UESTC 3018 - Communication Systems and Principles

Lecture 18 — Digital Passband Communication

Dr Hasan Abbas

Hasan.abbas@glasgow.ac.uk

Passband Communication

Why Modulate?

- Signal $m(t)$ is at low frequencies (near DC).
- Requires huge antennas ($\lambda/4 \approx 25$ km for voice).
- We shift the signal to a Carrier Frequency f_c.

$$s(t) = A(t)\cos(2\pi f_c t + \phi(t))$$

We can vary:

1. **Amplitude** (A) $\rightarrow$ ASK
2. **Frequency** (f) $\rightarrow$ FSK
3. **Phase** (ϕ) $\rightarrow$ PSK

Binary Digital Modulation

We switch a parameter of a sinusoidal carrier in accordance with the binary symbols `0` and `1`.

The Carrier:

$$c(t) = A_c \cos(2\pi f_c t + \phi_c)$$

We can vary:

1. **Amplitude (A_c):** ASK (Amplitude Shift Keying).
2. **Frequency (f_c):** FSK (Frequency Shift Keying).
3. **Phase (ϕ_c):** PSK (Phase Shift Keying).

- Assumption: Carrier frequency $f_c \gg$ Bit rate R_b.

Geometric Representation of Signals

- To understand modern comms, we don't draw waves; we draw **Vectors**.
- We define two **Basis Functions** (Axes) that are Orthogonal:

1. In-Phase (I):

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$$

2. Quadrature (Q):

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$$

Any signal $s_i(t)$ is a point:

$$\mathbf{s}_i = [s_{i1}, s_{i2}]$$

- Energy: Distance from origin squared.

$$E = ||\mathbf{s}_i||^2$$

Geometric Representation of Signals

To analyse performance (Probability of Error), we use **Vector Space Analysis**.

- 💡 Any set of M energy signals $\{s_i(t)\}$ can be represented as a linear combination of N orthonormal **Basis Functions** $\{\phi_j(t)\}$.

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \quad 0 \le t \le T$$

- Orthonormal Condition:

$$\int_0^T \phi_i(t)\phi_j(t)dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Binary Digital Modulation ($M = 2$)

We transmit **1 bit per symbol**.
We need 2 distinct signals: $s_1(t)$ and $s_2(t)$.

1. Binary Amplitude Shift Keying (BASK)

"On-Off Keying"

- **Bit 1:** Send Carrier ($\sqrt{E_b}$).
- **Bit 0:** Send Nothing (0).

Constellation:

- Points at $[0]$ and $[\sqrt{E}]$.
- Simple (Light bulb on/off).
- Susceptible to noise (Amplitude varies naturally).

2. Binary Frequency Shift Keying (BFSK)

We use frequency to distinguish bits.

- **Bit 1:** Send $f_1 = f_c + \Delta f$.
- **Bit 0:** Send $f_2 = f_c - \Delta f$.

Orthogonality Condition:

- To detect these independently, the frequencies must be spaced by $\Delta f = \frac{1}{2T_b}$.

Constellation:

- Vectors are orthogonal (90 degrees apart).
- Points at $[1, 0]$ and $[0, 1]$ in frequency space.
- Uses **more Bandwidth** than ASK/PSK.

3. Binary Phase Shift Keying (BPSK)

The most robust binary scheme. We flip the phase by 180°.

$$s(t) = \pm A \cos(2\pi f_c t)$$

- Constellation:
- Points at $+\sqrt{E_b}$ and $-\sqrt{E_b}$ on the I-axis.
- **Antipodal:** Max separation distance $d = 2\sqrt{E_b}$.
- **Q-Component:** Zero.

Bandwidth Efficiency: 1 bit / Hz.

