UESTC 3018 - Communication Systems and Principles

Lecture 14 — Frequency Modulation Generation

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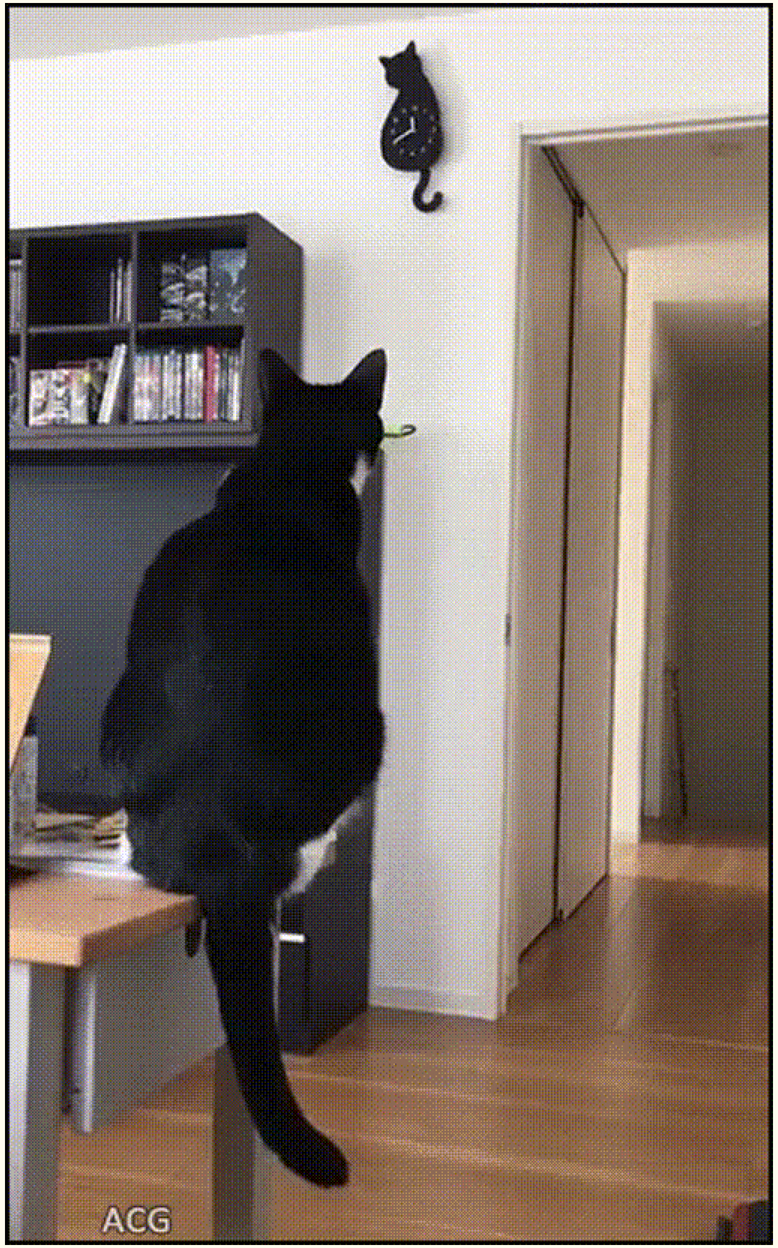
From the Morning … ⏳

- Simplest Message Signal
- Frequency Modulation Detection



Now 📆

- FM Generation

The Engineering Dilemma ⚖️

To build a commercial FM station, we need two conflicting things:

1. **High Frequency Stability**

- The center frequency (f_c) must not drift (FCC allows ± 2 kHz tolerance).
- Crystal Oscillator is the best solution here, but they are stiff and hard to modulate.

2. **Wide Frequency Deviation**

- Need $\Delta f = 75$ kHz for noise suppression.
- LC Tank Oscillator is flexible enough to do this, but they drift with temperature.

