Text as Data

Justin Grimmer

Professor
Department of Political Science
Stanford University

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Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2019)

1) Discovery: a hypothesis or view of the world
2) Measurement according to some organization
3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Text as Data Methods for Discovery

Text as Data Methods for Discovery Goal: Automatically Discover Organization (Similar Groups)

Texts and Geometry

Consider a document-term matrix

$$\boldsymbol{X} = \begin{pmatrix} 1 & 2 & 0 & \ldots & 0 \\ 0 & 0 & 3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \ldots & 3 \end{pmatrix}$$

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- Provides a geometry $\leadsto$ modify with word weighting
- Natural notions of distance
- Building block for clustering, supervised learning, and scaling

Texts in Space

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$$\text{Doc1} \quad = \quad (1, 1, 3, \ldots, 5)$$

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\begin{aligned}
\text{Doc1} &= (1, 1, 3, \ldots, 5) \\
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$$\begin{aligned} \text{Doc1} &= (1, 1, 3, \ldots, 5) \\ \text{Doc2} &= (2, 0, 0, \ldots, 1) \\ \textbf{Doc1}, \textbf{Doc2} &\in \Re^J \end{aligned}$$

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Inner Product between documents:

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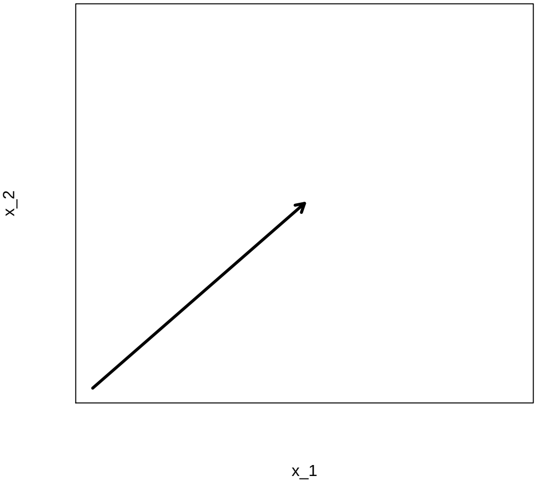
&= 7
\end{aligned}$$

Vector Length



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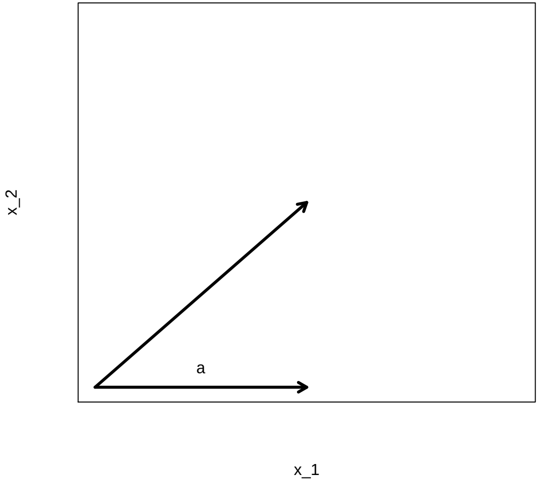
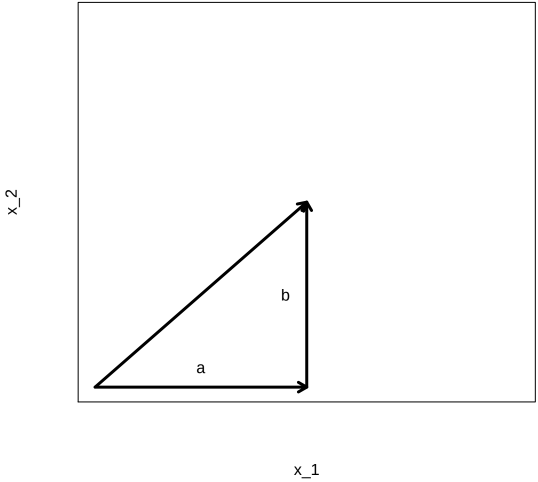


- Pythagorean Theorem: Side with length a

Vector Length



- Pythagorean Theorem: Side with length *a*
- Side with length *b* and right triangle

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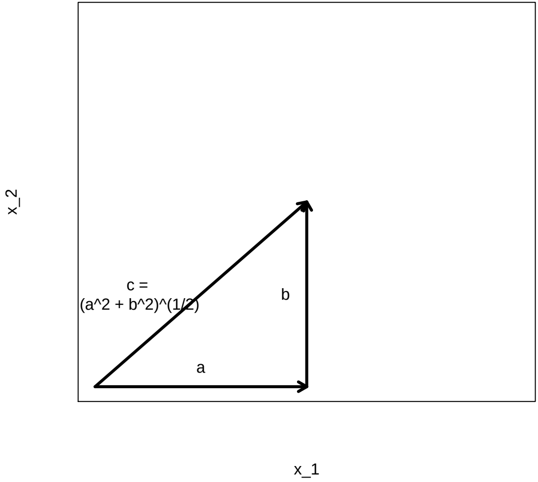

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- $c = \sqrt{a^2 + b^2}$

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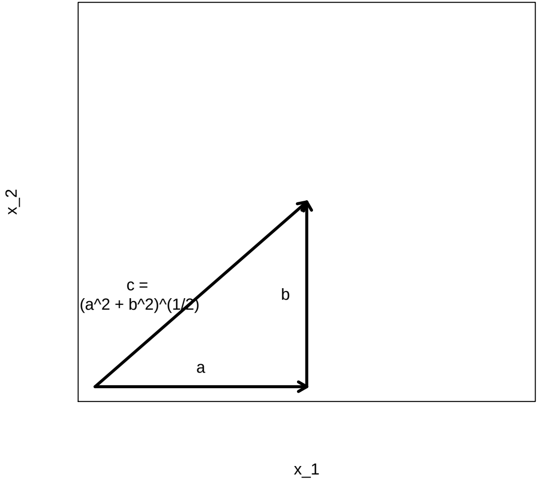

- Pythagorean Theorem: Side with length a
- Side with length b and right triangle
- $c = \sqrt{a^2 + b^2}$
- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\boldsymbol{v} \in \Re^J$. *Then, we will define its* *length* *as*

$$
\begin{aligned}
||\boldsymbol{v}|| &= (\boldsymbol{v} \cdot \boldsymbol{v})^{1/2} \\
&= (v_1^2 + v_2^2 + v_3^2 + \ldots + v_J^2)^{1/2}
\end{aligned}
$$

Measures of Dissimilarity

Initial guess$\rightsquigarrow$ Distance metrics

Properties of a metric: (distance function) $d(\cdot, \cdot)$. Consider arbitrary documents $\boldsymbol{X}_i$, $\boldsymbol{X}_j$, $\boldsymbol{X}_k$

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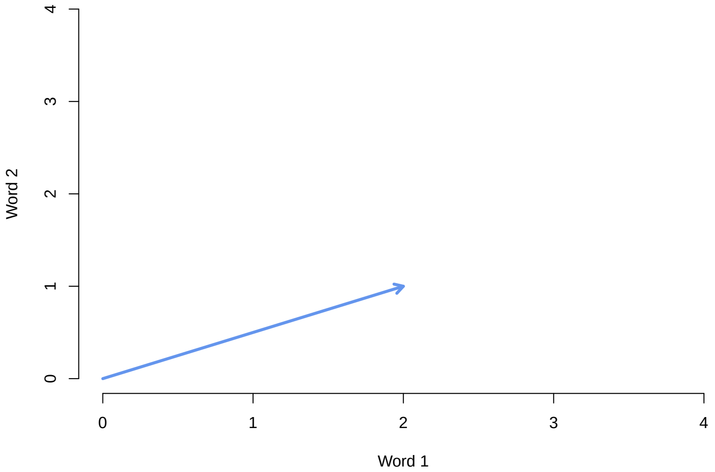
Explore distance functions to compare documents $\rightsquigarrow$ Do we want additional assumptions/properties?

Measuring the Distance Between Documents

Euclidean Distance



Measuring the Distance Between Documents

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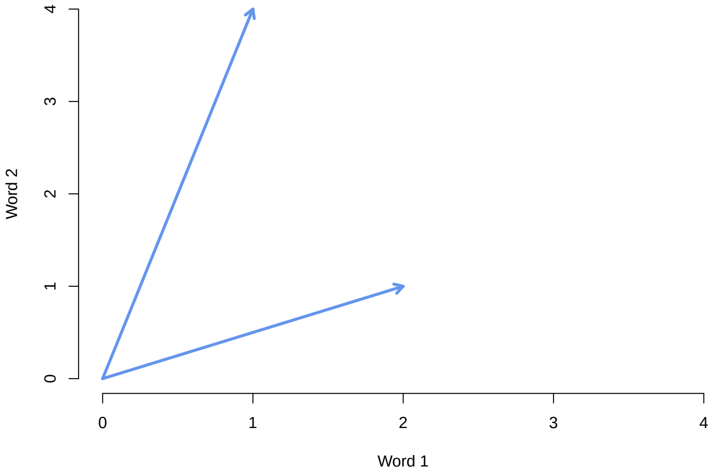

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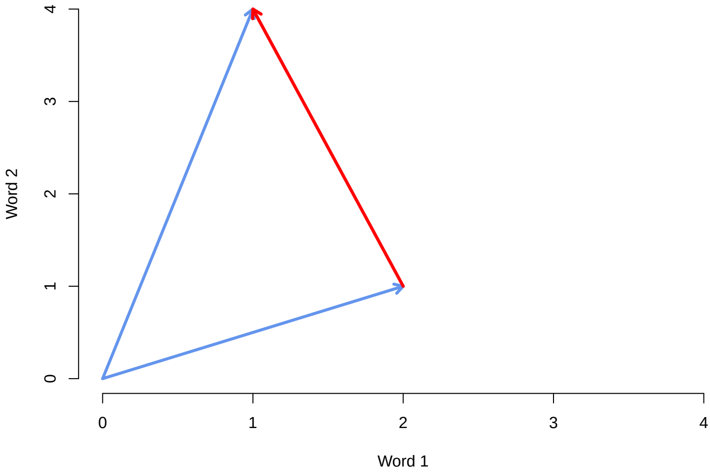

Measuring the Distance Between Documents

Definition

The Euclidean distance between documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$ *as*

$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

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The Euclidean distance between documents $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ as

$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $\boldsymbol{X}_i = (1, 4)$ and $\boldsymbol{X}_j = (2, 1)$. The distance between the documents is:

$$\begin{aligned} ||(1,4) - (2,1)|| &= \sqrt{(1-2)^2 + (4-1)^2} \\ &= \sqrt{10} \end{aligned}$$

Measuring Similarity (and removing document length)

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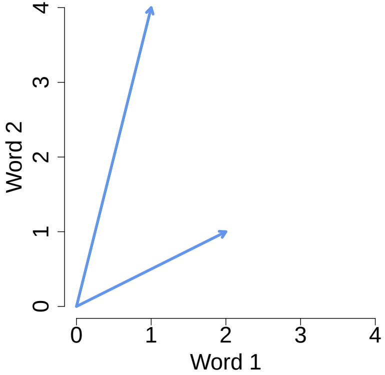
- ? $s(a, b) = s(b, a)$.

