3018 UESTC

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UESTC 3018 - Communication Systems and Principles

Lecture 11 — Angle Modulation and Zero Crossings

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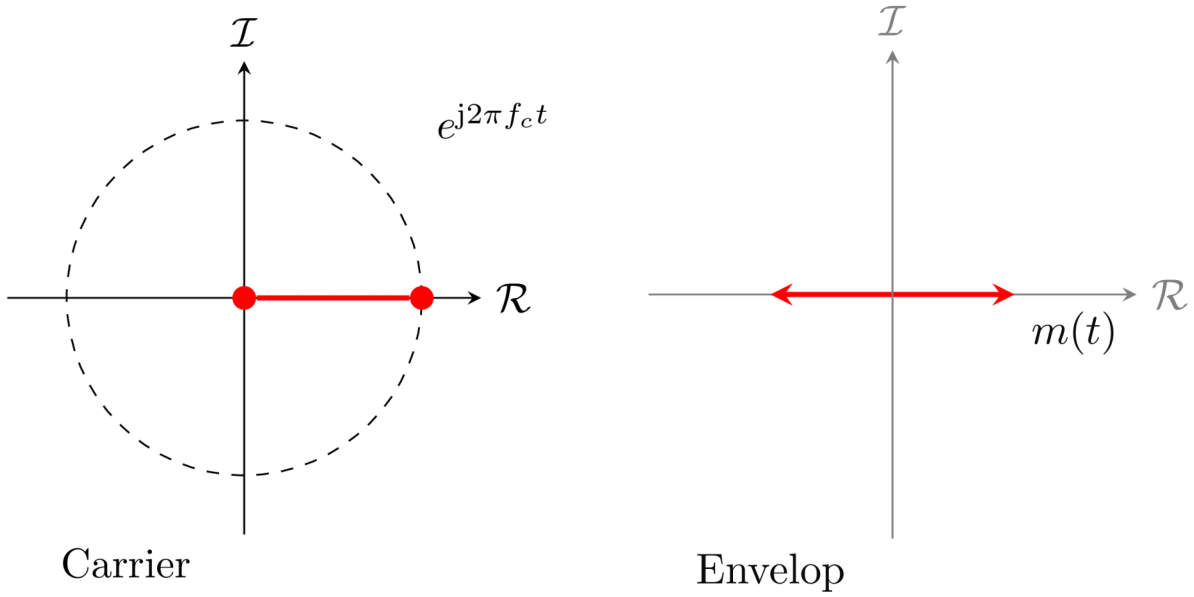
Reflection: The Limits of AM 🤔 💡

- Why Modulation?
- Amplitude Modulation (AM) is a linear process
- Information in the amplitude (envelope)
- Noise adds to the amplitude
- AM is **not** resilient to noise
- AM wave is a band limited signal

💡 Hide Information in the Timing ...

First, AM ...

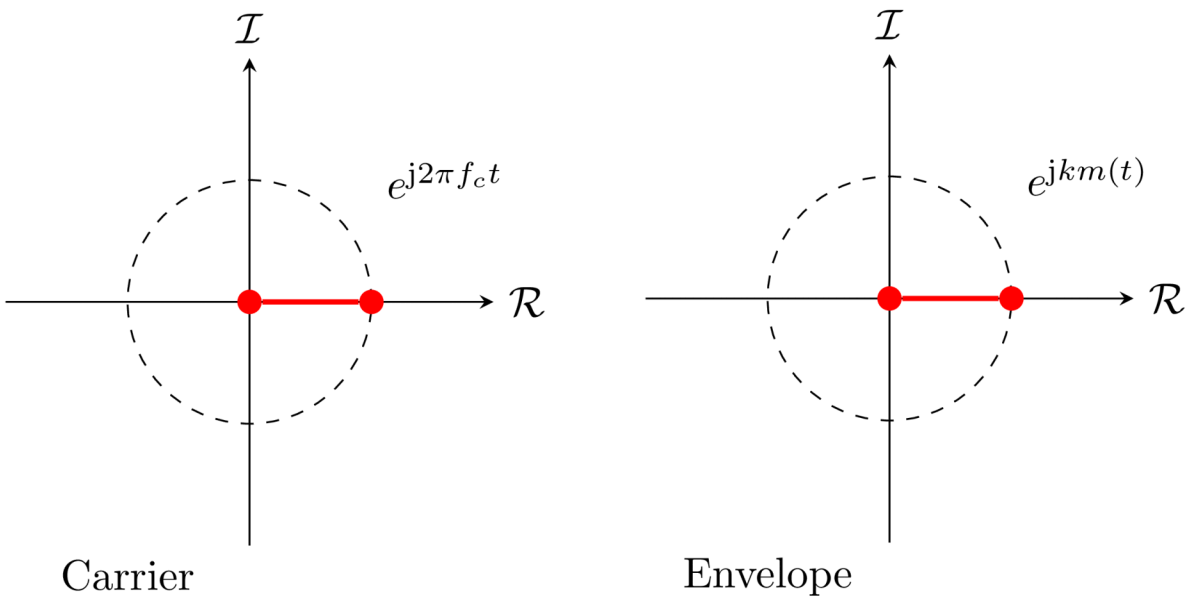
- For an AM signal, $m(t)\cos(2\pi f_c t)$
- The envelope changes along the real axis with $m(t)$
- Or $m(t)$ scales the carrier signal $\cos(2\pi f_c t)$



