STAT 410

Spring 2026
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Homework #01

(due Friday, January 30, by 11:59 p.m.)

No credit will be given without supporting work.

1. Consider a continuous random variable X with the probability density function

$$f_X(x) = \frac{2x+3}{C}, \qquad 1 < x < 6, \qquad \text{zero elsewhere.}$$

a) Find the value of C that makes $f_X(x)$ a valid probability density function.

b) Find the cumulative distribution function of X, $F_X(x) = P(X \le x)$.

"Hint": To double-check your answer: should be $F_X(1) = 0$, $F_X(6) = 1$.

c) Find the expected value of X, $E(X) = \mu_X$.

1. (continued)
Consider $Y = g(X) = \sqrt{X+3}$. Find the probability distribution of Y:

d) Find the support (the range of possible values) of the probability distribution of Y.

e) Use part (b) and the c.d.f. approach to find the c.d.f. of Y, $F_Y(y)$.

"Hint": $F_Y(y) = P(Y \le y) = P(g(X) \le y) = \ldots$.

f) Use the change-of-variable technique to find the p.d.f. of Y, $f_Y(y)$.

"Hint": $f_Y(y) = f_X(g^{-1}(y)) \left| \dfrac{dx}{dy} \right|$.

"Hint": To double-check your answer: should be $f_Y(y) = F_Y'(y)$.

1. (continued)

Consider $W = h(X) = \dfrac{10}{X+4}$. Find the probability distribution of W:

g) Find the support (the range of possible values) of the probability distribution of W.

h) Use part (b) and the c.d.f. approach to find the c.d.f. of W, $F_W(w)$.

"Hint": $F_W(w) = P(W \le w) = P(h(X) \le w) = \ldots$.

i) Use the change-of-variable technique to find the p.d.f. of W, $f_W(w)$.

"Hint": $f_W(w) = f_X(h^{-1}(w)) \left| \dfrac{dx}{dw} \right|$.

"Hint": To double-check your answer: should be $f_W(w) = F_W'(w)$.

2. Consider a discrete random variable X with the probability mass function

$$p_X(x) = \frac{2x+3}{C}, \qquad x = 2, 3, 4, 5, \qquad \text{zero elsewhere.}$$

a) Find the value of C that makes $p_X(x)$ a valid probability mass function.

b) Consider $Y = \dfrac{12}{X-1}$. Find the probability distribution of Y.

You are welcome to use a calculator and/or computer on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function inside and all the bounds) and the answer. For example,

$$\int_0^x u^2\, du = \frac{x^3}{3}, \qquad \int_0^4 \left(\int_0^{\sqrt{x}} x^2 y\, dy \right) dx = 32, \qquad \int_1^\infty \left(\int_0^y \frac{1}{(2x+y)^3}\, dx \right) dy = \frac{2}{9}.$$