Text as Data

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Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2019)

1) Discovery: a hypothesis or view of the world
2) Measurement according to some organization
3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Measurement

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- Positions on Court Cases
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- Taunting in floor statements

  ⇒ { Partisan Taunt, Intra party taunt, Agency taunt, ... }
- Negative campaigning

  ⇒ { Negative ad, Positive ad}

Pre-existing word weights$\rightsquigarrow$ Dictionaries

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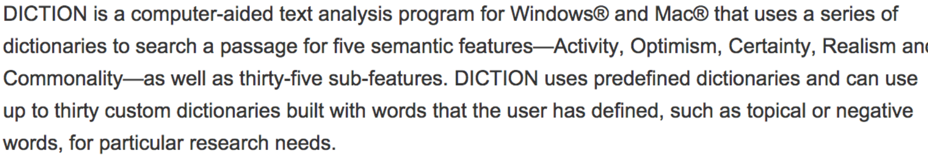
`DICTION`



DICTION is a computer-aided text analysis program for Windows® and Mac® that uses a series of dictionaries to search a passage for five semantic features—Activity, Optimism, Certainty, Realism an Commonality—as well as thirty-five sub-features. DICTION uses predefined dictionaries and can use up to thirty custom dictionaries built with words that the user has defined, such as topical or negative words, for particular research needs.

Pre-existing word weights $\rightsquigarrow$ Dictionaries

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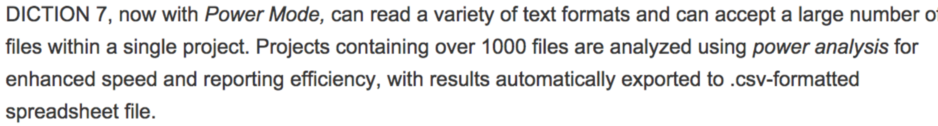

DICTION 7, now with *Power Mode,* can read a variety of text formats and can accept a large number o files within a single project. Projects containing over 1000 files are analyzed using *power analysis* for enhanced speed and reporting efficiency, with results automatically exported to .csv-formatted spreadsheet file.

Pre-existing word weights⇝ Dictionaries

`DICTION`

On an average computer, DICTION can process over 20,000 passages in about five minutes. DICTION requires 4.9 MB of memory and 38.4 MB of hard disk space.

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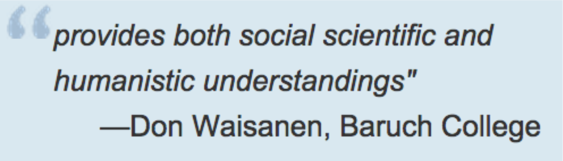

> *provides both social scientific and humanistic understandings"*
>
> —Don Waisanen, Baruch College

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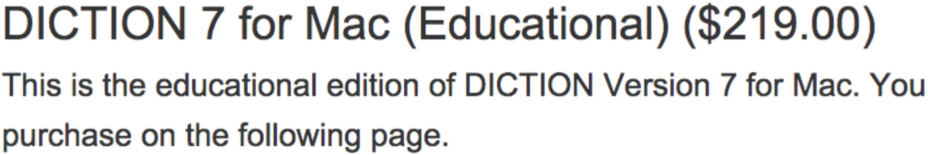
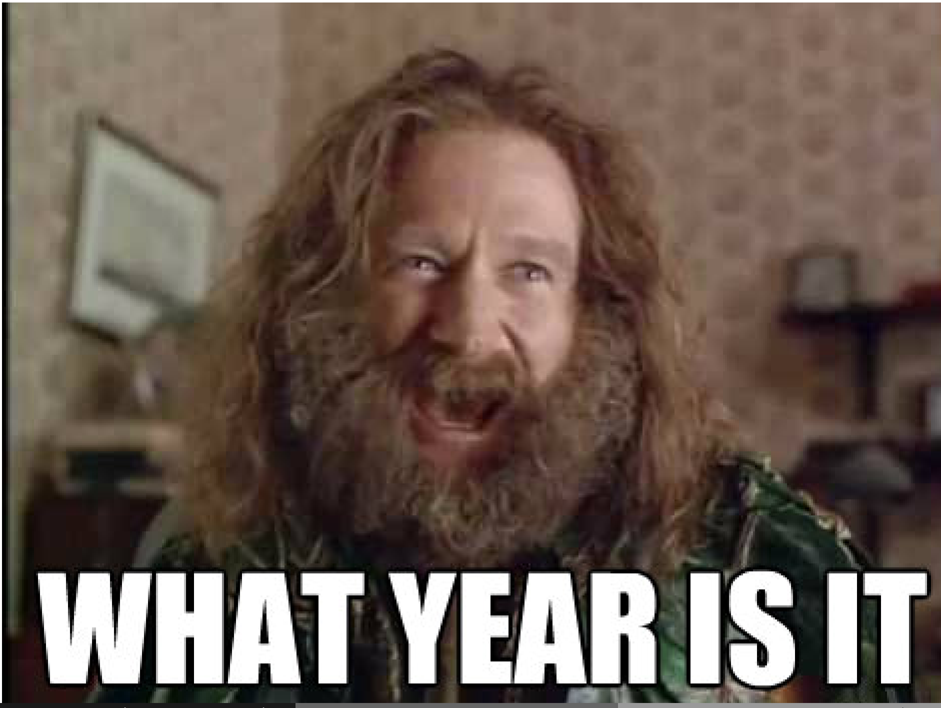


DICTION 7 for Mac (Educational) ($219.00)

This is the educational edition of DICTION Version 7 for Mac. You
purchase on the following page.


WHAT YEAR IS IT

Dictionary Methods

Many Dictionary Methods (like DICTION)

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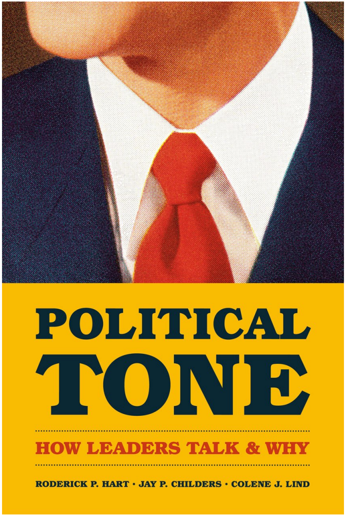
POLITICAL
TONE
HOW LEADERS TALK & WHY
RODERICK P. HART · JAY P. CHILDERS · COLENE J. LIND

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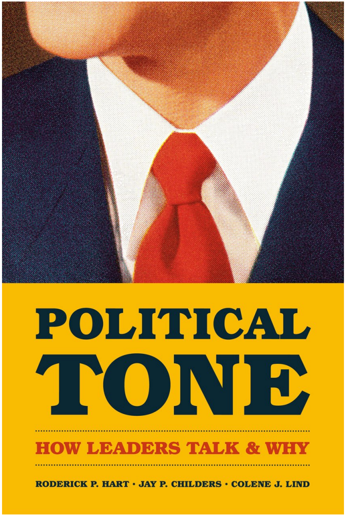

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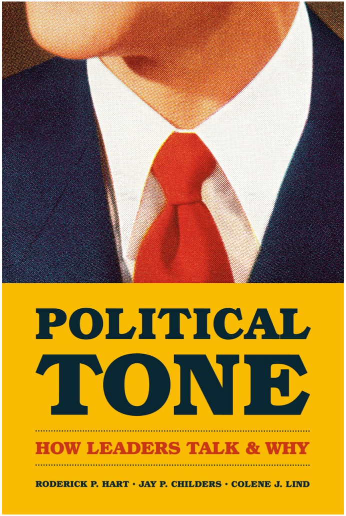

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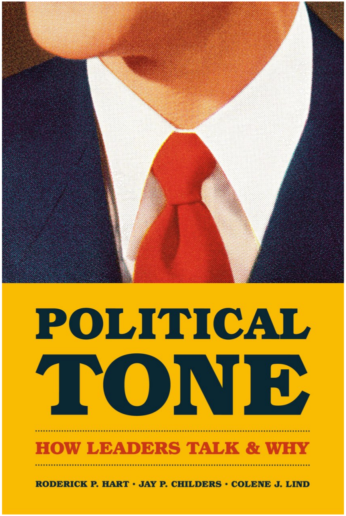

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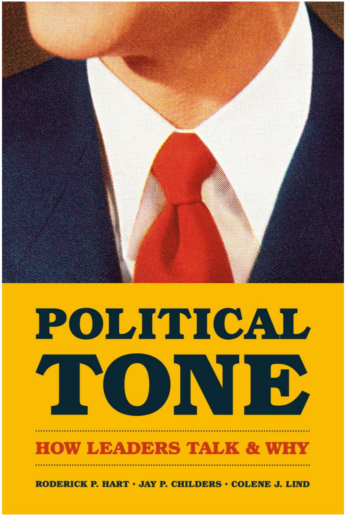

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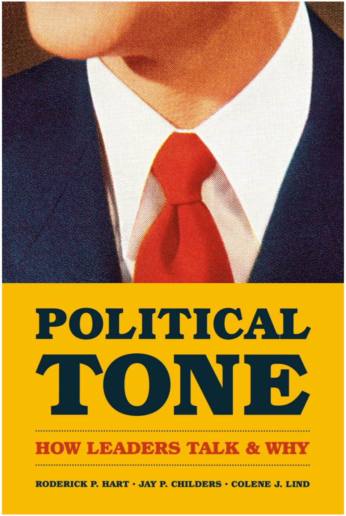

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Examine specific periods of American political history

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For each document i calculate score for document

$$\begin{aligned} Y_i &= \frac{\sum_{k=1}^{K} \theta_k X_{ik}}{\sum_{k=1}^{K} X_k} \\ Y_i &= \frac{\boldsymbol{\theta}^{'} \boldsymbol{X}_i}{\boldsymbol{X}_i^{'} \mathbf{1}} \end{aligned}$$

$Y_i \approx$ continuous $\rightsquigarrow$ Classification

$Y_i > 0 \Rightarrow$ Positive Category

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Applying Methods to Documents

Applying the model:

- Vector of word counts: $\boldsymbol{X}_i = (X_{i1}, X_{i2}, \ldots, X_{iK})$, $(i = 1, \ldots, N)$
- Weights attached to words $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_K)$
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`Python` code and press releases

Examining Positive and Negative Statements in Press Releases

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Least positive members of Congress:

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Legislators who are more extreme $\rightsquigarrow$ less positive in press releases

