UESTC 3018 - Communication Systems and Principles

Lecture 13 — Frequency Modulation Detection

Dr Hasan Abbas

Hasan.abbas@glasgow.ac.uk

From Last Time ⌛

- Angle Modulation Bandwidth Dilemma
- Narrowband $|k_f a(t)| \ll 1$ FM has bandwidth of $2B$
- For Wideband FM, we use the Carson's rule
- We improved the formula to:

$$B^{\mathrm{FM}} = 2B(\beta + 1)$$

where β is the deviation ratio $\Delta f / B$

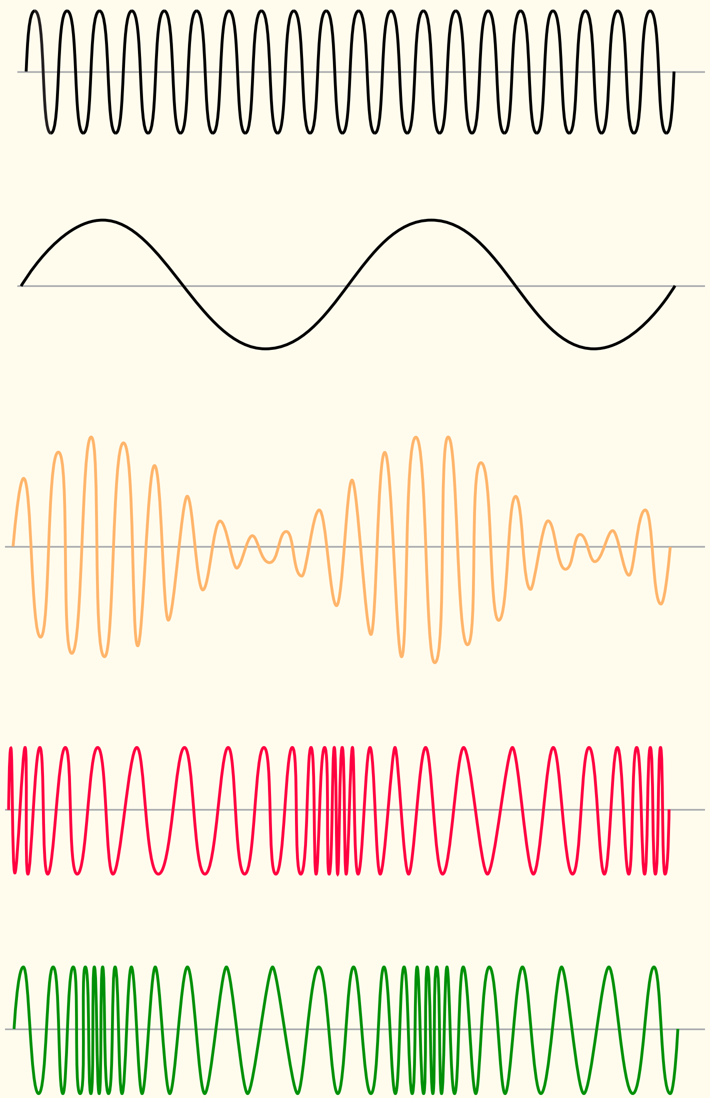
We should know ...

- Constancy of Power in FM/PM signals
- Nonlinearity of FM/PM modulation
- Irregularity of Zero Crossings
- Difficult to Visualise Message Waveform
- Increase Transmission Bandwidth

Today's Lecture 📅

- FM using simplest message signal
- FM Demodulation Techniques

Guess the 🟡, 🔴 and 🟢 signals



FM with a Tone

- Due to inherent non-linearity, FM is hard to analyse
- Lets start off with a tone i.e., a sinusoidal signal, $m(t) = \cos(2\pi f_m t)$

$$a(t) = \int_{-\infty}^{t} m(u)du = \frac{1}{2\pi f_m} \sin(2\pi f_m t)$$

- From last time,

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\, e^{j[\omega_c t + k_f a(t)]} = A e^{j k_f a(t)} e^{j\omega_c t}$$

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\, e^{j[\omega_c t + \frac{k_f}{2\pi f_m} \sin(2\pi f_m t)]} = A e^{j(\omega_c t + \beta \sin(2\pi f_m t))}$$

- Here we assume $a(-\infty) = 0$ (causality)
- For tone only, $B = f_m$
- Frequency deviation ratio β = Modulation index

Remember the Fourier Series 🌊

Any periodic signal $E(t)$ can be written as a sum of complex exponentials:

$$E(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}$$

To find the coefficients c_n, we use the Fourier Analysis integral over one period ($T = 2\pi$ in angle):

