Common Mathematical Relationship.

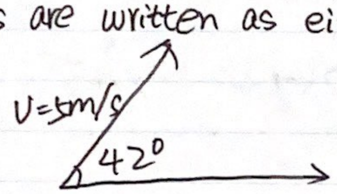
1. Linear Relationship:

$y = mx + b$ slope =

Kinematics – The physics of Motion

- There are two kinds of Physical Quantities: Vector and scalars.
- Scalars are quantities that have an amount (magnitude) but no direction.
  - Examples of scalars are time, ~~temperature~~, energy and speed.

- length of arrow represents the magnitude of vector
- Vector directions can be communicated with words (east, left, up) sign (+, −) or angles (220°], [30° E of N].
- Vectors are written as either bold font or with an arrow on top.



Describe a vector "$\vec{A}$" "$\rightarrow$" denotes a vector.

"$\vec{A}$" = 3.0N [40° N of west] / $\vec{B}$ = +30N [

Reference Frames / Points

- A reference frame is any space considered stationary for the purposes of measurement.

- A reference point is a fixed point within that space that we consider the 'starting' point of the 'zero' point.

Position, Distance, Displacement.

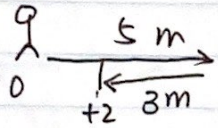
- position ($\vec{d}$ ~~$\vec{x}$~~ or $\vec{x}$) describes exactly where located

- Distance (d) – a scalar that describes how far you have traveled, regardless of direction
- Displacement ($\Delta \vec{d}$ or $\Delta \vec{x}$) – a vector that describes your change in position (where your are in relation to where you started).

symbol : $\Delta \vec{d} = \vec{d}_f - \vec{d}_i$

① distance : all the length traveled
② displacement : distance between positions.

ex



① $d = 5+3 = 8m$
② $\Delta \vec{d} = +2 - 0 = 2m$

- speed (v) – a scalar that describes how fast something moves. Measured in m/s.
  - Velocity ($\vec{v}$) – a vector that describes how fast and in which direction something moves. Again measured in m/s. describes "the ratio of change".
- Average velocity ($\vec{V}_{avg}$) given by displacement / time.

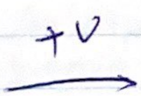
$$V_{avg} = \frac{\Delta \vec{d}}{\Delta t}$$

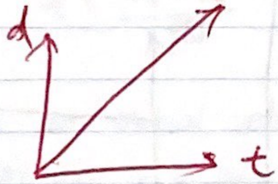
- If average velocity is the same for all time intervals then we have contact v.

$$v = \frac{d}{t}$$

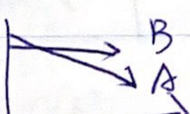
- Notice how we can calculate constant v with just the position and clock-time

Constant velocity :



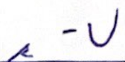


slope $= \frac{\Delta y}{\Delta x} = \frac{\Delta \bar{d}}{\Delta t} = \vec{v}$

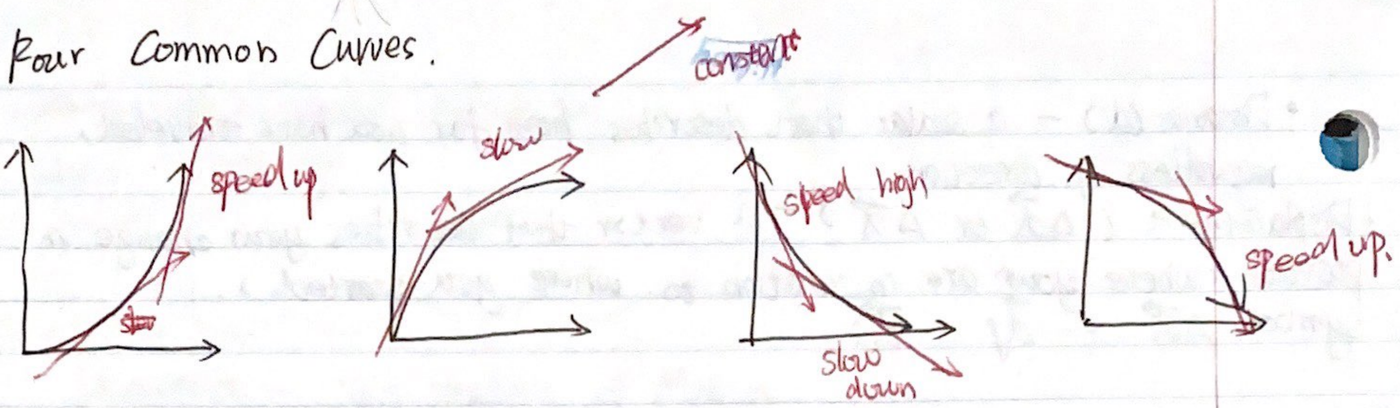


A > B

F

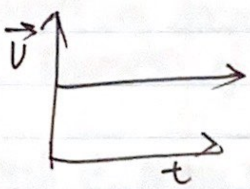


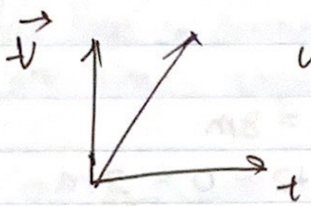
Four Common Curves.