Now ...

- Include $m(t)$ as the **phase** of the carrier

$$2\cos\left(2\pi f_c t + km(t)\right) = e^{j(2\pi f_c t + km(t))} + e^{-j(2\pi f_c t + km(t))}$$



Instantaneous Frequency 🥴

- In high school, we learnt **speed** $v(t)$ is the time derivative of distance, $x(t)$
- For signals, **frequency**, $\omega(t)$ is the time derivative of phase, $\theta(t)$.
- Time derivatives describe the **instantaneous** phenomenon
- We can vary the *instantaneous frequency* of

Angle Modulation

The Historical Irony: The "Bandwidth Fallacy" 🤯

FM was originally designed to SAVE space!

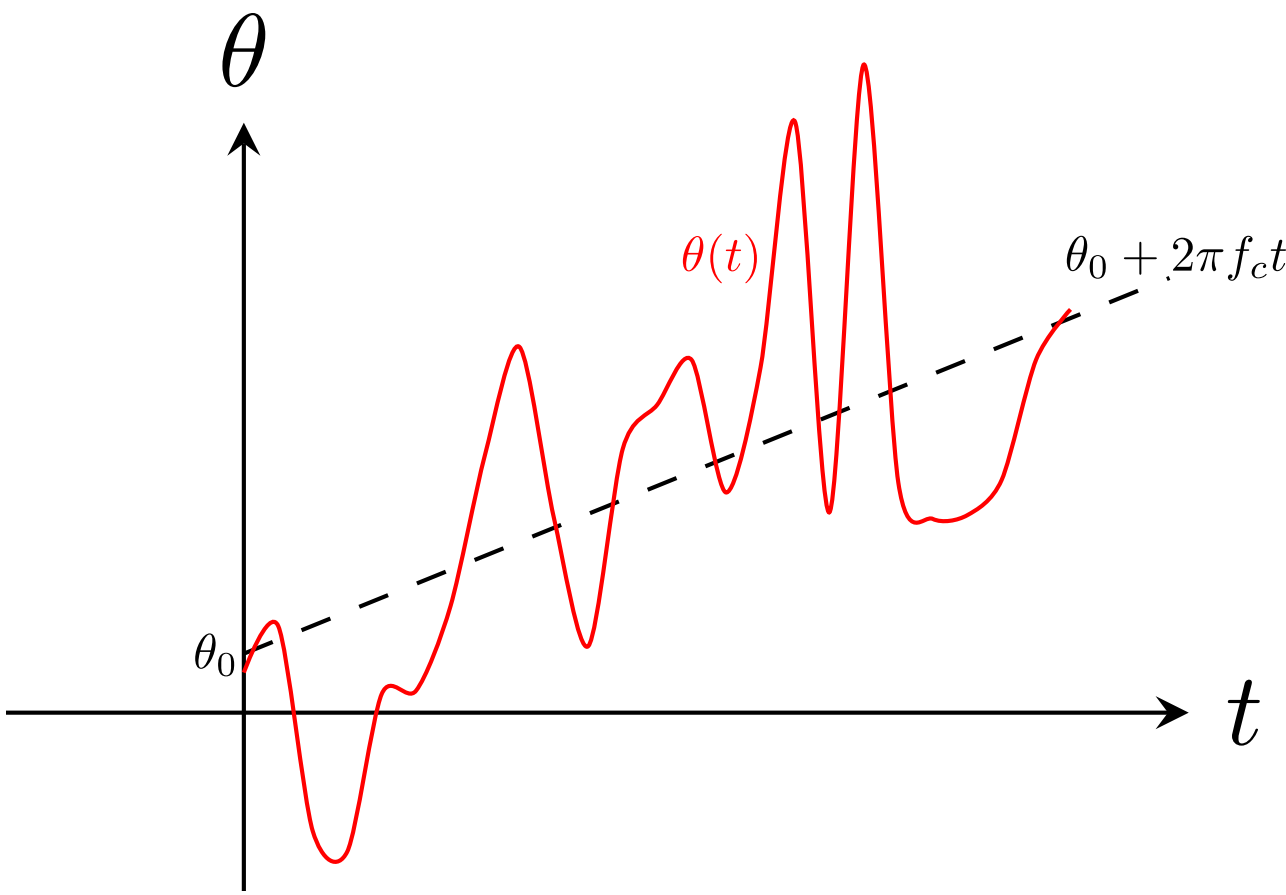
- **The 1920s Goal:** AM radios were crowded. Engineers wanted a "Narrowband" system to squeeze more stations onto the dial.
- **The Intuition:** "If I only wiggle the frequency by ± 50 Hz, surely the bandwidth is tiny!"
- **The Mathematical Reality:**
  - In 1922, mathematician **John Carson** proved that FM actually generates **infinite sidebands**.
  - He famously declared FM "a nuisance" and static.

