In FM, be quick, precise, and know your angles! 📻

UESTC 3018 - Communication Systems and Principles

Lecture 13 — Angle Modulation Detection

Dr Hasan Abbas

From Last Time ⏳

- Angle Modulation Bandwidth Dilemma

Today's Lecture 📆

- FM with a Tone
- FM Demodulation Techniques

FM with a Tone

- Due to inherent non-linearity, FM is hard to analyse
- Lets start off with a tone i.e., a sinusoidal signal, $m(t) = \cos(2\pi f_m t)$

$$a(t) = \int_{-\infty}^{t} m(u)du = \frac{1}{2\pi f_m}\sin(2\pi f_m t)$$

- From last time, $a(t) = \int_{-\infty}^{t} m(u)\, du$

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\, e^{j[\omega_c t + k_f a(t)]} = Ae^{jk_f a(t)} e^{j\omega_c t}$$

from where,

$$\hat{\varphi}^{\mathrm{FM}}(t) = A\, e^{j[\omega_c t + \frac{k_f}{2\pi f_m}\sin(2\pi f_m t)]} = Ae^{j(\omega_c t + \beta \sin(2\pi f_m t))}$$

- Here we assume $a(-\infty) = 0$ (causality)

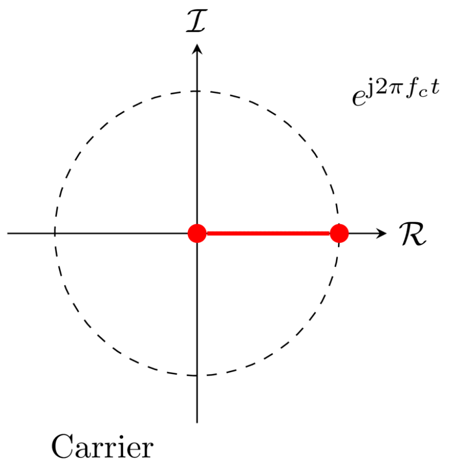
FM with a Tone (contd.) - The Bessel Function

- $e^{j\beta \sin(2\pi f_m t)}$ is a periodic function with frequency f_m
- We can have a Fourier series representation,

$$e^{j\beta \sin(2\pi f_m t)} = \sum_n c_n(\beta) e^{2n\pi j f_m t}$$

- The complex Fourier coefficient

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin(2\pi f_m t)} e^{-2n\pi j f_m t} dt$$



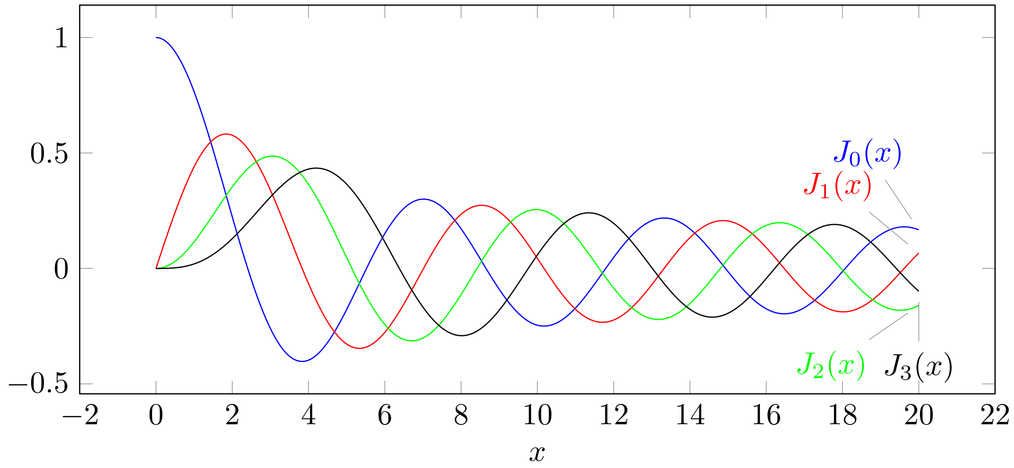
FM with a Tone (contd.) - The Bessel Function

- Defining a new substitute variable,

  $x = 2\pi f_m t$

  $$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta \sin x - njx} dx = J_n(\beta)$$