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} E(x) e^{-jnx} dx$$

Substituting our envelope $E(x) = e^{j\beta \sin x}$:

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

The Bessel Function

- Defining a new substitute variable,

  $x = 2\pi f_m t$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin x - njx} dx = J_n(\beta)$$

Power

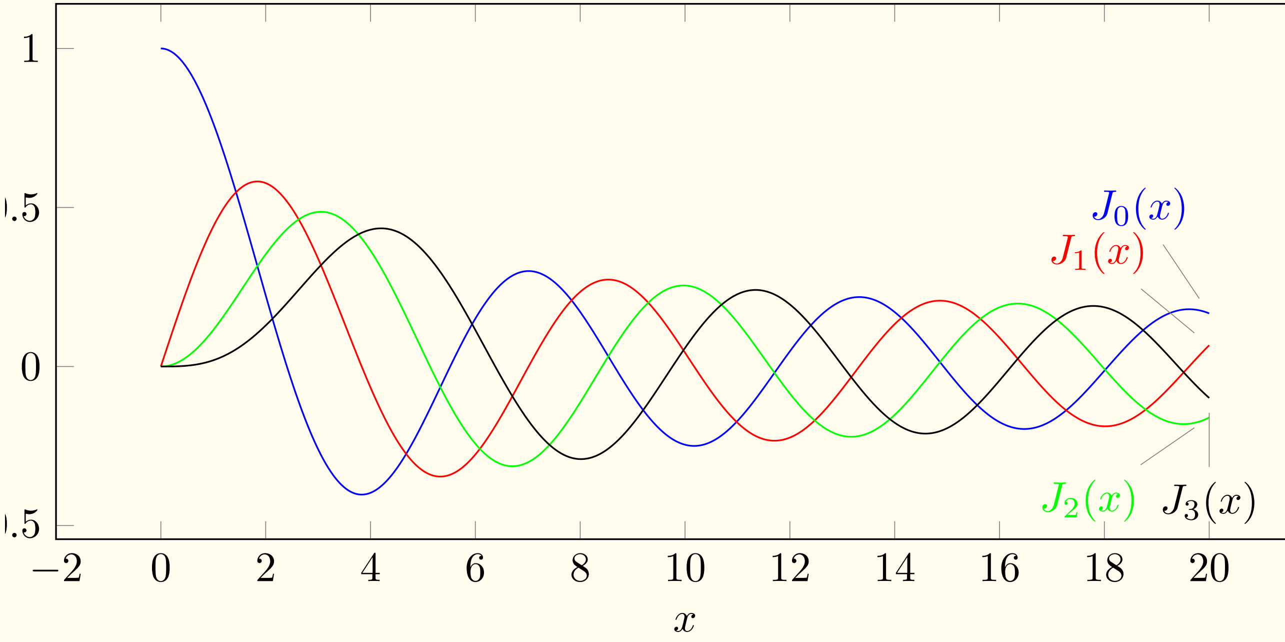
(%),0.5,β=1,β=2,β=5,β=10,β=15

80,—,1,2,4,7,14

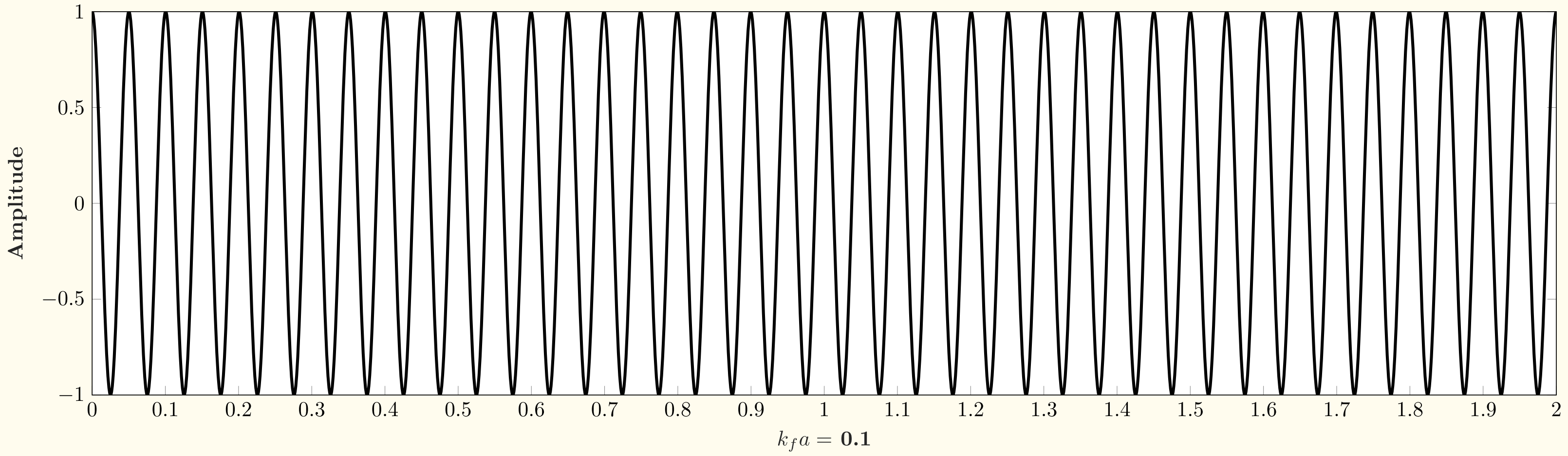
90,1,2,5,8,10,15

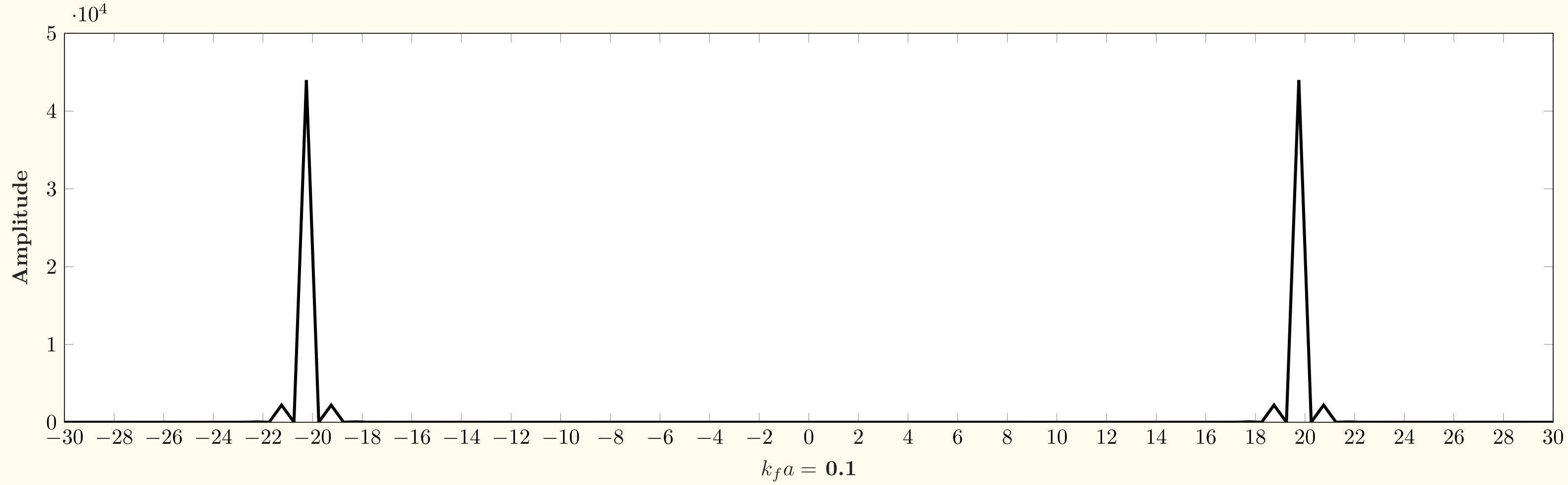
98,1,3,6,9,11,16

- It is the exact mathematical definition of the **Bessel Function of the First Kind (J_n).**