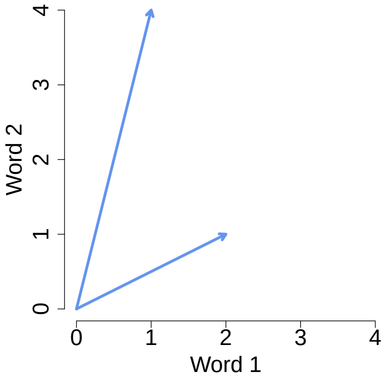
How should additional words be treated?

Measuring Similarity



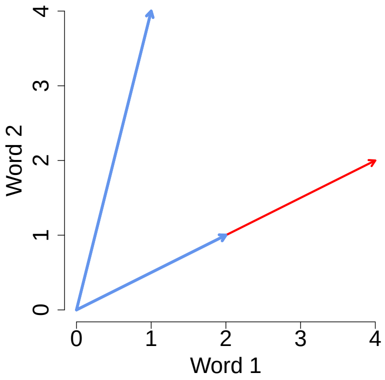
Measure 1: Inner product

Measuring Similarity

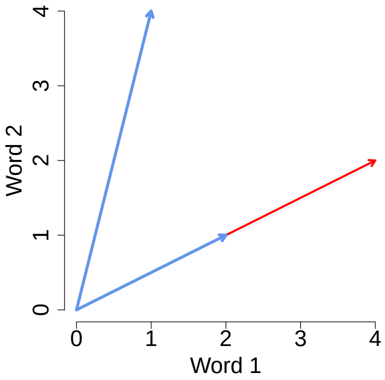


Measure 1: Inner product

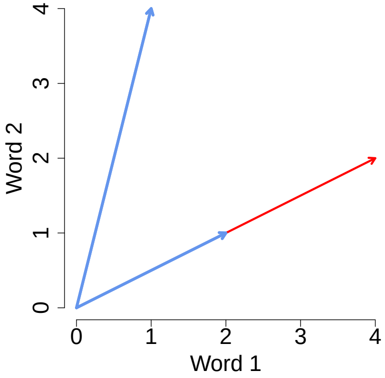
$$(2, 1)' \cdot (1, 4) \quad = \quad 6$$



Word 2
0
1
2
3
4
Word 1

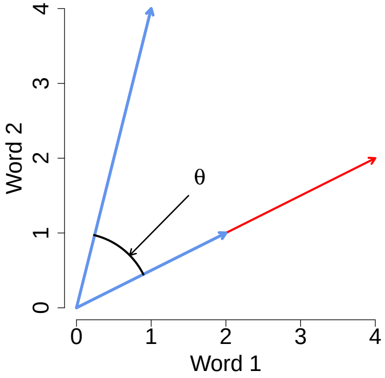


Problem(?): length dependent



Problem(?): length dependent

$$(4,2)^{'}(1,4) \quad = \quad 12$$



Problem(?): length dependent

$$
\begin{aligned}
(4,2)^{'}(1,4) &= 12 \\
a \cdot b &= ||a|| \times ||b|| \times \cos\theta
\end{aligned}
$$

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(0.89, 0.45)^{'}(0.24, 0.97) &= 0.65
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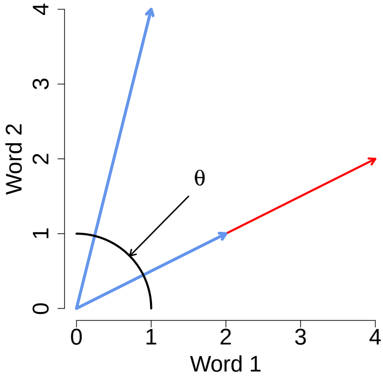

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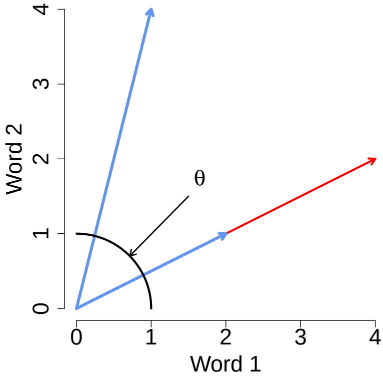

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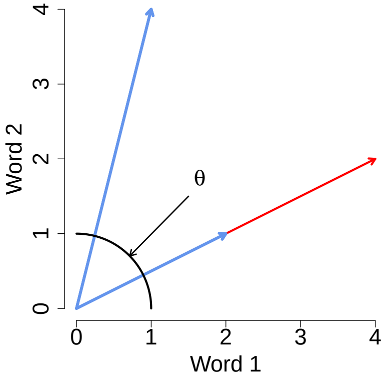

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How to generate weights?

- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

Weighting Words: TF-IDF Weighting

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Why log ?

- Maximum at $n_j = 1$
- Decreases at rate $\frac{1}{n_j} \Rightarrow$ diminishing "penalty" for more common use
- Other functional forms are fine, embed assumptions about penalization of common use

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Inner Product

Weighting Words: TF-IDF

$$
\begin{aligned}
\mathbf{X}_{i,\text{idf}} \equiv \underbrace{\mathbf{X}_i}_{\text{tf}} \times \mathbf{idf} &= (X_{i1} \times \text{idf}_1, X_{i2} \times \text{idf}_2, \ldots, X_{iJ} \times \text{idf}_J) \\
\mathbf{X}_{j,\text{idf}} \equiv \mathbf{X}_j \times \mathbf{idf} &= (X_{j1} \times \text{idf}_1, X_{j2} \times \text{idf}_2, \ldots, X_{jJ} \times \text{idf}_J)
\end{aligned}
$$

How Does This Matter For Measuring Similarity/Dissimilarity?
Inner Product

$$
\mathbf{X}_{i,\text{idf}} \cdot \mathbf{X}_{j,\text{idf}} = (\mathbf{X}_i \times \mathbf{idf})'(\mathbf{X}_j \times \mathbf{idf})
$$

Weighting Words: TF-IDF

$$\mathbf{X}_{i,\text{idf}} \equiv \underbrace{\mathbf{X}_i}_{\text{tf}} \times \mathbf{idf} = (X_{i1} \times \text{idf}_1, X_{i2} \times \text{idf}_2, \ldots, X_{iJ} \times \text{idf}_J)$$

$$\mathbf{X}_{j,\text{idf}} \equiv \mathbf{X}_j \times \mathbf{idf} = (X_{j1} \times \text{idf}_1, X_{j2} \times \text{idf}_2, \ldots, X_{jJ} \times \text{idf}_J)$$

How Does This Matter For Measuring Similarity/Dissimilarity?

Inner Product

$$\begin{aligned} \mathbf{X}_{i,\text{idf}} \cdot \mathbf{X}_{j,\text{idf}} &= (\mathbf{X}_i \times \mathbf{idf})^{\prime}(\mathbf{X}_j \times \mathbf{idf}) \\ &= (\text{idf}_1^2 \times X_{i1} \times X_{j1}) + (\text{idf}_2^2 \times X_{i2} \times X_{j2}) + \\ &\quad \ldots + (\text{idf}_J^2 \times X_{iJ} \times X_{jJ}) \end{aligned}$$

Weighting Words: Inner Product

Define:

Weighting Words: Inner Product

Define:

$$\boldsymbol{\Sigma} = \begin{pmatrix} \text{idf}_1^2 & 0 & 0 & \ldots & 0 \\ 0 & \text{idf}_2^2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \text{idf}_J^2 \end{pmatrix}$$

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If we use tf-idf for our documents, then

Weighting Words: Inner Product

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If we use tf-idf for our documents, then

$$\begin{aligned} d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) &= \sqrt{\sum_{m=1}^{J} (x_{im,\text{idf}} - x_{jm,\text{idf}})^2} \\ &= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)} \end{aligned}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \dots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \dots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \dots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \dots & 0 \end{pmatrix}$$

Lower Triangle contains unique information $N(N-1)/2$

Clustering

Fully Automated Clustering

1) Distance metric $\rightsquigarrow$ when are documents close?
2) Objective function $\rightsquigarrow$ how do we summarize distances?
3) Optimization method $\rightsquigarrow$ how do we find optimal clustering?