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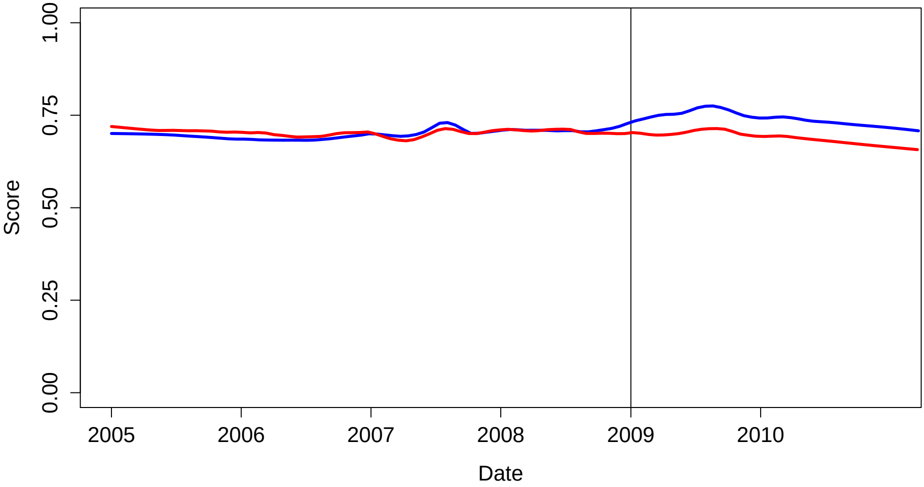

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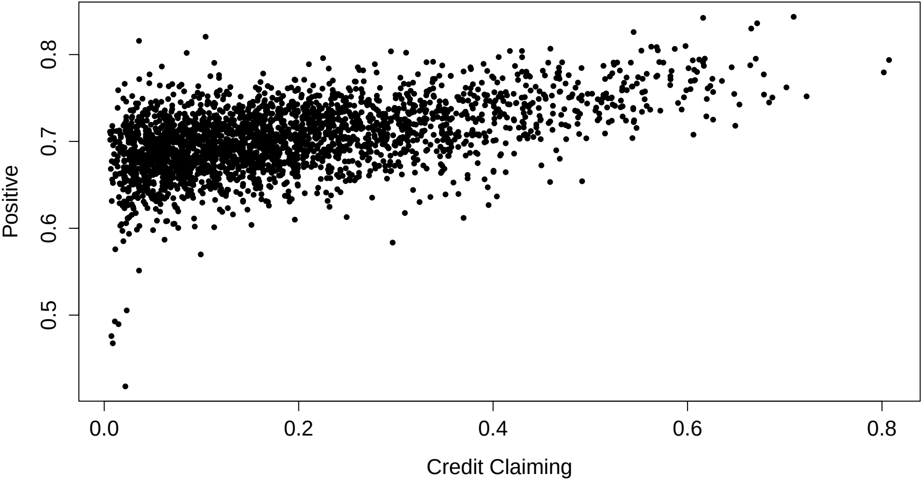

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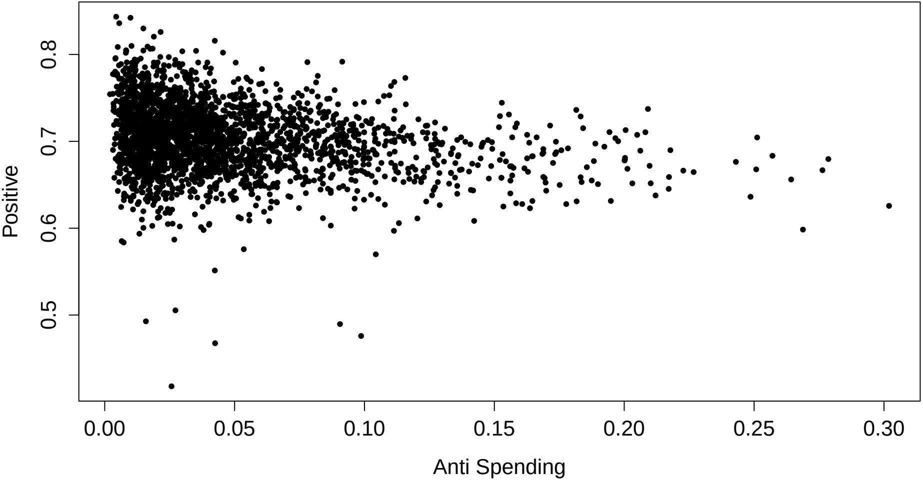

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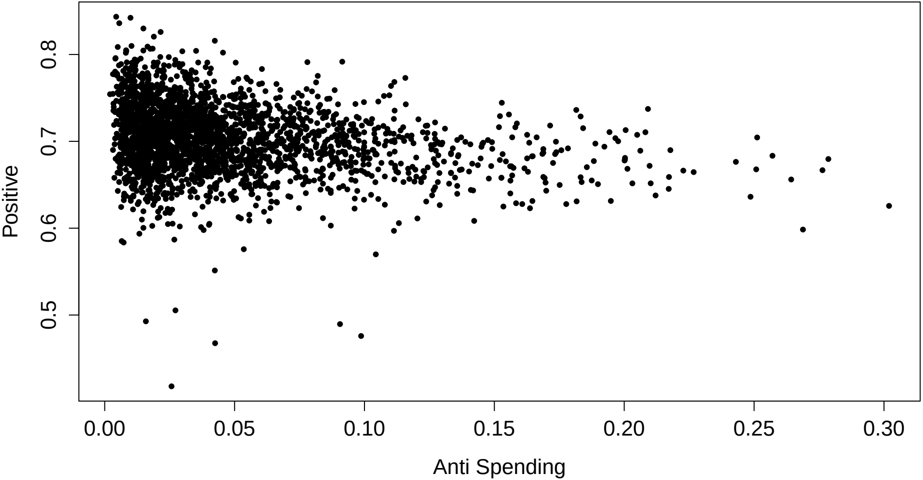

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  - Ambiguity in classification rules
- A procedure for training coders:
  1) Coding rules
  2) Apply to new texts
  3) Assess coder agreement (we'll discuss more in a few weeks)
  4) Using information and discussion, revise coding rules

Assessing Classification

Measures of classification performance

	Actual Label	
Guess	Liberal	Conservative
Liberal	True Liberal	False Liberal
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Under reported for dictionary classification

What about continuous measures?

$\rightsquigarrow$

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Necessarily more complicated

⇝

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- Go back to hand coding exercise

⇝

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$\leadsto$

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Modifiable areal unit problem in texts ⇝ aggregating destroys information, conclusion may depend on level of aggregation

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Accounting Research: measure tone of 10-K reports

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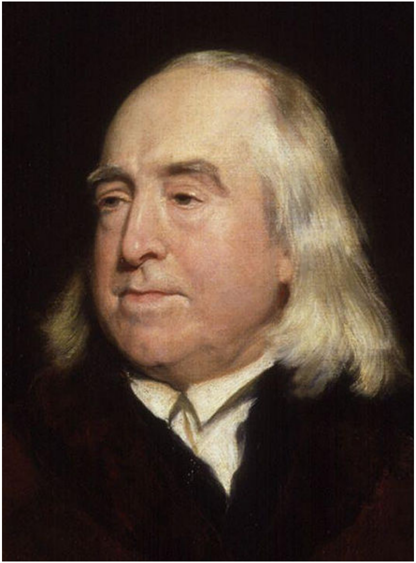
Measuring Happiness



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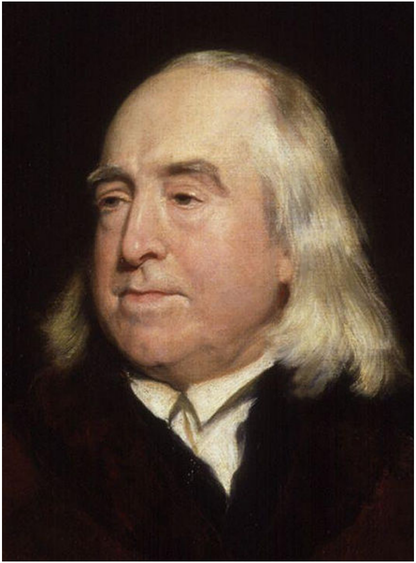

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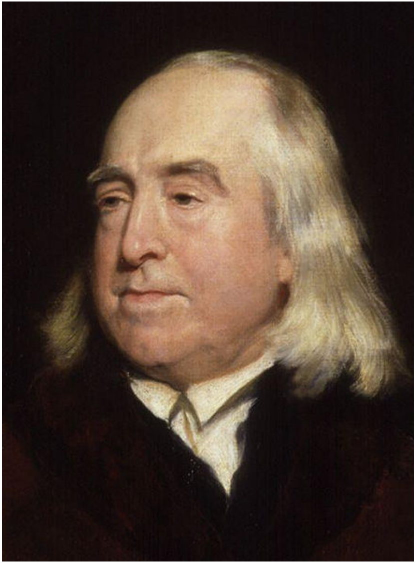

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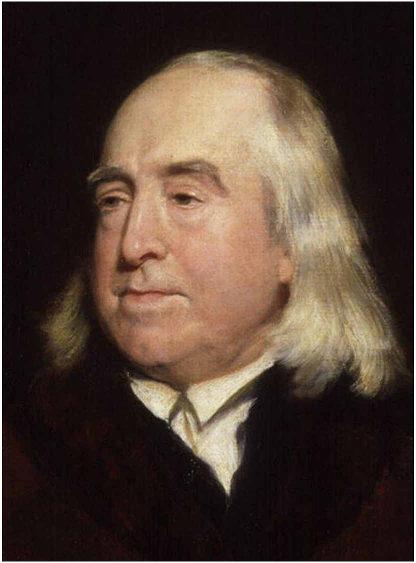

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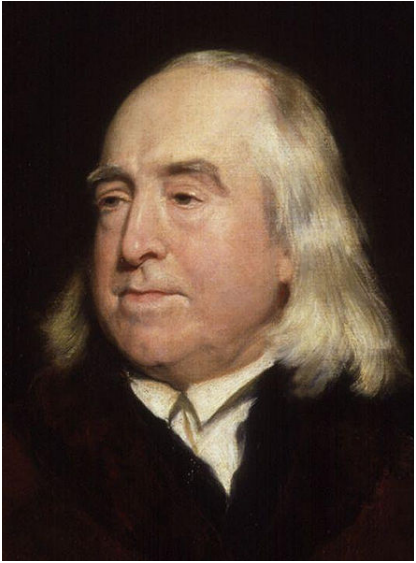

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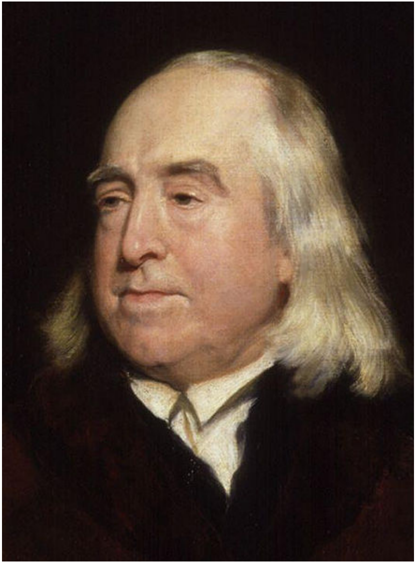

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$$\text{Happiness}_i = \frac{\sum_{k=1}^{K} \theta_k X_{ik}}{\sum_{k=1}^{K} X_{ik}}$$

Lyrics for
Michael Jackson's Billie Jean

"She was more like a beauty queen
from a movie scene.
⋮
And mother always told me,
be careful who you love.
And be careful of what you do
'cause the lie becomes the truth.