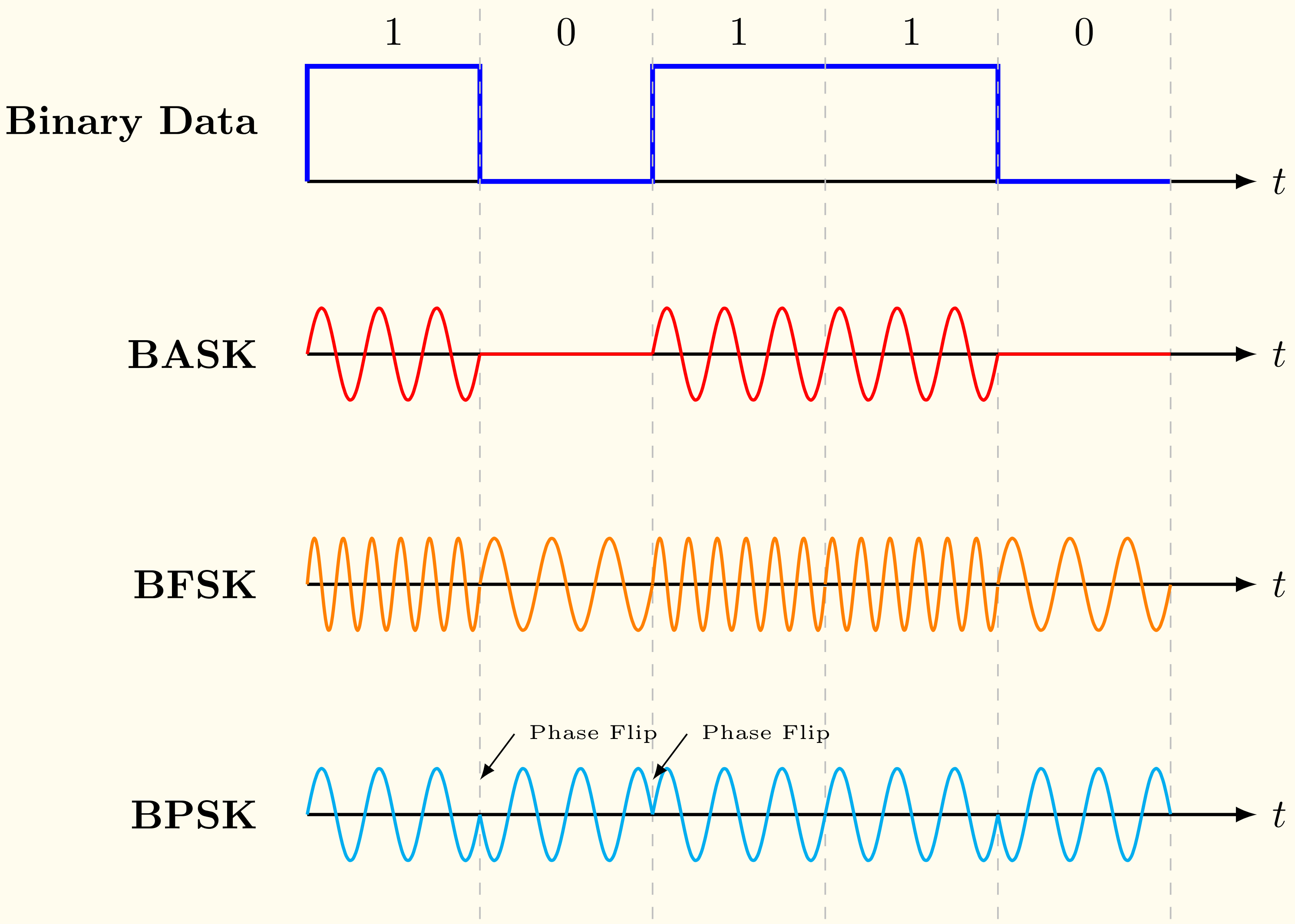
Part 3: Quadrature Modulation (QPSK)

The Engineering Breakthrough:
Since $\cos(t)$ and $\sin(t)$ are orthogonal, we can transmit two separate BPSK signals on the same frequency **simultaneously**.

Quadrature PSK:

- **I-Channel:** Carries Bit 1 ($\cos$).
- **Q-Channel:** Carries Bit 2 ($\sin$).