Angle Modulation 🔑

- Keep the modulated wave amplitude the same
- We can modulate a carrier by changing the **phase** and **frequency**
- Former is PM and latter is FM
- Angle Modulation is much more complex than AM
- A non-linear process

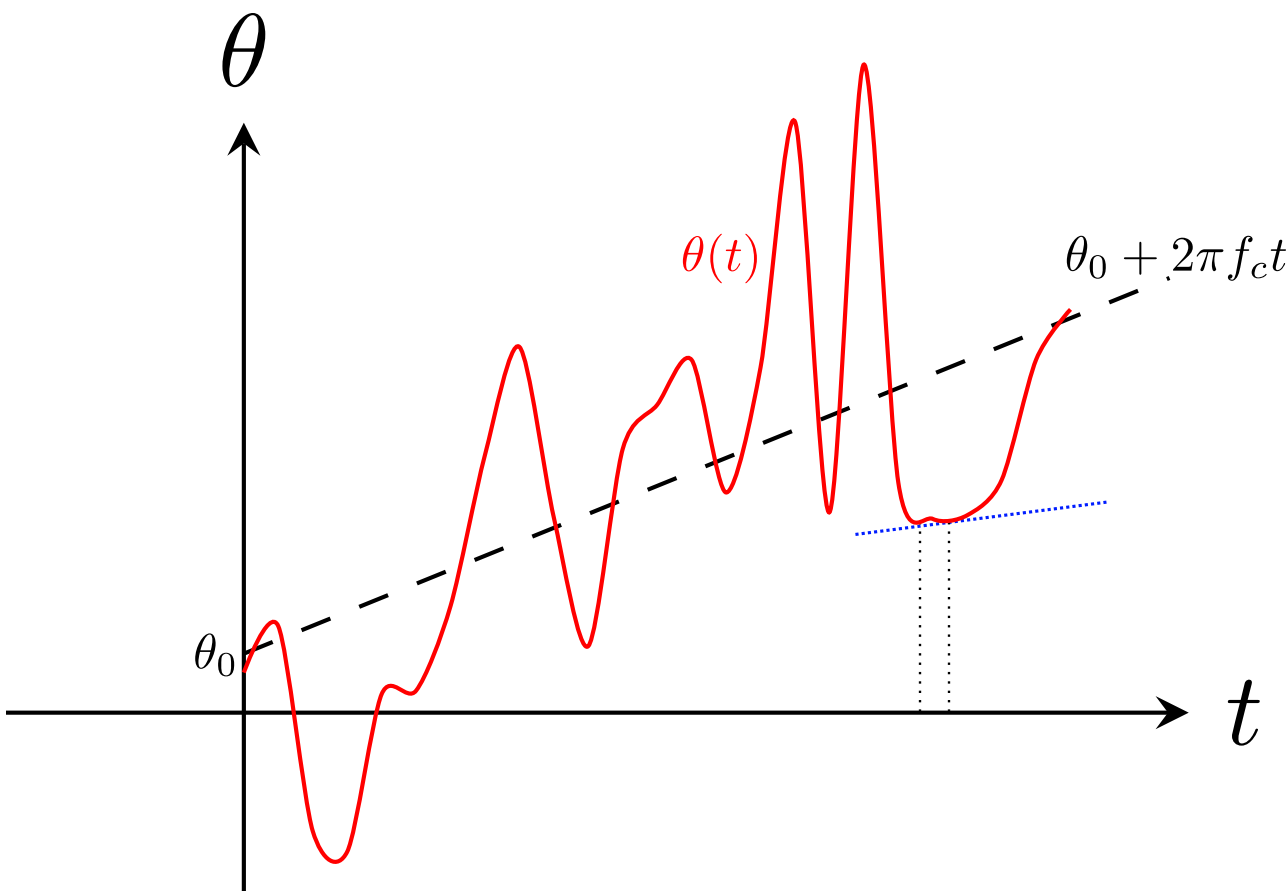
Instantaneous Frequency 😵‍💫

- Remember instantaneous velocity from high school days
- Use of derivatives to describe the **instantaneous** phenomenon
- We can vary the *instantaneous frequency* of the signal through $m(t)$



Instantaneous Frequency (contd.)

- At any time instant t_0, we can find the instantaneous frequency by taking the derivative



A Generalised Sinusoid

Lets look at,

$$\varphi(t) = A_c \cos \theta(t)$$

Here $\theta(t)$ is the generalised angle that can have any value

- For a true sinusoidal signal,

$$\theta(t) = \omega_c t + \theta_0$$

- $\theta(t)$: **Total Instantaneous Phase**
- θ_0: **Phase Deviation** (carries the message)

Finding the Instantaneous Frequency

- The derivative of the generalised angle,

$$\omega_i(t) = \frac{d\theta}{dt} = \theta'(t)$$

- The other way, phase is the integral of the instantaneous frequency,

$$\theta(t) = \int_{-\infty}^{t} \omega_i(u)du = \theta_0 + \int_{0}^{t} \omega_i(u)du$$

- We can modulate by either varying $\omega_i(t)$ or $\theta(t)$

Phase Modulation (PM)

- As the name suggests, we vary the phase $\theta(t)$ **linearly** with $m(t)$

$$\theta^{PM}(t) = \omega_c t + k_p m(t)$$

- The generalised or in fact the transmitted signal is thus,

$$\varphi^{\mathrm{PM}}(t) = A_c \cos(\omega_c t + k_p m(t))$$

We get the instantaneous frequency as,

$$\omega_i^{PM}(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

- 🔑 $\omega_i^{PM}(t)$ is proportional to the rate of change of $m(t)$
- Rapid changes in $m(t)$ results in higher value of $\dot{m}(t)$
- ❓ What about the bandwidth of the signal?

Frequency Modulation (FM)

- In FM, we vary the frequency $\omega(t)$ **linearly** with $m(t)$,

$$\omega_i^{FM}(t) = \omega_c + k_f m(t)$$

- The phase θ^{FM} is,

$$\theta^{FM}(t) = \int_{-\infty}^{t} \omega_i^{FM}(u)du = \omega_c t + k_f \int_{-\infty}^{t} m(u)du$$