Method 1: Direct FM Generation ⚡

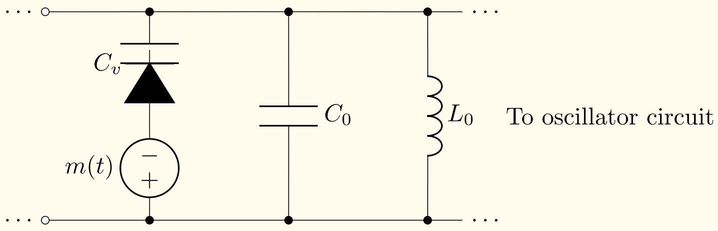
- The direct method uses a Voltage-Controlled Oscillator (VCO), where the frequency is controlled by an external voltage.
- The oscillation frequency of the VCO varies linearly with the control voltage.
- An FM wave can be generated by incorporating the modulating signal $m(t)$ into the control signal.
- This results in an instantaneous angular frequency represented as:

$$\omega_i(t) = \omega_c + k_f m(t)$$

The Varactor Diode FM Generator ⚡

- Need an oscillator whose frequency changes with voltage.
- Use an LC Tank Circuit where the capacitor is variable.
- Varactor Diode is a reverse-biased PN junction diode acts as a capacitor.
- Increasing the Reverse Bias voltage (V) widens the depletion region (d).

$$C = \frac{\epsilon A}{d}$$



Voltage to Frequency 📐

The capacitance of a varactor under bias V_0 and message $m(t)$ is:

$$C(t) = C_0 \left(1 - \frac{m(t)}{V_0}\right)^{-1/2}$$

The oscillator frequency is determined by the tank circuit:

$$\omega_i(t) = \frac{1}{\sqrt{LC(t)}} = \frac{1}{\sqrt{LC_0}} \left(1 - \frac{m(t)}{V_0}\right)^{1/4}$$

Linearising the Derivation

We define the carrier frequency as $\omega_c = \frac{1}{\sqrt{LC_0}}$.

If the message is small ($m(t) \ll V_0$), we use the Binomial Series $(1+x)^n \approx 1 + nx$:

$$\omega_i(t) \approx \omega_c \left(1 + \frac{1}{4}\frac{m(t)}{V_0}\right)$$

This matches our definition of FM:

$$\omega_i(t) = \omega_c + k_f m(t)$$

Indirect FM (Armstrong) 📻

Instead of forcing an unstable oscillator to wiggle, we take a different approach,

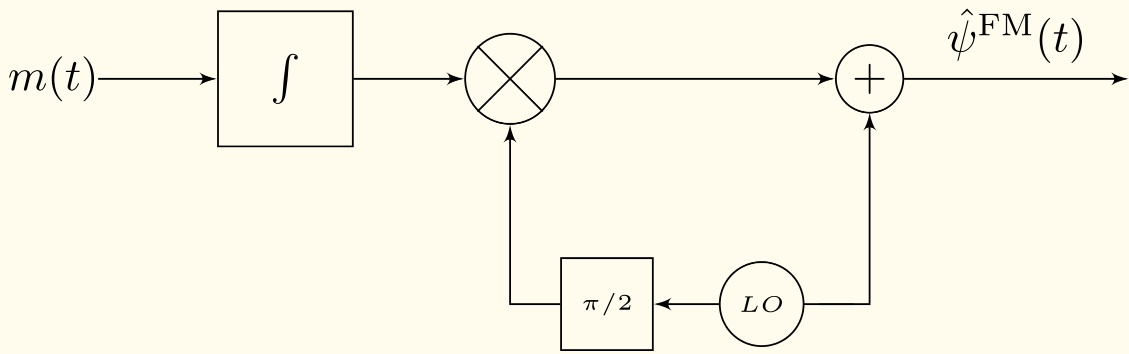
1. Start with a **Crystal Oscillator**
2. Generate **Narrowband FM (NBFM)**
3. Use math (multipliers) to "stretch" it into **Wideband FM**