Result:
We send **2 bits per symbol**.


1
0
1
1
0
Binary Data
t
BASK
t
BFSK
t
Phase Flip
Phase Flip
BPSK
t

Quadriphase-Shift Keying (QPSK)

We use **4 Phases** to transmit **2 bits** (a dibit) per symbol.

Phases: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], \quad i = 1, 2, 3, 4$$

Trigonometric Expansion:

$$s_i(t) = \underbrace{\sqrt{\frac{2E}{T}} \cos(\theta_i)\cos(2\pi f_c t)}_{\text{In-Phase (I)}} - \underbrace{\sqrt{\frac{2E}{T}} \sin(\theta_i)\sin(2\pi f_c t)}_{\text{Quadrature (Q)}}$$

QPSK Derivation - Expansion

Using the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$:

Let $A = 2\pi f_c t$ and $B = (2i - 1)\frac{\pi}{4}$.

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left((2i-1)\frac{\pi}{4}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left((2i-1)\frac{\pi}{4}\right) \sin(2\pi f_c t)$$

This decomposes the signal into two orthogonal components:

1. **In-Phase Component:** Multiplies $\cos(2\pi f_c t)$
2. **Quadrature Component:** Multiplies $\sin(2\pi f_c t)$

QPSK Derivation: Basis Projection

We recall our basis functions:

$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$ and $\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$.

Substituting these back into our expanded equation:

$$s_i(t) = \underbrace{\sqrt{E}\cos(\theta_i)}_{\text{Scalar } s_{i1}}\phi_1(t) - \underbrace{\sqrt{E}\sin(\theta_i)}_{\text{Scalar } s_{i2}}\phi_2(t)$$

Thus, the signal vector is:

$$\mathbf{s}_i = \left[\sqrt{E}\cos(\theta_i), \quad -\sqrt{E}\sin(\theta_i)\right]$$

QPSK as two BPSK signals

The QPSK signal is literally the sum of two orthogonal BPSK signals.

1. **Odd Bits** modulate the In-Phase carrier (ϕ_1).
2. **Even Bits** modulate the Quadrature carrier (ϕ_2).

$$s_i(t) = \pm\sqrt{\frac{E}{2}}\phi_1(t) \pm \sqrt{\frac{E}{2}}\phi_2(t)$$

- ❗ Since ϕ_1 and ϕ_2 are orthogonal, we can detect them separately. This doubles the data rate without increasing bandwidth.