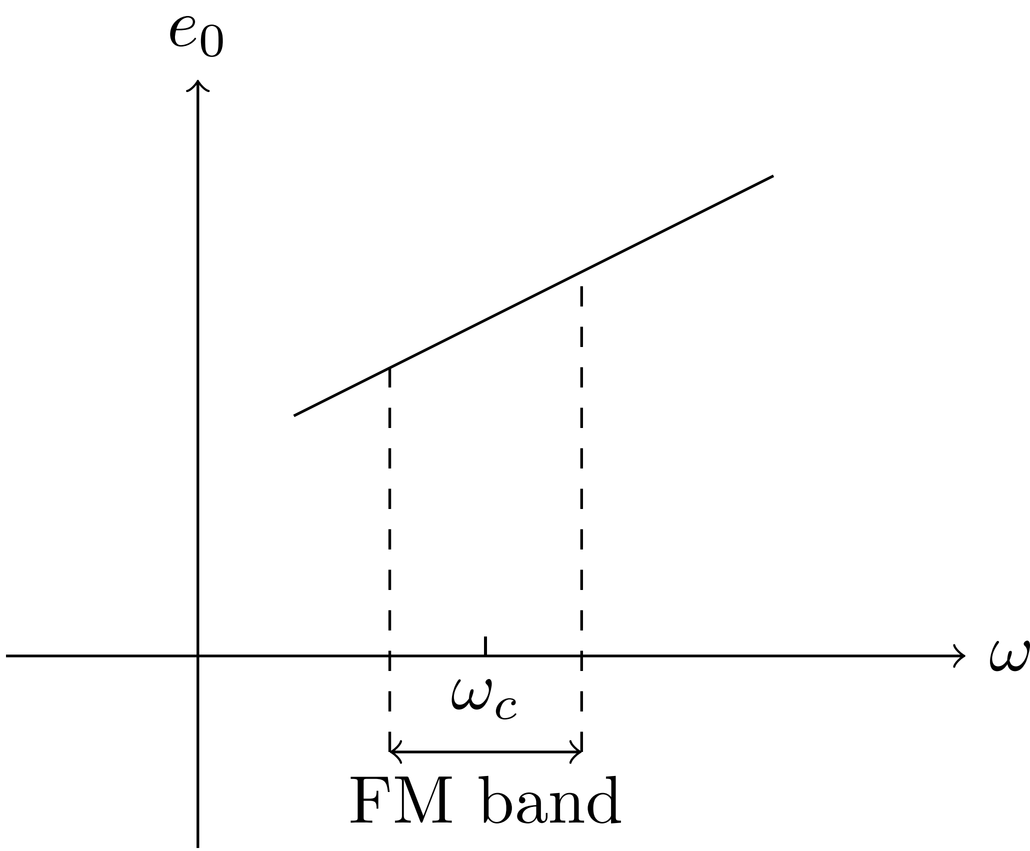
- This is the Bessel function of 1st kind and order n
- β is the deviation ratio



Demodulation

FM Demodulation

- Recall, the instantaneous frequency changes with signal amplitude,
- $\omega_i^{FM}(t) = \omega_c + k_f m(t)$
- We need a system where output is proportional to the input.
- The simplest is an ideal differentiator ($j\omega$)
- Need to convert frequency variations into amplitude variations
- Then use envelope detection.

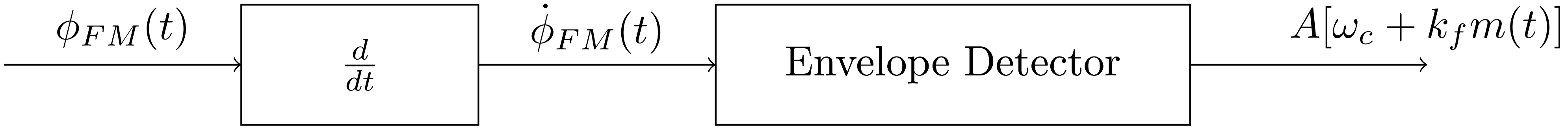


FM Demodulation (contd.)

$$\dot{\varphi}^{\mathrm{FM}}(t) = \frac{d}{dt}\left\{A\cos\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)d\alpha\right]\right\}$$
$$= A\left[\omega_c + k_f m(t)\right]\sin\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)d(\alpha) - \pi\right]$$

- Note the signal $m(t)$ is present both in the envelope and frequency
- Because $\omega = k_f m_p < \omega_c$, we have $\omega_c + k_f m(t) > 0$
- We can simply perform envelope detection (as in AM).