Billie Jean is not my lover,
She's just a girl who claims
that I am the one.
⋮

ANEW words	v_k	f_k
k=1. love	8.72	1
2. mother	8.39	1
3. baby	8.22	3
4. beauty	7.82	1
5. truth	7.80	1
6. people	7.33	2
7. strong	7.11	1
8. young	6.89	2
9. girl	6.87	4
10. movie	6.86	1
11. perfume	6.76	1
12. queen	6.44	1
13. name	5.55	1
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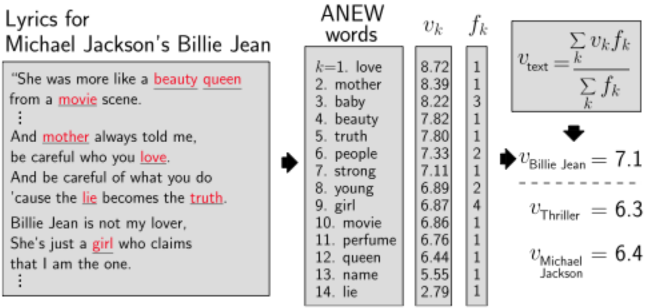
$$v_{\text{text}} = \frac{\sum_k v_k f_k}{\sum_k f_k}$$

$$v_{\text{Billie Jean}} = 7.1$$

$$v_{\text{Thriller}} = 6.3$$

$$v_{\text{Michael Jackson}} = 6.4$$



Lyrics for
Michael Jackson's Billie Jean

"She was more like a beauty queen
from a movie scene.
⋮
And mother always told me,
be careful who you love.
And be careful of what you do
'cause the lie becomes the truth.
Billie Jean is not my lover,
She's just a girl who claims
that I am the one.
⋮

ANEW words	v_k	f_k
k=1. love	8.72	1
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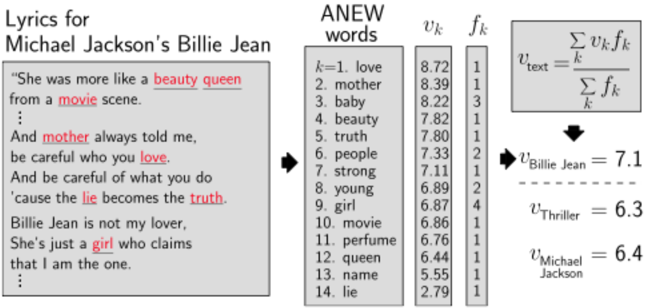

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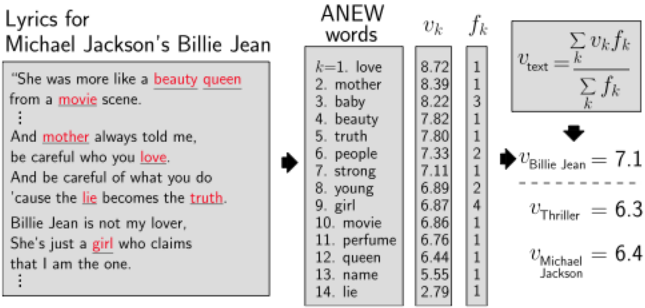

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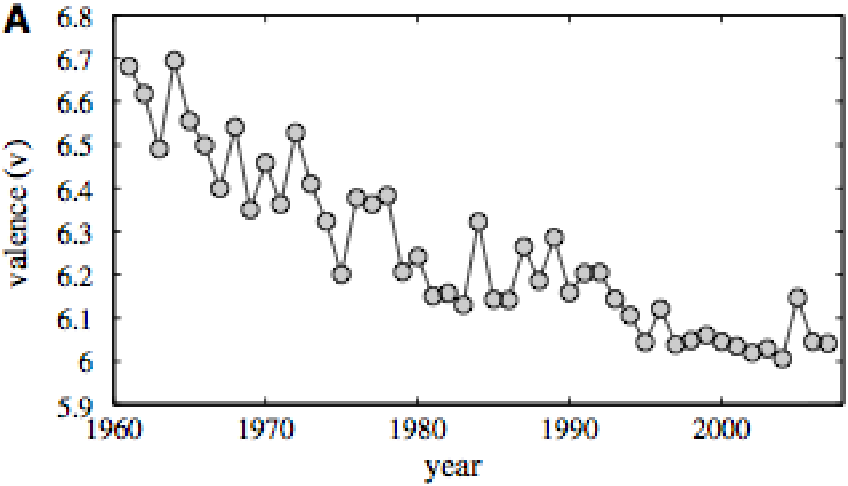
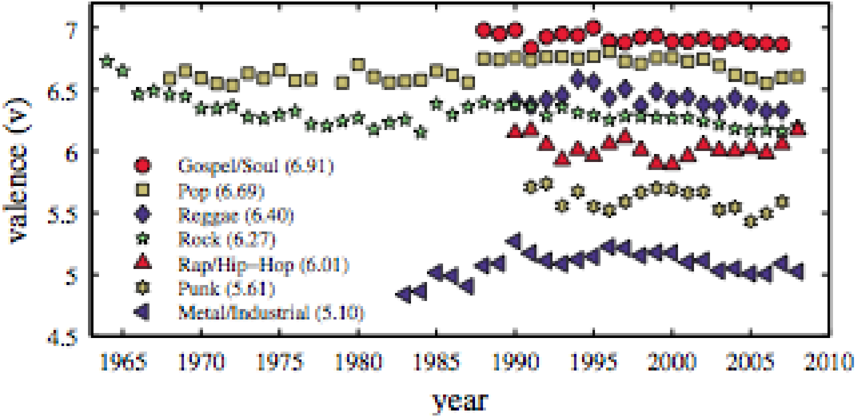
Happiest Song on Thriller?

P.Y.T. (Pretty Young Thing) (This is the right answer!)

Happiness in Society



Happiness in Society



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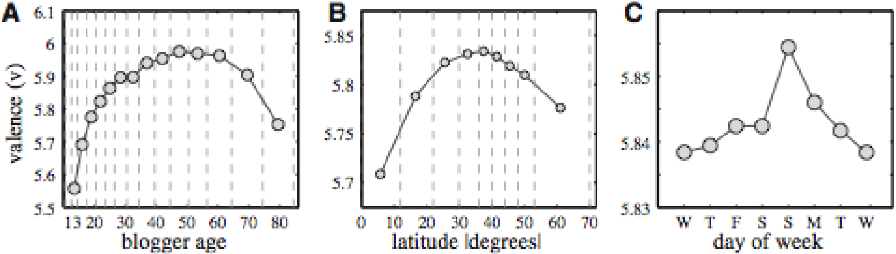

Supervised Learning

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Supervised Methods:

Supervised Learning

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- Models for categorizing texts

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    - Know (develop) categories before hand

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Supervised Methods:

- Models for categorizing texts
    - Know (develop) categories before hand
    - Hand coding: assign documents to categories
    - Infer: new document assignment to categories (distribution of documents to categories)

Supervised Learning

Supervised Learning

- How to generate valid hand coding categories

Supervised Learning

- How to generate valid hand coding categories
    - Assessing coder performance
    - Assessing disagreement among coders
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Methods generalize beyond text

Components to Supervised Learning Method

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3) Set of unlabeled documents
4) Method to extrapolate from hand coding to unlabeled documents

Three categories of documents

Hand labeled

- Training set (what we'll use to estimate model)
- Validation set (what we'll use to assess model)

Unlabeled

- Test set (what we'll use the model to categorize)

Label more documents than necessary to train model

Regression models

Suppose we have N documents, with each document i having label $y_i \in \{-1, 1\} \rightsquigarrow \{\text{not, credit claiming}\}$

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Predictions will be variable

Mean Square Error

Suppose θ is some value of the true parameter

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To reduce MSE, we are willing to induce bias to decrease variance⇝ methods that shrink coefficeints toward zero

Ridge Regression

Penalty for model complexity

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$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y})$$

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$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \quad = \quad \sum_{i=1}^{N}\left(y_i - \ \beta_0 - \sum_{j=1}^{J}\beta_j x_{ij}\right)^2$$

Ridge Regression

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where:

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where:

- $\beta_0 \rightsquigarrow$ intercept
- $\lambda \rightsquigarrow$ penalty parameter
- Standardized $\boldsymbol{X}$ (coefficients on same scale)

Ridge Regression$\rightsquigarrow$ Optimization

$$\boldsymbol{\beta}^{\text{Ridge}} = \text{arg min}_{\boldsymbol{\beta}} \left\{ f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \right\}$$

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Demean the data and set $\beta_0 = \bar{y} = \sum_{i=1}^{N} \frac{y_i}{N}$

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\end{aligned}$$

Ridge Regression$\leadsto$ Intuition (1)

Suppose $\boldsymbol{X}^{'}\boldsymbol{X} = \boldsymbol{I}_J$.

$$\begin{aligned}
\widehat{\boldsymbol{\beta}} &= \left(\boldsymbol{X}^{'}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\
&= \boldsymbol{X}^{'}\boldsymbol{Y} \\
\boldsymbol{\beta}^{\text{ridge}} &= \left(\boldsymbol{X}^{'}\boldsymbol{X} + \lambda\boldsymbol{I}_J\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\
&= (\boldsymbol{I}_j + \lambda\boldsymbol{I}_j)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\
&= (\boldsymbol{I}_j + \lambda\boldsymbol{I}_j)^{-1}\widehat{\boldsymbol{\beta}}
\end{aligned}$$

Ridge Regression$\rightsquigarrow$ Intuition (1)