THERE IS NO A PRIORI OPTIMAL METHOD

Computer Assisted Clustering (Grimmer and King, 2011)

- crucial to combine human and computer insights

K-Means$\rightsquigarrow$ Objective Function

N documents $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iJ})$ (normalized)

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Two parameters to estimate

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   Cluster k has center

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$$\boldsymbol{\theta}_k \quad = \quad (\theta_{1k}, \theta_{2k}, \ldots, \theta_{Jk})$$

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Hard Assignment

K-Means$\rightsquigarrow$ Objective Function

Assume squared euclidean distance

K-Means$\rightsquigarrow$ Objective Function

Assume squared euclidean distance

$$f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = \sum_{i=1}^{N} \sum_{k=1}^{K} \overbrace{\tau_{ik}}^{\text{cluster indicator}} \underbrace{\left(\sum_{j=1}^{J} (x_{ij} - \theta_{kj})^2 \right)}_{\text{Squared Euclidean Distance}}$$

K-Means $\rightsquigarrow$ Objective Function

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- Calculate squared euclidean distance from center

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- Calculate squared euclidean distance from center
- Only for the assigned cluster

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- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions

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- Calculate squared euclidean distance from center
- Only for the assigned cluster
- Two trivial solutions
    - If $K = N$ then $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = 0$ (Minimum)

K-Means$\leadsto$ Objective Function

Assume squared euclidean distance

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- Calculate squared euclidean distance from center
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  - If $K = N$ then $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = 0$ (Minimum)
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- Calculate squared euclidean distance from center
- Only for the assigned cluster
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    - $\boldsymbol{\theta}_i = \boldsymbol{x}_i$

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  - If $K = 1$, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$

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  - If $K = 1$, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$
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  - If $K = 1$, $f(\boldsymbol{X}, \boldsymbol{T}, \boldsymbol{\Theta}) = N \times \sigma^2$
    - Each observation in same cluster
    - $\boldsymbol{\theta}_1 =$ Average across documents

K-Means $\rightsquigarrow$ Optimization

Coordinate descent

K-Means$\rightsquigarrow$ Optimization

Coordinate descent$\rightsquigarrow$ iterate between labels and centers.

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Iterative algorithm: each iteration t

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- Conditional on $\boldsymbol{\Theta}^{t-1}$ (from previous iteration), choose $\boldsymbol{T}^t$

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Coordinate descent $\rightsquigarrow$ iterate between labels and centers.

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Repeat until convergence $\rightsquigarrow$ as measured as change in f dropping below threshold ϵ

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$$\text{Change} = f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) - f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$$

K-Means $\rightsquigarrow$ Optimization

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1) initialize K cluster centers $\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \ldots, \boldsymbol{\theta}_K^t$.

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2) Choose $\boldsymbol{T}^t$

$$\tau_{im}^t = \begin{cases} 1 \text{ if } m = \arg\min_k \sum_{j=1}^{J} (x_{ij} - \theta_{kj}^t)^2 \\ 0 \text{ otherwise }, \end{cases} .$$

K-Means$\rightsquigarrow$ Optimization

1) initialize K cluster centers $\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \ldots, \boldsymbol{\theta}_K^t$.

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$$\tau_{im}^t = \begin{cases} 1 \text{ if } m = \arg\min_k \sum_{j=1}^J (x_{ij} - \theta_{kj}^t)^2 \\ 0 \text{ otherwise }, \end{cases} .$$

In words: Assign each document $\boldsymbol{x}_i$ to the closest center $\boldsymbol{\theta}_m^t$

K-Means$\rightsquigarrow$ Optimization

K-Means$\rightsquigarrow$ Optimization

3) Choose $\Theta^t \rightsquigarrow$ Focus on the center for cluster k

K-Means $\rightsquigarrow$ Optimization

3) Choose Θ^t $\rightsquigarrow$ Focus on the center for cluster k

$$f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k \quad = \quad \sum_{i=1}^{N} \tau_{ik}^t \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^2 \right)$$

K-Means$\rightsquigarrow$ Optimization

3) Choose $\Theta^t \rightsquigarrow$ Focus on the center for cluster k

$$\begin{aligned}
f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k &= \sum_{i=1}^{N} \tau_{ik}^t \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^2 \right) \\
\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k}{\partial \theta_{kj}} &= -2 \sum_{i=1}^{N} \tau_{ij}^t \left(x_{ij} - \theta_{jk} \right)
\end{aligned}$$

K-Means $\rightsquigarrow$ Optimization

3) Choose Θ^t $\rightsquigarrow$ Focus on the center for cluster k

$$
\begin{aligned}
f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k &= \sum_{i=1}^{N} \tau_{ik}^{t} \left(\sum_{j=1}^{J} (x_{ij} - \theta_{jk})^2 \right) \\
\frac{\partial f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k}{\partial \theta_{kj}} &= -2 \sum_{i=1}^{N} \tau_{ij}^{t} \left(x_{ij} - \theta_{jk} \right) \\
0 &= -2 \sum_{i=1}^{N} \tau_{ij}^{t} \left(x_{ij} - \theta_{jk}^{*} \right)
\end{aligned}
$$

K-Means$\rightsquigarrow$ Optimization

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\begin{aligned}
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0 &= -2 \sum_{i=1}^{N} \tau_{ij}^t \left(x_{ij} - \theta_{jk}^* \right) \\
&= \sum_{i=1}^{N} \tau_{ij}^t x_{ij} - \theta_{jk}^* \sum_{i=1}^{N} \tau_{ij}^t
\end{aligned}
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&= \sum_{i=1}^{N} \tau_{ij}^t x_{ij} - \theta_{jk}^* \sum_{i=1}^{N} \tau_{ij}^t \\
\frac{\sum_{i=1}^{N} \tau_{ik}^t x_{ij}}{\sum_{i=1}^{N} \tau_{ik}^t} &= \theta_{jk}^*
\end{aligned}
$$

K-Means$\rightsquigarrow$ Optimization

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}}$$

K-Means$\rightsquigarrow$ Optimization

$$\boldsymbol{\theta}^{t+1} = \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i$$

K-Means $\rightsquigarrow$ Optimization

$$\boldsymbol{\theta}^{t+1} \;=\; \frac{\sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i}{\sum_{i=1}^{N} \tau_{ik}} \propto \sum_{i=1}^{N} \tau_{ik} \boldsymbol{x}_i$$

In words: $\boldsymbol{\theta}^{t+1}$ is the average of the documents assigned to k.

K-Means$\leadsto$ Optimization

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K-Means$\leadsto$ Optimization

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Optimization algorithm:

- Initialize centers
- Do until converged:

K-Means$\rightsquigarrow$ Optimization

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  - For each center, take average of assigned documents$\rightsquigarrow$ $\boldsymbol{\theta}_k^t$

K-Means$\rightsquigarrow$ Optimization

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Optimization algorithm:

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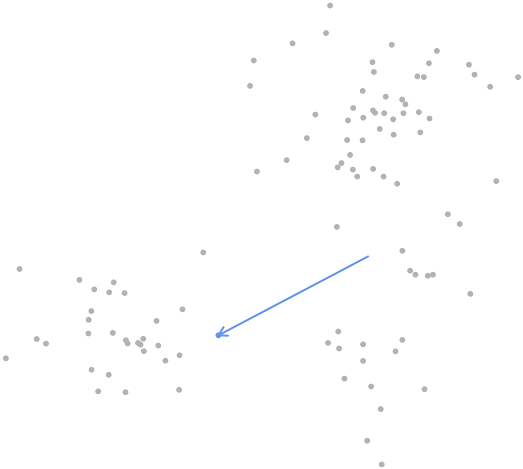
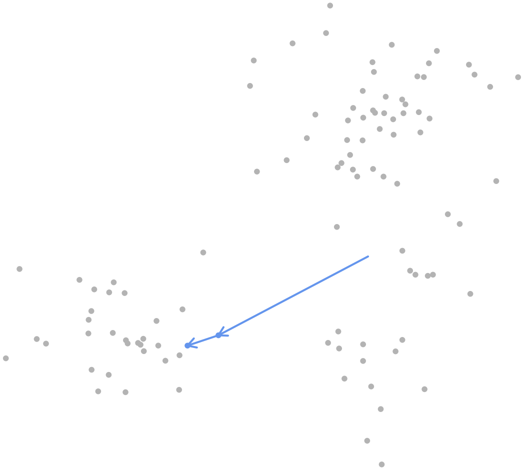
- Do until converged:
  - For each document, find closest center$\rightsquigarrow$ $\boldsymbol{\tau}_i^t$
  - For each center, take average of assigned documents$\rightsquigarrow$ $\boldsymbol{\theta}_k^t$
  - Update change $f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) - f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$

Visual Example

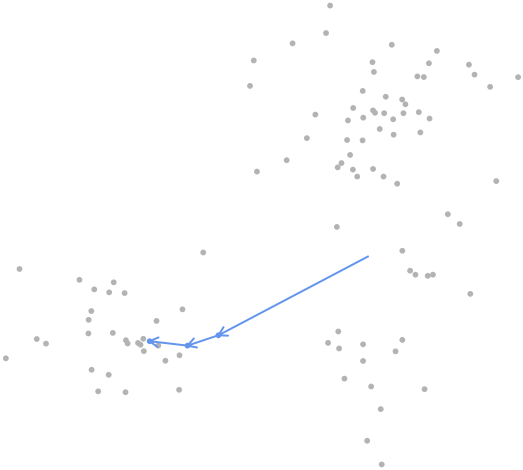
Visual Example



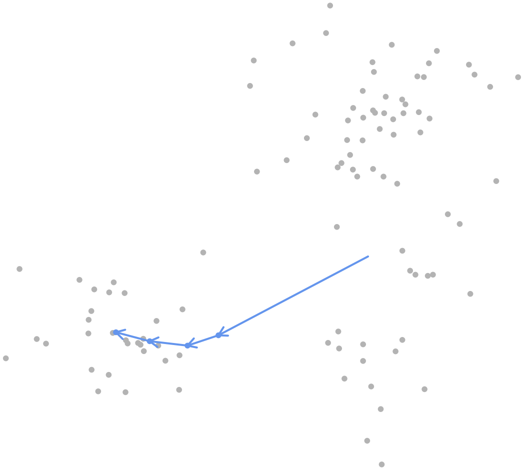
Visual Example



Visual Example



Visual Example



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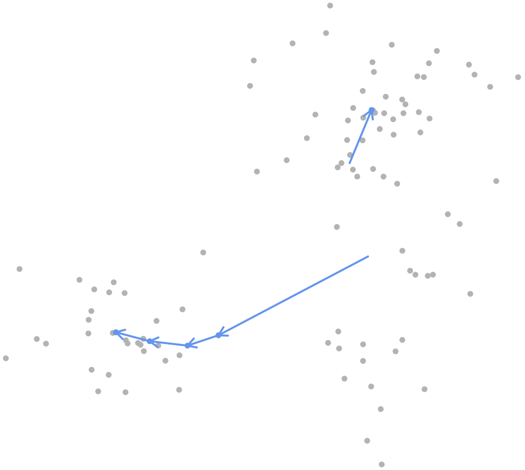