FM Demodulation

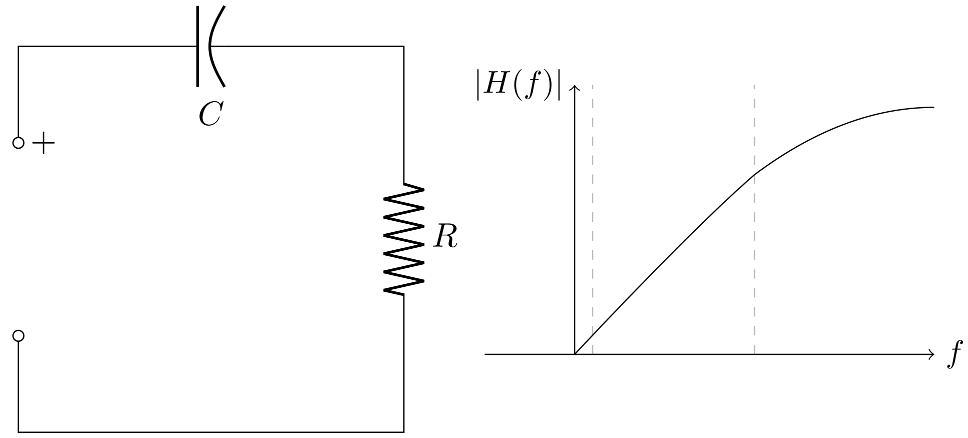


A Simple RC Circuit

- A Simple RC high-pass circuit can be used to detect the slope
- The transfer function (voltage across the resistor) is

$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} \approx j\omega RC$$

- The approximation is true when $\omega_c RC \ll 1$
- We have a differentiator
- This is one of many possibilities



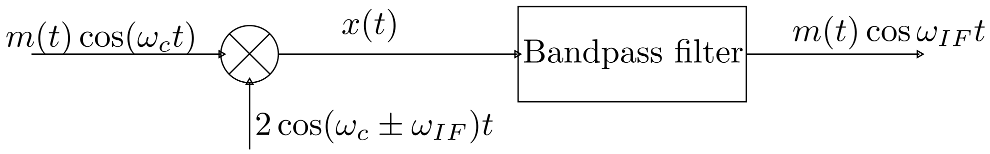
Superheterodyne Receivers

- Frequency Conversion or mixing is done to change the carrier frequency from ω_c to ω_{IF}
- We call IF as intermediate frequency

$$\begin{aligned} x(t) &= 2m(t)\cos\omega_c t\cos\omega_{mix}t \\ &= m(t)\left[\cos(\omega_c+\omega_{mix})t+\cos(\omega_c-\omega_{mix})t\right] \end{aligned}$$

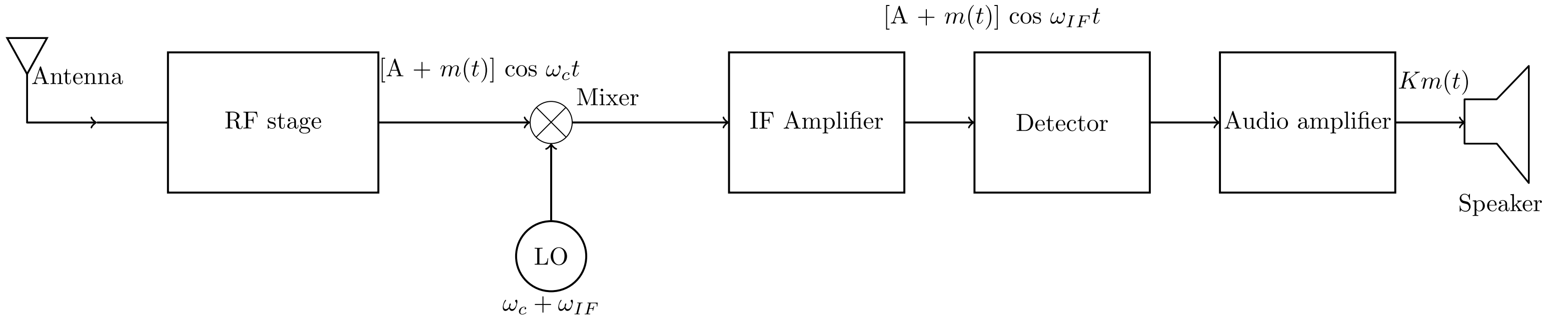
- Setting $\omega_{mix} = \omega_c \pm \omega_{IF}$