Narrowband FM Generation

- Recall, for narrowband FM, $|k_f a(t)| \ll 1$

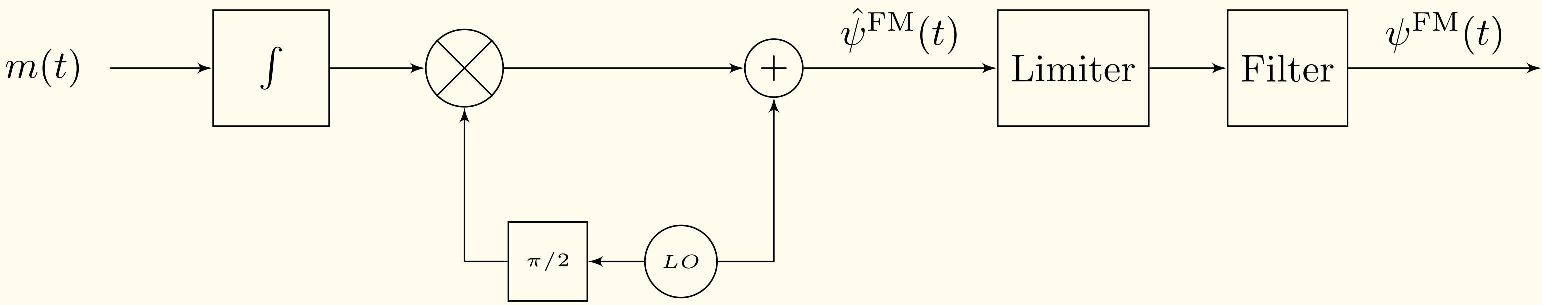
$$\psi^{\mathrm{NBFM}}(t) \simeq A\left[\cos\omega_c t - k_f a(t)\sin\omega_c t\right]$$

- This is a linear (simpler) system
- We can use an AM modulator (DSB-SC) with a phase shifter ($\sin\omega_c t$)



Narrowband FM Generation

- The above is simple, yet there are some artefacts
- Some amplitude modulation remains in the system

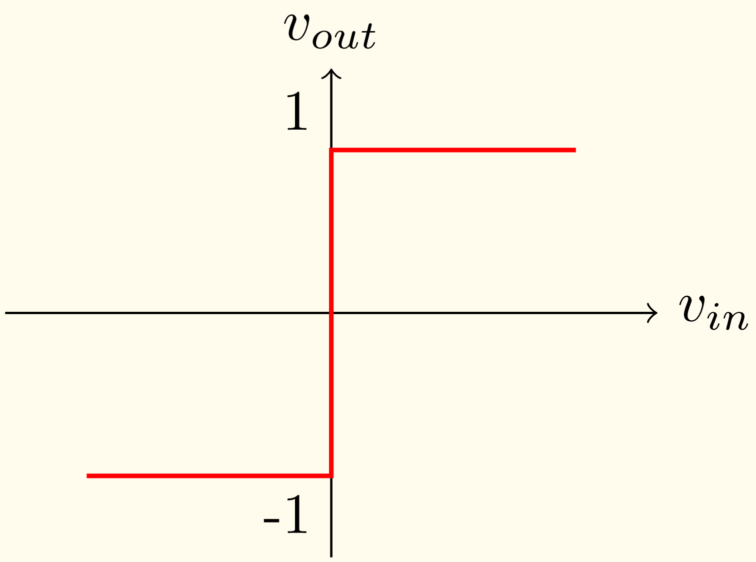


The Limiter and Bandpass Filter

- Amplitude variations create noise and interference 👎
- We first use a limiter circuit to limit any amplitude fluctuations

$$v_o(t) = \begin{cases} +1 & v_i(t) > 0 \\ -1 & v_i(t) < 0 \end{cases}$$

- A non-linear circuit that "clips" the signal.
- It turns the wobbly sine wave into a constant-amplitude **Square Wave**.



The Limiter and Bandpass Filter (contd.)

- Selects the fundamental frequency of the square wave and rejects the harmonics ($3\omega_c, 5\omega_c \ldots$).
- The input to the limiter and bandpass filter blocks is,

$$v_i(t) = A(t)\cos\theta(t)\,,\ \theta(t) = 2\pi f_c t + k_f a(t)$$

- $A(t)$ is ideally constant but usually very slowly varying

$$v_o(\theta) = \begin{cases} +1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

$$
\begin{aligned}
v_o(\theta) &= \frac{4}{\pi}\left(\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta + \cdots\right) \\
&= \frac{4}{\pi}\Big(\cos\left(2\pi f_c t + k_f a(t)\right) - \frac{1}{3}\cos 3\left(2\pi f_c t + k_f a(t)\right) + \cdots\Big)
\end{aligned}
$$

Next: The Multiplier Chain 🔗

We now have a stable NBFM signal, but Δf is tiny (e.g., 25 Hz). We need $75kHz$.

Frequency Multiplier ($\times N$)

Passes signal through non-linear amp (Class C) to generate harmonics.

- Freq $\times N$
- Deviation $\times N$
- $\beta \times N$

Mixer / Heterodyne ($\pm f_{LO}$)

Multiplies by local oscillator $\cos(\omega_{LO} t)$.

