# Perfect Reconstruction with 2B

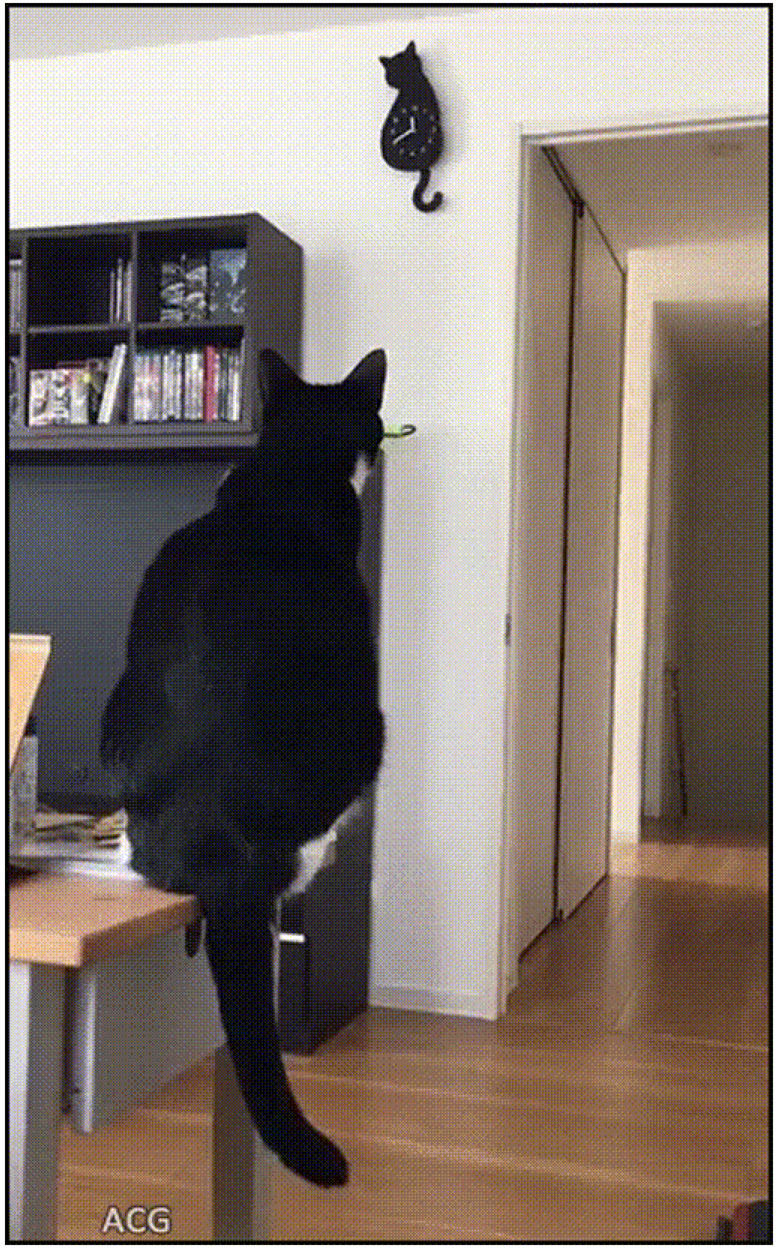
# UESTC 3018 — Communication Systems and Principles

Lecture 16 — Precursor to Digital Communications

Dr Hasan Abbas

# From Last Time ⏳

- Frequency Modulation Detection



## Today's Lecture 📆

- The (im)Pulse train
- Sampling Theorem
- Interpolation
- Pulse Train

# Sampling Theorem

- A signal $g(t)$ with bandwidth $< B$ can be reconstructed exactly from samples taken at any rate $R > 2B$.
- Sampling can be achieved mathematically by multiplying by an impulse train.