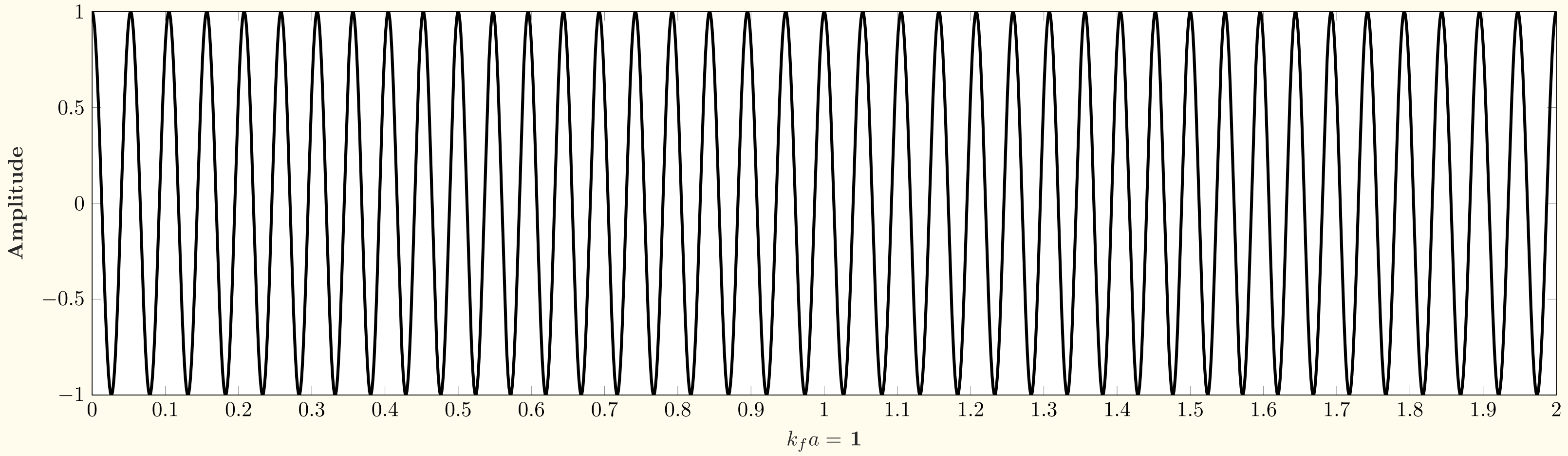


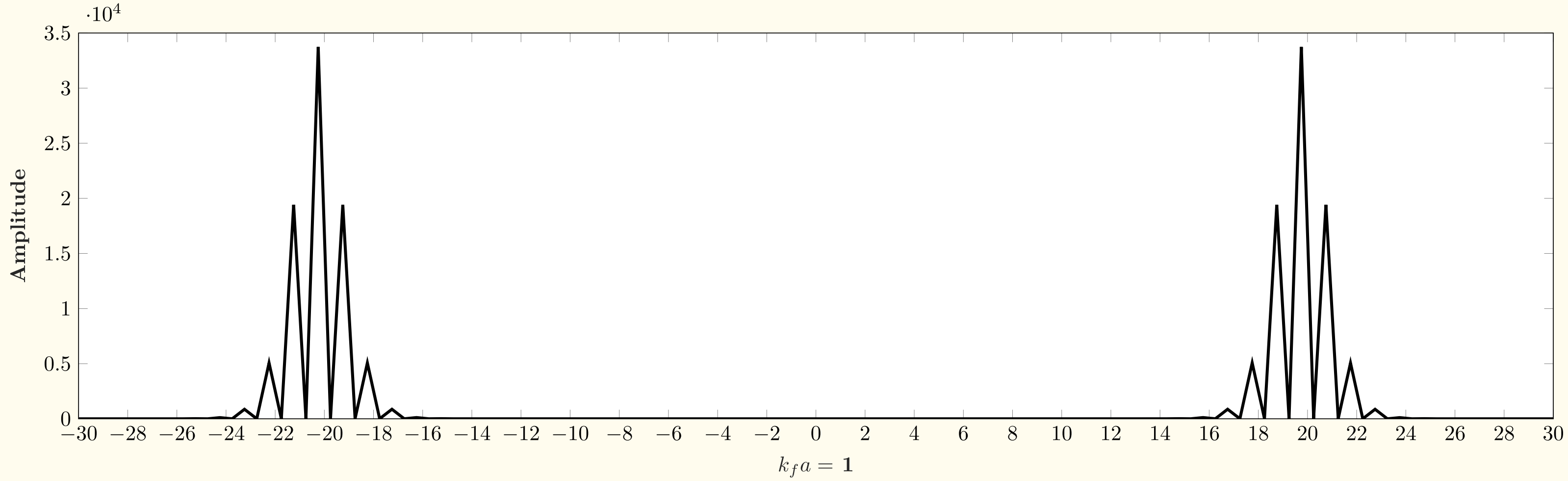
Playing with the tones



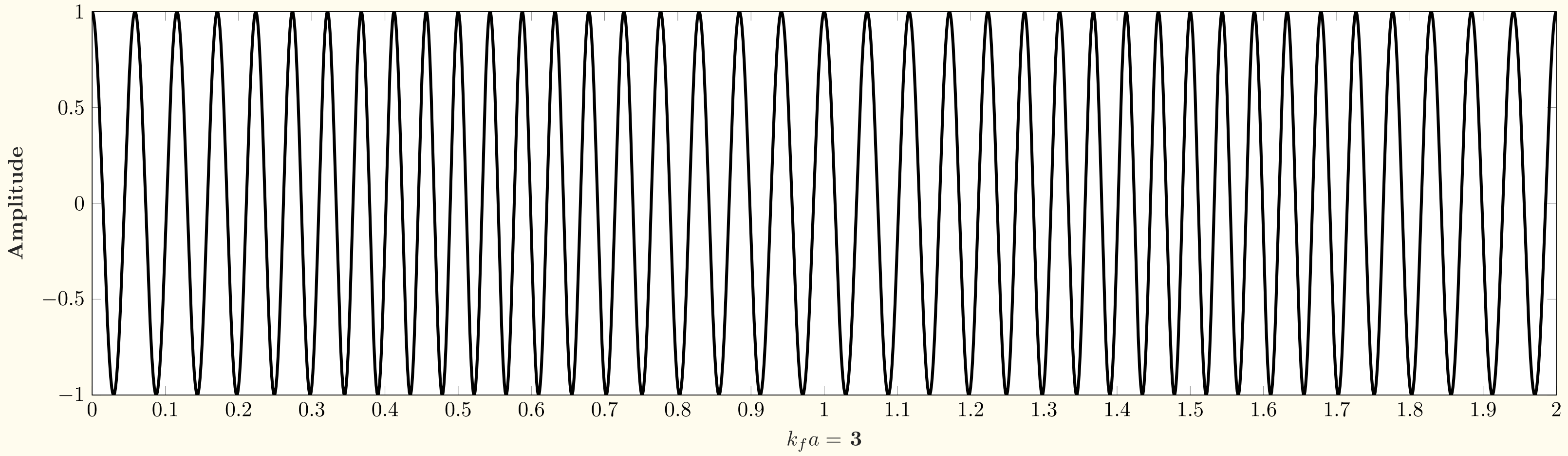


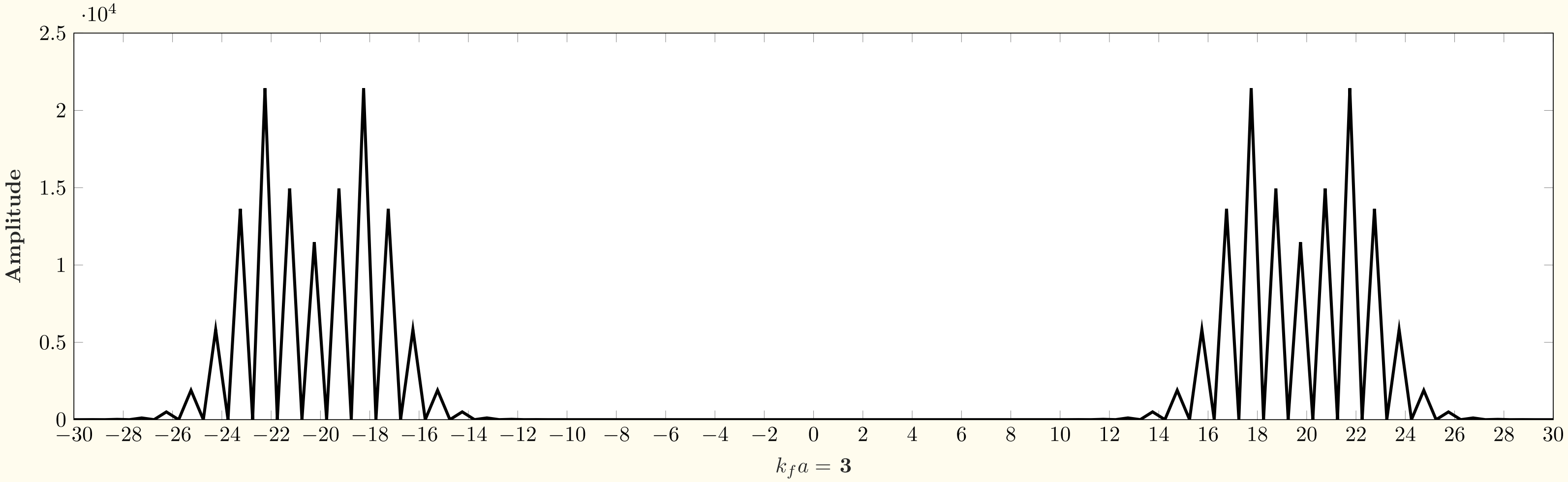
From yesterday … Playing with the tones





Playing with the tones





Required Number of Harmonics (N)

Power (%)	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 10$	$\beta = 15$
80	—	1	2	4	7	14
90	1	2	5	8	10	15
98	1	3	6	9	11	16

From Message to Signal

The Demodulation Challenge 🧩

- In AM, the message $m(t)$ was sitting right there on the amplitude envelope. We just used a diode to "grab" it.
- In FM, the message is buried inside the phase integral:

$$\varphi^{\mathrm{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(u)du\right)$$

- We need a system where the Output Voltage is proportional to the Instantaneous Frequency.