Velocity vs time

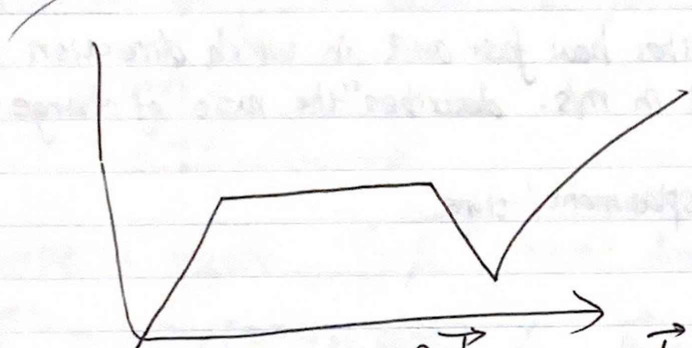
constant





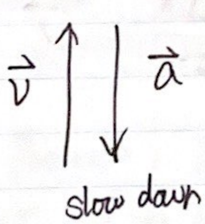
uniformly change.

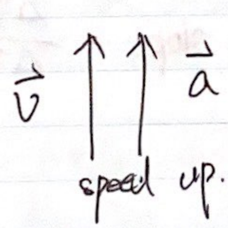
Acceleration: $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ $(v_f - v_0)$ a(vector)



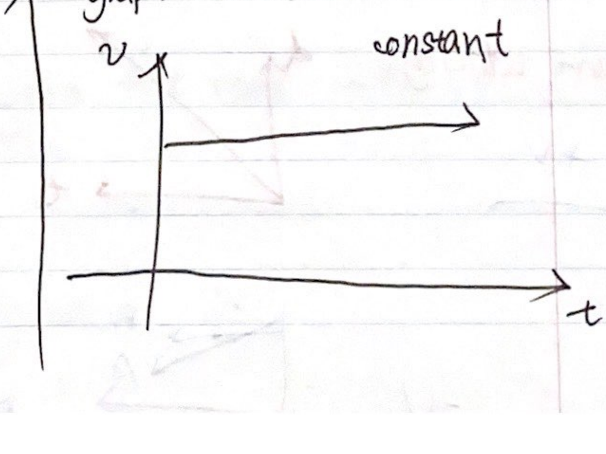
$$\vec{v}_{avg} = \frac{\Delta \vec{d}}{\Delta t} = \frac{\vec{d_f} - \vec{d_i}}{t_f - t_i}$$

$\vec{g} = 9.80 m/s^2$ (gravity)

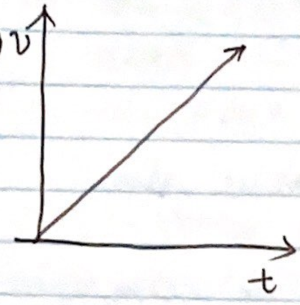




graph acceleration $\vec{v}$ vs t

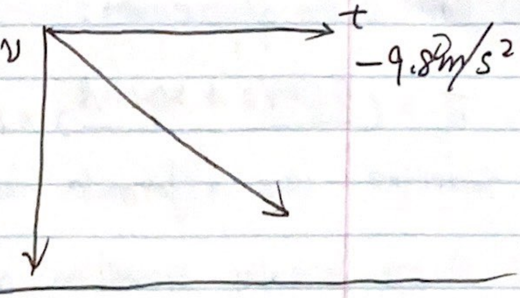


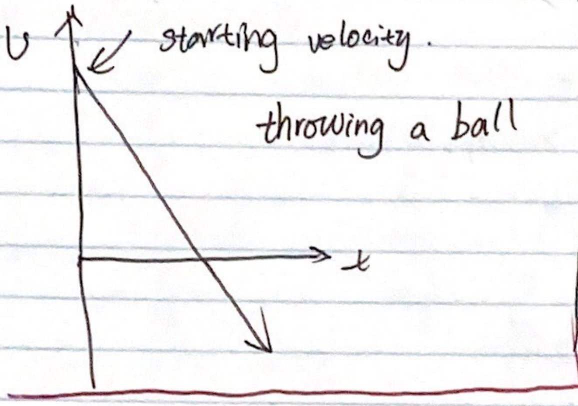
uniform change



slope $= \frac{\Delta y}{\Delta x} = \frac{\Delta \vec{v}}{\Delta t}$

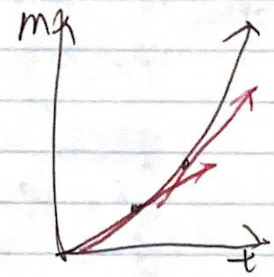
$\vec{g}$ (gravity)

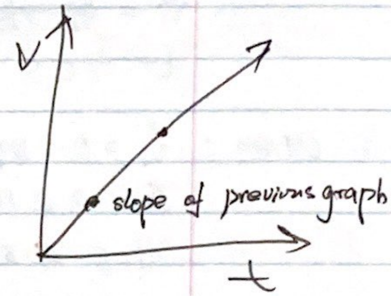




Translation :

position vs time → v vs t.





Equation For Accleration.

① $\vec{v}_{avg} = \left[\frac{\vec{v}_i - \vec{v}_f}{2}\right]$ + $\vec{v} = \frac{\Delta \vec{d}}{t}$ = $\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)t$

② $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ + $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$ = $\vec{v}_f = \vec{v}_i + \vec{a}t$

③ $\vec{v}_f = \vec{v}_i + \vec{a}t$ + $\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)t$ = $\vec{d} = \vec{v}_i \cdot t + \frac{1}{2}\vec{a}t^2$

④ $\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)t$ + $t = \frac{\vec{v}_f - \vec{v}_i}{\vec{a}}$ = $v_f^2 = v_i^2 + 2\vec{a}\vec{d}$

ex. An airplane with an initial velocity of 50.0m/s acceleration at -5.0m/s² until coming to a stop. Find time required to stop.