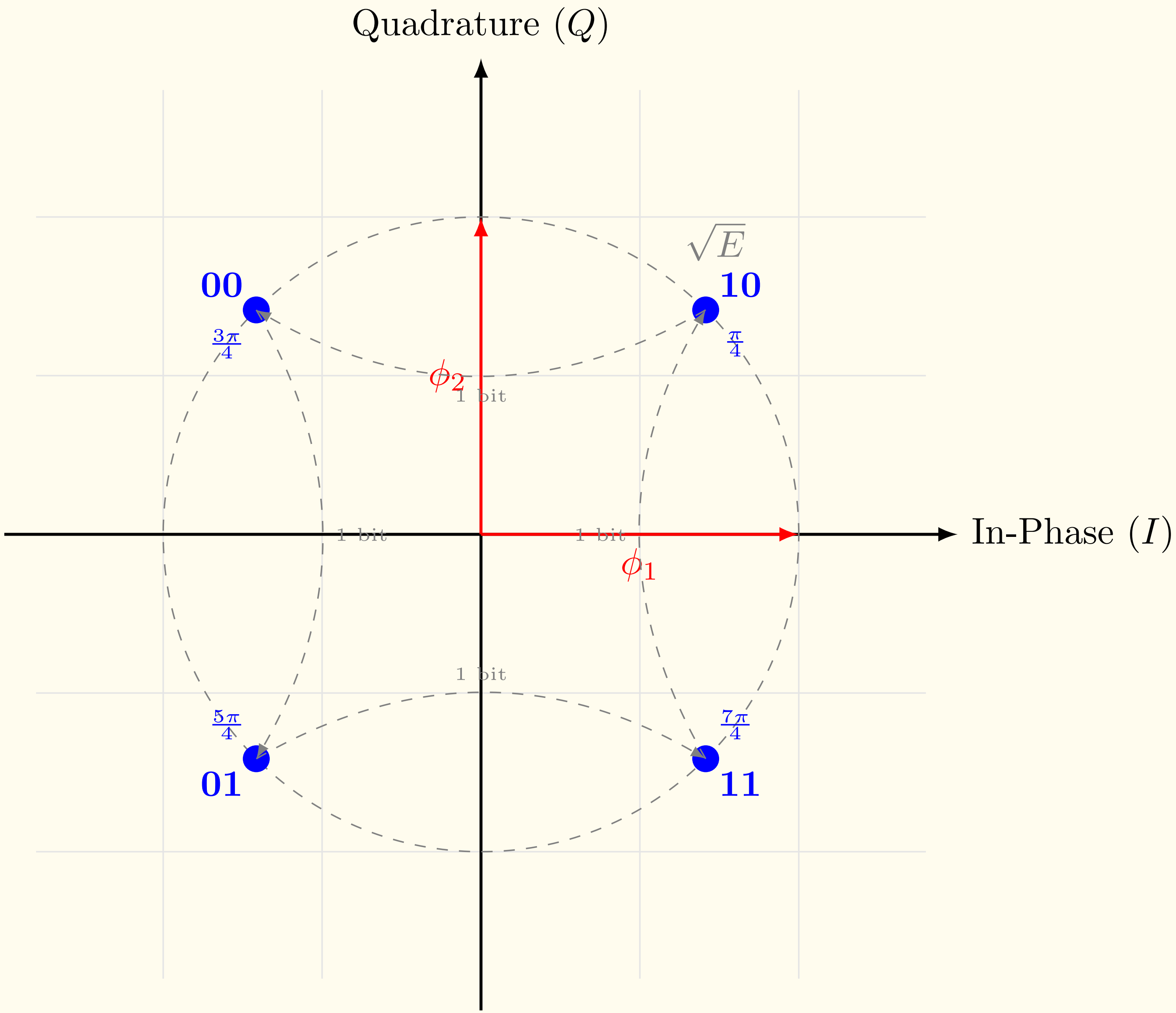
QPSK Constellation

Gray Coding:

- Adjacent points differ by 1 bit.
- $10 \to 00 \to 01 \to 11.$

Bandwidth:

- $R_{sym} = R_b/2.$
- Bandwidth = $R_b/2$ Hz.
- Half the bandwidth of BPSK for the same bit rate.



Quadrature (Q)
In-Phase (I)
$\sqrt{E}$
00
10
01
11
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
ϕ_2
ϕ_1
1 bit
1 bit
1 bit
1 bit

Example

You are designing a digital radio link to stream high-quality audio.
Required Data Rate (R_b): 3 Mbps.Available Channel Bandwidth (B): 2 MHz.

Can you use BPSK for this link? If not, will QPSK work?

Solution

BPSK Efficiency:

BPSK transmits 1 bit/symbol (or 1 bit/Hz).

Capacity: With 2 MHz bandwidth, the maximum speed is:

$$R_{\text{BPSK}} = 1 \times 2\ \text{MHz} = \mathbf{2}\ \text{Mbps}$$

$2\ \text{Mbps} < 3\ \text{Mbps}$, hence BPSK Fails

Solution

QPSK Efficiency:

QPSK transmits 2 bits/symbol (or 2 bits/Hz).Capacity: With 2 MHz bandwidth, the maximum speed is:

$$R_{\text{QPSK}} = 2 \times 2\ \text{MHz} = \mathbf{4}\ \text{Mbps}$$

$4\ \text{Mbps} > 3\ \text{Mbps}$. QPSK Works!