The transmitted signal is thus,

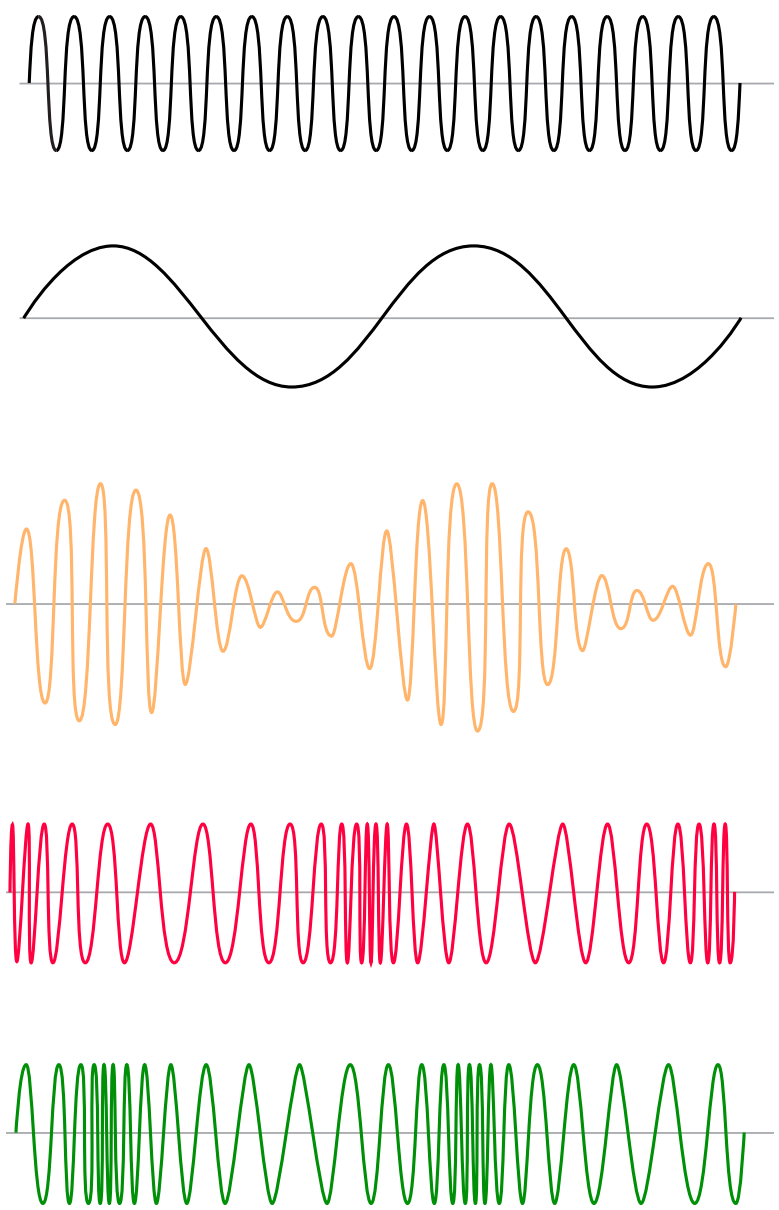
$$\varphi^{\mathrm{FM}}(t) = A_c \cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