Given: $\vec{v}_i = 50.0 m/s$
$\vec{v}_f = 0.0 m/s$
$\vec{a} = -5.0 m/s^2$
$\vec{t} = ?$

$v_f = v_i + \vec{a}t \rightarrow t = \frac{v_f - v_i}{\vec{a}}$

$t = \frac{0.0m/s - 50.0m/s}{-5.0m/s^2} = 10.\ s$

b) What is the displacement: $\vec{d} = (\frac{\vec{v}_i + \vec{v}_f}{2}) \times t$

$\vec{d} = (\frac{0.0 m/s + 50.0 m/s}{2}) \times 10.s = 250 m.$

technique: use a formula that doesn't involve the previous value.

2. A car initially travelling at 90.0 Km/h accelerates at 2.0 m/s for 7.0 s. find final velocity.

Given: $\vec{a} = 2.0 m/s^2$

$\vec{v}_i = \cancel{90.0 km/h}$ 25 m/s

$t = 7.0 s$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_f = \cancel{90.0}^{25} + 2.0 m/s \times 7.0 s$

$v_f = \cancel{104}$ m/s 39.

3. Given: $\vec{v}_i = 0.0 m/s^2$

$\vec{a} = 3.0 m/s^2$

$v_f = 27.8 m/s$

$\cancel{\vec{d} = v_i}$

$v_f^2 = v_i^2 + 2\vec{a}\vec{d}$

$\vec{d} = \frac{v_f^2 - v_i^2}{2\vec{a}}$

$\vec{d} = \frac{(27.8 m/s)^2 - 0^2}{2 \times 3.0}$

$\vec{d} = 130 m / 129 m$

keep 2 or 3 sig figs

An object is dropped and free fall for 4.0 s, find velocity / displacement

Given: $v_i = 0.0 m/s$

$\cancel{t} = 4.0 s$

$a = -9.80 m/s^2$

$\vec{v}_f = \vec{v}_i + \vec{a}t$

$\vec{v}_f = 0.0 m/s + -9.8 m/s^2 \times 4.0 s$

$\vec{v}_f = -39.2 m/s$ / $-39 m/s$

$\vec{d} = \vec{v}_i \cdot t + \frac{1}{2}\vec{a}t^2$

$\vec{d} = 0.0 m/s \times 4.0 s + \frac{1}{2}(-9.8)^2 \times (4.0)^2$

$\vec{d} = -78.4 m$ / $-78 m$

4. throw up at initial v of 30.0 m/s, find max displacement / time

Given: $\vec{v}_i = 30.0 m/s$

$\vec{a} = -9.80 m/s^2$

$\vec{v}_f = 0.0 m/s$

$\cancel{\vec{d} = v}$

$\vec{v}_f^2 = \vec{v}_i^2 + 2\vec{a}\vec{d}$

$\vec{d} = \frac{v_f^2 - v_i^2}{2\vec{a}}$

$\vec{d} = \frac{0 - (30.0)^2}{2 \times (-9.80)}$

$\vec{d} \approx 46 m$ or 45.9 m

② $\vec{v_f} = \vec{v_i} + \vec{a}t$

$0.0 m/s = 30.0 m/s + (-9.8 m/s^2) \times t$

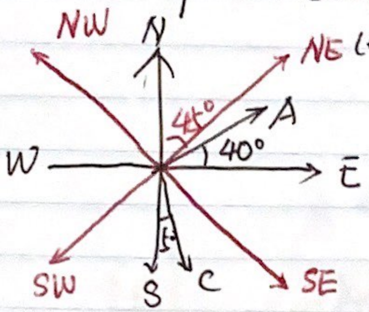
$t = 3.06 s$

Vector Additions:

Directions conventions:

▲ — Horizontal plain

- Compass / cardinal → with respect to (W.R.T) N/W/E/S



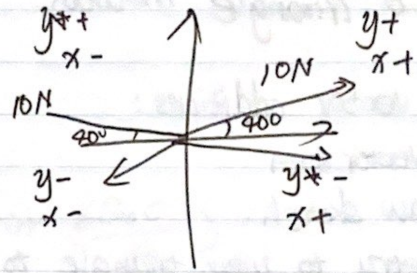
$\vec{A} = 10N [40° N of E]$ / $\vec{A} = 10N [5° E of NE]$

prefer: $\vec{C} = 7N$ [E of S]

(since it's closer)

- RCS
- ~~clockwise~~ counterclockwise from + x-axis

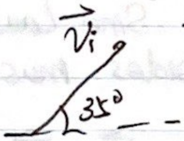
$0° \leq \theta \leq 360°$



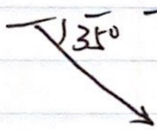
- Bearing:
- clockwise from North.

▲ — Vertical plain.

- W.R.t horizon(tal)



$\vec{v_i} = 25 m/s$ [35° a.h] [above horizontal]

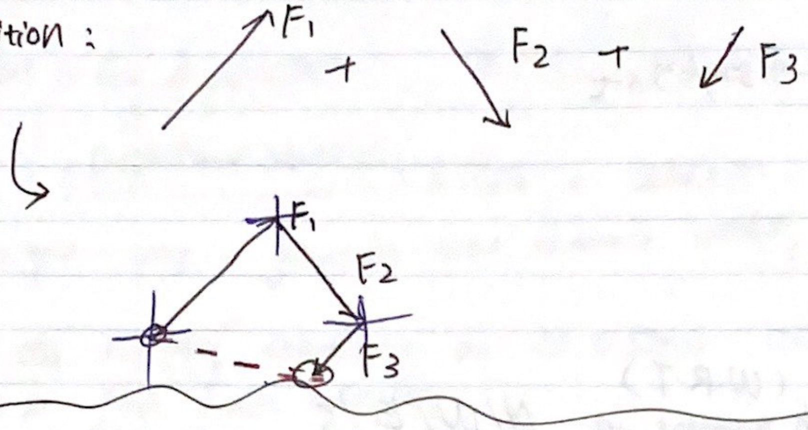


$\vec{v_f} = 25 m/s$ [35° b.h]

☆ Methods:

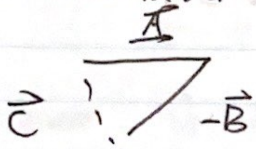
- Graphical methods:
- tip to tail
  - to the tip of the first vector goes the tail of the next vector.
  - from the tail of the first vector to the tip of the last vector goes the resultant. (sum)

condition :



Equality of vectors. (three vector rule

Vector subtraction :



$\vec{C} = \vec{A} - \vec{B}$

Two Methods: o Trinangle Method. o Component Method.