M-ary Modulation

Binary Modulation ($M = 2$):

- Sends 1 bit per symbol ($T = T_b$).
- Simple, robust, but bandwidth inefficient.

M-ary Modulation:

- We group m bits into one Symbol.
- Number of Symbols: $M = 2^m$.
- Symbol Duration: $T = mT_b$.
- 🙂 We send more bits in the same amount of time/spectrum.
- 🙁 Requires more Power and Complexity.

Defining Error Rates

Before comparing schemes, we must define how we measure "failure".

1. Bit Error Rate (BER):

The probability that a single bit is corrupted (P_b).

- Typically 10^{-3} for voice, 10^{-6} for data.

2. Symbol Error Rate (SER):

The probability that the receiver mistakes one symbol for another (P_M).

For Gray coding (where errors usually flip only 1 bit):

$$BER \approx \frac{SER}{\log_2 M}$$

The Metric: E_b/N_0

How do we compare apples (BPSK) to oranges (16-QAM)?

We normalise everything to the **Energy per Bit (E_b)**.

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} \propto \frac{E_b}{N_0}$$

- As we increase M (more bits/symbol), the points on the constellation get closer.
- $\implies$ We need a higher E_b/N_0 to maintain the same BER.

M-ary Phase Shift Keying (M-PSK)

We keep the amplitude constant (A_c) and vary the phase.

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \quad i = 1, \ldots, M$$

- **QPSK ($M = 4$):** Phases separated by 90°.
- **8-PSK ($M = 8$):** Phases separated by 45°.
- **16-PSK ($M = 16$):** Phases separated by 22.5°.
- All points lie on a circle of radius $\sqrt{E}$.

8-PSK Constellation

- **3 bits per symbol** ($m = 3$).
- 8 points on the circle.
- Phase step: $2\pi/8 = 45^\circ$.
- 3 times more efficient than BPSK.
- Bandwidth = $R_b/3$ Hz.

M-ary Quadrature Amplitude Modulation (QAM)

Phase modulation has a limit. As M increases, points get too close together on the circle.

🤔 Change **Amplitude** AND **Phase**.

$$s_i(t) = a_i\phi_1(t) + b_i\phi_2(t)$$

We usually construct M-QAM as a **Square Grid**.

- **16-QAM:** 4×4 grid ($m = 4$ bits).
- **64-QAM:** 8×8 grid ($m = 6$ bits).