(u)du\right)$$

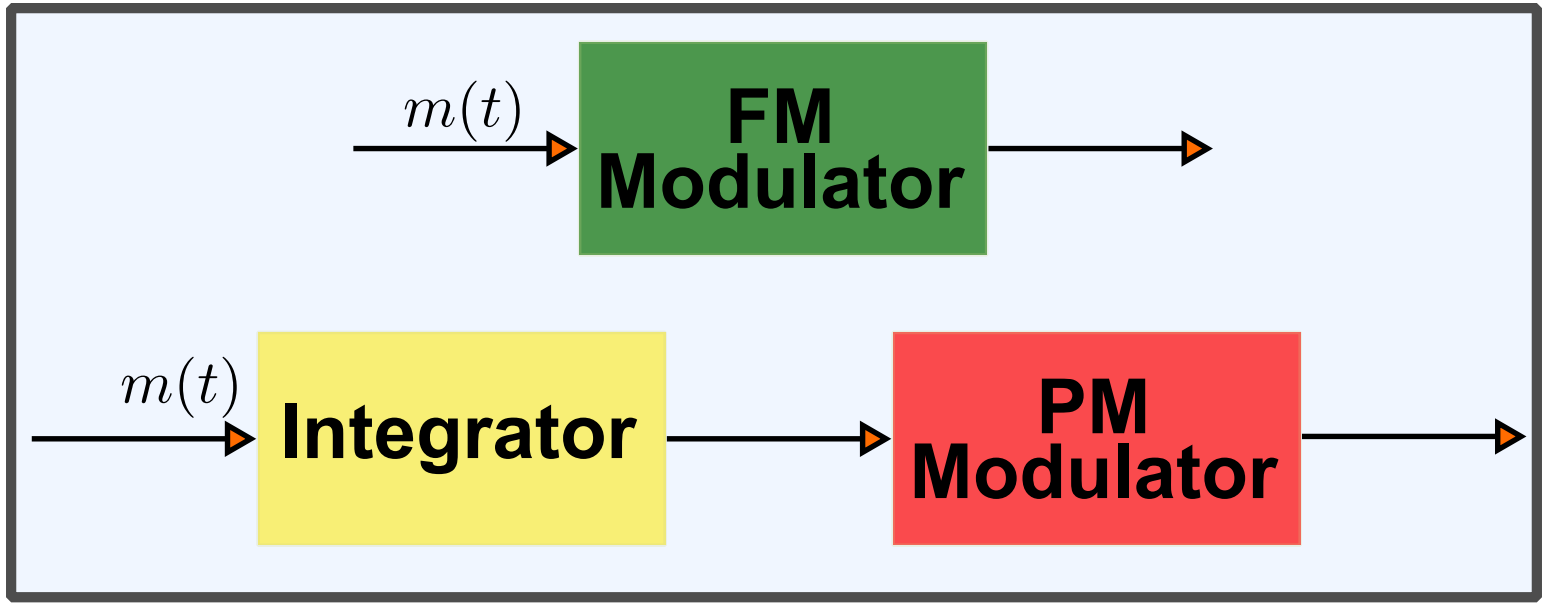
Note the parameter k_f which along with $m(t)$ determines the bandwidth