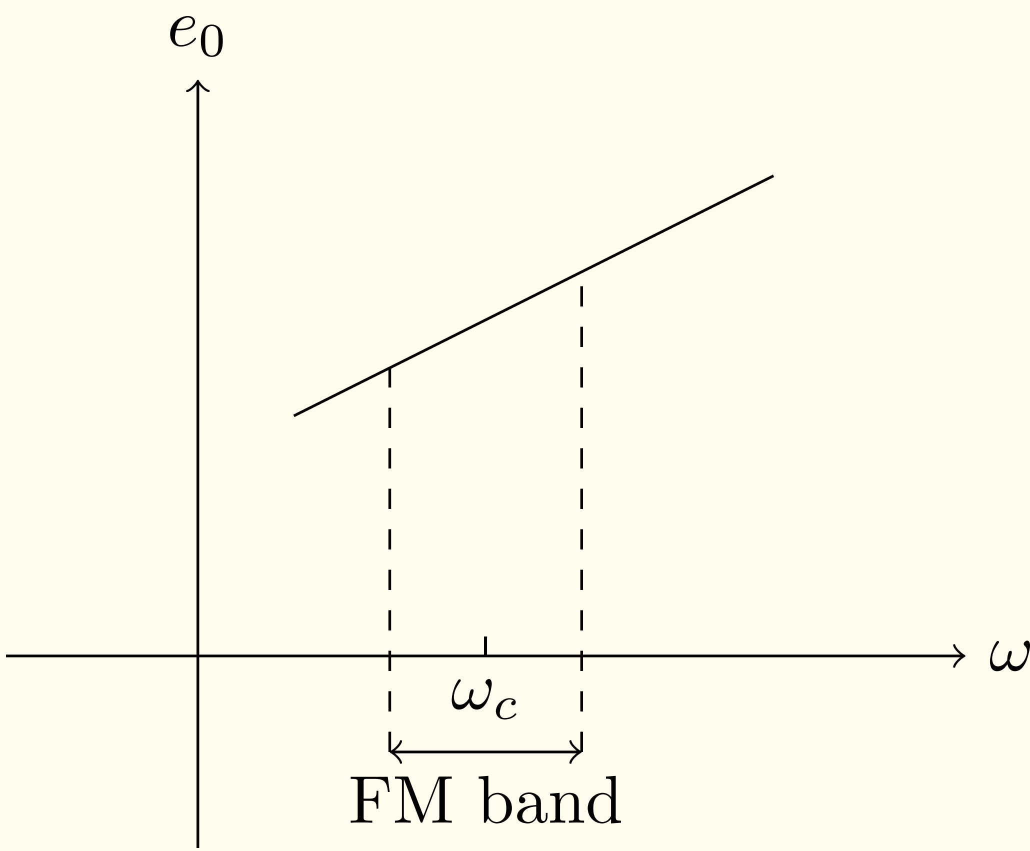
$$V_{out}(t) \propto \omega_i(t) \propto m(t)$$

FM Demodulation

- Recall, the instantaneous frequency changes with signal amplitude,

$$\omega_i^{FM}(t) = \omega_c + k_f m(t)$$

- We need a system where output is proportional to the input.
- Need to convert frequency variations into amplitude variations
- Then use envelope detection.



Demodulation Strategy: Slope Detection 📉

- A system where the Output Voltage is proportional to the Input Frequency.

$$V_{out} \propto \omega_{in}$$

- If we differentiate a sinusoid $\sin(\omega t)$, the frequency ω pops out as a multiplier:

$$\frac{d}{dt}[A\sin(\omega t)] = A \cdot \omega \cdot \cos(\omega t)$$

But How? 🤔

1. **Ideal Differentiator:** A circuit with transfer function $H(j\omega) = j\omega$ (e.g., a Capacitor).
2. **FM to AM Conversion:** The frequency variations are now converted into *Amplitude* variations ($A \cdot \omega$).
3. **Envelope Detection:** We can now use a simple AM Diode detector to recover $m(t)$.