- Freq $\pm f_{LO}$
- Deviation Unchanged
- β Unchanged

Design Example: The "Radio Station" 🧮

- Transmit at **91.2 MHz** with $\Delta f = 75$ kHz.
- We have a source: Crystal NBFM at **200 kHz** with $\Delta f = 25$ Hz.
- i. Calculate Total Multiplication:

$$N_{total} = \frac{\Delta f_{target}}{\Delta f_{start}} = \frac{75,000}{25} = 3000$$

2. The Carrier Problem:

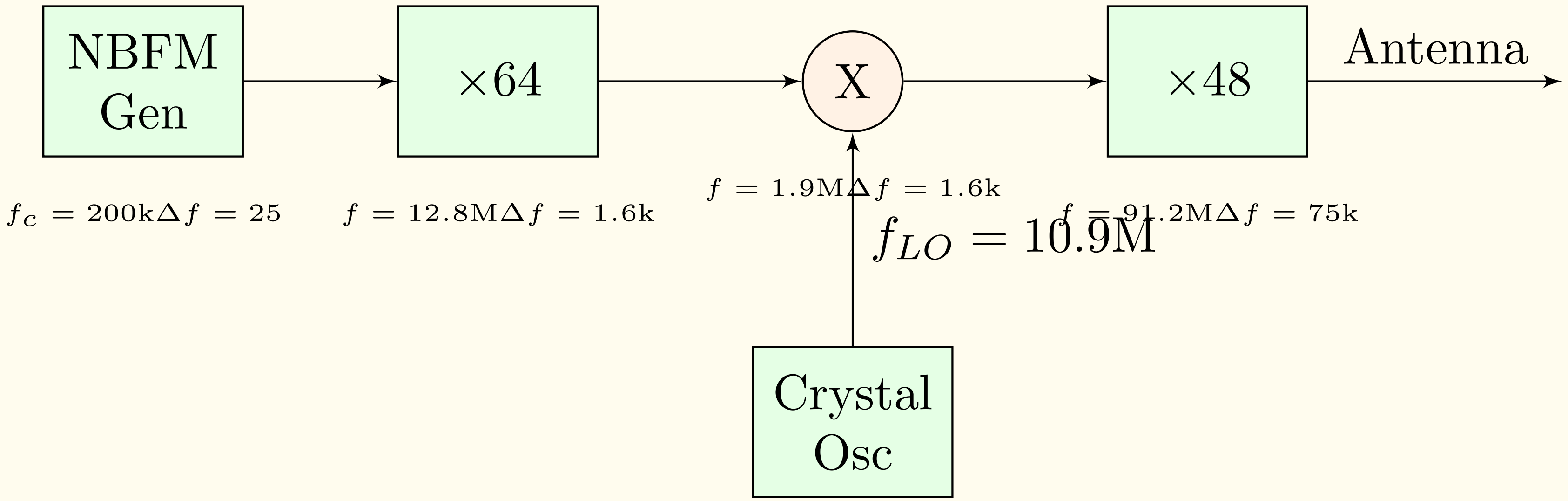
   If we just multiply by 3000:

$$f_c = 200 \text{ kHz} \times 3000 = 600 \text{ MHz}$$

This is way too high! We need a Mixer.

The Full Armstrong Diagram

We split the multiplication ($64 \times 48 \approx 3000$) and insert a **Mixer** in the middle to down-shift the carrier.



Logic Trace:

1. $\times 64$: Carrier $\rightarrow$ 12.8 MHz. Deviation $\rightarrow$ 1.6 kHz.

Summary: Direct vs Indirect

Feature	Direct FM (VCO)	Indirect FM (Armstrong)
Stability	Poor (drifts with Temp)	**Excellent** (Crystal)
Bandwidth	Wide deviation is easy	Needs multipliers to get Wide
Complexity	Simple (1 Transistor)	Complex (dozens of stages)
Use Case	Cheap Radios, Toys	**Broadcast Stations**