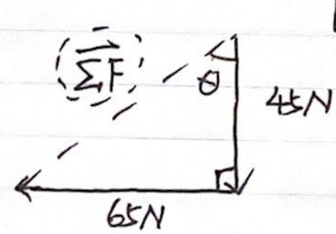
Steps solving vector additions:
- Quote the vector sum
- If 2d, draw sketch.
- Do any geometry to your triangle to learn as much about it as possible
- If RtΔ, use Pythagoras and trig.
- If not Rt. Assign coefficient to angles and sides.
  Use Cosine Law and Sine Law.
- known angle between two known sides must be C.

Two Dimensional Vector sum $\Sigma \vec{F} = \vec{F_1} + \vec{F_2}$

ex. $\vec{F_1} = 45N[S]$ $\Sigma \vec{F} = ?$

$\vec{F_2} = 65N[W]$

Find $|\Sigma \vec{F}|$



$|\Sigma \vec{F}| = \sqrt{F_1^2 + F_2^2}$

$= \sqrt{45^2 + 65^2}$

$= 79.0569 \ldots$

$\theta = \cancel{Tan} \; Tan^{-1}\left(\frac{F_2}{F_1}\right) = tan^{-1}\left(\frac{65}{45}\right) = 55.3°$

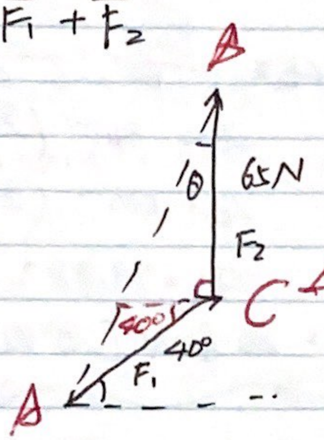
$\Sigma \vec{F} = 79N[55° \text{ W of S}]$

ex2. $\vec{F_1} = 45N$ [40° N of E]

$\vec{F_2} = 65$ [N]

$\sum \vec{F} = ?$

$\Rightarrow \sum \vec{F} = \vec{F_1} + \vec{F_2}$



Given formula: $c^2 = a^2 + b^2 - 2ab\cos C$.

known angle between two known sides

$|\sum \vec{F}| = \sqrt{a^2 + b^2 - 2ab\cos C}$

$= \sqrt{65^2 + 45^2 - 2 \times 65 \times 45 \cos 130°}$

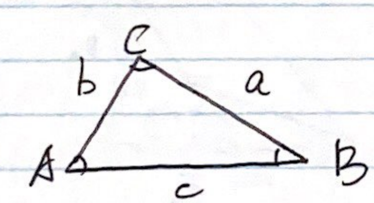
$= 100.051524$

$\angle A = \frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow A = \sin^{-1}\left(\frac{a\sin C}{c}\right)$

$A = \sin^{-1}\left(\frac{65 \sin 130}{100.051524}\right)$

$= 29.84°$

$\sum \vec{F} = 10 \times 10N$ [50° N of E]



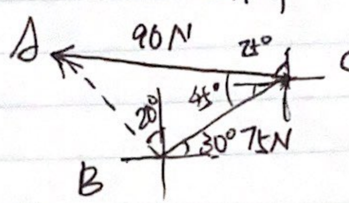
① $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ ② $c^2 = a^2 + b^2 - 2ab\cos\angle C$

ex. $\vec{F_1} = 75\,N\,[30^\circ\,N\text{ of }E]$

$\vec{F_2} = 90\,N\,[75^\circ\,W\text{ of }N]$

$\Sigma\vec{F} = \vec{F_1} + \vec{F_2}$

$|\Sigma\vec{F}|$



$|\Sigma\vec{F}| = c = \sqrt{a^2+b^2-2ab\cos\angle C}$

$= \sqrt{75^2+90^2-2\times75\times90\times\cos 45^\circ}$

$= \sqrt{6975}$

$= 64.645$

$\angle B = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

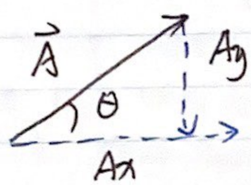
$\angle B = \sin^{-1}\left(\dfrac{\sin C \times b}{c}\right)$

$\angle B = \sin^{-1}\left(\dfrac{\sin 45^\circ \times 90}{64.645}\right)$

$\angle B = 79.88^\circ$

Answer: $\Sigma\vec{F} = 65N\,[110^\circ]$ / $\Sigma\vec{F} = \cancel{80}\ 65N\,[20^\circ\,W\text{ of }N]$

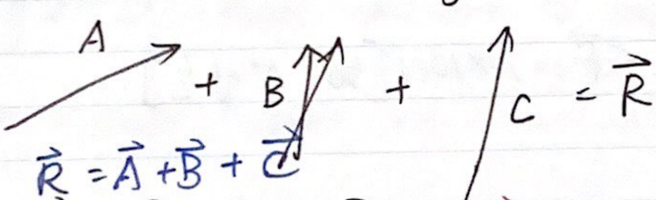
Component Method:

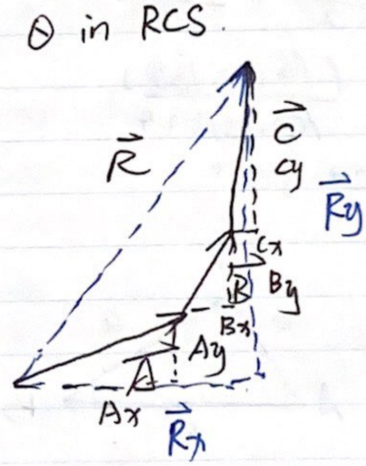


calculate the A_x, and A_y for the sum, which gives vector $\vec{A}$

$\vec{A} = \vec{A_x} + \vec{A_y}$

$\left.\begin{array}{l}\vec{A_x} = \vec{A}\cos\theta \\ \vec{A_y} = \vec{A}\sin\theta\end{array}\right\}$ θ in RCS.





$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$\vec{R_x} = \vec{A_x} + \vec{B_x} + \vec{C_x}$ $\quad \Sigma\vec{x}$

$\vec{R_y} = \vec{A_y} + \vec{B_y} + \vec{C_y}$ $\quad \Sigma\vec{y}$

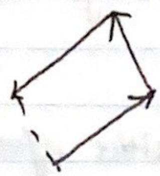
~~mag~~ magnitude is exact, sum is based on definition.

ex. $\vec{F_1} = 150N[45°]$

$\vec{F_2} = 90N[100°]$

$\vec{F_3} = 110N[220°]$

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$



$\vec{R_x} = \vec{F_{1x}} + \vec{F_{2x}} + \vec{F_{3x}}$

$= \vec{F_1}\cos\theta_1 + \vec{F_2}\cos\theta_2 + \vec{F_3}\cos\theta_3$

$= 150\cos 45° + 90\cos 100° + 110\cos 220°$

$= 66.80$

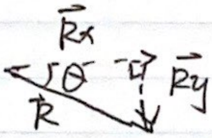
$\vec{R_y} = \vec{F_{1y}} + \vec{F_{2y}} + \vec{F_{3y}}$

$= \vec{F_1}\sin\theta_1 + \vec{F_2}\sin\theta_2 + \vec{F_3}\sin\theta_3$

$= 150\sin 45° + 90\sin 100° + 110\sin 220°$

$= -42.58\ldots$

$\vec{R} = \vec{R_x} + \vec{R_y}$



$|\vec{R}| = \sqrt{\vec{R_x}^2 + \vec{R_y}^2} = \sqrt{66.80^2 + (-42.58)^2} = 79.2$

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \cancel{325}° \quad \tan^{-1}\left(\frac{-42.58}{66.80}\right) = 32.5°$

$\vec{R} = 79.2N[327.5°]$

Test Points.

$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

$\vec{d} = \left(\frac{\vec{v_i} + \vec{v_2}}{2}\right) \times t$

$\vec{d} = \vec{v_i} \times t + \frac{1}{2}\vec{a}t^2$

$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\vec{d}$

$\vec{v_f} = \vec{v_i} + \vec{a}t$

Relativity:

A transformation is a mathematical function or formula used to tranform quality.

[Galilean transformation] use to describe velocities reference frames to called the Vector Equation.

Vector Equation:

$$\vec{V}_{AC} = \vec{V}_{AB} + \vec{V}_{BC}$$

object A relative to space C.

- Note how the subscripts line up in the vector equation
- First subscript are the same.
- Last subscripts are the same.
- Adjacent subscripts are the same

Provide a legend

ex. flip the object, flip the direction

$$\vec{V}_{FC} = -\vec{V}_{CF}$$

class relative to Fulop.

Fulop relative to class

ex 2. $\vec{V}_{PT} = -2.0m/s$ $\vec{V}_{TS} = 6.0m/s$ $\vec{V}_{LS} = 3.0m/s$

What is $\vec{V}_{PL}$?

$$\vec{V}_{PL} = \vec{V}_{PT} + \vec{V}_{TS} + \vec{V}_{SL}$$

$$\vec{V}_{PL} = (-2.0 + 6.0 - 3.0) m/s$$

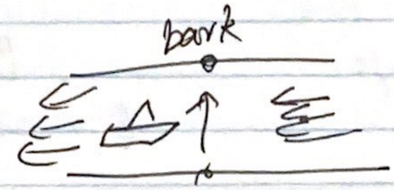
$$\vec{V}_{PL} = +1.0 m/s.$$

physics Language:

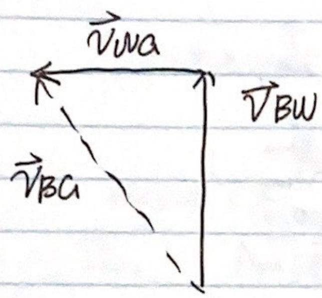
- Heading direction that a craft is pointing. Not necessarily the moving direction.

River Problems:

Water:
Boat:
Ground.

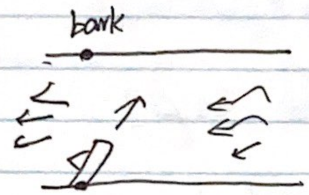


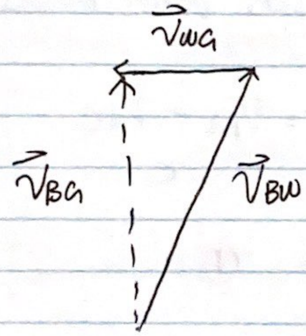
①



$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

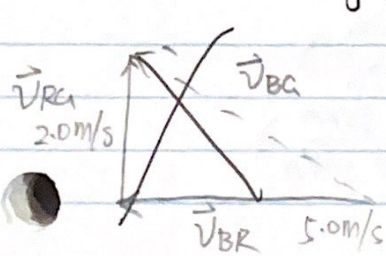
②





$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

ex. A boat capable of moving at 5.0m/s w.r.t water. The boat $\boxed{\text{wishes}}$ to travel directly west across a river flowing North at 2.0m/s. Determine
a) Heading of the boat
b) Time to cross if river is 500 m wide.



