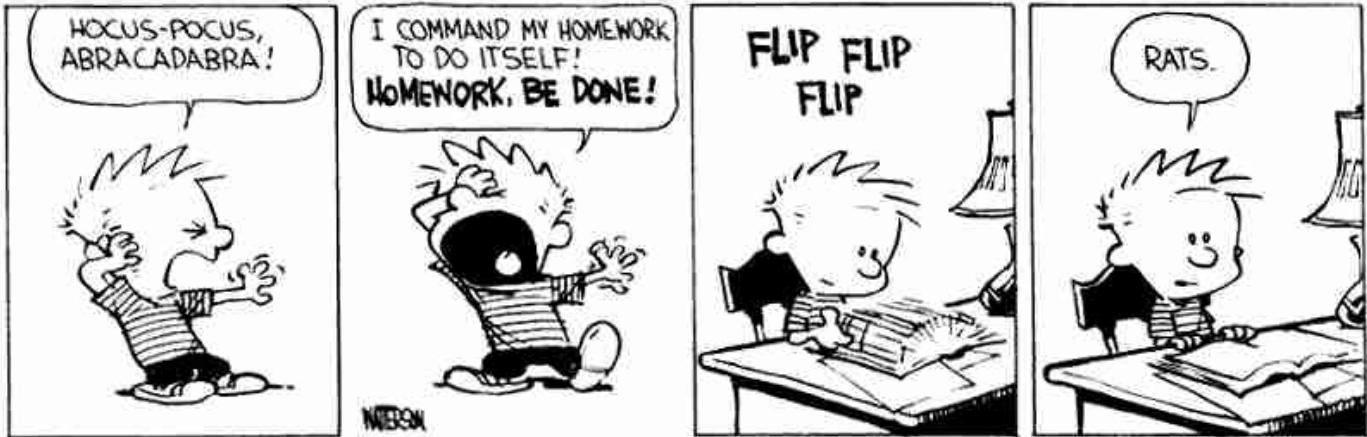


Homework #02

(due Friday, February 6, by 11:59 p.m.)

Spring 2026
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No credit will be given without supporting work.



It is a really good idea to start every problem, every question, every sub-problem, every sub-question with a fresh sketch of the support of the joint probability distribution.

3. Let the joint probability density function for (X, Y) be

$$f(x, y) = \frac{8x+5y}{C}, \quad 0 < x < 2, \quad 0 < y < 2 + x, \quad \text{zero otherwise.}$$

- a) Sketch the support of (X, Y) .

That is, sketch $\{(x, y) : 0 < x < 2, 0 < y < 2 + x\}$.

- b) Find the value of C so that $f(x, y)$ is a valid joint probability density function.

- c) Find the marginal probability density function of X , $f_X(x)$.

Be sure to include its support.

d) Find the marginal probability density function of Y , $f_Y(y)$.

Be sure to include its support.

“Hint”: It would be **wise** to break this question into **pieces**.

e) Find the probability $P(X + Y \leq 1.8)$.

f) Find the probability $P(X + Y \leq 3.0)$.

g) Find the probability $P(X \cdot Y \leq 3.0)$.

h) Find the probability $P\left(\frac{Y}{X} \leq 1.8\right)$.

i) Find the probability $P\left(\frac{Y}{X} \leq 3.0\right)$.

j) Are X and Y independent? *Justify your answer.*

You are welcome to use a calculator and/or computer on any problem to evaluate any integral. For the supporting work, you should include the full integral (with the function inside and all the bounds) and the answer. For example,

$$\int_0^x u^2 du = \frac{x^3}{3}, \quad \int_0^4 \left(\int_0^{\sqrt{x}} x^2 y dy \right) dx = 32, \quad \int_1^\infty \left(\int_0^y \frac{1}{(2x+y)^3} dx \right) dy = \frac{2}{9}.$$