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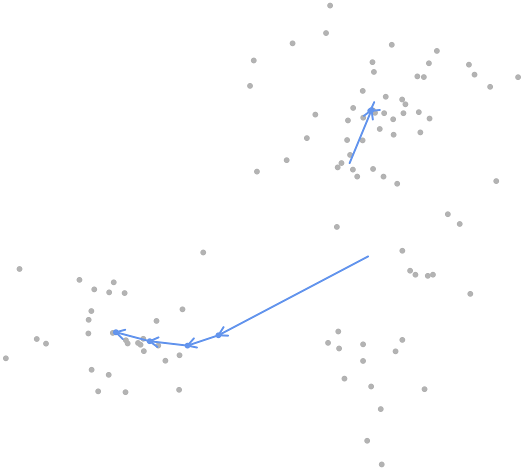

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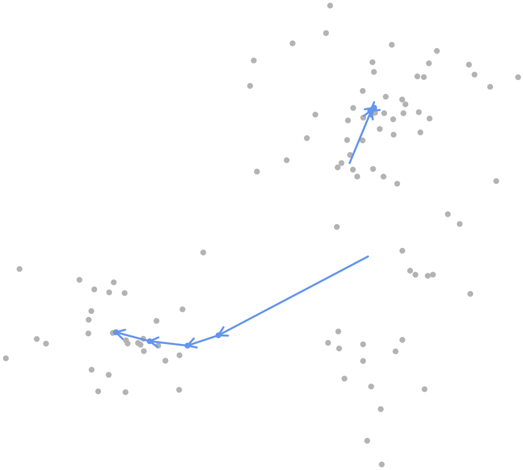

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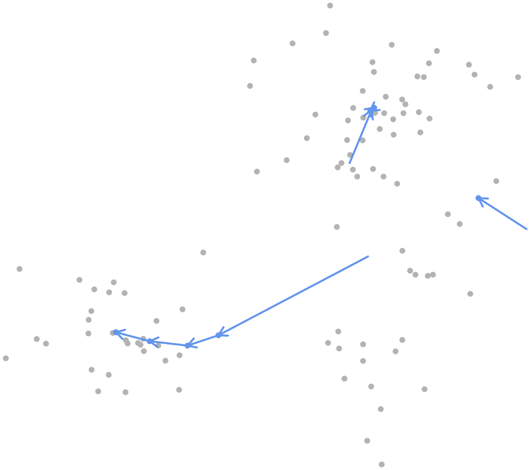

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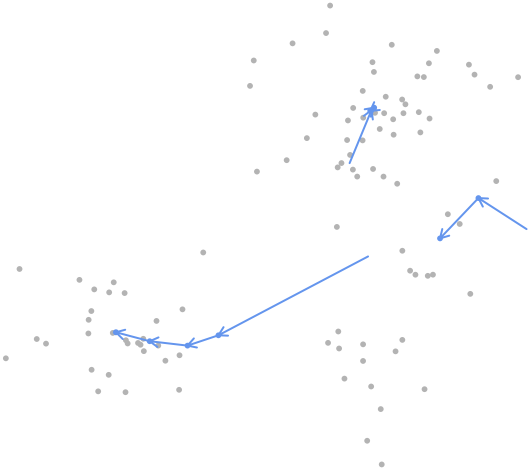

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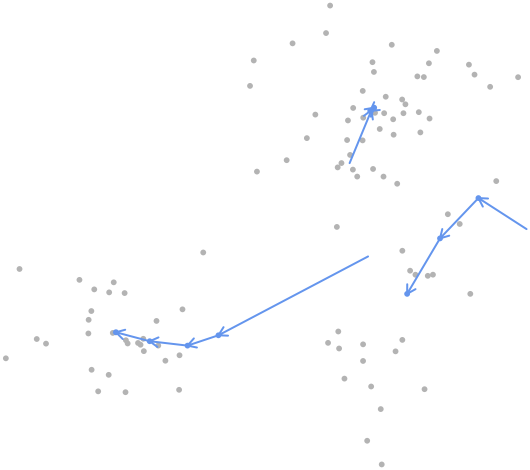

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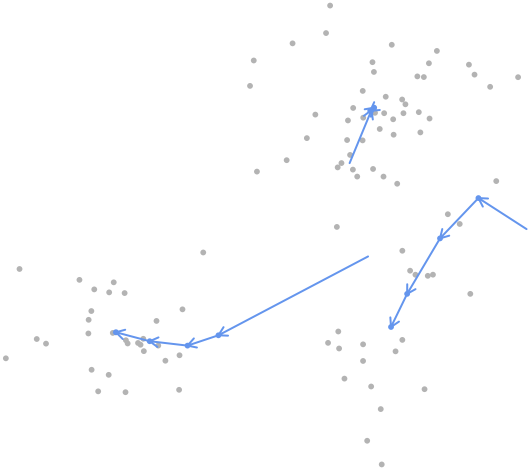

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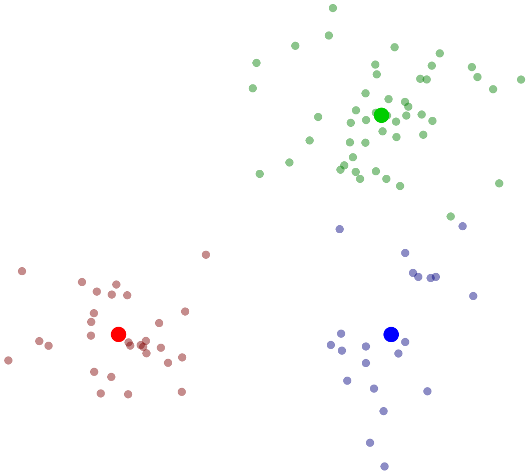

An Example: Jeff Flake

`To the R Code!`

Interpreting Cluster Components

Unsupervised methods

Interpreting Cluster Components

Unsupervised methods $\rightsquigarrow$ low startup costs, high post-model costs

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`back to the R code!`

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In words:

- Draw a cluster label
- Given distribution, draw realization

Mixture of Unigram Models (Mixture of Multinomials)

A mixture of unigram-language models

$$
\begin{aligned}
\boldsymbol{\pi} &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\theta} &\sim \text{Dirichlet}(\mathbf{1}) \\
\boldsymbol{\tau}_i | \boldsymbol{\pi} &\sim \text{Multinomial}(1, \boldsymbol{\pi}) \\
\boldsymbol{x}_i | \tau_{ik} = 1, \boldsymbol{\theta}_k &\sim \text{Multinomial}(N_i, \boldsymbol{\theta}_k)
\end{aligned}
$$

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This implies the following posterior distribution:

$$p(\boldsymbol{T}, \boldsymbol{\Theta}, \boldsymbol{\pi} | \boldsymbol{X})$$

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& \propto \underbrace{\prod_{i=1}^{N} p(\boldsymbol{\tau}_i, \boldsymbol{x}_i | \boldsymbol{\theta}, \boldsymbol{\pi})}_{\text{Complete data likelihood}}
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&\propto \prod_{i=1}^{N} p(\boldsymbol{\tau}_i|\boldsymbol{\pi})p(\boldsymbol{x}_i|\boldsymbol{\theta}, \boldsymbol{\tau}_i) \\
&\propto \prod_{i=1}^{N}\prod_{k=1}^{K}\left[\pi_k \prod_{j=1}^{J} \theta_{jk}^{x_{ik}}\right]^{\tau_{ik}}
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Obtain MAP estimates $\rightsquigarrow$ EM Algorithm

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Obtain $\boldsymbol{\Theta}^{t+1}$, $\boldsymbol{\pi}^{t+1}$

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$$\begin{aligned} \text{Change} &= \mathrm{E}[\log \text{Complete data} | \boldsymbol{\Theta}^{t+1}, \boldsymbol{\pi}^{t+1}] \\ &\quad - \mathrm{E}[\log \text{Complete data} | \boldsymbol{\Theta}^{t}, \boldsymbol{\pi}^{t}] \end{aligned}$$

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Our update steps will be strikingly similar to the K-Means algorithm

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$$p(\tau_{ik}|\boldsymbol{\Theta}^t, \boldsymbol{\pi}^t, \boldsymbol{X})$$

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2) E-Step

$$p(\tau_{ik}|\boldsymbol{\Theta}^t, \boldsymbol{\pi}^t, \boldsymbol{X}) \quad = \quad \overbrace{\frac{p(\tau_{ik}|\boldsymbol{\pi}^t)p(\boldsymbol{x}_i|\boldsymbol{\theta}_k^t)}{\sum_{m=1}^{K}\left(p(\tau_{im}|\boldsymbol{\pi}^t)p(\boldsymbol{x}_i|\boldsymbol{\theta}_m^t)\right)}}^{\text{general form}}$$

Mixture of Unigram Models (Mixture of Multinomials)

