Discrete Distributions

Bernoulli

$0 < p < 1$

$$f(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1$$
$$M(t) = 1 - p + pe^t$$
$$\mu = p, \qquad \sigma^2 = p(1-p)$$

Binomial

$b(n, p)$

$0 < p < 1$

$$f(x) = \frac{n!}{x!(n-x)!}\, p^x(1-p)^{n-x}, \qquad x = 0, 1, 2, \ldots, n$$
$$M(t) = (1 - p + pe^t)^n$$
$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Geometric

$0 < p < 1$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, \ldots$$
$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \qquad t < -\ln(1-p)$$
$$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

Hypergeometric

$N_1 > 0, \;\; N_2 > 0$

$N = N_1 + N_2$

$$f(x) = \frac{\dbinom{N_1}{x}\dbinom{N_2}{n-x}}{\dbinom{N}{n}}, \qquad x \le n, x \le N_1, n - x \le N_2$$
$$\mu = n\left(\frac{N_1}{N}\right), \qquad \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Negative Binomial

$0 < p < 1$

$r = 1, 2, 3, \ldots$

$$f(x) = \binom{x-1}{r-1}p^r(1-p)^{x-r}, \qquad x = r, r+1, r+2, \ldots$$
$$M(t) = \frac{(pe^t)^r}{[1-(1-p)e^t]^r}, \qquad t < -\ln(1-p)$$
$$\mu = r\left(\frac{1}{p}\right), \qquad \sigma^2 = \frac{r(1-p)}{p^2}$$

Poisson

$0 < \lambda$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \qquad x = 0, 1, 2, \ldots$$
$$M(t) = e^{\lambda(e^t - 1)}$$
$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Uniform

$m > 0$

$$f(x) = \frac{1}{m}, \qquad x = 1, 2, \ldots, m$$
$$\mu = \frac{m+1}{2}, \qquad \sigma^2 = \frac{m^2-1}{12}$$