Submission 2.3 (with LaTeX)

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1 Translate into English (2.8.1)

None for this submission.

2 Translate into quantificational logic (2.8.2)

None for this submission.

3 Problems (2.8.3)

1. Nothing.
2. Find an ϵ where there is no σ such that $|f(x) - 1| < \epsilon$ when $|x| < \sigma$.
3. Show that there exists a sequence of real numbers, x_n that converge to zero, but $f(x_n)$ does not converge to 1.
4. Convergence is not yet defined, so I will take the liberty. A sequence x_n converges to c if and only if for all ϵ, there exists an integer, call it δ, such that for all $n > \delta$, $|x_n - c| < \epsilon$.

   More intuitively, the limit as $n \to \infty$ of x_n is c. Saying something is continuous is equivalent to stating that the limit as $x \to c$ of $f(x)$ equals $f(c)$. I won't prove this rigorously since this is just a sketch, but if we can show $\lim_{x \to c} f(x) = f(c)$, then $f(x)$ is continuous at c.

   Now, pick any ϵ. Let

$$a = \begin{cases} \sqrt{8 - \epsilon + 1} & \epsilon \leq 9 \\ 0 & \epsilon > 9 \end{cases}$$

$$b = \sqrt{8 + \epsilon + 1}$$

   Notice, I have chosen values for a and b such that $8 - \epsilon \leq f(a)$ and $f(b) \leq 8 + \epsilon$ (I would show this more rigorously for a real proof). We also know that a and b are both non-negative.

   For any x in between a and b, $f(x)$ is in between $f(a)$ and $f(b)$.

$$\begin{array}{ccccccl} a & < & x & < & b & & \\ a^2 & < & x^2 & < & b^2 & & \text{since everything is non-negative} \\ a^2 - 1 & < & x^2 - 1 & < & b^2 - 1 & & \\ f(a) & < & f(x) & < & f(b) & & \end{array}$$

$$8 - \epsilon \leq f(a) < f(x) < f(b) \leq 8 + \epsilon$$

.

In summary if $a < x < b$ then $8 - \epsilon < f(x) < 8 + \epsilon$. So we set δ to be which ever error bound is more restrictive:

$$\delta = \min\{|3 - a|, |3 - b|\}$$

. For any ϵ, I found a δ such that, for any x with $3 - \delta < x < 3 + \delta$, $8 - \epsilon < f(x) < 8 + \epsilon$, therefore the limit as $x \to 3$ of $f(x)$ equals $f(3)$. Therefore the function is continuous at 3.

4 Arguments in English (2.8.4)

1. (a) Cxy: x causes y

   $\forall x \exists y Cyx$

   $\overline{\exists y \forall x Cyx}$

   (b) Valid, soundness is indeterminate.

   (c) consider the interpretation where $UD = a, b$ and $C = \{(a, b), (b, a)\}$.

2. (a) Hxy: x hates y

   a: Al

   f: Fred

   $\forall x(Hxa \to Hfx)$

   $\forall x Hax$

   $\overline{Haf \wedge Hfa}$

   (b) Valid, soundness is indeterminate.

   (d)

1.	$\forall x(Hxa \to Hfx)$	Premise
2.	$\forall x Hax$	Premise
3.	Haf	UI(2)
4.	$Haa \to Hfa$	UI(1)
5.	Haa	UI(2)
6.	Hfa	MP(4,5)
7.	$Haf \wedge Hfa$	Conj(3,6)

3. (a) Hx: x is an insect in this house

   Lx: x is a large hostile insect

   Ix: x is impervious to pesticides

   $\forall x(Hx \to Lx)$

   $\exists x(Hx \wedge Ix)$

   $\overline{\exists(Lx \wedge Ix)}$

   (b) Valid, soundness is indeterminate.

(d)

1.	$\forall x(Hx \to Lx)$	Premise
2.	$\exists x(Hx \land Ix)$	Premise
3.	$Hc \land Ic$	UI(2)
4.	$Hc \to Lc$	UI(1)
5.	Hc	Simp(3)
6.	Lc	MP(4,5)
7.	Ic	Simp(3)
8.	$Lc \land Ic$	Conj(6,7)
9.	$\exists x(Hx \land Ix)$	EG(8)

4. (a) Sx: x can succeed
Bx: x is bright
Mx: x is mature
$\exists x \neg Sx$
$\forall x((Bx \land Mx) \to Sx)$

$\exists x(\neg Bx \land \neg Mx)$

(b) Valid, soundness is indeterminate.

(d)

1.	$\exists x \neg Sx$	Premise
2.	$\forall x((Bx \land Mx) \to Sx)$	Premise
3.	$\neg Sc$	UI(1)
4.	$(Bc \land Mc) \to Sc$	UI(2)
5.	$\neg Sc \to \neg(Bc \land Mc)$	ContraPos(4)
6.	$\neg(Bc \land Mc)$	MP(5,3)
7.	$\exists x \neg(Bx \land Mx)$	EG(6)

5. (a) Px: x is a pig
$\exists x \exists y \exists z(Px \land Py \land Pz \land x \neq y \land y \neq z \land x \neq z)$

$\exists x \exists y(Px \land Py \land x \neq y)$

(b) Valid, sound.