1) Initialize parameters $\boldsymbol{\Theta}^t$ and $\boldsymbol{\pi}^t$
2) E-Step

$$
\begin{aligned}
p(\tau_{ik}|\boldsymbol{\Theta}^t, \boldsymbol{\pi}^t, \boldsymbol{X}) &= \overbrace{\frac{p(\tau_{ik}|\boldsymbol{\pi}^t)p(\boldsymbol{x}_i|\boldsymbol{\theta}_k^t)}{\sum_{m=1}^{K}\left(p(\tau_{im}|\boldsymbol{\pi}^t)p(\boldsymbol{x}_i|\boldsymbol{\theta}_m^t)\right)}}^{\text{general form}} \\
&= \frac{\pi_k^t \prod_{j=1}^{J}(\theta_{jk}^t)^{x_{ij}}}{\sum_{m=1}^{K}\left(\pi_m^t \prod_{j=1}^{J}(\theta_{jm}^t)^{x_{ij}}\right)}
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$$

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Define:

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\end{aligned}
$$

Define:

$$
r_{ik}^t \equiv \frac{\pi_k^t \prod_{j=1}^{J}(\theta_{jk}^t)^{x_{ij}}}{\sum_{m=1}^{K}\left(\pi_m^t \prod_{j=1}^{J}(\theta_{jm}^t)^{x_{ij}}\right)}
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\end{aligned}
$$

Define: Avoid underflow

$$
r_{ik}^t = \left[1 + \sum_{k' \neq k} \frac{\pi_{k'} \prod_{j=1}^{J}(\theta_{jk'}^t)^{x_{ij}}}{\pi_k \prod_{j=1}^{J}(\theta_{jk}^t)^{x_{ij}}}\right]^{-1}
$$

Mixture of Unigram Models (Mixture of Multinomials)

3) M-Step:

Mixture of Unigram Models (Mixture of Multinomials)

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$$\text{E}[\log \text{Complete data}|\boldsymbol{\theta}, \boldsymbol{\pi}] = \sum_{i=1}^{N} \sum_{k=1}^{K} E[\tau_{ik}] \log \left(\pi_k \prod_{j=1}^{J} \theta_{jk}^{x_{ik}} \right)$$

Mixture of Unigram Models (Mixture of Multinomials)

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&= \sum_{i=1}^{N} \sum_{k=1}^{K} r_{ik}^{t} \log \pi_k + \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{j=1}^{J} r_{ik}^{t} x_{ij} \log \theta_{jk}
\end{aligned}
$$

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Introducing constraints, differentiating, setting equal to zero and algebra yields:

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Introducing constraints, differentiating, setting equal to zero and algebra yields:

$$
\pi_k^{t+1} = \frac{\sum_{i=1}^{N} r_{ik}^{t}}{N}
$$

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\end{aligned}
$$

Example: Jeff Flake Again!

```
To the R Code!
```

Fully Automated Clustering

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- Notion of similarity and "good" partition $\rightsquigarrow$ clustering

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- Many clustering methods:

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YOU DON'T! $\rightsquigarrow$ **And never will**

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Topic and Mixed Membership Models

Clustering

Document $\rightsquigarrow$ One Cluster

Doc 1

Doc 2

Doc 3

$\vdots$

Doc N

Cluster 1

Cluster 2

$\vdots$

Cluster K

Topic and Mixed Membership Models

Clustering

Document $\rightsquigarrow$ One Cluster

Doc 1	
	Cluster 1
Doc 2	
	Cluster 2
Doc 3	
	$\vdots$
$\vdots$	
	Cluster K
Doc N	

Topic and Mixed Membership Models

Clustering

Document $\rightsquigarrow$ One Cluster

Doc 1	
	Cluster 1
Doc 2 $\rightarrow$ Cluster 1	
	Cluster 2
Doc 3	
	$\vdots$
$\vdots$	
	Cluster K
Doc N	

Topic and Mixed Membership Models

Clustering
Document $\rightsquigarrow$ One Cluster

Doc 1
Doc 2
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Doc N

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Topic and Mixed Membership Models

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Doc 1	
	Cluster 1
Doc 2	
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$\vdots$	
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Topic and Mixed Membership Models

Topic Models (Mixed Membership)

Document $\rightsquigarrow$ Many clusters

Doc 1

Cluster 1

Doc 2

Cluster 2

Doc 3

$\vdots$

$\vdots$

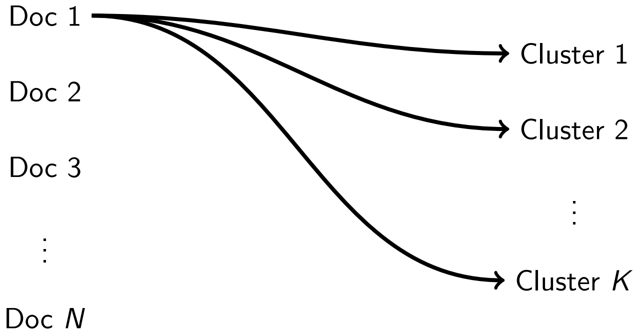
Cluster K

Doc N

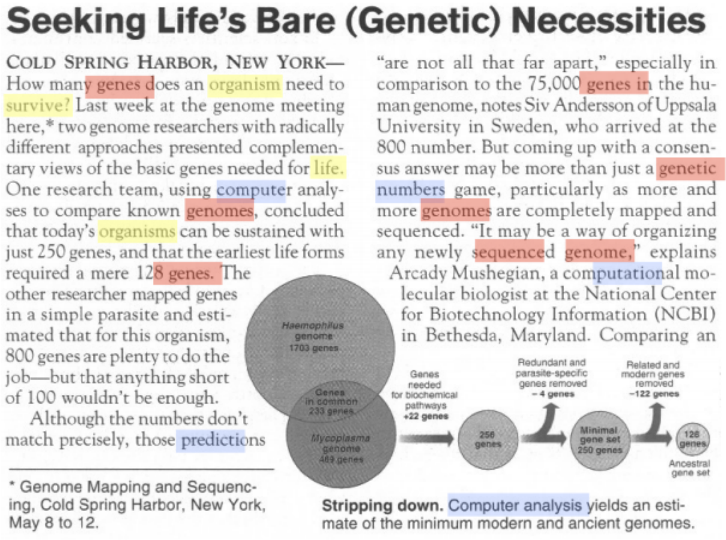
Topic and Mixed Membership Models

Topic Models (Mixed Membership)

Document $\rightsquigarrow$ Many clusters



A Statistical Highlighter (With Many Colors)



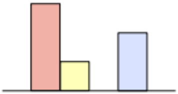
Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK— How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions "are not all that far apart," especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an

Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.



AdMixture: A General Approach to Modeling

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Latent Dirichlet Allocation: an admixture model

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Intuition: each unit is a mixture of latent components

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Latent Dirichlet Allocation: an admixture model

Intuition: each unit is a mixture of latent components

Mixture Model

$$
\begin{aligned}
\boldsymbol{\pi} & \sim \text{Dirichlet}(\boldsymbol{\alpha}) \\
\tau_i & \sim \text{Multinomial}(1, \boldsymbol{\pi}) \\
\boldsymbol{x}_i | \tau_{ik} = 1 & \sim \text{Distribution}(\boldsymbol{\theta}_k)
\end{aligned}
$$

AdMixture: A General Approach to Modeling

Latent Dirichlet Allocation: an admixture model
Intuition: each unit is a mixture of latent components
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AdMixture Model

$$\begin{aligned} \boldsymbol{\pi}_i & \sim \text{Dirichlet}(\boldsymbol{\alpha}) \\ \tau_{im} & \sim \text{Multinomial}(1, \boldsymbol{\pi}) \\ \boldsymbol{x}_{im} | \tau_{imk} = 1 & \sim \text{Distribution}(\boldsymbol{\theta}_k) \end{aligned}$$

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Vanilla Latent Dirichlet Allocation $\leadsto$ Objective Function

- Consider document i, $(i = 1, 2, \ldots, N)$.

Vanilla Latent Dirichlet Allocation $\rightsquigarrow$ Objective Function

- Consider document i, $(i = 1, 2, \ldots, N)$.
- Suppose there are M_i total words and $\boldsymbol{x}_i$ is an $M_i \times 1$ vector, where x_{im} describes the m^{th} word used in the document*.

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*Notice: this is a different representation than a document-term matrix. x_{im} is a number that says which of the J words are used. The difference is for clarity and we'll this representation is closely related to document-term matrix

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$$\boldsymbol{\theta}_k \sim \text{Dirichlet}(\mathbf{1})$$

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$$
\begin{aligned}
\boldsymbol{\theta}_k &\sim \text{Dirichlet}(\mathbf{1}) \\
\alpha_k &\sim \text{Gamma}(\alpha, \beta) \\
\boldsymbol{\pi}_i | \boldsymbol{\alpha} &\sim \text{Dirichlet}(\boldsymbol{\alpha}) \\
\boldsymbol{\tau}_{im} | \boldsymbol{\pi}_i &\sim \text{Multinomial}(1, \boldsymbol{\pi}_i) \\
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Vanilla Latent Dirichlet Allocation $\rightsquigarrow$ Objective Function

Together the model implies the following posterior:

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$$p(\boldsymbol{\pi}, \boldsymbol{T}, \boldsymbol{\Theta}, \alpha | \boldsymbol{X}) \quad \propto \quad p(\boldsymbol{\alpha})p(\boldsymbol{\pi}|\alpha)p(\boldsymbol{T}|\boldsymbol{\pi})p(\boldsymbol{X}|\theta, \boldsymbol{T})$$

Vanilla Latent Dirichlet Allocation $\rightsquigarrow$ Objective Function

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$$
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p(\boldsymbol{\pi}, \boldsymbol{T}, \boldsymbol{\Theta}, \alpha | \boldsymbol{X}) &\propto p(\boldsymbol{\alpha}) p(\boldsymbol{\pi}|\alpha) p(\boldsymbol{T}|\boldsymbol{\pi}) p(\boldsymbol{X}|\theta, \boldsymbol{T}) \\
&\propto p(\boldsymbol{\alpha}) \prod_{i=1}^{N} \left[p(\boldsymbol{\pi}_i|\boldsymbol{\alpha}) \prod_{m=1}^{M_i} p(\boldsymbol{\tau}_{im}|\boldsymbol{\pi}) p(x_{im}|\boldsymbol{\theta}_k, \tau_{imk} = 1) \right]
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&\propto p(\boldsymbol{\alpha}) \prod_{i=1}^{N} \left[\frac{\Gamma(\sum_{k=1}^{K} \alpha_k)}{\prod_{k=1}^{K} \Gamma(\alpha_k)} \prod_{k=1}^{K} \pi_{ik}^{\alpha_k - 1} \prod_{m=1}^{M} \prod_{k=1}^{K} \left[\pi_{ik} \prod_{j=1}^{J} \theta_{jk}^{x_{imj}} \right]^{\tau_{ikm}} \right]
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$$

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Together the model implies the following posterior:

$$
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Vanilla Latent Dirichlet Allocation $\rightsquigarrow$ Objective Function

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- Variational Approximation $\rightsquigarrow$ Find "closest" distribution

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Described in the slides appendix

Running a Topic Model with STM

`to the STM Code`

Why does this work $\rightsquigarrow$ Co-occurrence

Where's the information for each word's topic?