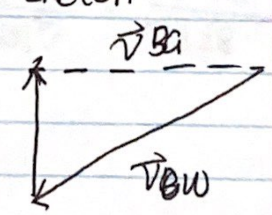
$\vec{v}_{BG} = \vec{v}_{BR} + \vec{v}_{RG}$

Given: $\vec{v}_{BW} = 5.0$ m/s [?] $\vec{v}_{WG}$

$\vec{v}_{WG} = 2.0$ m/s [N]

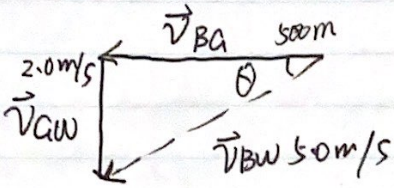
$\vec{v}_{BG} = ?$ [W]

Sketch



Alternate Method.

$\vec{V}_{BW} = \vec{V}_{BG} + \vec{V}_{GW}$ a) Heading: $\theta = \sin^{-1}\left(\frac{\vec{V}_{GW}}{\vec{V}_{BW}}\right)$



$\theta = \sin^{-1}\left(\frac{2.0\,m/s}{5.0\,m/s}\right)$

$\theta = 23.6°$

$\theta = 23.6°$ [S of W]

b) $|\vec{V}_{BG}| = \sqrt{\cancel{\vec{V}_{GW}^2 + \vec{V}_{BW}^2}\ \vec{V}_{BW}^2 - \vec{V}_{GW}^2}$

$|\vec{V}_{BG}| = \sqrt{\cancel{2.0^2+5}\ 5.0^2 m/s^2 - 2.0^2}$

$|\vec{V}_{BG}| = \sqrt{(25-4)\,m^2/s^2}$

$|\vec{V}_{BG}| = \sqrt{21}$ m/s

$|\vec{V}_{BG}| = 4.58$ m/s

$\vec{d} = \vec{V}_{avg} \times t$

$t = \frac{\vec{d}}{\vec{V}_{avg}}$

$t = \frac{500m}{\sqrt{21}\,m/s}$

$t = 109.1s$

Air Plane Problem [Don't assume Rt △]

A plane flies at 450 km/h 40° [W of N] w.r.t the air, wind blows at 85 km/h west. Find Ground speed.

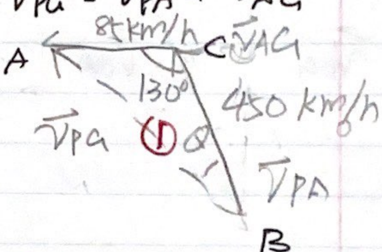
Given: ① $\vec{V}_{PA}$ = 450 km/h [40° W of N]

$\vec{V}_{AG}$ = 85 km/h [W]

$\vec{V}_{PG}$ = ? [?]

① Sketch : $\vec{V}_{PG} = \vec{V}_{PA} + \vec{V}_{AG}$

Sketch.



① $|\vec{V}_{PG}| = \sqrt{a^2+b^2-2ab\cos\angle C}$

$= \sqrt{450^2 + 85^2 - 2 \times 450 \times 85\cos 130°}$

$= 508.8$ km/h

① Legend:

P = plane G = ground.

A = Air

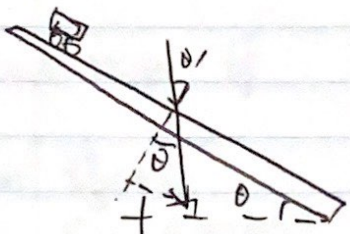
① $\angle\theta$: $\frac{\sin\angle\theta}{85km} = \frac{\sin\angle C}{\vec{V}_{PG}} \rightarrow \angle\theta = \sin^{-1}\left(\frac{\sin\angle C \cdot \vec{V}_{AG}}{\cancel{\sin\angle}\ \vec{V}_{PG}}\right)$

$\angle\theta = \sin^{-1}\left(\frac{\sin 130° \times 85km}{508.8\ km/h}\right)$

$\angle\theta = 7.35°$

① $\vec{V}_{PG}$ = 509 km/h [47.4° W of N] ①

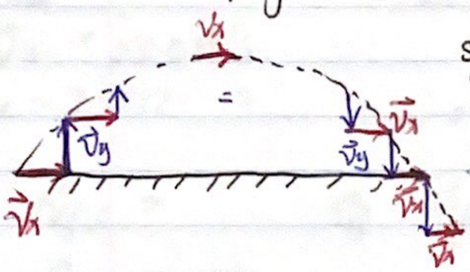
Marks Distribution ↑ Dwnlod (sparkvue) (Pasco)



$g_{pll} = g \sin\theta$

Projectiles

①

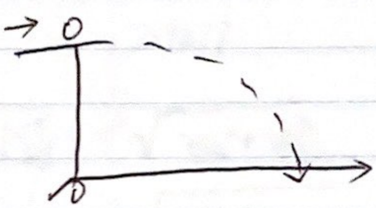


symmetric projectile

$\vec{d}_y = 0$

→ $\vec{v}_x$ constant ↑$\vec{v}_y$

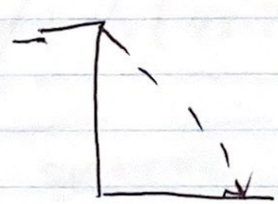
② Horizontal projectile $(\vec{v}_{yi} = 0)$



③ Asymmetric Projectile: $(\vec{d}_y < 0, \vec{v}_{yi} > 0)$ or $(\vec{d}_y < 0, \vec{v}_{yi} < 0)$

①

②



Some ~~Trigonom~~ Terminology and math.

