Examples for Week 02 (1)

6.* Let Z be a $N(0, 1)$ standard normal random variable.
Show that $X = Z^2$ has a chi-square distribution with 1 degree of freedom.

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Consider a continuous random variable X, with p.d.f. f and c.d.f. F, where F is strictly increasing on some interval I, $F = 0$ to the left of I, and $F = 1$ to the right of I. I may be a bounded interval or an unbounded interval such as the whole real line. $F^{-1}(u)$ is then well defined for $0 < u < 1$.

Fact 1: Let $U \sim \text{Uniform}(0, 1)$, and let $X = F^{-1}(U)$. Then the c.d.f. of X is F.

Proof: $P(X \le x) = P(F^{-1}(U) \le x) = P(U \le F(x)) = F(x)$.

Fact 2: Let $U = F(X)$; then U has a $\text{Uniform}(0, 1)$ distribution.

Proof: $P(U \le u) = P(F(X) \le u) = P(X \le F^{-1}(u)) = F(F^{-1}(u)) = u$.