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Reconsider document-term matrix

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Reconsider document-term matrix

	Word_1	Word_2	$\ldots$	Word_J
Doc_1	0	1	$\ldots$	0
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Latent Semantic Analysis: Reduce information in matrix using linear algebra (provides similar results, difficult to generalize)

Biclustering: Models that partition documents and words simultaneously

Why does this work $\rightsquigarrow$ Co-occurrence logic (h/t Colorado Reed Tutorial)

$$p(\boldsymbol{\pi}, \boldsymbol{T}, \boldsymbol{\Theta}, \alpha | \boldsymbol{X}) \propto p(\boldsymbol{\alpha}) p(\boldsymbol{\pi} | \boldsymbol{\alpha}) p(\boldsymbol{T} | \boldsymbol{\pi}) p(\boldsymbol{X} | \boldsymbol{\theta}, \boldsymbol{T})$$

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1) $\boldsymbol{\theta} \rightsquigarrow$ Greater weight on terms that occur together

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1) $\boldsymbol{\theta} \leadsto$ Greater weight on terms that occur together
2) $\boldsymbol{\pi} \leadsto$ Greater weight on indicators that appear more regularly

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$$p(\boldsymbol{\pi}, \boldsymbol{T}, \boldsymbol{\Theta}, \alpha | \boldsymbol{X}) \quad \propto \quad p(\boldsymbol{\alpha}) \underbrace{p(\boldsymbol{\pi}|\boldsymbol{\alpha})}_{3} \underbrace{p(\boldsymbol{T}|\pi)}_{2} \underbrace{p(\boldsymbol{X}|\boldsymbol{\theta}, \boldsymbol{T})}_{1}$$

1) $\boldsymbol{\theta} \rightsquigarrow$ Greater weight on terms that occur together
2) $\boldsymbol{\pi} \rightsquigarrow$ Greater weight on indicators that appear more regularly
3) $\boldsymbol{\alpha} \rightsquigarrow$ Emphasis on $\boldsymbol{\pi}$ with greater weight

Validation $\rightsquigarrow$ Topic Intrusion

Discussed several validations

- Labeling paragraphs
  - Identify separating words automatically
  - Label topics manually (read!)
- Statistical methods
  1) Entropy
  2) Exclusivity
  3) Cohesiveness
- Experiment Based Methods
  - Word intrusion $\rightsquigarrow$ topic validity
  - Topic intrusion $\rightsquigarrow$ model fit

Validation $\rightsquigarrow$ Topic Intrusion

1) Ask research assistant to read paragraph
2) Construct experiment
    - For the document, select top three topics
    - Select a fourth topic
    - Show participant, ask her/him to identify intruder

Higher identification $\rightsquigarrow$ topics are a better model of text

Measuring Cluster Performance: Out of Sample Prediction

How well does our model perform?

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How well does our model perform? $\rightsquigarrow$ predict new documents?

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How well does our model perform? $\rightsquigarrow$ predict new documents?
Problem $\rightsquigarrow$ in sample evaluation leads to overfit.
Solution $\rightsquigarrow$ evaluate performance on held out data with perplexity

What's Prediction Got to Do With It?

- Prediction $\rightsquigarrow$ One Task

(Roberts, et al 2017 Forthcoming)

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Different strategy $\rightsquigarrow$ measure quality in topics and clusters

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Different strategy$\rightsquigarrow$ measure quality in topics and clusters

- Statistics: measure cohesiveness and exclusivity (Roberts, et al 2017 Forthcoming)
- Experiments: measure topic and cluster quality

Experimental Approaches

Mathematical approaches

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assumes we're in the model $\rightsquigarrow$ including text representation

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Do humans think the model is performing well?

1) Topic Quality
2) Cluster Quality

Experimental Approaches

1) Take M top words for a topic
2) Randomly select a top word from another topic
   2a) Sample the topic number from l from $K - 1$ (uniform probability)
   2b) Sample word j from the M top words in topic l
   2c) Permute the words and randomly insert the intruder:
      - List:

$$\text{test} \quad = \quad (v_{k,3}, v_{k,1}, v_{l,j}, v_{k,2}, v_{k,4}, v_{k,5})$$

Example Experiment: Word Intrusion (Weiss and Grimmer, In Progress)

`bowl, flooding, olympic, olympics, nfl, coach`

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Higher rate of intruder identification $\rightsquigarrow$ more exclusive/cohesive topics

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Deploy on Mechanical Turk

Stylometry⇝ Who Wrote Disputed Federalist Papers?

Federalist papers ⇝ Mosteller and Wallace (1963)

- Persuade citizens of New York State to adopt constitution
- Canonical texts in study of American politics
- 77 essays
  - Published from 1787-1788 in Newspapers
  - And under the name Publius, anonymously

Who Wrote the Federalist papers?

- Jay wrote essays 2, 3, 4,5, and 64
- Hamilton: wrote 43 papers
- Madison: wrote 12 papers

Disputed: Hamilton or Madison?

- Essays: 49-58, 62, and 63
- Joint Essays: 18-20

Task: identify authors of the disputed papers.

Task: Classify papers as Hamilton or Madison using dictionary methods

Setting up the Analysis

Training $\rightsquigarrow$ papers Hamilton, Madison are known to have authored
Test $\rightsquigarrow$ unlabeled papers
Preprocessing:

- Hamilton/Madison both discuss similar issues
- Differ in extent they use stop words
- Focus analysis on the stop words

Setting up the Analysis

- $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_N) = (\text{Hamilton}, \text{Hamilton}, \text{Madison}, ..., \text{Hamilton})$
  $N \times 1$ matrix with author labels
- Define the number of words in federalist paper i as num_i

$$\boldsymbol{X} = \begin{pmatrix} \frac{1}{\text{num}_1} & \frac{2}{\text{num}_1} & \frac{0}{\text{num}_1} & \cdots & \frac{3}{\text{num}_1} \\ \frac{0}{\text{num}_2} & \frac{1}{\text{num}_2} & \frac{0}{\text{num}_2} & \cdots & \frac{0}{\text{num}_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{0}{\text{num}_N} & \frac{0}{\text{num}_N} & \frac{1}{\text{num}_N} & \cdots & \frac{0}{\text{num}_N} \end{pmatrix}$$

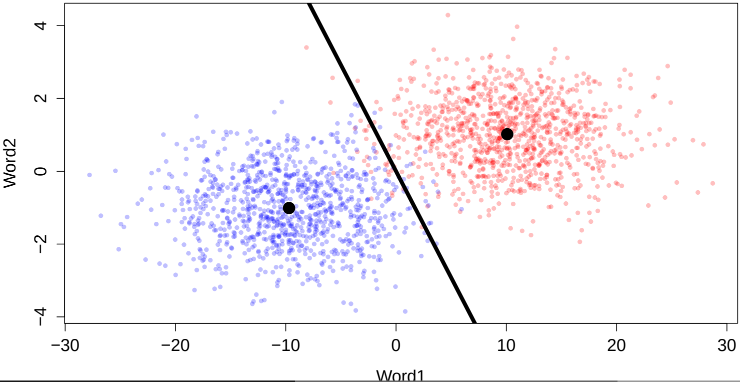
  $N \times J$ counting stop word usage rate
- $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_J)$
  Word weights.