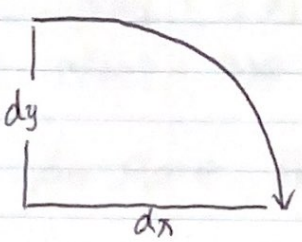
Trajectory: the path of projectile
Range: the x - displacement of a projectile.

$\vec{v}_x = \vec{v} \times \cos\theta$ $\quad$ $\vec{v}_y = \vec{v}\sin\theta$

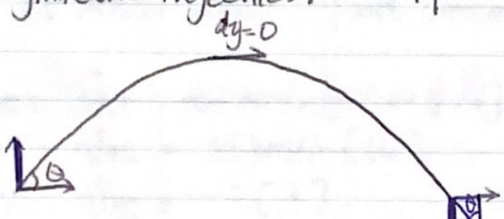
Time of flight: $\vec{d}_y = \vec{v}_{yi}t + (1/2)\vec{a}_y t^2$

Types of projectiles:

① Horizontal projectiles: kinametic discriptor $\vec{v}_{yi} = 0$



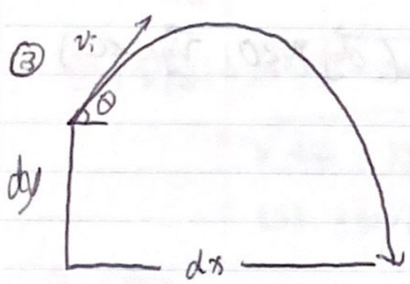
② Symmetric Projectiles: discriptor: $\vec{d}_y = 0$



$|\vec{v}_{i}| = |\vec{v}_{f}|$

$|\vec{v}_i| = |\vec{v}_f|$

③



$\vec{d}_y \neq 0$, $\vec{v}_i \neq 0$

$\vec{d}_y < 0$
$\vec{v}_{yi} < 0$

$\vec{d}_y > 0$
$v_{yi} > 0$

Horizontal

ex. A car drives straight off a 420m high cliff at 28.0m/s.
a) determine time of flight
b) determine how far from base of cliff is car hitting.
c) Determine $\vec{v}_f$.

a) Given $\vec{v}_{yi} = 0$
$\vec{a}_y = -9.80\,m/s^2$
$t = ?$
$\vec{d}_y = -420\,m$.

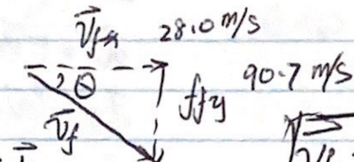
$$\vec{d}_y = \cancel{\vec{v}_{yi} t}^{\,0} + \frac{1}{2}\vec{a}_y t^2$$
$$\sqrt{\frac{2\vec{d}_y}{\vec{a}_y}} = t$$
$$t = \sqrt{\frac{2\times(-420)m}{-9.80\,m/s^2}} =$$

b) $\vec{d}_x = ?$
$\vec{v}_x = 28.0\,m/s$
$t = 2.9277\ldots s$

$$\vec{v}_x = \frac{\vec{d}_x}{t} \Rightarrow \vec{d}_x = \vec{v}_x \times t$$
$$d_x = 28.0\,m/s \times 2.92 = 82.0\,m.$$

c) $\vec{v}_f = ?$
$\vec{v}_{xf} = 28.0\,m/s$
$v_{yf} = ?$
$\vec{a}_y = -9.80\,m/s^2$
$t = 2.9277$

$$\vec{v}_f = \vec{v}_{fx} + \vec{v}_{fy}$$



$$\boxed{\vec{v}_{fy} = \vec{v}_i + \cancel{2}\vec{a}t \qquad \vec{v}_{fy} = \vec{v}_i + a(-9.8)}$$

$$\vec{v}_{yf}^{\,2} = v_{iy}^2 + 2\vec{a}\vec{d}$$
$$\vec{v}_{yf}^{\,2} = \sqrt{0^2 + 2\times(-9.80)\times \cancel{2.9277}(-420)}$$
$$v_{yf} = 90.73$$

$$|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2}$$
$$= \sqrt{28^2 +}$$
$$= 40.1\,m/s$$

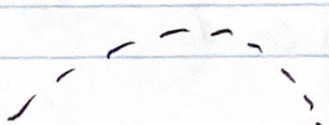
$$\theta = \tan^{-1}\left(\frac{\vec{v}_{fy}}{\vec{v}_{fx}}\right)$$
$$= \tan^{-1}\left(\frac{78.69}{28.0}\right)$$
$$= 45.7^\circ$$

$\therefore \vec{v}_f = 40.1\,m/s$ [45.7° b.h]

---

symmetric

ex. A cannon fires a ball over level ground at 245 m/s [45° a.h]



a) determine the time of flight
b) Determine range.
c) $\vec{v}_f$ By symmetry, $\vec{v}_f = 245\,m/s$ [45°, b.h]
d) maximum height ($\vec{d}_y$ max)

a) $\cancel{\vec{d}_y}^{\,0} = \vec{v}_{yi}\, t + \frac{1}{2}\vec{a}_y t^2$

$0 = \vec{v}_{yi}\, t + \frac{1}{2}\vec{a}_y t^2$

$\frac{-2\vec{v}_{yi}\,\cancel{t}^{\times 1}}{\vec{a}_y} = \cancel{t^2}\, t \Rightarrow \frac{-2\vec{v}_{yi}}{\vec{a}_y} = t$