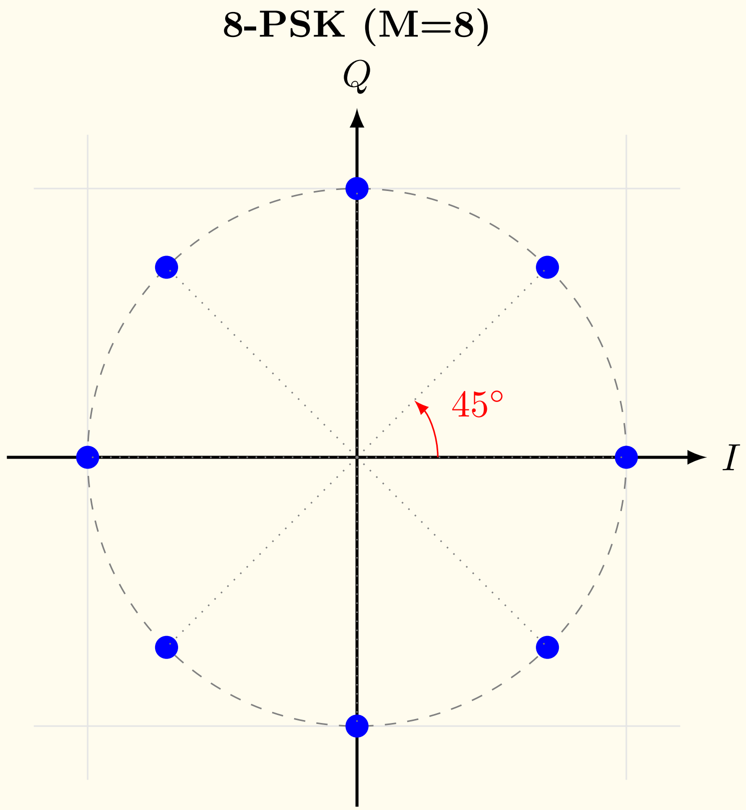
16-QAM Signal Space

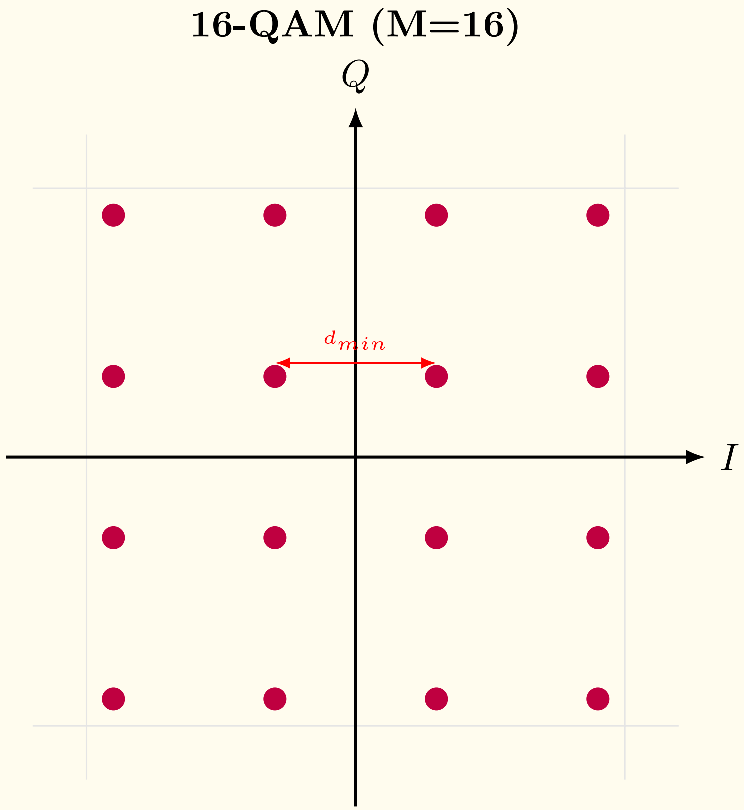
Constellation:

- 16 Points.
- Each point represents **4 bits**.
- Amplitudes are not constant!

Efficiency:

- 4 bits/sec per Hz.
- Widely used in 4G LTE and Wi-Fi.


8-PSK (M=8)
Q
I
45°


16-QAM (M=16)
Q
I
d_{min}

The Great Trade-off: Power vs Bandwidth

As we increase M (e.g., QPSK $\rightarrow$ 16-QAM $\rightarrow$ 64-QAM):

1. **Bandwidth Efficiency (η) Increases:** $\eta = \log_2 M$ bits/s/Hz.
2. **Euclidean Distance Decreases:** Points are packed tighter.
3. **Error Rate Increases:** Noise easily causes confusion.

- **Result:** To maintain the same Bit Error Rate (BER), M-ary schemes require **Higher Signal-to-Noise Ratio (SNR)**.

Performance Comparison

Scheme	Bits/Sym	Bandwidth Req.	Power Req. (SNR)
BPSK	1	$1 \times B$	Low (Robust)
QPSK	2	$0.5 \times B$	Low (Efficient)
8-PSK	3	$0.33 \times B$	Medium
16-QAM	4	$0.25 \times B$	High
64-QAM	6	$0.16 \times B$	Very High

- QPSK is unique because it doubles efficiency without increasing Power.

Questions ❓

- You can ask on Menti

Further Reading

- Sections 6.9

  Modern Digital and Analog Communication Systems, 5^{th} Edition
- B P Lathi and Zhi Ding

Get in touch

Hasan.Abbas@glasgow.ac.uk