$$III(t) = \sum_{k=-\infty}^{\infty} \delta(t-k)$$

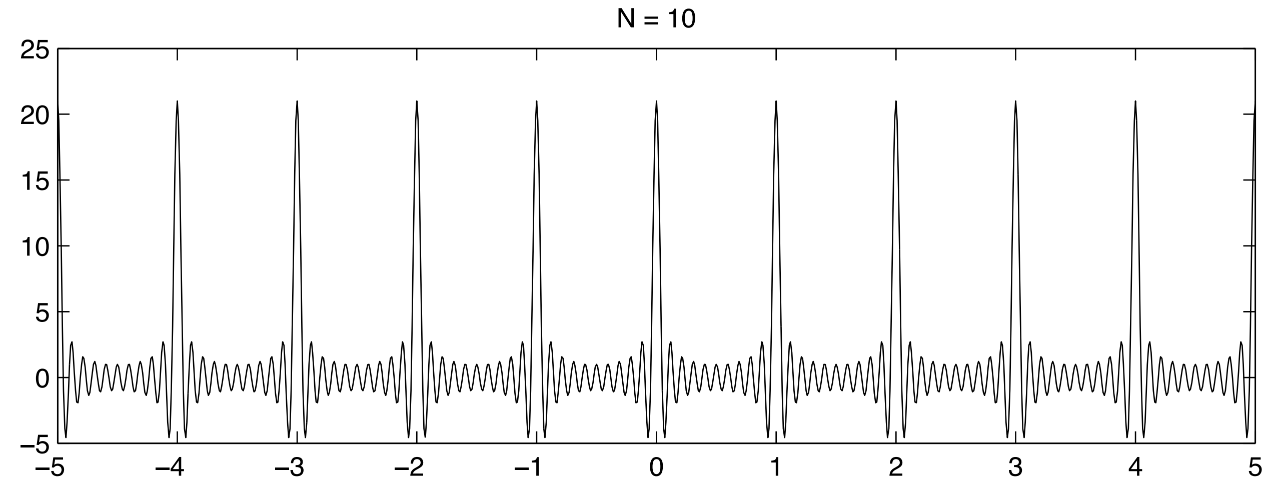
- Also called a comb function

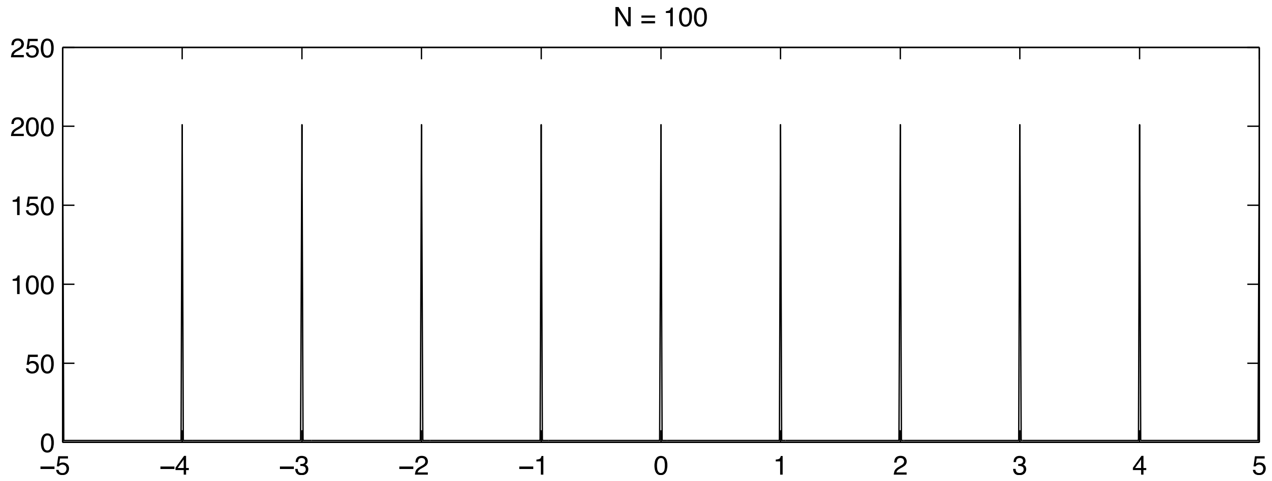
$$\overline{g}(t) = III(t)g(t) = \sum_{k=-\infty}^{\infty} g(t)\delta(t-kT) = \sum_{k=-\infty}^{\infty} g(kT_s)\delta(t-kT_s)$$

# The Impulse Train

- Interesting the Fourier Transform of an impulse train is also an impulse train
- The complex exponentials cancel at non-integer frequencies and add up to an impulse at integer frequencies

$$\mathcal{F}III(t) = \mathcal{F}\sum_{k=-\infty}^{\infty} g(t)\delta(t-k) = \int_{-\infty}^{\infty} e^{-j2\pi n f} = III(f)$$