Given: $\vec{d}_y = 0$

$\vec{v}_{yi} = \vec{v}_y \sin\theta$

~~$\vec{v}_{yi}$~~ $\vec{v}_i = 245 m/s$

$\theta = 45°$

$\vec{a}_y = -9.80 m/s^2$

$\frac{-2\vec{v}_i \sin\theta}{\vec{a}_y} = t$

$t = \frac{-2(245 \sin 45°)}{\cancel{\vec{a}_y}\ -9.8 m/s^2}$

$t = 35.4 s$

b) $\vec{d}_x = ?$

$\vec{v}_{ix} = \vec{v}_i \cos\theta$

$\vec{v}_i = 245 m/s$

$\theta = 45°$

$t = 35.4 s$

~~$\vec{d}_x = \vec{v}_{ix} \times t + \frac{1}{2}\vec{a} t^2$~~

$\vec{d}_x = (\vec{v}_{ix} + \cancel{\vec{a}t}\ \vec{v}_{fx}) \times \frac{1}{2} \times t$

$\vec{d}_x = (\vec{v}_i \cos\theta + \cancel{\vec{a} \times 35.4 s}\ \vec{v}_i \cos\theta) \times \frac{1}{2} \times t$

$\vec{d}_x = 6125 m.$

d) $\boxed{\vec{d}_{ymax} = \vec{v}_{yi} \times t_{\frac{1}{2}} + \frac{1}{2}\vec{a}_y t^2}$

or $\vec{v}_{fy}^2 = \vec{v}_{yi}^2 + 2\vec{a}\vec{d}_{ymax}$

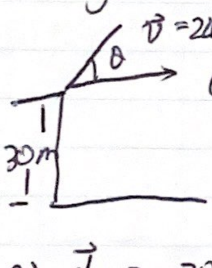
or $\vec{d}_{ymax} = \left(\frac{\vec{v}_{iy} + \vec{v}_{fy}}{2}\right) \times t_{\frac{1}{2}}$

$\vec{d}_{ymax} = \left(\frac{0 + 245 \sin 45°}{2}\right) \times 35.4 \times \frac{1}{2}$

$= 1531.25$

$\approx 1530 m.$

ex. Asymmetric Projectile.



$\theta = 42°$

a) Determine time of flight.

b) Determine $\vec{d}_{ymax}$.

c) Determine time for cannon ball come back down with 40.0m of the ground.

a) $\vec{d}_y = -30.0 m$

$\vec{a}_y = -9.8 m/s^2$

$t = ?$

$\vec{v}_{yi} = \vec{v} \sin\theta$

$\vec{v}_i = 245 m/s$

$\theta = 42°$

$\vec{d}_y = v_{iy} \times t + \frac{1}{2}\vec{a}_y t^2$

$t = \frac{-v_{iy} \pm \sqrt{v_{iy}^2 + 4 \times \frac{1}{2}\vec{a}_y \times \vec{d}_y}}{2 \times \frac{1}{2}\vec{a}_y}$

$t = \frac{-v_{iy} \pm \sqrt{v_{iy}^2 + 2\vec{a}_y \vec{d}_y}}{\vec{a}_y}$

$A = \frac{1}{2}\vec{a}_y = \cancel{4}.9$

$B = \vec{v}_i \sin\theta = 163.93$

$C = -\vec{d}_y = -(-30.0) = 30$

By quadratics $t = \dfrac{-163.93\cdots \pm \sqrt{(163.93)^2 - 4\times 4.9\times 30}}{2\times(4.9)}$

$t = 33.6\,s$ or $\cancel{1.74\,s}$ $-0.18\,s$

c)

$A = -4.9$

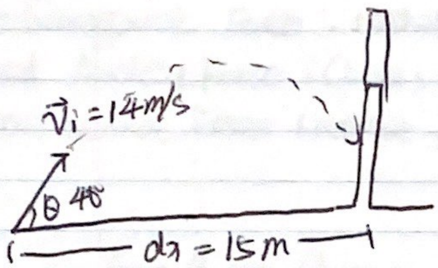
$B = 163.93\cdots$

$C = -d_y = -10\,m$

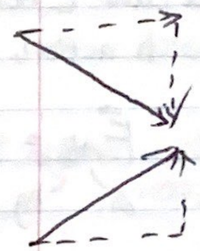
$t = \dfrac{-163.93 \pm \sqrt{(163.93)^2 + 4\times 4.9\times 10}}{-2\times 4.9}$

$t = 33.5\,s \qquad t = 0.006\,s.$

Determine $\vec{v}_f$



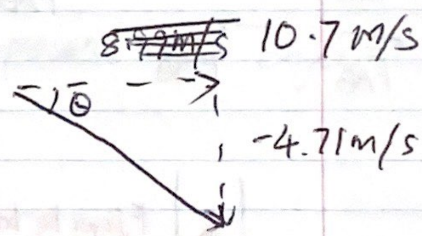
$\vec{v}_f = \vec{v}_{xf} + \vec{v}_{yf}$ ①



$v_{ix} \cdot t = \vec{d}_x$

$t = \dfrac{d_x}{v_{ix}}$

$t = \dfrac{d_x}{v\cos 40^\circ} \qquad t = 1.39\,s$



$v_{fy} = v_{iy} + \vec{a}_y t$

$v_{fy} = \vec{v}\sin 40^\circ + (-9.8)\times 1.39$

$v_{fy} = -4.718\,m/s$

$|\vec{v}_f| = \sqrt{\vec{v}_{fx} + \vec{v}_{fy}}$

$= \sqrt{\cancel{8.99^2 + 4.71^2}\ 10.7^2 + 4.71^2}$

$= 11.5$