Submission 3.1 (with $\LaTeX$)

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1. Lxy is not a function. One person can love two people. So $\exists x \exists y \exists z(Lxy \wedge Lxz \wedge y \neq z$
2. Mxy is a function mapping from people to people. Each person has exactly one mother, so we know Mxy is a function
3. Cxy is not a function. I am the child of Mark and I am the child of Terri, therefore $\exists x \exists y \exists z(Cxy \wedge Cxz \wedge y \neq z$
4. Hxy is not a function. Multiple people can share the same height.
5. Hxy is a function from people to lengths. Each person has exactly one height therefore we can say Hxy is a function.
6. Fxy is a function from married men to women. Each married person has exactly one first wife.
7.

1.	$\forall x f(x,0) = x$	Premise
2.	$\forall x f(x, g(x)) = 0$	Premise
3.	$\forall x \forall y f(x,y) = f(y,x)$	Premise
4.	$f(g(0),0) = g(0)$	UI(1)
5.	$f(0,g(0)) = 0$	UI(2)
6.	$f(0,g(0)) = f(g(0),0)$	UI(3)
7.	$g(0) = 0$	Transitive Property (4,6,5) (Used thrice)
8.	$\exists g(x) = x$	EG(7)

8.

1.	$\forall x f(x,0) = x$	Premise
2.	$\forall x f(x,g(x)) = 0$	Premise
3.	$f(0,0) = 0$	UI(1)
4.	$\exists x f(x,x) = x$	EG(3)

9. Neither surjective nor injective.
10. Not surjective but injective.
11. Both surjective and injective.
12. Surjective but not injective.

13. Both Surjective and injective.

    Every natural number has a representation in Roman Numerals. Therefore f is surjective.

    No two natural numbers have the same representation in Roman Numerals. Therefore f is injective.

14. Surjective but not injective

    Every element in B is pointed to by at at least one element in A. Therefore f is surjective.

    $f(1) = a$ and $f(27) = a$, but $1 \neq 27$. Also the domain is larger than the image, so every element can not map to a unique element. Therefore f is not injective.

15. Not surjective but injective.

    $\neg\exists x(f(x) = 1 \land x \in Dom(f))$. Therefore f is not surjective.

    If $f(x_1) = y = f(x_2)$, then $2x_1 = 2x_2$ so $x_1 = x_2$. Therfore f is injective.

16. Surjective and injective.

    Given any $y \in Cod(f)$, $f(y-1) = y$, therefore $\forall y\exists x(y \in Cod(f) \rightarrow (f(x) = y \land x \in Dom(f)))$. Therefore f is surjective.

    If $f(x_1) = y = f(x_2)$, then $x_1 - 1 = x_2 - 1$ so $x_1 = x_2$. Therfore f is injective.

17. Neither surjective nor injective.

    $\neg\exists x(f(x) = -1 \land x \in Dom(f))$. Therefore f is not surjective.

    $f(-2) = 4 = f(2)$, but $2 \neq -2$. Therefore f is not injective.

18. Surjective and injective.

    Given any $y \in Cod(f)$, $f(\sqrt[3]{y}) = y$. Therefore $\forall y\exists x(y \in Cod(f) \rightarrow (f(x) = y \land x \in Dom(f)))$. Therefore f is surjective.

    If $f(x_1) = y = f(x_2)$, then $x_1^3 = x_2^3$ so $x_1 = x_2$. Therfore f is injective.

19. Not surjective but injective

    $\neg\exists x(f(x) = -1 \land x \in Dom(f))$. Therefore f is not surjective.

    $f(x_1) = f(x_2) \rightarrow e^{x_1} = e^{x_2} \rightarrow x_1 = x_2$. Therefore f is injective.

20. Neither surjective nor injective.

    $\neg\exists x(f(x) = 2 \land x \in Dom(f))$. Therefore f is not surjective.

    $f(0) = 0 = f(2\pi)$. Therefore f is not injective.

21. Surjective but not injective.

    Given any $y \in Cod(f)$, $f(\sin^{-1} y) = y$. Therefore $\forall y\exists x(y \in Dom(f) \rightarrow (f(x) = y \land x \in Dom(f)))$. Therefore f is surjective.

    $f(0) = 0 = f(2\pi)$. Therefore f is not injective.

22. Neither surjective nor injective.

    Given any $y \in Cod(f)$, $f(\sin^{-1} y) = y$. Therefore $\forall y\exists x(y \in Cod(f) \rightarrow (f(x) = y \land x \in Dom(f)))$. Therefore f is surjective.

    $f(0) = 0 = f(2\pi)$. Therefore f is not injective.

23. Surjective but not injective.

    Given any $y \in Cod(f)$, $f(y) = y$. Therefore $\forall y \exists x (y \in Cod(f) \rightarrow (f(x) = y \wedge x \in Dom(f)))$. Therefore f is surjective.

    $f(\frac{1}{2}) = 0 = f(\frac{1}{3})$, but $\frac{1}{2} \neq \frac{1}{3}$.