Suppose $\boldsymbol{X}^{'}\boldsymbol{X} = \boldsymbol{I}_J$.

$$\begin{aligned}
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&= \left(\boldsymbol{I}_j + \lambda\boldsymbol{I}_j\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\
&= \left(\boldsymbol{I}_j + \lambda\boldsymbol{I}_j\right)^{-1}\widehat{\boldsymbol{\beta}} \\
\beta_j^{\text{Ridge}} &= \frac{\widehat{\beta}_j}{1+\lambda}
\end{aligned}$$

Ridge Regression$\rightsquigarrow$ Intuition (2)

$$
\begin{aligned}
\boldsymbol{\beta}_j & \sim \text{Normal}(0, \tau^2) \\
y_i & \sim \text{Normal}(\beta_0 + \boldsymbol{x}_i^{'}\boldsymbol{\beta}, \sigma^2)
\end{aligned}
$$

Ridge Regression$\rightsquigarrow$ Intuition (2)

$$\begin{aligned} \boldsymbol{\beta}_j &\sim \text{Normal}(0, \tau^2) \\ y_i &\sim \text{Normal}(\beta_0 + \boldsymbol{x}_i^{'}\boldsymbol{\beta}, \sigma^2) \end{aligned}$$

$$p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) \quad \propto \quad \prod_{j=1}^{J} p(\beta_j) \prod_{i=1}^{N} p(y_i|\boldsymbol{x}_i, \boldsymbol{\beta})$$

Ridge Regression⇝ Intuition (2)

$$\begin{aligned}\boldsymbol{\beta}_j &\sim \text{Normal}(0, \tau^2)\\ y_i &\sim \text{Normal}(\beta_0 + \boldsymbol{x}_i^{'}\boldsymbol{\beta}, \sigma^2)\end{aligned}$$

$$\begin{aligned}p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) &\propto \prod_{j=1}^{J} p(\beta_j) \prod_{i=1}^{N} p(y_i|\boldsymbol{x}_i, \boldsymbol{\beta})\\ &\propto \prod_{j=1}^{J} \frac{1}{\sqrt{2\pi}\tau} \exp\left(-\frac{\beta_j^2}{2\tau^2}\right) \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y_i - \beta_0 - \boldsymbol{x}_i^{'}\boldsymbol{\beta})^2}{2\sigma^2}\right)\end{aligned}$$

Ridge Regression $\rightsquigarrow$ Intuition (2)

$$\log p(\beta|\boldsymbol{X}, \boldsymbol{Y}) = -\sum_{j=1}^{J} \frac{\beta_j^2}{2\tau^2} - \sum_{i=1}^{N} \frac{(y_i - \beta_0 - \boldsymbol{x}^{'}\boldsymbol{\beta})^2}{2\sigma^2}$$

Ridge Regression$\rightsquigarrow$ Intuition (2)

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\log p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) &= -\sum_{j=1}^{J} \frac{\beta_j^2}{2\tau^2} - \sum_{i=1}^{N} \frac{(y_i - \beta_0 - \boldsymbol{x}^{'}\boldsymbol{\beta})^2}{2\sigma^2} \\
-2\sigma^2 \log p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) &= \sum_{i=1}^{N}(y_i - \beta_0 - \boldsymbol{x}^{'}\boldsymbol{\beta})^2 + \sum_{j=1}^{J} \frac{\sigma^2}{\tau^2}\beta_j^2
\end{aligned}$$

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where:

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\end{aligned}$$

where:

- $\lambda = \frac{\sigma^2}{\tau^2}$

Ridge Regression $\rightsquigarrow$ Intuition (3)

Definition

Suppose $\boldsymbol{X}$ is an $N \times J$ matrix. Then $\boldsymbol{X}$ can be written as:

$$\boldsymbol{X} = \underbrace{\boldsymbol{U}}_{N\times N} \underbrace{\boldsymbol{S}}_{N\times J} \underbrace{\boldsymbol{V}^{'}}_{J\times J}$$

Where:

$$\begin{aligned} \boldsymbol{U}^{'}\boldsymbol{U} &= \boldsymbol{I}_N \\ \boldsymbol{V}^{'}\boldsymbol{V} &= \boldsymbol{V}\boldsymbol{V}^{'} = \boldsymbol{I}_J \end{aligned}$$

$\boldsymbol{S}$ *contains* $\min(N, J)$ *singular values,* $\sqrt{\lambda_j} \geq 0$ *down the diagonal and then 0's for the remaining entries*

Ridge Regression $\rightsquigarrow$ Intuition (3)

Recall: PCA:

$$\frac{1}{N}\boldsymbol{X}^{'}\boldsymbol{X} = \underbrace{\boldsymbol{W}}_{\text{eigenvectors}} \begin{pmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_J \end{pmatrix} \underbrace{\boldsymbol{W}^{'}}_{\text{eigenvectors}}$$

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Using SVD:

Ridge Regression ⇝ Intuition (3)

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Using SVD:

$$\frac{1}{N}\boldsymbol{X}^{'}\boldsymbol{X} \quad = \quad \boldsymbol{V}\boldsymbol{S}^{'}\underbrace{\left(\boldsymbol{U}^{'}\boldsymbol{U}\right)}_{\boldsymbol{I}_J}\boldsymbol{S}\boldsymbol{V}^{'}$$

Ridge Regression $\rightsquigarrow$ Intuition (3)

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Ridge Regression ⇝ Intuition (3)

We can write the predicted values for a regular regression as

$$
\begin{aligned}
\hat{Y} &= \boldsymbol{X}\hat{\beta} \\
&= \boldsymbol{X}\left(\boldsymbol{X}^{'}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\
&= \boldsymbol{U}\boldsymbol{U}^{'}\boldsymbol{Y} = \sum_{j=1}^{J}\boldsymbol{u}_j\boldsymbol{u}_j^{'}\boldsymbol{Y}
\end{aligned}
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Ridge Regression $\rightsquigarrow$ Intuition (3)

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$$\hat{Y}^{\text{ridge}} = \boldsymbol{X}\left(\boldsymbol{X}^{'}\boldsymbol{X} + \lambda\boldsymbol{I}_J\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y}$$

Ridge Regression $\rightsquigarrow$ Intuition (3)

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$$\begin{aligned}\hat{Y}^{\text{ridge}} &= \boldsymbol{X}\left(\boldsymbol{X}^{'}\boldsymbol{X} + \lambda\boldsymbol{I}_J\right)^{-1}\boldsymbol{X}^{'}\boldsymbol{Y} \\ &= \boldsymbol{U}\tilde{\boldsymbol{S}}\boldsymbol{U}^{'}\boldsymbol{Y}\end{aligned}$$

Where

$$\tilde{\boldsymbol{S}} = \left[\boldsymbol{S}(\boldsymbol{S}^{'}\boldsymbol{S} + \lambda\boldsymbol{I}_J)^{-1}\boldsymbol{S}\right]$$

Ridge Regression $\rightsquigarrow$ Intuition (3)

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Where

$$\tilde{\boldsymbol{S}} = \left[\boldsymbol{S}(\boldsymbol{S}^{'}\boldsymbol{S} + \lambda\boldsymbol{I}_J)^{-1}\boldsymbol{S}\right]$$

Which we can write as:

$$\hat{Y}^{\text{ridge}} = \sum_{j=1}^{J}\boldsymbol{u}_j\frac{\lambda_j}{\lambda_j + \lambda}\boldsymbol{u}_j^{'}\boldsymbol{Y}$$

Degrees of Freedom for Ridge

We will say that the degrees of freedom for Ridge regression with penalty λ is

$$\text{dof}(\lambda) = \sum_{j=1}^{J} \frac{\lambda_j}{\lambda_j + \lambda}$$

Lasso Regression Objective Function

Different Penalty for Model Complexity

$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{J} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{J} \underbrace{|\beta_j|}_{\text{Penalty}}$$

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Lasso Regression Optimization

Definition

Coordinate Descent Algorithms:
Consider $g : \Re^J \rightarrow \Re$. *Our goal is to find* $\boldsymbol{x}^* \in \Re^J$ *such that* $g(\boldsymbol{x}^*) \leq g(\boldsymbol{x})$ *for all* $\boldsymbol{x} \in \Re$.
To find $\boldsymbol{x}^*$*:*
Until convergence: for each iteration t *and each coordinate* j

$$x_j^{t+1} = arg\ min_{x_j \in \Re} g(x_1^{t+1}, x_2^{t+1}, \ldots, x_{j-1}^{t+1}, x_j, x_{j+1}^{t}, \ldots, x_J^{t})$$

Lasso Regression Optimization: Coordinate Descent

$$\tilde{f}(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{2N}\sum_{i=1}^{N}\left(y_i - \beta_0 - \sum_{j=1}^{J}\beta_j x_{ij}\right)^2 + \lambda\sum_{j=1}^{J}|\beta_j|$$

Lasso Regression Optimization: Coordinate Descent

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- Case 1: If $\beta_j = 0 \rightsquigarrow$ not differentiable. But $\beta_j = 0$

Lasso Regression Optimization: Coordinate Descent

$$\tilde{f}(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \quad = \quad \frac{1}{2N}\sum_{i=1}^{N}\left(y_i - \beta_0 - \sum_{j=1}^{J}\beta_j x_{ij}\right)^2 + \lambda \sum_{j=1}^{J}|\beta_j|$$

- Case 1: If $\beta_j = 0 \rightsquigarrow$ not differentiable. But $\beta_j = 0$
- Case 2: If $\beta_j > (<)0 \rightsquigarrow$ differentiable $\rightsquigarrow$ differentiate and solve for β_j

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Define $\tilde{y}_i^j = \beta_0 + \sum_{l \neq j} x_{il}\beta_l$

Lasso Regression Optimization: Coordinate Descent

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Define $\tilde{y}_i^j = \beta_0 + \sum_{l \neq j} x_{il}\beta_l$
$r^j \equiv \frac{1}{N}\sum_{i=1}^{N} x_{ij}(y_i - \tilde{y}_i^j)$

Lasso Regression Optimization: Coordinate Descent

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Update step for β_j is

$$\beta_j \leftarrow \text{sign}(r^j)\max(|r^j| - \lambda, 0)$$

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

Suppose again $\boldsymbol{X}^{'}\boldsymbol{X} = \boldsymbol{I}_J$

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

Suppose again $\boldsymbol{X}^{'}\boldsymbol{X} = \boldsymbol{I}_J$

$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \;\; = \;\; (Y - \boldsymbol{X}\boldsymbol{\beta})^{'}(Y - \boldsymbol{X}\boldsymbol{\beta}) + \lambda \sum_{j=1}^{J} |\beta_j|$$

