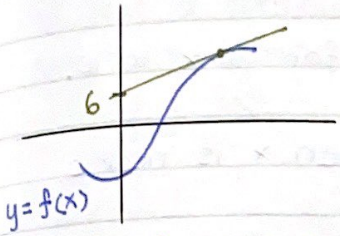
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \frac{dz}{dt}$$

$$\frac{15.2(-1.3) + 32.4(-0.3)}{35.78826623} = \frac{dz}{dt}$$

202 #1, 3, 6, 7, 13,

Linear Aproximation:



At point $(a, f(a))$, tangent line is $y = mx + b$

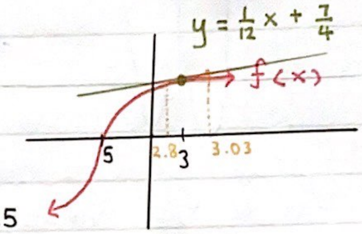
$y = f'(a) + b \rightarrow b = f(a) - f'(x)a$

$\hookrightarrow y = f'(x) + f(a) - f'(x)a \rightarrow f(a) + f'(x)(x-a)$

★: the approximation $f(x) \approx f(a) + f'(x)(x-a)$ is the linear or tangent line appoximation of $f(x)$ at a.

★: The function $L(x) = f(a) + f'(x)(x-a)$ is the linearization of $f(x)$ at a.

ex: $f(x) = \sqrt[3]{x+5}$. Find the linearization of $f(x)$ at $a=3$, and use it to estimate $\sqrt[3]{8.03}$ and $\sqrt[3]{7.96}$. Are these overestimates or underestimates?



$L(x) = f(a) + f'(a)(x-a)$

$f(a) = f(3) = \sqrt[3]{3+5} = 2 \qquad f'(x) = \frac{1}{3}(x+5)^{\frac{-2}{3}} = \frac{1}{3\sqrt[3]{(x+5)^2}}$

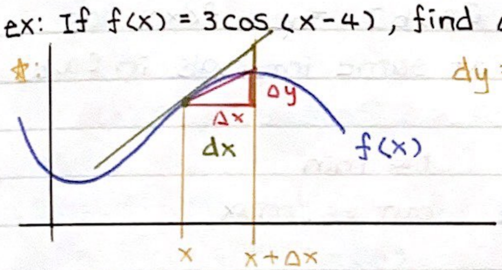
$L(x) = 2 + \frac{1}{3\sqrt[3]{(3+5)^2}}(x-3) \rightarrow f(x) \approx \frac{1}{12}x + \frac{7}{4}$

$8.03 = 3.03 + 5 \qquad \sqrt[3]{8.03} = \sqrt[3]{3.03+5} \approx \frac{1}{12}(3.03) + \frac{7}{4} = 2.0025$

$7.96 = 2.96 + 5 \qquad \sqrt[3]{7.96} = \sqrt[3]{2.96+5} \approx \frac{1}{12}(2.96) + \frac{7}{4} = 1.99\overline{6}$

ex: If $f(x) = 3\cos(x-4)$, find Δy and dy where x changes from $\frac{\pi}{16}$ to $\frac{\pi}{15}$.

★:



$dy = f'(x)\,dx$

$\Delta x = \frac{\pi}{15} - \frac{\pi}{16} = \frac{\pi}{240} = dx$

$f'(x) = 3(-\sin(x-4))(1) = -3\sin(x-4)$

$\Delta y = f(\frac{\pi}{15}) - f(\frac{\pi}{16}) = 3\cos(\frac{\pi}{15}) - 3\cos(\frac{\pi}{16} - 4) = -0.023937424$

$dy = f'(x)dx \rightarrow -3\sin(\frac{\pi}{16} - 4)(\frac{\pi}{240}) = -0.024140831$ close!

- We can use linear approximation to estimate the value of a function at a value a using $f(x) \approx f(a) + f'(a)(x-a)$ [x close to a]
- The corresponding linear function $L(x) = f(a) + f'(a)(x-a)$ is the linearization of $f(x)$ at a.
- If we increment x by Δx in a function $y = f(x)$, we define the differential dx as that increment (Δx). The dependent variable dy is another differential, representing the change in linearization of $f(x)$ over the increment Δx. Thus $dy = f'(x)dx$