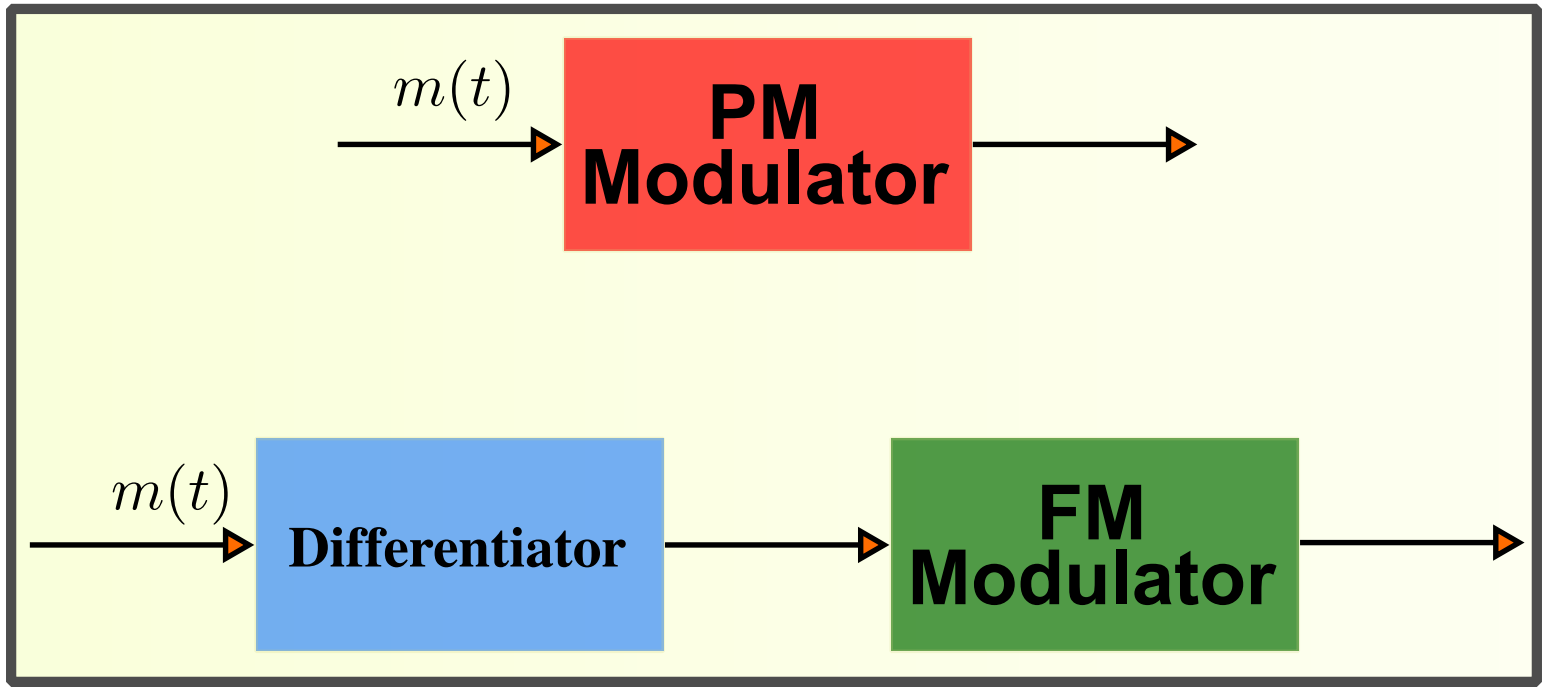
- 📄 🟡 $\theta^{FM}(t)$ to a phase modulator $= \omega^{PM}(t)$ to a frequency