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

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The coefficient is

$$\beta_j^{\text{LASSO}} = \text{sign}\left(\widehat{\beta}_j\right)\left(|\widehat{\beta}_j| - \lambda\right)_{+}$$

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- $\text{sign}(\cdot) \rightsquigarrow 1$ or -1

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

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- $\text{sign}(\cdot) \rightsquigarrow 1$ or -1
- $\left(|\widehat{\beta}_j| - \lambda\right)_+ = \max(|\widehat{\beta}_j| - \lambda, 0)$

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

Compare soft assignment

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

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Lasso Regression⇝ Intuition 1, Soft Thresholding

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With hard assignment, selecting M biggest components

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

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$$\beta_j^{\text{subset}} = \widehat{\beta}_j \cdot I\left(|\widehat{\beta}_j| \geq |\widehat{\beta}_{(M)}|\right)$$

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Intuition 2: Prior on coefficients $\rightsquigarrow$ Laplace “The Bayesian LASSO”

Lasso Regression$\rightsquigarrow$ Intuition 1, Soft Thresholding

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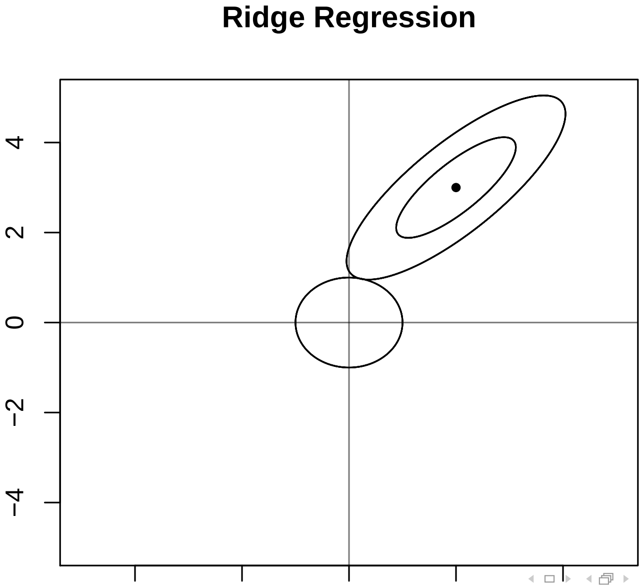
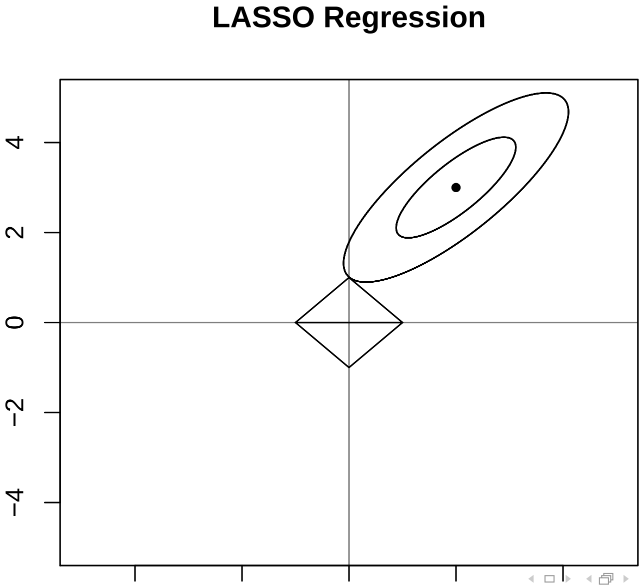
Intuition 2: Prior on coefficients $\rightsquigarrow$ Laplace "The Bayesian LASSO"
Why does LASSO induce sparsity?

Comparing Ridge and LASSO



Comparing Ridge and LASSO



Comparing Ridge and LASSO

Contrast $\beta = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $\tilde{\beta} = (1, 0)$

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Under ridge:

Comparing Ridge and LASSO

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Under ridge:

$$\sum_{j=1}^{2} \beta_j^2 \quad = \quad \frac{1}{2} + \frac{1}{2} = 1$$

Comparing Ridge and LASSO

Contrast $\beta = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $\tilde{\beta} = (1, 0)$

Under ridge:

$$\sum_{j=1}^{2} \beta_j^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$\sum_{j=1}^{2} \tilde{\beta}_j^2 = 1 + 0 = 1$$

Comparing Ridge and LASSO

Contrast $\beta = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and $\tilde{\beta} = (1, 0)$

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Ridge and LASSO: The Elastic-Net

Combining the two criteria ⇝ Elastic-Net

$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \quad = \quad \frac{1}{2N} \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{J} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{J} \left(\frac{1}{2} (1 - \alpha) \beta_j^2 + \alpha |\beta_j| \right)$$

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$$

Training and Test Sets

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Even if no division, useful to think about systematic components of data.

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$$=$$

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$$\text{Error} = \text{E}\left[\text{E}[L(\boldsymbol{Y}, f(\hat{\beta}, \boldsymbol{X}))|\mathcal{T}]\right]$$

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&= \text{Irreducible error} + \text{Bias}^2 + \text{Variance}
\end{aligned}$$

Probit Regression (for motivational purposes)

Suppose:

$$\begin{aligned} Y_i &\sim \text{Bernoulli}(\pi_i) \\ \pi_i &= \Phi(\boldsymbol{\beta}^{'}\boldsymbol{x}_i) \end{aligned}$$

where $\Phi(\cdot)$ is the cumulative normal distribution.
Implies log-likelihood

$$\log \text{L}(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{N} \left[Y_i \log \Phi(\boldsymbol{\beta}^{'}\boldsymbol{x}_i) + (1 - Y_i) \log(1 - \Phi(\boldsymbol{\beta}^{'}\boldsymbol{x}_i)) \right]$$

Log-likelihood is a loss function $\rightsquigarrow$ overly optimistic: improves with more parameters

How Do We Build A Model?

There are many ways to fit models

And many choices made when performing model fit

How do we choose?

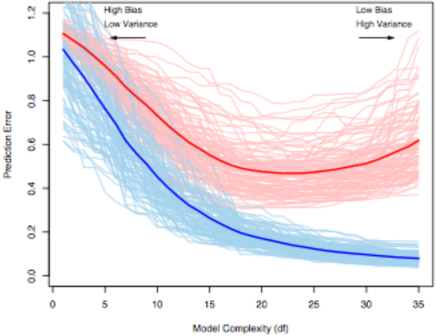

FIGURE 7.1. *Behavior of test sample and training sample error as the model complexity is varied. The light blue curves show the training error* $\overline{\text{err}}$*, while the light red curves show the conditional test error* $\text{Err}_{\mathcal{T}}$ *for 100 training sets of size 50 each, as the model complexity is increased. The solid curves show the expected test error* Err *and the expected training error* $\text{E}[\overline{\text{err}}]$.

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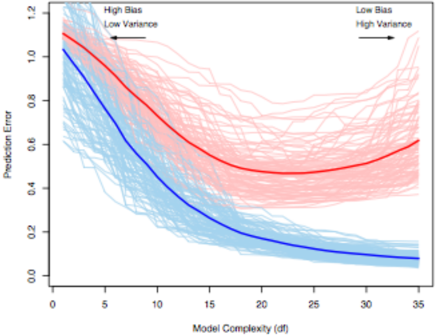

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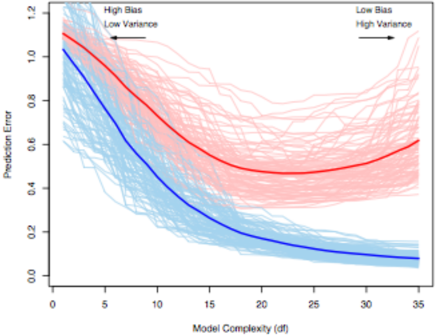
Bad way to choose: within sample model fit (HTF Figure 7.1)



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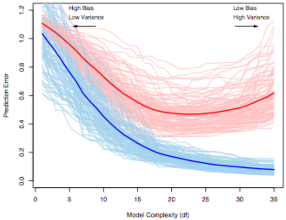

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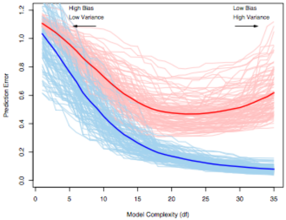

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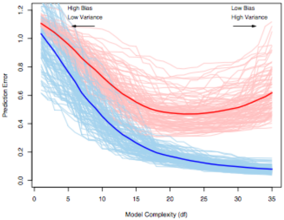

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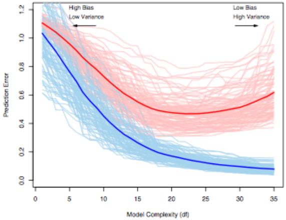

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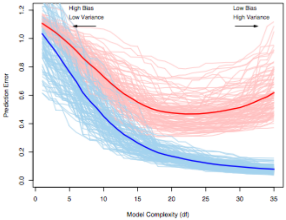

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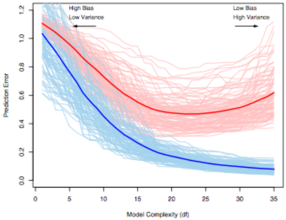

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How Do We Choose Covariates?