Objective Function

Heuristically: find $\boldsymbol{\theta}^* = (\theta_1^*, \theta_2^*, \ldots, \theta_J^*)$ used to create score

$$p_i = \sum_{j=1}^{J} \theta_j^* X_{ij}$$

that maximally discriminates between categories



Objective Function

Define:

$$
\begin{aligned}
\boldsymbol{\mu}_{\text{Madison}} &= \frac{1}{N_{\text{Madison}}} \sum_{i=1}^{N} I(Y_i = \text{Madison}) \boldsymbol{X}_i \\
\boldsymbol{\mu}_{\text{Hamilton}} &= \frac{1}{N_{\text{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \text{Hamilton}) \boldsymbol{X}_i
\end{aligned}
$$

Objective Function

We can then define functions that describe the "projected" mean and variance for each author

$$
\begin{aligned}
g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Madison}) &= \frac{1}{N_{\text{Madison}}} \sum_{i=1}^{N} I(Y_i = \text{Madison})\boldsymbol{\theta}^{'}\boldsymbol{X}_i = \boldsymbol{\theta}^{'}\boldsymbol{\mu}_{\text{Madison}} \\
g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Hamilton}) &= \frac{1}{N_{\text{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \text{Hamilton})\boldsymbol{\theta}^{'}\boldsymbol{X}_i = \boldsymbol{\theta}^{'}\boldsymbol{\mu}_{\text{Hamilton}} \\
s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Madison}) &= \sum_{i=1}^{N} I(Y_i = \text{Madison})(\boldsymbol{\theta}^{'}\boldsymbol{X}_i - \boldsymbol{\theta}^{'}\boldsymbol{\mu}_{\text{Madison}})^2 \\
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\end{aligned}
$$

Objective Function$\leadsto$ Optimization

$$
\begin{aligned}
f(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}) &= \frac{(g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Hamilton}) - g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Madison}))^2}{s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Hamilton}) + s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \text{Madison})} \\
&= \frac{\left(\boldsymbol{\theta}^{'}(\boldsymbol{\mu}_{\text{Hamilton}} - \boldsymbol{\mu}_{\text{Madison}})\right)^2}{\text{Scatter}_{\text{Hamilton}} + \text{Scatter}_{\text{Madison}}}
\end{aligned}
$$

Optimization$\leadsto$ find $\boldsymbol{\theta}^{*}$ to maximize $f(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y})$, assuming independence across dimensions.
(Fisher's) Linear Discriminant Analysis

Optimization$\rightsquigarrow$ Word Weights

For each word j, construct weight θ_j^*,

$$
\begin{aligned}
\mu_{j,\text{Hamilton}} &= \frac{\sum_{i=1}^{N} I(Y_i = \text{Hamilton})X_{ij}}{\sum_{j=1}^{J}\sum_{i=1}^{N} I(Y_i = \text{Hamilton})X_{ij}} \\
\mu_{j,\text{Madison}} &= \frac{\sum_{i=1}^{N} I(Y_i = \text{Madison})X_{ij}}{\sum_{j=1}^{J}\sum_{i=1}^{N} I(Y_i = \text{Madison})X_{ij}} \\
\sigma^2_{j,\text{Hamilton}} &= \text{Var}(X_{i,j}|\text{Hamilton}) \\
\sigma^2_{j,\text{Madison}} &= \text{Var}(X_{i,j}|\text{Madison})
\end{aligned}
$$

We can then generate weight θ_j^* as

$$
\theta_j^* = \frac{\mu_{j,\text{Hamilton}} - \mu_{j,\text{Madison}}}{\sigma^2_{j,\text{Hamilton}} + \sigma^2_{j,\text{Madison}}}
$$

Optimization$\rightsquigarrow$ Trimming the Dictionary

- Trimming weights: Focus on discriminating words (very simple regularization)
- Cut off: For all $|\theta_j^*| < 0.025$ set $\theta_j^* = 0$.

Classification ⇝ Determining Authorship

For each disputed document i, compute discrimination statistic

$$p_i = \sum_{j=1}^{J} \theta_j^{*} X_{ij}$$

$p_i \rightsquigarrow$ classification (linear discriminator)

- Above midpoint in training set $\rightarrow$ Hamilton text
- Below midpoint in training set $\rightarrow$ Madison text

Findings: Madison is the author of the disputed federalist papers.

Inferring Separating Words

Classification$\rightsquigarrow$ Custom Dictionaries

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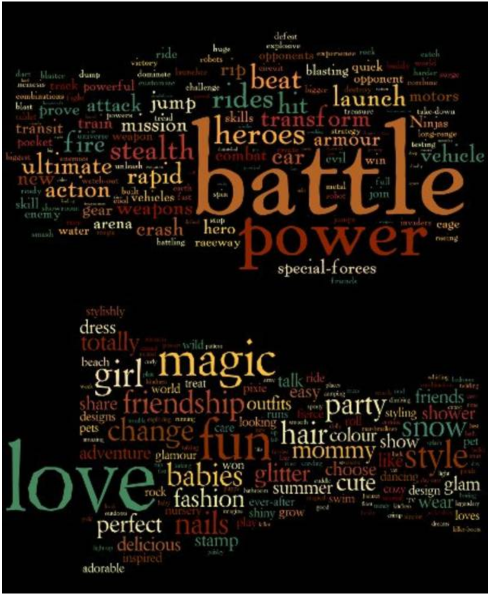

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Vague and Difficult to derive before hand

Congressional Language Across Sources

Congregational Language Across Sources

Congressional Press Releases and Floor Speeches

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- One Answer: texts used for different purposes

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- Deeper question: what does it mean for two text collections to be different?
- One Answer: texts used for different purposes
- Partial answer: identify words that distinguish press releases and floor speeches

A Method for Identifying Distinguishing Words

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Mutual Information

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    - Minimum: $0 \rightarrow X_j$ fails to separate speeches and floor statements

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Bigger mutual information $\Rightarrow$ better discrimination

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Objective function and optimization$\rightsquigarrow$ estimate probabilities that we then place in mutual information

A Method for Identifying Distinguishing Words

Formula for mutual information
(based on ML estimates of probabilities)

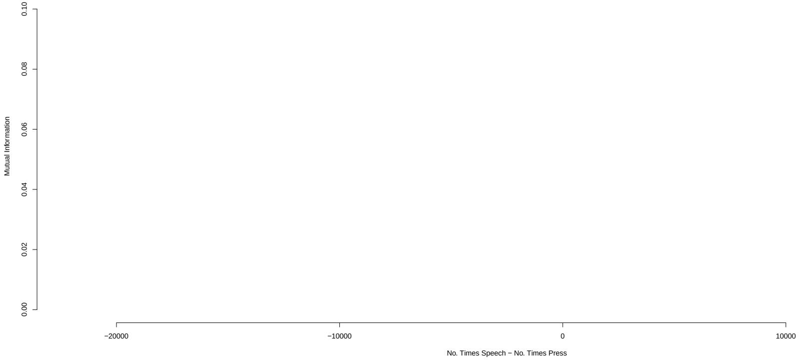
$$
\begin{aligned}
n_p &= \text{Number Press Releases} \\
n_s &= \text{Number of Speeches} \\
D &= n_p + n_s \\
n_j &= \sum_{i=1}^{D} X_{i,j} \qquad \text{(No. docs } X_j \text{ appears)} \\
n_{-j} &= \text{ No. docs } X_j \text{ does not appear} \\
n_{j,p} &= \text{No. press and } X_j \\
n_{j,s} &= \text{No. speech and } X_j \\
n_{-j,p} &= \text{No. press and not } X_j \\
n_{-j,s} &= \text{No. speech and not } X_j
\end{aligned}
$$

A Method for Identifying Distinguishing Words

Formula for Mutual Information

$$
\begin{aligned}
\mathrm{MI}(X_j) &= \frac{n_{j,p}}{D}\log_2\frac{n_{j,p}D}{n_j n_p} + \frac{n_{j,s}}{D}\log_2\frac{n_{j,s}D}{n_j n_s} \\
&\quad + \frac{n_{-j,p}}{D}\log_2\frac{n_{-j,p}D}{n_{-j} n_p} + \frac{n_{-j,s}}{D}\log_2\frac{n_{-j,s}D}{n_{-j} n_s}.
\end{aligned}
$$

What's Different About Press Releases



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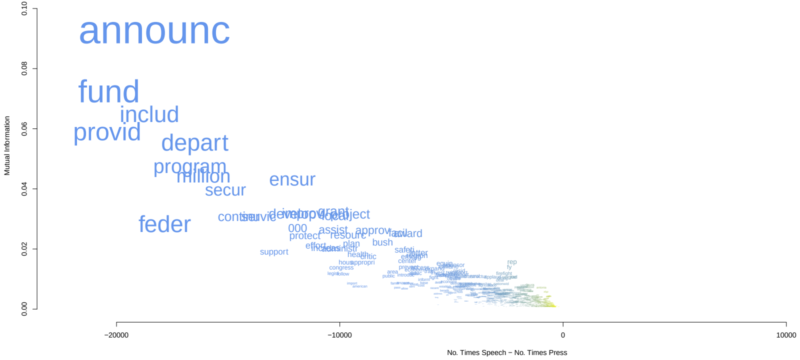

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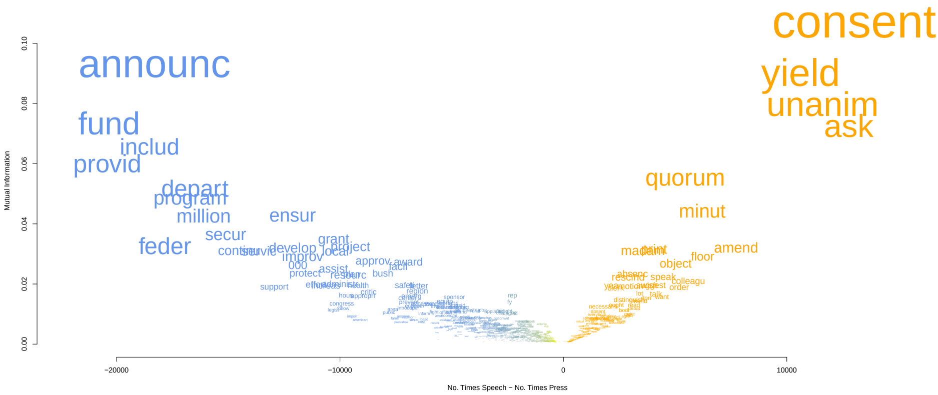

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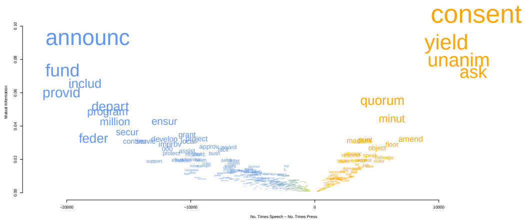

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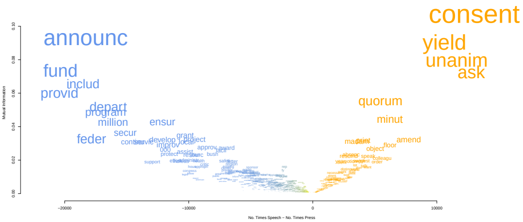

What's Different?