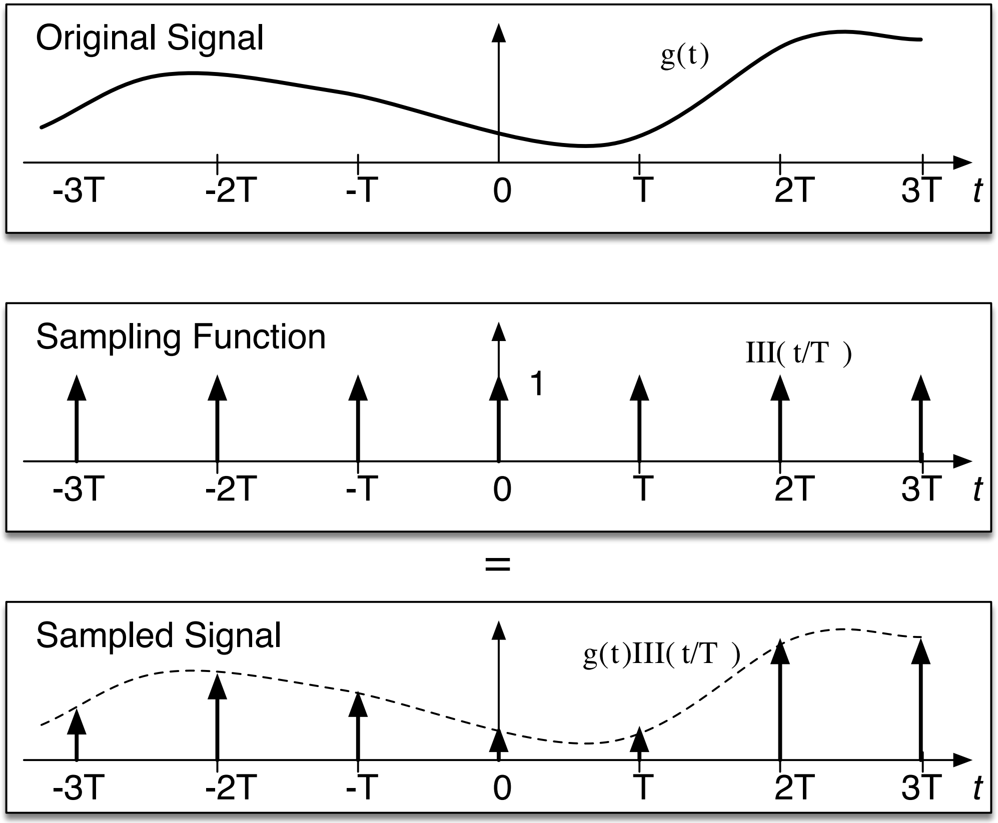
# Fourier Transform of a Sampled Signal

The impulse train $III(t/Ts)$ is periodic with period $T_s$ and can be represented as the sum of complex exponentials of all multiples of the fundamental frequency,

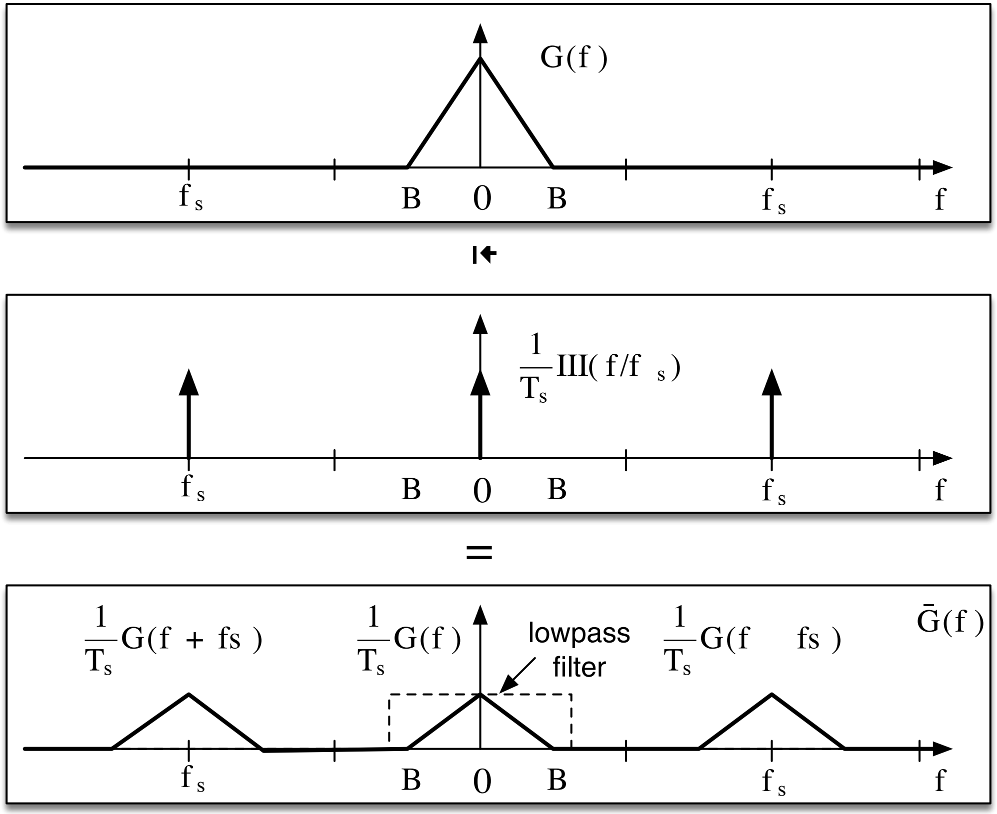
$$III(t/Ts) = 1/Ts \sum_{k=-\infty}^{\infty} e^{-j2\pi f_s t}$$