Best model depends on task

- Causal inference observational study: make treatment assignment ignorable
- Prediction: improve predictive performance

Stepwise Regression

Suppose we have P covariates.
2^P potential models

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Stepwise procedures

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Problematic:

1) Not optimal model selection (path dependent)
2) P-value $\neq$ objective of model

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$$-2\text{E}[\log P_{\hat{\boldsymbol{\beta}}}(Y)] = -2\left[\text{E}[\log \text{L}(\hat{\beta}|\boldsymbol{X}, \boldsymbol{Y})] - d\right]$$

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As $N \rightarrow \infty$

$$
\begin{aligned}
-2\mathrm{E}[\log P_{\hat{\boldsymbol{\beta}}}(Y)] &= -2\left[\mathrm{E}[\log \mathrm{L}(\hat{\beta}|\boldsymbol{X}, \boldsymbol{Y})] - d\right] \\
\mathrm{AIC} &= -2\left[\log \mathrm{L}(\hat{\beta}|\boldsymbol{X}, \boldsymbol{Y}) - d\right]
\end{aligned}
$$

Analytic Solutions

Approximate optimism and compensate in loss function.
Akaike Information Criterion (AIC) $\rightsquigarrow$ Minimize
As $N \to \infty$

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where d is the number of parameters in the model

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As $N \rightarrow \infty$

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where d is the number of parameters in the model

- Intuition: balances model fit with penalty for complexity
- Derived from method to estimate optimism in likelihood based models
- Derived from a method to compute similarity between estimated model and true model (under assumptions of course)
- Can be extended to general models, though requires estimate of irresolvable error

Analytic Solutions

Bayesian Information Criterion (BIC) [Schwarz Criterion]

Analytic Solutions

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$$\text{BIC} = -2 \log \text{L}(\widehat{\beta}|\boldsymbol{X}, \boldsymbol{Y}) + (\log N)d$$

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- Derived from Bayesian approach to model selection

Analytic Solutions

Bayesian Information Criterion (BIC) [Schwarz Criterion]

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where d is again the effective number of parameters

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- Approximation to Bayes' factor

Analytic Solutions

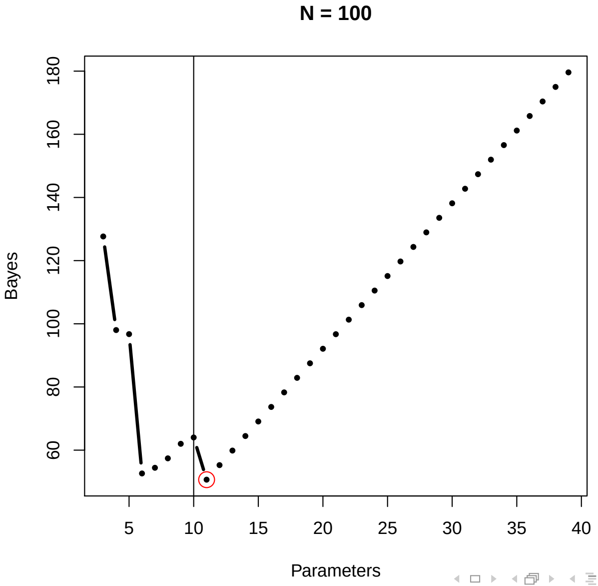
Bayesian Information Criterion (BIC) [Schwarz Criterion]

$$\text{BIC} = -2\log \text{L}(\widehat{\beta}|\boldsymbol{X}, \boldsymbol{Y}) + (\log N)d$$

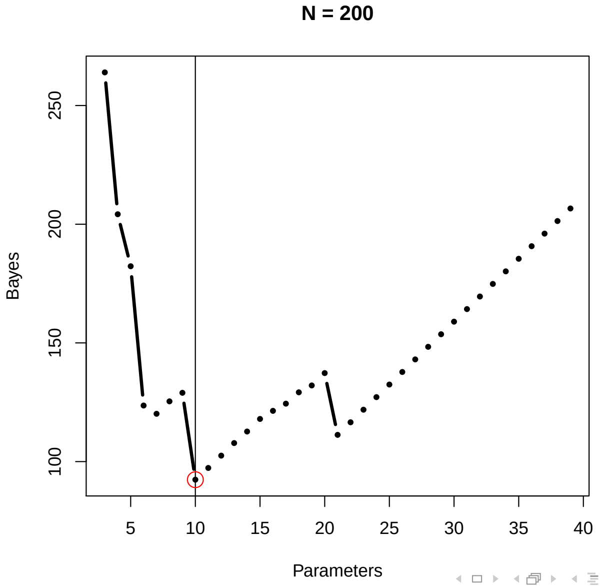
where d is again the effective number of parameters

- Intuition: balances model fit with penalty for complexity
- Derived from Bayesian approach to model selection
- Approximation to Bayes' factor
- Penalizes more heavily than AIC

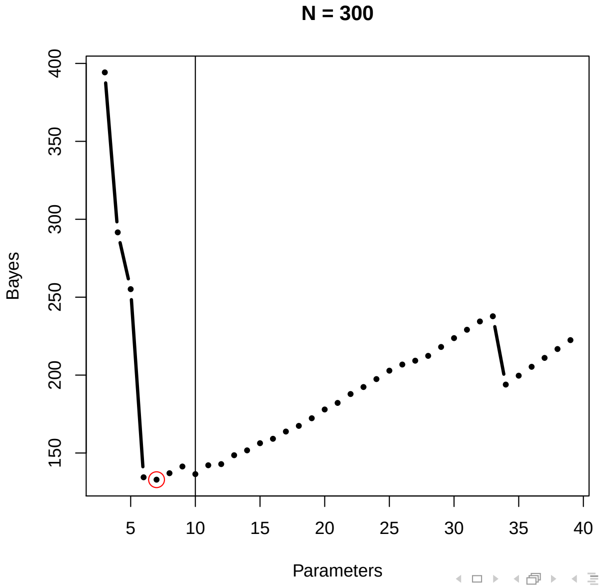
BIC or AIC?


N = 100
Bayes
60
80
100
120
140
160
180
5
10
15
20
25
30
35
40
Parameters

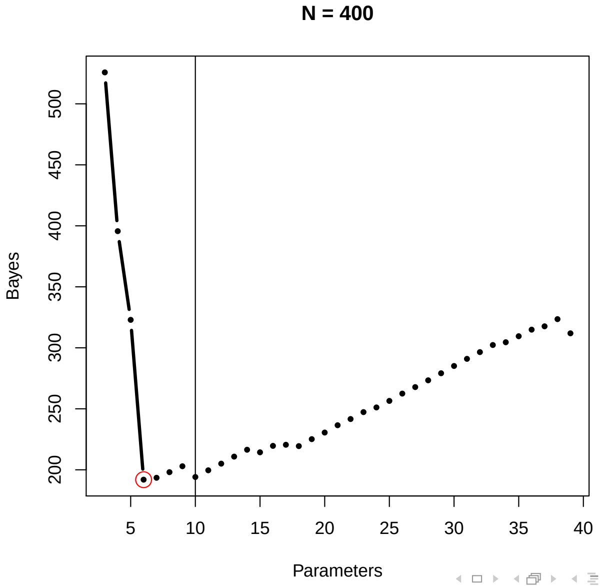
BIC or AIC?



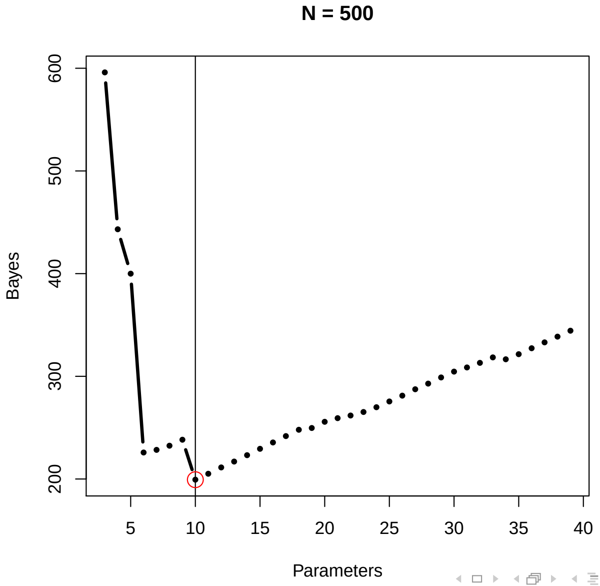
BIC or AIC?


N = 300
Bayes
400
350
300
250
200
150
Parameters
5
10
15
20
25
30
35
40

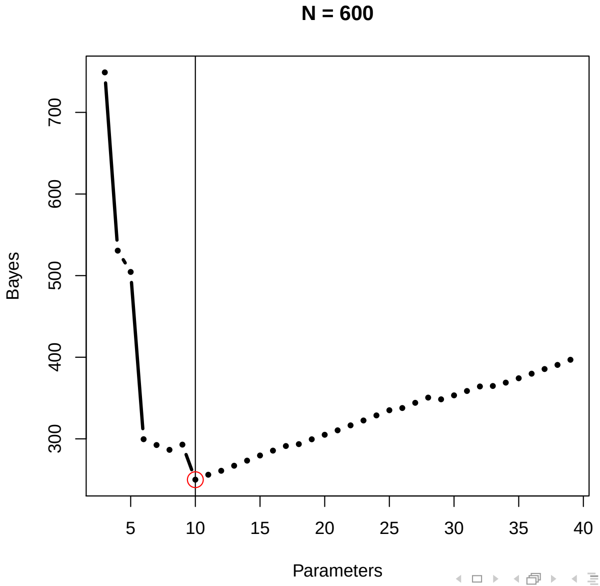
BIC or AIC?


N = 400
Bayes
Parameters
200
250
300
350
400
450
500
5
10
15
20
25
30
35
40

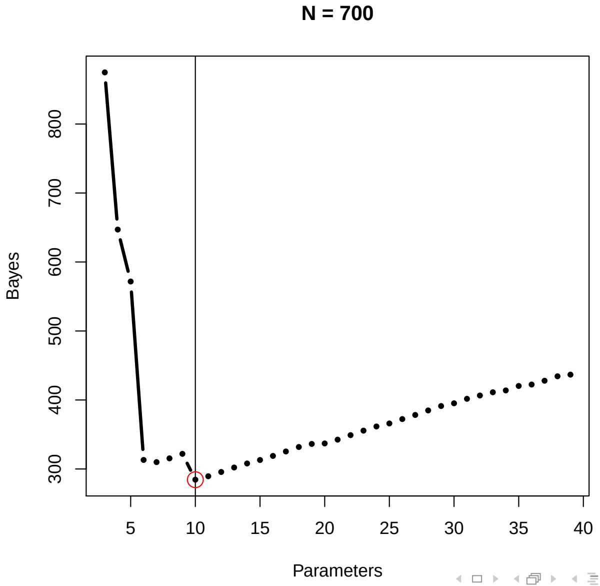
BIC or AIC?



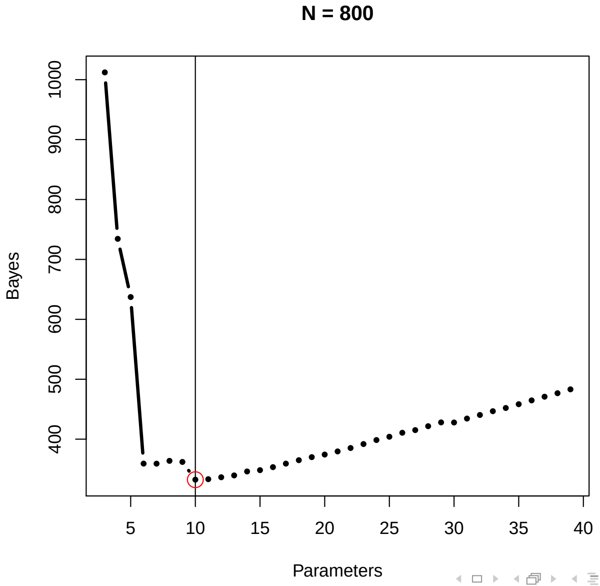
BIC or AIC?



