

Functions of One Random Variable

4. The p.d.f. of X is $f_X(x) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$. Let $Y = -2\theta \ln X$. How is Y distributed?

- a) Determine the probability distribution of Y by finding the c.d.f. of Y

$$F_Y(y) = P(Y \leq y) = P(-2\theta \ln X \leq y).$$

“Hint”: It may be helpful to find $F_X(x)$ first.

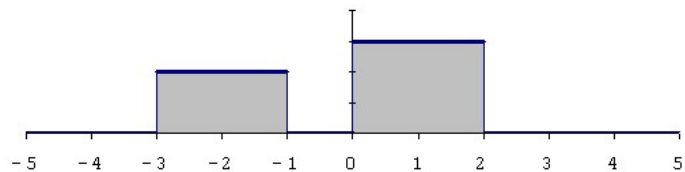
- b) Determine the probability distribution of Y by finding the m.g.f. of Y

$$M_Y(t) = E(e^{Y \cdot t}) = E(e^{-2\theta \ln X \cdot t}).$$

- c) Determine the probability distribution of Y by finding the p.d.f. of Y , $f_Y(y)$, using the change-of-variable technique.

5. Consider a continuous random variable X with p.d.f.

$$f_X(x) = \begin{cases} 0.2 & -3 < x < -1 \\ 0.3 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$



Find the probability distribution of $Y = X^2$.