Advantages of FM

- FM is less susceptible to amplifier non-linearities

$$x(t) = A\cos(\omega_c t + \psi(t))$$

- The output that we get is,

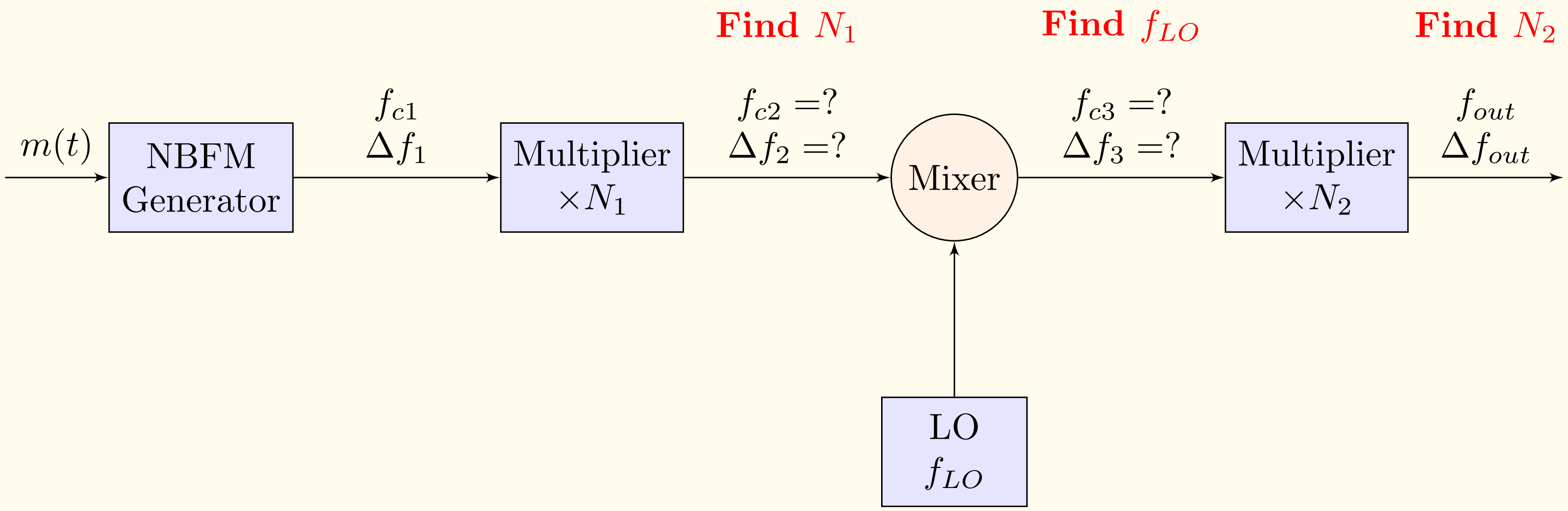
$$\begin{aligned} y(t) &= a_0 + a_1 x(t) + a_2 x^2(t) + \cdots \\ &= c_0 + c_1 \cos(\omega_c t + \psi(t)) + c_2 \cos(2\omega_c t + 2\psi(t)) + \cdots \end{aligned}$$

- For amplitude modulation, the output is, $y(t) = ax(t) + bx^3(t)$

$$\begin{aligned} y(t) &= am(t)\cos\omega_c t + bm^3(t)\cos^3\omega_c t \\ &= \left(am(t) + \frac{3}{4}bm^3(t)\right)\cos\omega_c t + \frac{1}{4}b\cos 3\omega_c t \end{aligned}$$

Exercise

Design an Armstrong indirect FM transmitter system that takes a Narrowband FM (NBFM) signal with a known starting carrier frequency $f_{c1} = 200kHz$ and frequency deviation $\Delta f_1 = 25Hz$, and converts it to a Wideband FM signal with a target carrier frequency $f_{out} = 96MHz$ and target frequency deviation greater than $\Delta f_{out} = 75KHz$. The design task is to calculate the required multiplication factors N_1 and N_2, and the local oscillator frequency f_{LO} for the mixer stage to achieve these target specifications.



Find N_1
Find f_{LO}
Find N_2
$m(t)$
NBFM Generator
f_{c1}
Δf_1
Multiplier $\times N_1$
$f_{c2} =?$
$\Delta f_2 =?$
Mixer
$f_{c3} =?$
$\Delta f_3 =?$
Multiplier $\times N_2$
f_{out}
Δf_{out}
LO
f_{LO}

Further Reading

- Section 4.9 - Generating FM Signals
  Modern Digital and Analog Communication Systems, 5^{th} Edition
- B P Lathi and Zhi Ding

Get in touch

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