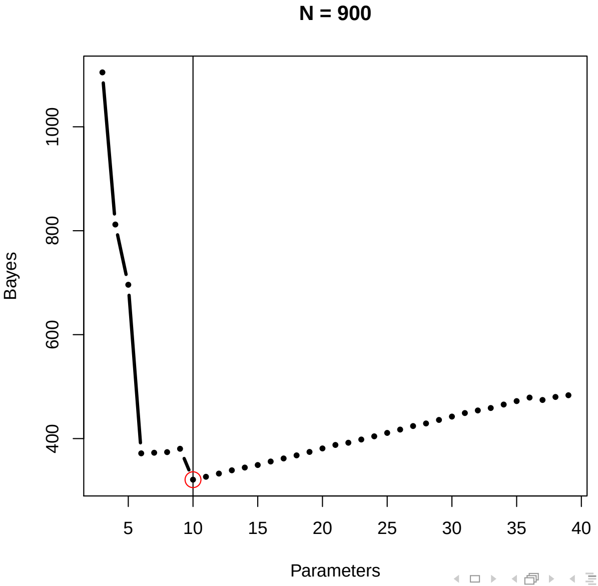
BIC or AIC?


N = 700
Bayes
800
700
600
500
400
300
5
10
15
20
25
30
35
40
Parameters

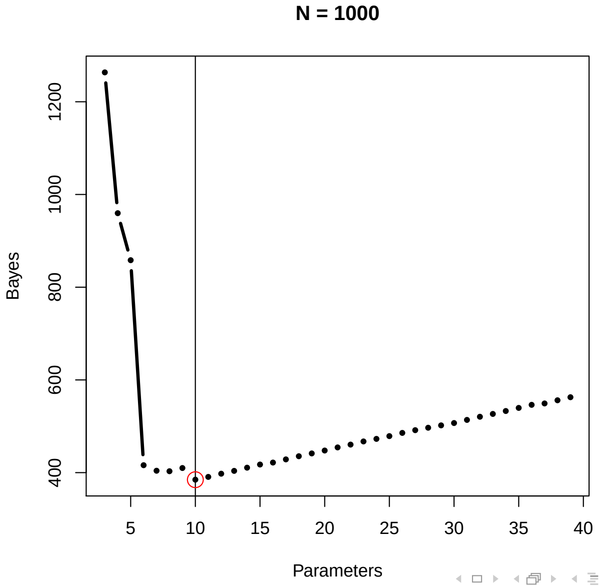
BIC or AIC?


N = 800
Bayes
Parameters
1000
900
800
700
600
500
400
5
10
15
20
25
30
35
40

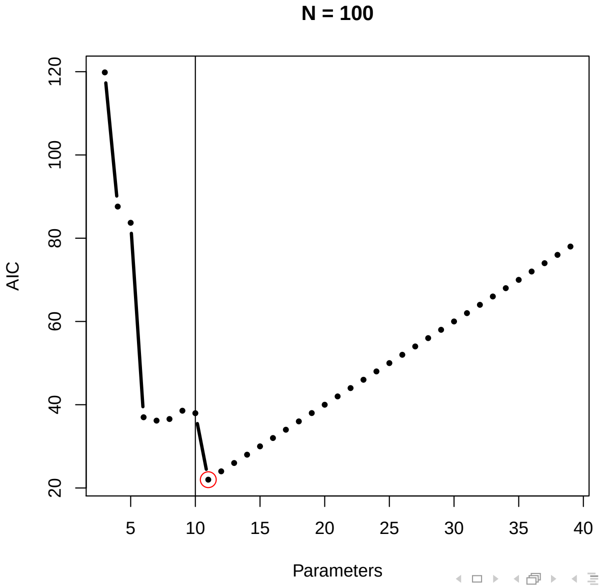
BIC or AIC?



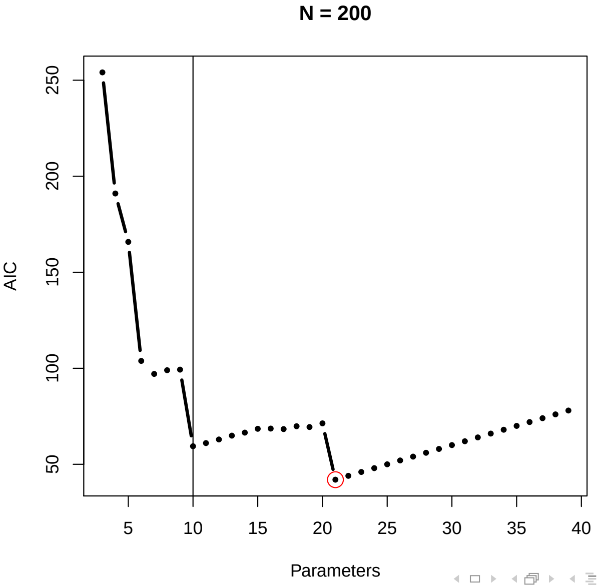
BIC or AIC?


N = 1000
Bayes
Parameters
400
600
800
1000
1200
5
10
15
20
25
30
35
40

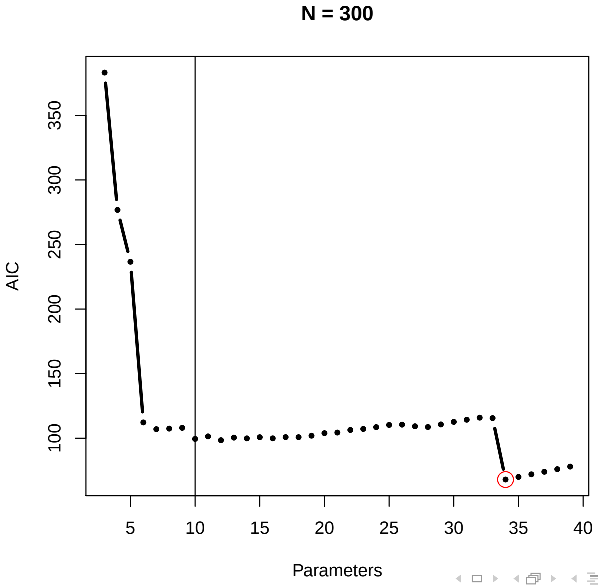
BIC or AIC?



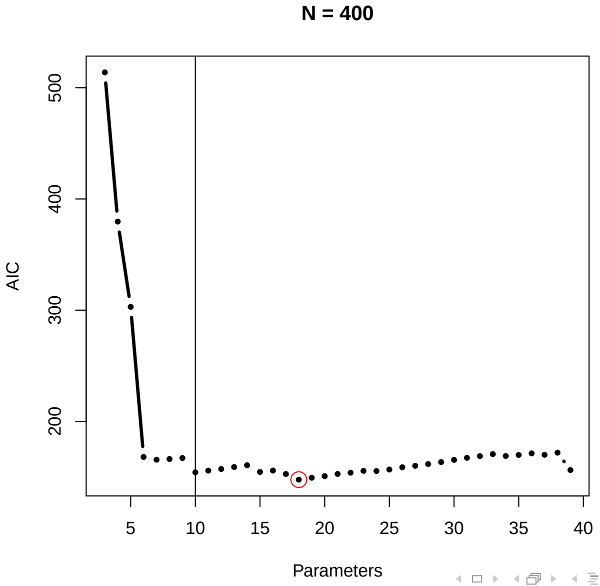
BIC or AIC?



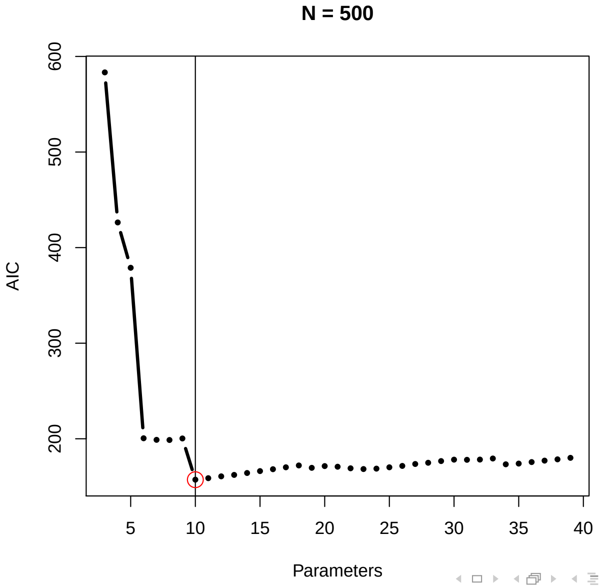
BIC or AIC?


N = 300
AIC
350
300
250
200
150
100
5
10
15
20
25
30
35
40
Parameters

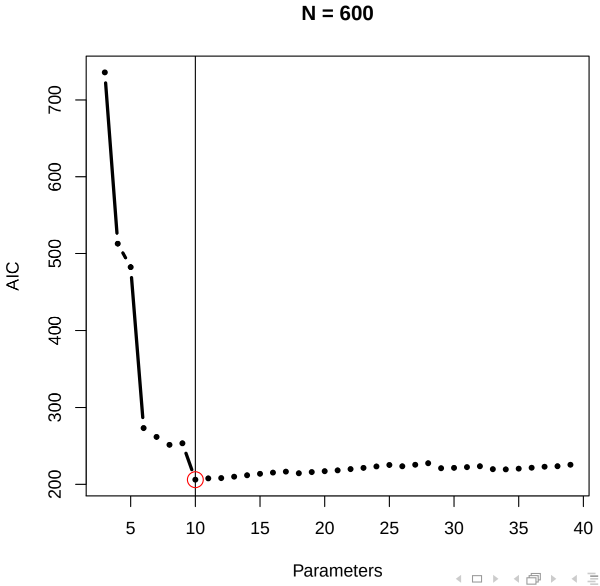
BIC or AIC?