- $f_s = 1/T_s$

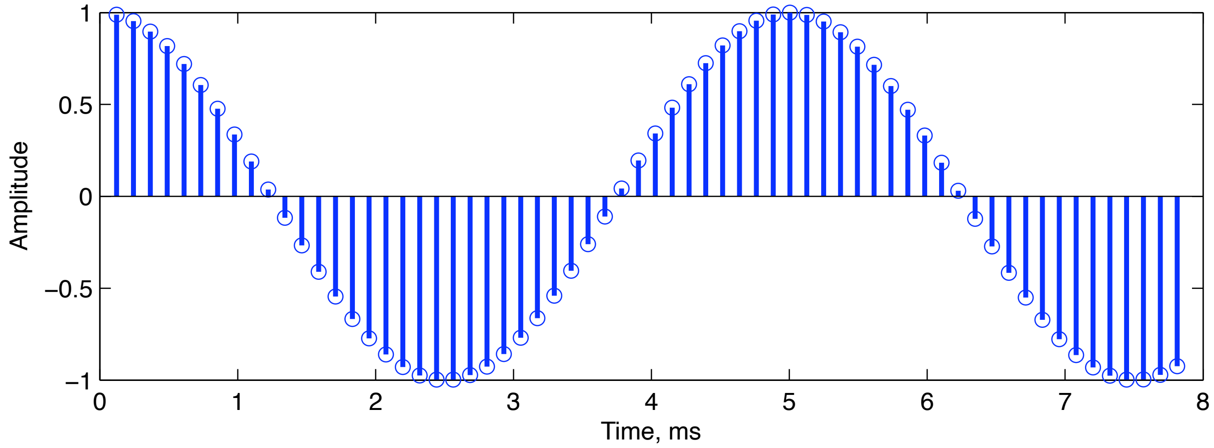
# A Sampled Signal

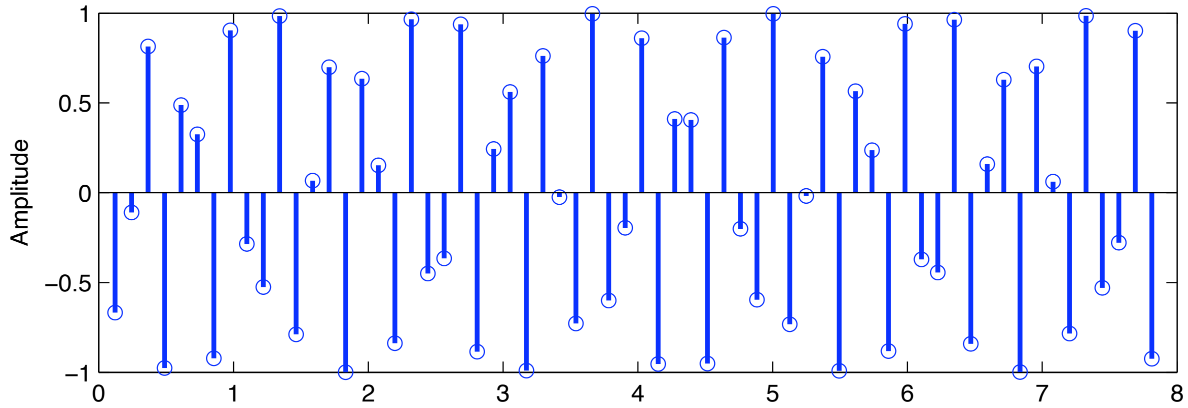


# Sampled Signal and the