FM: Where the sound is always sweeter! 📻

UESTC 3018 - Communication Systems and Principles

Lecture 12 — Angle Modulation II

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Somewhere (just) Familiar …

From Last Time ⏳

- Angle Modulation is a non-linear process
- We don't change the amplitude
- PM and FM are very similar - a 90° phase-shift

This Lecture 📅

- Bandwidth in FM
- Carson's Rule
- Spectral Analysis

Down the History Lane

- Due to inherent non-linearity, FM is hard to analyse
- Can't really apply Fourier transform tools
- Motivation was to reduce the bandwidth
- Turns out FM has infinite theoretical bandwidth

First an Example - FSK/PSK

- Given a signal $m(t)$, sketch the FM and PM waves.
- $k_f = 2\pi \times 10^5$, $k_p = \pi/2$ and $f_c = 100$ MHz.

Bandwidth Analysis

Analysing Bandwidth

- To simplify the expression, $a(t) = \int_{-\infty}^{t} m(u)\, du$
- Let's define,

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\, e^{j[\omega_c t + k_f a(t)]} = A e^{jk_f a(t)} e^{j\omega_c t}$$

from where,

$$\varphi^{\mathrm{FM}}(t) = \Re\left[\hat{\varphi}^{\mathrm{FM}}(t)\right].$$

- Expanding the complex exponential through the Maclaurin power series,

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\left[1 + jk_f a(t) - \frac{k_f^2}{2!} a^2(t) + \cdots + j^n \frac{k_f^n}{n!} a^n(t)\right] \times e^{j\omega_c t}$$

Some Observations

- If $m(t)$ or $M(\omega)$ has a bandwidth of B
- Then $ a(t)$ also has a bandwidth of B Hz (integration is a linear operator).
- The nth term, $\frac{k_f^n}{n!}a^n(t)$ will have a bandwidth of $n \times B$
- This is due to convolution principle, i.e.
- $A(\omega) * A(\omega)$ spreads the Fourier transform to $2B$
- Essentially, we have **infinite bandwidth**
- But...
- $\frac{k_f^n}{n!}a^n(t) \to 0$, meaning we only care about the first few terms.

FM Signal Representation

- Using the $\varphi^{\mathrm{FM}}(t) = \Re\left[\hat{\varphi}^{\mathrm{FM}}(t)\right]$ representaion, we get,

$$\begin{aligned}
\varphi^{\mathrm{FM}}(t) &= \Re\left\{A\left[1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \cdots + j^n\frac{k_f^n}{n!}a^n(t)\right] \times \left[\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right]\right\} \\
&= A\left(\cos(2\pi f_c t) - k_f a(t)\sin(2\pi f_c t) - \frac{k_f^2}{2!}a^2(t)\cos(2\pi f_c t) + \ldots\right) \\
&\approx A\left(\cos(2\pi f_c t) - k_f a(t)\sin(2\pi f_c t)\right)
\end{aligned}$$

- This is a narrowband FM signal representation
- The approximation is good when $\left|k_f a(t)\right| \ll 1$
- Generally, we consider 2B bandwidth as narrowband
- PM has a similar expression

Playing with the tones

Playing with the tones

Playing with the tones

A Dilemma 😵‍💫

- To make the best of FM (or PM), we need make the frequency deviation large enough
- Need to choose a large enough k_f to break the $|k_f a(t)| \ll 1$ condition
- This is the **wideband** FM case
- ⚠️ We can't ignore the higher order terms in the power series anymore
- We need to rely on empirical formulas to estimate the bandwidth

Wideband FM

- **Context** We need frequency deviation for the FM signal to be meaningful
- But what is the bandwidth of an FM signal?
- *Answer* We use empirical methods / estimations

An Example

- For an FM signal $m(t)$, the max and min centre frequencies are $\omega_c + k_f m_p$ and $\omega_c - k_f m_p$ respectively
- Taking into account the bandwidth of the sinc lobe ($4\pi B$)
- Total Bandwidth is the difference

Carson's Rule 🔑

- The estimated bandwidth is,

$$B^{\mathrm{FM}} = \frac{1}{2\pi}(2k_f m_p + 8\pi B)$$

- The frequency deviation Δf is given by,

$$\Delta f = k_f \frac{m_{\max} - m_{\max}}{2 \times 2\pi} \ \mathrm{Hz}$$

- The estimated bandwidth (Hz) is,

$$B^{\mathrm{FM}} \approx 2(\Delta f + 2B)$$

Carson's Rule - Narrowband Formula

- Remember for narrowband FM, $|k_f a(t)| \ll 1$
- Δf is also very small

$$B^{\mathrm{FM}} \approx 4B$$

- Earlier we analysed that for narrowband, the bandwidth is 2B Hz.
- A better estimate is then,

$$B^{\mathrm{FM}} = 2\left(\Delta f + B\right) = 2\left(\frac{k_f m_p}{2\pi} + B\right)$$

- A commonly used expression is in terms of deviation ratio,

$$B^{\mathrm{FM}} = 2B(\beta + 1)$$

where β is the deviation ratio $\Delta f / B$

Further Reading

- Section 4.6 - Bandwidth Analysis of Angle Modulations
  Modern Digital and Analog Communication Systems, 5th Edition
- B P Lathi and Zhi Ding

Get in touch

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