- Press Releases: Credit Claiming

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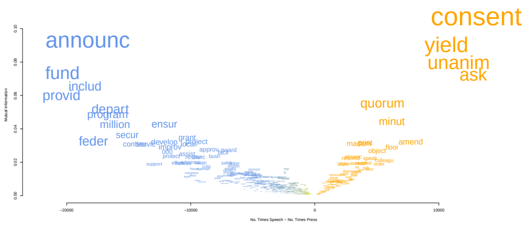

What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words

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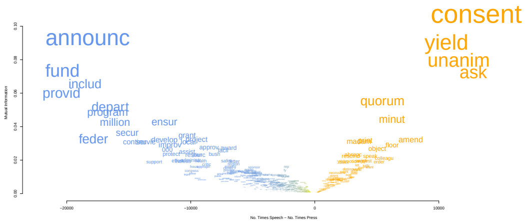

What's Different?

- Press Releases: Credit Claiming
- Floor Speeches: Procedural Words
- Validate: Manual Classification

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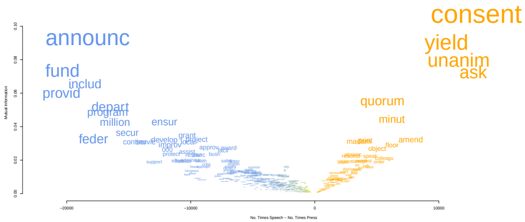

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- Sample 500 Press Releases, 500 Floor Speeches

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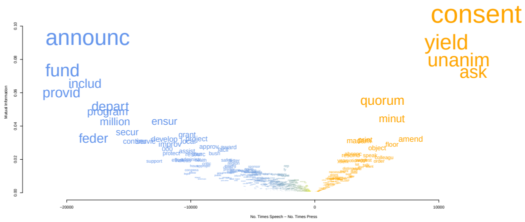

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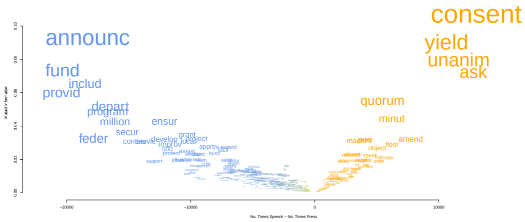

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- Procedural: 0% Press Releases, 44% Floor Speeches

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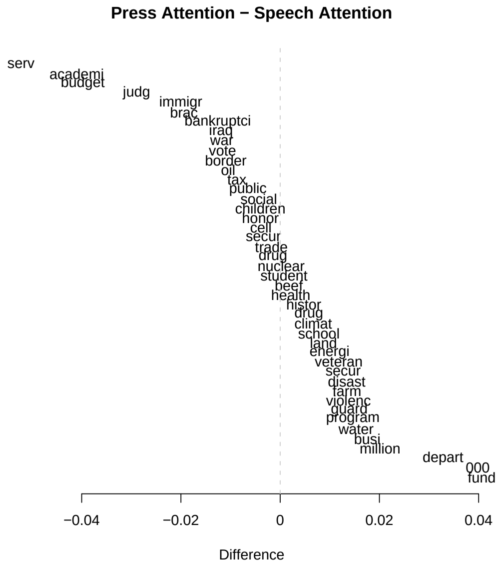

Fightin' Words $\rightsquigarrow$ An Introduction to Regularization

Monroe, Colaresi, and Quinn (2009) $\rightsquigarrow$ what makes a word partisan?

Fightin' Words $\leadsto$ An Introduction to Regularization

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\text{Log Odds Ratio}(E, F) &= \log\left(\frac{P(E)}{1 - P(E)}\right) - \log\left(\frac{P(F)}{1 - P(F)}\right)
\end{aligned}
$$

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Argue for using Log Odds Ratio, weighted by variance
Recall: For some event E and F

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\begin{aligned}
P(E) &= 1 - P(E^c) \\
\text{Odds}(E) &= \frac{P(E)}{1 - P(E)} \\
\text{Odds Ratio}(E, F) &= \frac{\frac{P(E))}{(1-P(E))}}{\frac{P(F)}{1-P(F)}} \\
\text{Log Odds Ratio}(E, F) &= \log\left(\frac{P(E)}{1 - P(E)}\right) - \log\left(\frac{P(F)}{1 - P(F)}\right)
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Strategy$\leadsto$ Construct objective function on *proportions* (and then calculate log-odds)

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Objective Function

Suppose we're interested in how a word separates partisan speech.
$\boldsymbol{Y} = (\text{Republican}, \text{Republican}, \text{Democrat}, \ldots, \text{Republican})$
$\boldsymbol{X} = \text{Unnormalized matrix of word counts } N \times J$
Define

$$\begin{aligned}
\boldsymbol{x}_{\text{Republican}} &= (\sum_{i=1}^{N} I(Y_i = \text{Republican})X_{i1}, \sum_{i=1}^{N} I(Y_i = \text{Republican})X_{i2}, \\
& \ldots, \sum_{i=1}^{N} I(Y_i = \text{Republican})X_{iJ})
\end{aligned}$$

with $N_{\text{Republican}} = \text{Total number of Republican words}$

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$p(\boldsymbol{\pi}|\boldsymbol{\alpha}, \boldsymbol{X}, \boldsymbol{Y})$ is a Dirichlet distribution:

$$\pi^{*}_{\text{Republican},j} = \frac{x_{\text{Republican},j} + \alpha_j}{N_{\text{Republican}} + \sum_{j=1}^{J} \alpha_j}$$

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Calculating Log Odds Ratio

Define $\log \text{Odds Ratio}_j$ as

$$\log \text{Odds Ratio}_j = \log\left(\frac{\pi_{\text{Republican},j}}{1-\pi_{\text{Republican},j}}\right) - \log\left(\frac{\pi_{\text{Democratic},j}}{1-\pi_{\text{Democratic},j}}\right)$$

$$\text{Var}(\log \text{Odds Ratio}_j) \approx \frac{1}{x_{jD}+\alpha_j} + \frac{1}{x_{jR}+\alpha_j}$$

$$\text{Std. Log Odds}_j = \frac{\log \text{Odds Ratio}_j}{\sqrt{\text{Var}(\log \text{Odds Ratio}_j)}}$$

Applying the Model

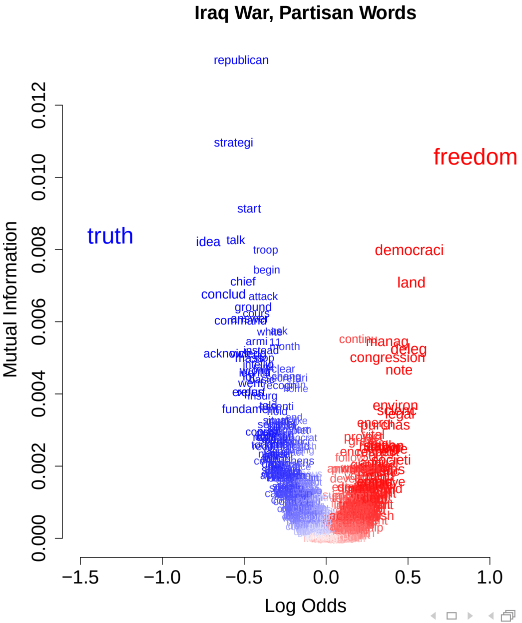
https://gist.github.com/thiagomarzagao/5851207

How do Republicans and Democrats differ in debate?

Condition on topic and examine word usage

- Press Releases (64,033)
- Topic Coded
- Given press release is about topic, what are the features that distinguish Republican and Democratic language?

Mutual Information, Standardized Log Odds



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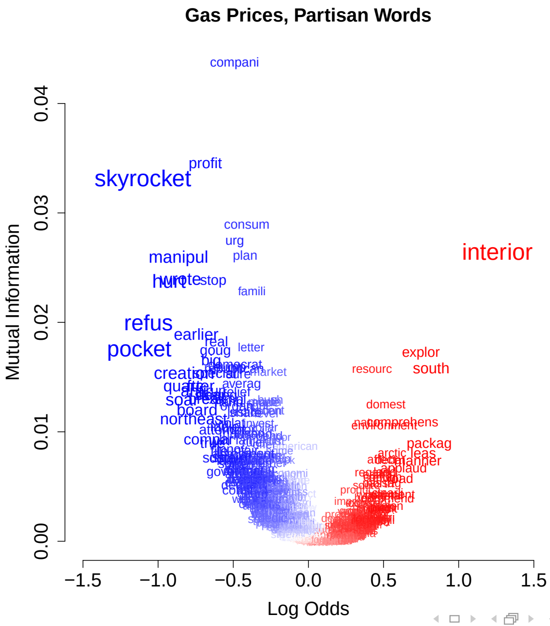
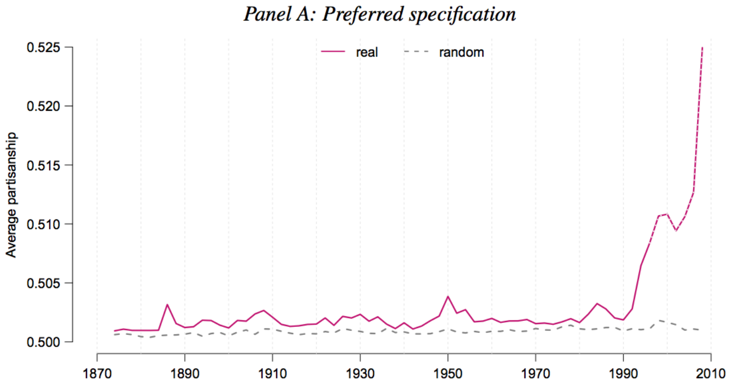


Gentzkow, Shapiro, and Taddy (2017): Rhetorical Polarization

Figure 3: Average partisanship of speech, penalized estimates



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Where do concepts/ideas/questions come from?

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  does.not.matter