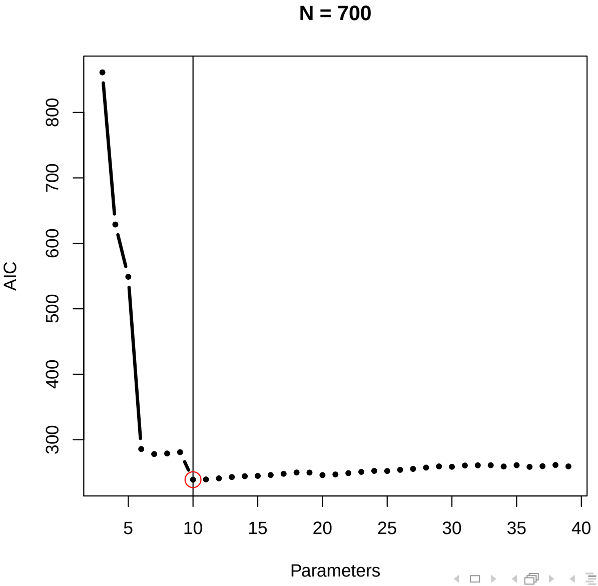
BIC or AIC?



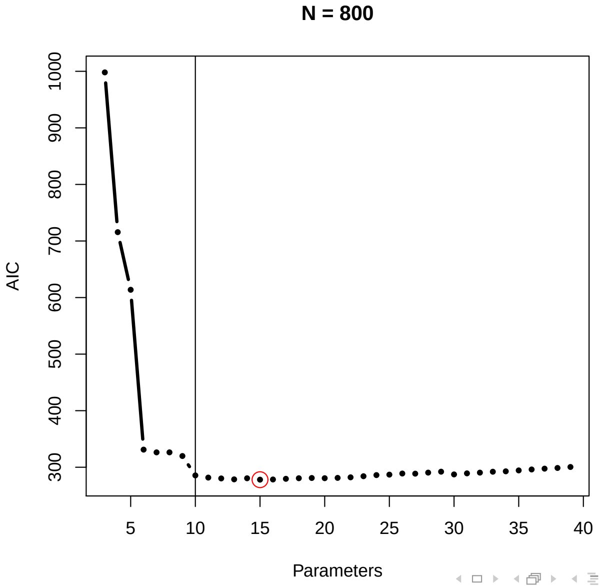
BIC or AIC?


N = 600
AIC
200
300
400
500
600
700
5
10
15
20
25
30
35
40
Parameters

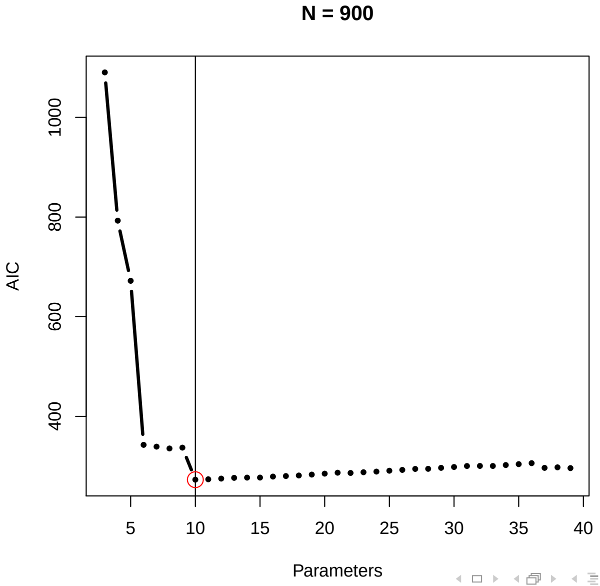
BIC or AIC?


N = 700
AIC
800
700
600
500
400
300
Parameters
5
10
15
20
25
30
35
40

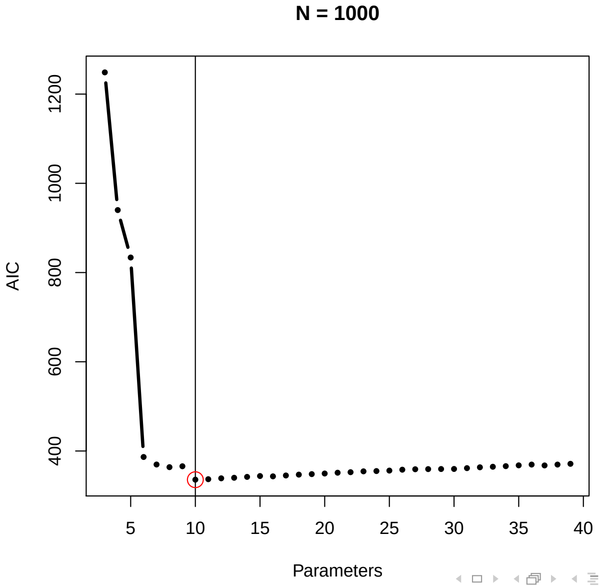
BIC or AIC?


N = 800
AIC
1000
900
800
700
600
500
400
300
5
10
15
20
25
30
35
40
Parameters

BIC or AIC?



BIC or AIC?


N = 1000
AIC
1200
1000
800
600
400
Parameters
5
10
15
20
25
30
35
40

BIC or AIC?

- BIC
  - Asymptotically consistent if true model is in choice set
  - As $N \rightarrow \infty$ will choose correct model with probability 1 (if available)
  - Small samples$\rightsquigarrow$ overpenalize
- AIC
  - No asymptotic guarantees $\rightsquigarrow$ derivation doesn't require truth in set. (KL-criteria)
  - In large samples$\rightsquigarrow$ favors complexity
  - Small samples$\rightsquigarrow$ avoids over penalization

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

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- AIC : Akaka Information Criterion

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Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

How Do We Select A Model?

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Can work well, but...

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- AIC : Akaka Information Criterion
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Can work well, but...

- Rely on specific loss function

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- Rely on asymptotic argument
- Rely on estimate of number of parameters

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- Rely on asymptotic argument
- Rely on estimate of number of parameters
- Extremely model dependent

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- DIC: Deviance Information Criterion

Can work well, but...

- Rely on specific loss function
- Rely on asymptotic argument
- Rely on estimate of number of parameters
- Extremely model dependent

Need: general tool for evaluating models, replicates decision problem

Cross-Validation: Some Intuition

Optimal division of data for prediction:

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model

Cross-Validation: Some Intuition

Optimal division of data for prediction:

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- Validation: assess model
- Test: predict remaining data

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- Validation: assess model
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K-fold Cross-validation idea: create many training and test sets.

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- Idea: use observations both in training and test sets

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- Each step: use held out data to evaluate performance

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- Avoid overfitting and have context specific penalty

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- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

- Idea: use observations both in training and test sets
- Each step: use held out data to evaluate performance
- Avoid overfitting and have context specific penalty

Estimates:

$$\text{Error} \quad = \quad \text{E}\left[\text{E}[L(\boldsymbol{Y}, f(\hat{\beta}, \boldsymbol{X}))|\mathcal{T}]\right]$$

Cross-Validation: A How To Guide

Process:

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
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Step	Training	Validation (“Test”)

Cross-Validation: A How To Guide

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- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, . . ., Group K - 1	Group K

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, . . ., Group K - 1	Group K

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K - 1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K - 1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, . . ., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K - 1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
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    - Mean square error, Absolute error, Prediction error, ...

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, . . ., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K - 1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
  - Mean square error, Absolute error, Prediction error, ...

CV(ind. classification) $= \frac{1}{N}\sum_{i=1}^{N} L(\boldsymbol{Y}_i, f^{-k}(\boldsymbol{\beta}, \boldsymbol{X}_i))$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, . . ., Group K	Group 1
2	Group 1, Group3, Group 4, . . ., Group K	Group 2
3	Group 1, Group 2, Group 4, . . ., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, . . ., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
    - Mean square error, Absolute error, Prediction error, ...

CV(ind. classification) $= \frac{1}{N}\sum_{i=1}^{N} L(\boldsymbol{Y}_i, f^{-k}(\boldsymbol{\beta}, \boldsymbol{X}_i))$

CV(proportions) $=$
$\frac{1}{K}\sum_{j=1}^{K}$ Mean Square Error Proportions from Group j

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  $\frac{1}{K}\sum_{j=1}^{K}$ Mean Square Error Proportions from Group j
- Final choice: model with highest CV score

How Do We Select K? (HTF, Section 7.10)

Common values of K

- $K = 5$: Five fold cross validation
- $K = 10$: Ten fold cross validation
- $K = N$: Leave one out cross validation

Considerations:

- How sensitive are inferences to number of coded documents? (HTF, pg 243-244)
- 200 labeled documents
  - $K = N \rightarrow 199$ documents to train,
  - $K = 10 \rightarrow 180$ documents to train
  - $K = 5 \rightarrow 160$ documents to train
- 50 labeled documents
  - $K = N \rightarrow 49$ documents to train,
  - $K = 10 \rightarrow 45$ documents to train
  - $K = 5 \rightarrow 40$ documents to train
- How long will it take to run models?
  - $K-$fold cross validation requires $K \times$ One model run
- What is the correct loss function?

If you cross validate, you really need to cross validate (Section 7.10.2, ESL)

- Use CV to estimate prediction error
- All supervised steps performed in cross-validation
- Underestimate prediction error
- Could lead to selecting lower performing model

Example from Facebook Data

What do people say to legislators? (Franco, Grimmer, and Lee 2017)

1) Example: estimating classification error
    a) Accuracy in legislator posts: 75%
    b) Accuracy in public posts: 66.25%

Credit Claiming (Back to Ridge/Lasso, Grimmer, Westwood, and Messing 2014)

```
library(glmnet)
set.seed(8675309) ##setting seed
folds<- sample(1:10, nrow(dtm), replace=T) ##assigning to fold
out_of_samp<- c() ##collecting the predictions
```

Credit Claiming (Back to Ridge/Lasso, Grimmer, Westwood, and Messing 2014)

```
for(z in 1:10){
train<- which(folds!=z) ##the observations we will use to train the model

test<- which(folds==z) ##the observations we will use to test the model
part1<- cv.glmnet(x = dtm[train,], y = credit[train], alpha = 1, family =
b́inomial)́ ##fitting the LASSO model on the data.
## alpha = 1 -> LASSO
## alpha = 0 -> RIDGE
## 0<alpha<1 -> Elastic-Net
out_of_samp[test]<- predict(part1, newx= dtm[test,], s = part1$lambda.min,
type =ćlass)́ ##predicting the labels
print(z) ##printing the labels
}
conf_table<- table(out_of_samp, credit) ##calculating the confusion table
> round(sum(diag(conf_table))/len(credit), 3)
[1] 0.844
```

Generalized Cross Validation and Ridge Regression

In some special cases there are analytic solutions:

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Leave one out cross validation

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Leave one out cross validation

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&= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{Y_i - f(\boldsymbol{X}, \boldsymbol{Y}, \lambda, \hat{\boldsymbol{\beta}})}{1 - H_{ii}}\right)^2
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