$$x(t) = m(t)\left[\cos\omega_{IF}t+\cos(2\omega_c\mp\omega_{IF})t\right]$$



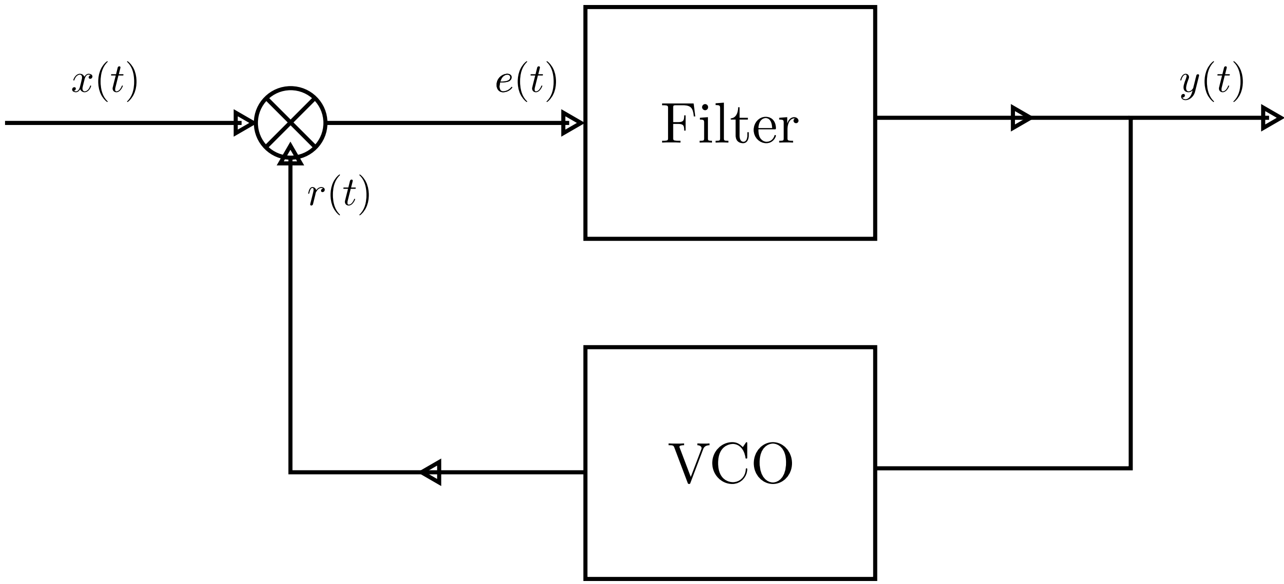
Superheterodyne Receivers

- Down converting to IF allows us to use sensitive amplifiers
- Bandpass filter is very hard to design at RF
- Commonly used in many broadcast systems



Phased-Locked Loop

- A negative feedback system used in FM demodulation
- Compares the phase of the FM signal with the phase of a locally generated reference signal.
- First generate a VCO output $r(t)$ Phase Comparison
- Check for errors with $e(t)$ Error Generated
- $e(t)$ controls the VCO frequency VCO function



Questions ❓

- You can ask on Menti

Further Reading

- Section 4.7 - Demodulation of FM Signals
  Modern Digital and Analog Communication Systems, 5^{th} Edition
- B P Lathi and Zhi Ding

Get in touch

Hasan.Abbas@glasgow.ac.uk