(d)

1.	$\exists x \exists y \exists z(Px \land Py \land Pz \land x \neq y \land y \neq z \land x \neq z)$	Premise
2.	$Pa \land Pb \land Pc \land a \neq b \land b \neq c \land a \neq c$	EI(1) (used three times)
3.	$Pa \land Pb \land a \neq b$	Simp(2) used three times

6. (a) Lxy: x likes y
p: Popeye
o: Olive Oyl
$\forall x(Lxo \to Lpx)$
$\forall x(Lox)$

$Lpo \land Lop$

(b) Valid, soundness is indeterminate.

(d)

1.	$\forall x(Lxo \to Lpx)$	Premise
2.	$\forall x(Lox)$	Premise
3.	Loo	UI(2)
4.	$Loo \to Lpo$	UI(1)
5.	Lpo	MP(5,4)
6.	Lop	UI(2)
7.	$Lpo \land Lop$	Conj(5,6)

7. (a) Sx: x is sound
   Cx: The conclusion of x is true
   a: This argument

$$\begin{array}{l} \neg Ca \\ \neg\exists x(Sx \land \neg Cx) \\ \hline \neg Sa \end{array}$$

   (b) Valid, soundness is indeterminate.

   (c) Consider, for the sake of contradiction, that an interpretation exists where the premises are true and the conclusion is false. The Sa. Because of the second premise, we have Ca. This contradicts our premise $\neg Ca$.

   (d)

1.	$\neg Ca$	Premise
2.	$\neg\exists x(Sx \land \neg Cx)$	Premise
3.	$\forall x \neg(Sx \land \neg Cx)$	QEx(2)
4.	$\neg(Sa \land \neg Ca)$	UI(3)
5.	$\neg Sa \lor \neg\neg Ca$	DeM(4)
6.	$\neg Ca \to \neg Sa$	CDis (5)
7.	$\neg Sa$	MP(6,1)

8. (a) Jx: x is Jones's killer
   Wx: x weighs more than 200 pounds
   s: Smith

$$\begin{array}{l} \forall x(Jx \implies Wx) \\ \neg Ws \\ \hline \neg Js \end{array}$$

   (b) Valid, soundness is indeterminate.

   (c) Consider the interpretation where all the premises are true and the conclusion is false. Then we have that Js. By our first premise, Wx. This contradicts our second premise $\neg Wx$, therefore the conclusion is false.

   (d)

1.	$\forall x(Jx \to Wx)$	Premise
2.	$\neg Ws$	Premise
4.	$Js \to Ws$	UI(1)
5.	$\neg Ws \to \neg Js$	ContraPos(4)
6.	$\neg Js$	MP(5,2)

9. (a) Lxy: x likes y
   m: Mandy
   a: Andy

$$\begin{array}{l} \forall x Lxm \\ \forall x(x = a \leftrightarrow Lmx) \\ \hline a = m \end{array}$$

   (b) Valid, soundness is indeterminate.

(d)

1.	$\forall x Lxm$	Premise
2.	$\forall x(x = a \leftrightarrow Lmx)$	Premise
3.	Lmm	UI(1)
4.	$m = a \leftrightarrow Lmm$	UI(2)
5.	$Lmm \rightarrow m = a$	Equiv(4)
6.	$m = a$	MP(5,3)

10. (a) j: Dr. Jekyll
h: Mr. Hyde
Axy: x is afraid of y
$\forall x Axh$
$\forall x(x = j \leftrightarrow Ahx)$
———
$h = j$

(b) Valid, soundness is indeterminate.

(d)

1.	$\forall x Axh$	Premise
2.	$\forall x(x = j \leftrightarrow Ahx)$	Premise
3.	Ahh	UI(1)
4.	$h = j \leftrightarrow Ahh$	UI(2)
5.	$Ahh \rightarrow h = j$	Equiv(4)
6.	$h = j$	MP(5,3)

5 Arguments in quantificational logic (2.8.5)

2. (a) Valid

(b) Consider, for the sake of contradiction, that an interpretation exists where the premises are true and the conclusion is false. Then there exists an element, a, for which $\neg(\neg Ga \rightarrow \neg Fa)$. This is equivalent to $\neg Ga$ and Fa (this is the only way to falsify the conclusion). Then, $Fa \rightarrow Ga$ must be false. This contradicts the premise, therefore the conclusion is valid

(c)

1.	$\forall x(Fx \rightarrow Gx)$	Premise
2.	$Fc \rightarrow Gc$	UI(1)
3.	$\neg Gc \rightarrow \neg Fc$	ContraPos(2)
4.	$\forall x(\neg Gx \rightarrow \neg Fx)$	UG(3)