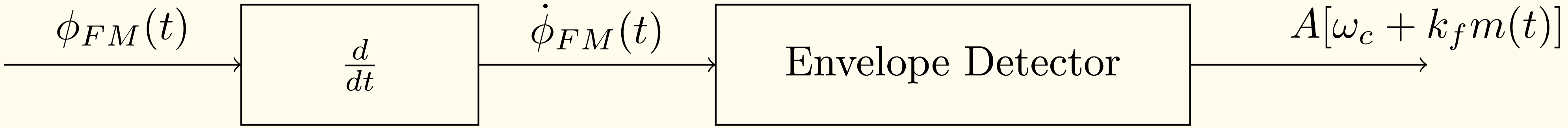
FM Demodulation (contd.)

$$\dot{\varphi}^{\mathrm{FM}}(t) = \frac{d}{dt}\left\{A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right]\right\}$$

$$= A\left[\omega_c + k_f m(t)\right]\sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d(\alpha) - \pi\right]$$

- Note the signal $m(t)$ is present both in the envelope and frequency
- Because $\omega = k_f m_p < \omega_c$, we have $\omega_c + k_f m(t) > 0$
- We can simply perform envelope detection (as in AM).

FM Demodulation

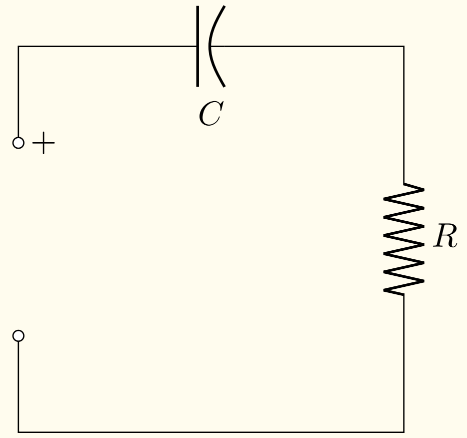


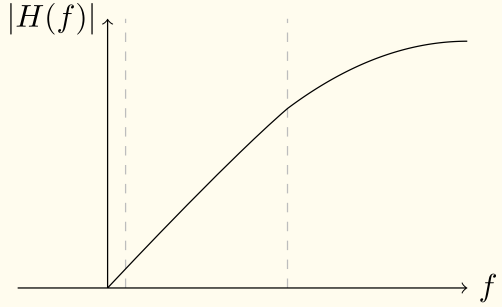
A Simple RC Circuit

- A Simple RC high-pass circuit can be used to detect the slope
- The transfer function (voltage across the resistor) is

$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} \approx j\omega RC$$

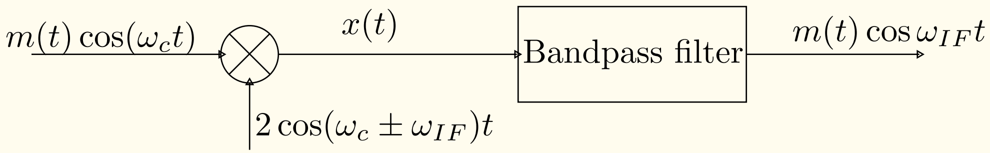
- The approximation is true when $\omega_c RC \ll 1$
- We have a differentiator
- This is one of many possibilities (LC tank circuit being a better one)





Superheterodyne Receivers

- Frequency Conversion or mixing is done to change the carrier frequency from ω_c to ω_{IF}
- We call IF as intermediate frequency



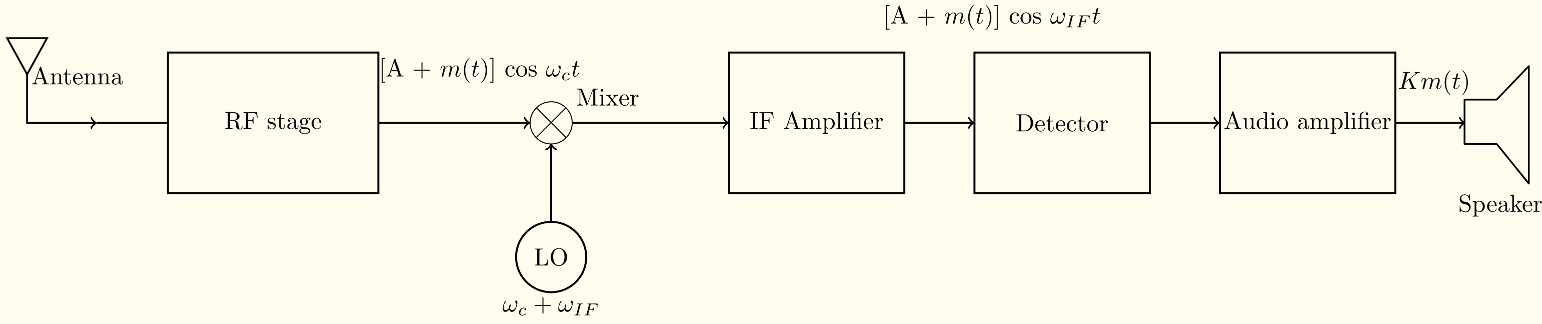
$$
\begin{aligned}
x(t) &= 2m(t)\cos\omega_c t\cos\omega_{mix}t \\
&= m(t)\left[\cos(\omega_c + \omega_{mix})t + \cos(\omega_c - \omega_{mix})t\right]
\end{aligned}
$$