FM vs PM

Guess the 🟡, 🔴 and 🟢 signals

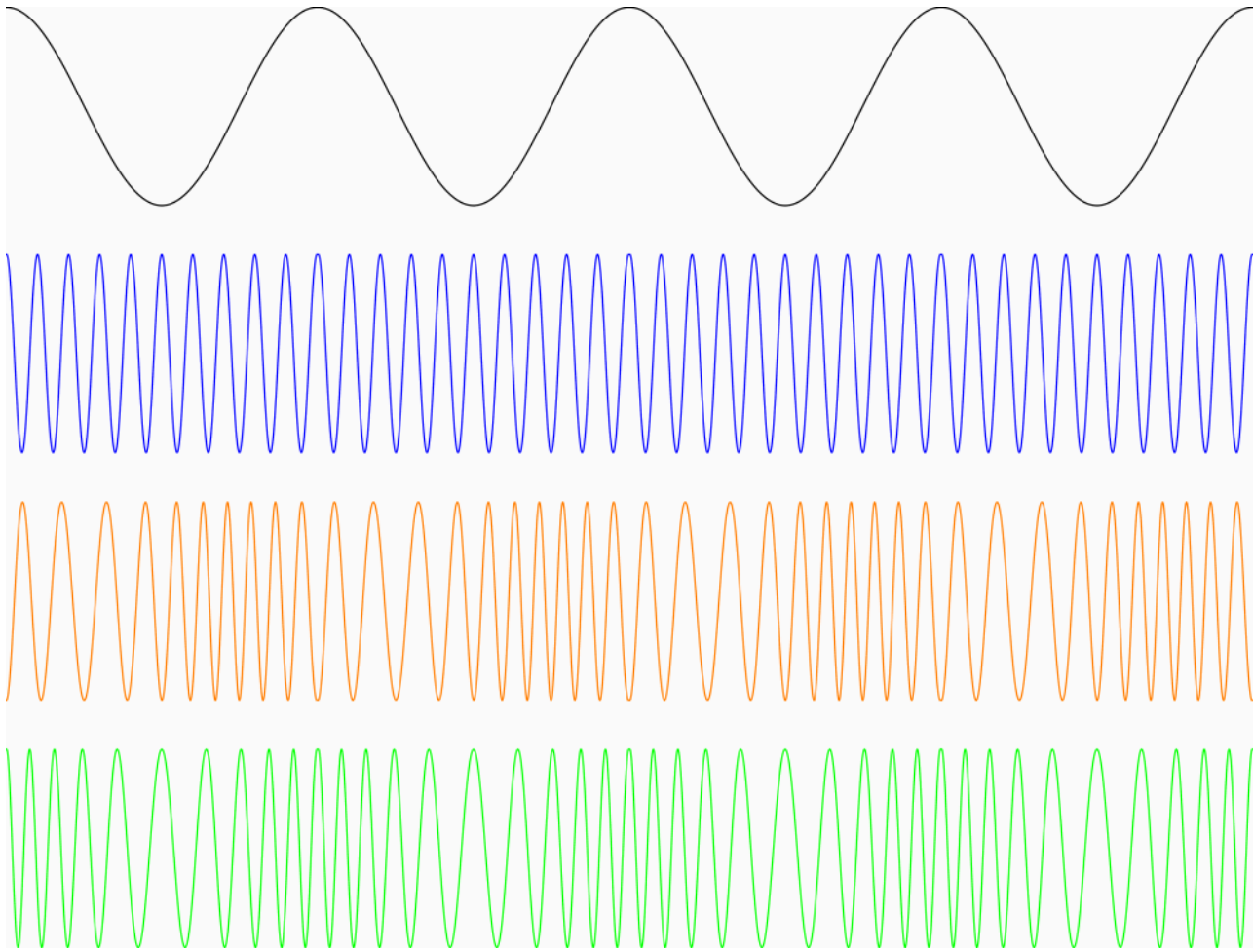



$m(t)$
FM Modulator
$m(t)$
Integrator
PM Modulator


$m(t)$
PM Modulator
$m(t)$
Differentiator
FM Modulator

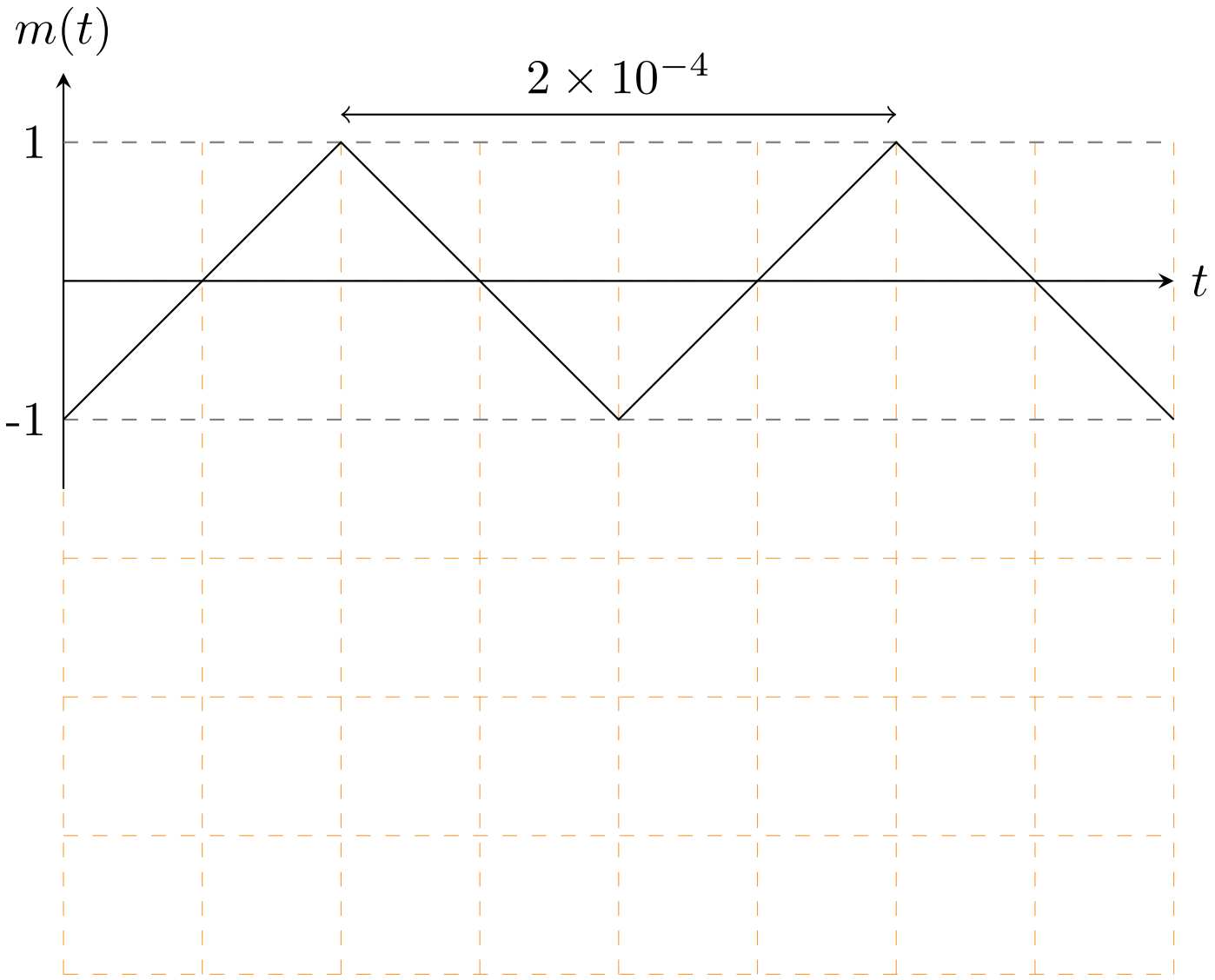
Some Observations

- Regardless of PM or FM, angle modulation signal has constant amplitude
- Power of the signal remains the same irrespective of k_p or k_f.
- PM is a 90° shifted version of FM and vice versa.



An Example

- Given a signal $m(t)$, sketch the FM and PM waves.
- $k_f = 2\pi \times 10^5$, $k_p = 10\pi$ and $f_c = 100$ MHz.



Questions 🤔

- You can ask on Menti

Further Reading

- Section 4.5 - Angle Modulation
  Modern Digital and Analog Communication Systems, 5th Edition
- B P Lathi and Zhi Ding

💡 Modulation is the art of transforming simple signals into complex forms!

Get in touch

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