- Setting $\omega_{mix} = \omega_c \pm \omega_{IF}$

$$x(t) = m(t)\left[\cos\omega_{IF}t + \cos(2\omega_c \mp \omega_{IF})t\right]$$

Superheterodyne Receivers

- Down converting to IF allows us to use sensitive amplifiers
- Bandpass filter is very hard to design at RF
- Commonly used in many broadcast systems



Phased-Locked Loop

- Slope detection is "Open Loop"—it just measures what comes in.
- Modern systems use Feedback.
- A negative feedback system used in FM demodulation
- Instead of measuring the frequency, let's generate our own local frequency and try to keep it synchronised ("Locked").
- Compares the phase of the FM signal with the phase of a locally generated reference signal.
- First generate a VCO output $r(t)$ Phase Comparison
- Check for errors with $e(t)$ Error Generated
- $e(t)$ controls the VCO frequency VCO function

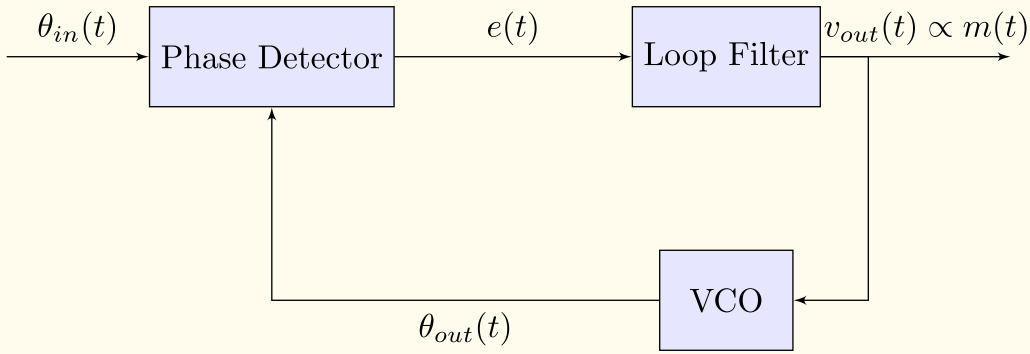
The Control Loop Logic 🧠

1. **Phase Detector:** Compares the Input angle θ_{in} vs the Local VCO angle θ_{out}.

$$e(t) = \theta_{in}(t) - \theta_{out}(t)$$

2. **Loop Filter:** Averages the error to produce a control voltage $v_c(t)$.
3. **VCO:** Adjusts its frequency based on $v_c(t)$.

If the loop is locked, $\omega_{out} = \omega_{in}$.
Since the VCO's frequency is set by $v_c(t)$, then the control voltage IS the demodulated message.



Noise Performance: The "Triangle" 🔺

- Why do FM radio stations boost the Treble (high-pitch sound)?
- Demodulation involves differentiation (multiplying spectrum by f).
- White Noise (flat) $\times f$ = Ramped Noise.
- The Noise Power Spectral Density increases quadratically (f^2).
- Result: High frequencies (Treble) suffer from much worse SNR than Bass.

The Solution: Pre-Emphasis & De-Emphasis 🎛️

- Rather than employing noise removal steps, we cheat the system.

1. **Tx (Pre-emphasis):** We know the channel kills treble quality. So we artificially BOOST the treble (High Frequencies) by ~6dB/octave before transmitting.
2. **Rx (De-emphasis):** We CUT the Treble at the receiver.

- This restores the original music balance.
- Crucially, it crushes the high-frequency triangle noise down to a flat floor.

Further Reading

- Section 4.7 - Demodulation of FM Signals
  Modern Digital and Analog Communication Systems, 5^{th} Edition
- B P Lathi and Zhi Ding

Get in touch

Hasan.Abbas@glasgow.ac.uk