Fourier Transform

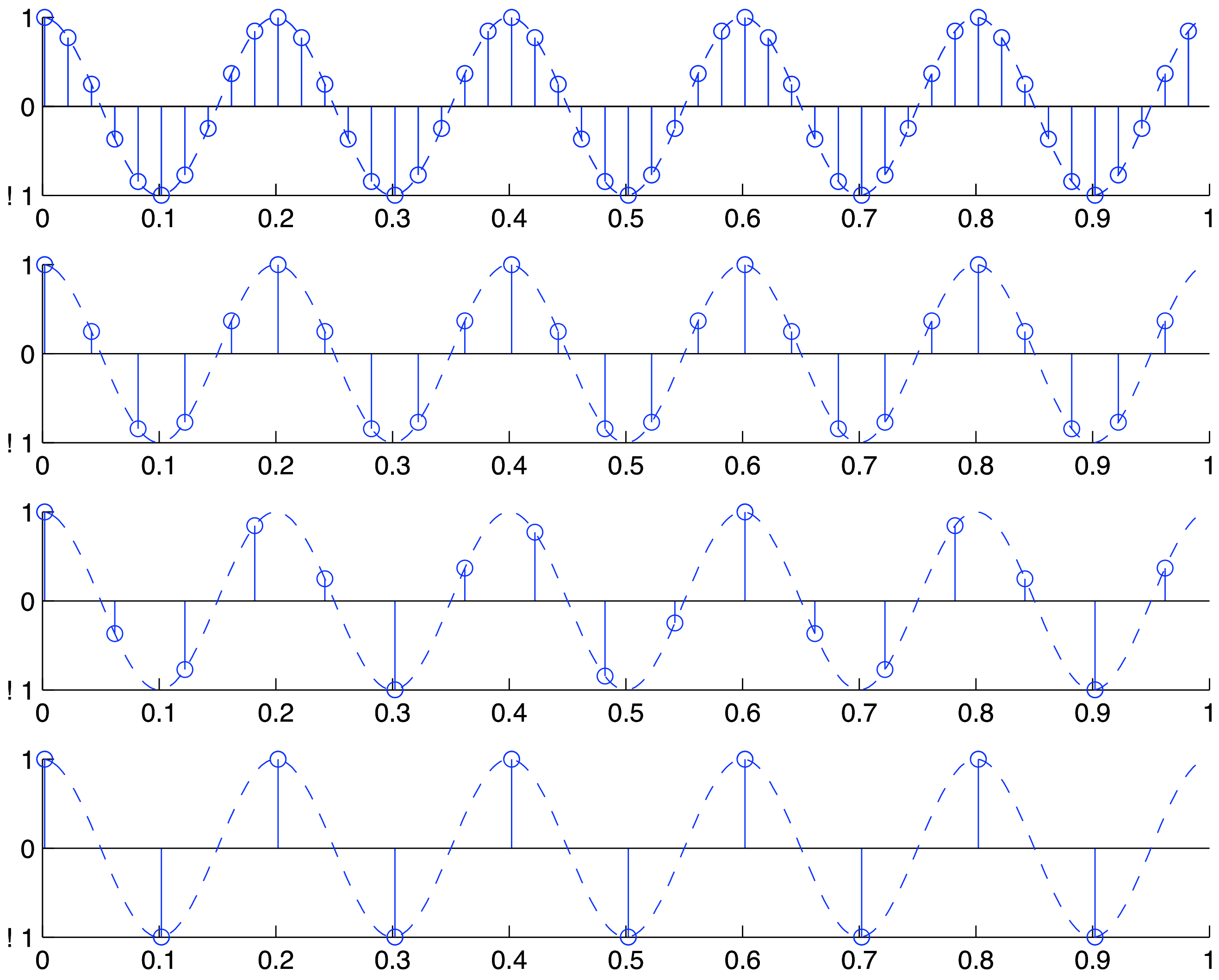


# Sampled Cosines



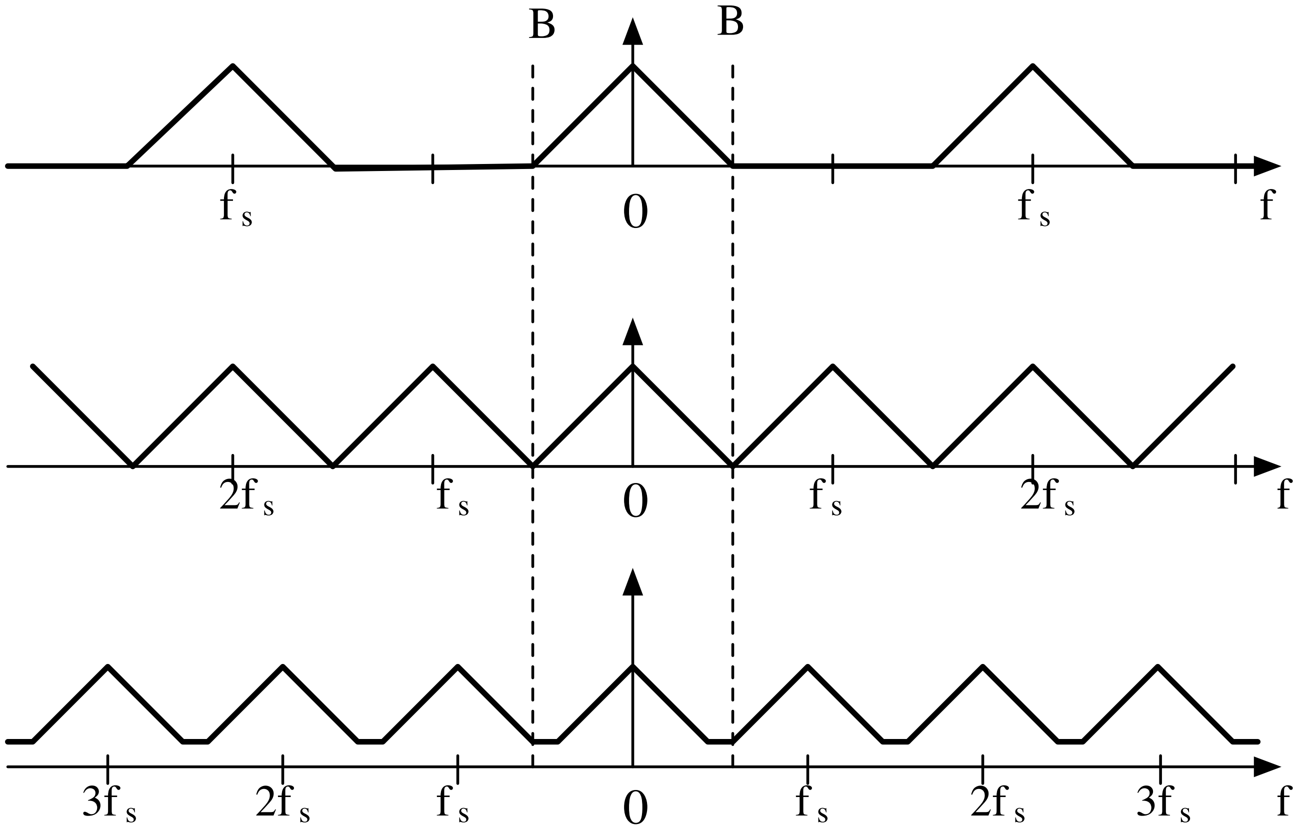



# Sampled Examples



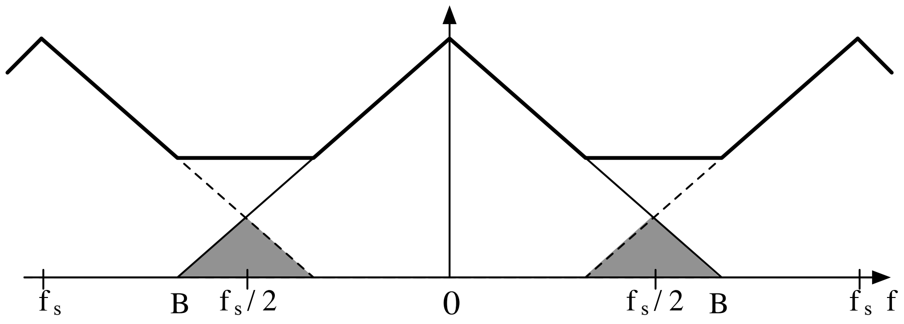
# The Minimum Sampling Rate

- When the sampling rate is too low, the spectral replicas overlap



# Aliasing

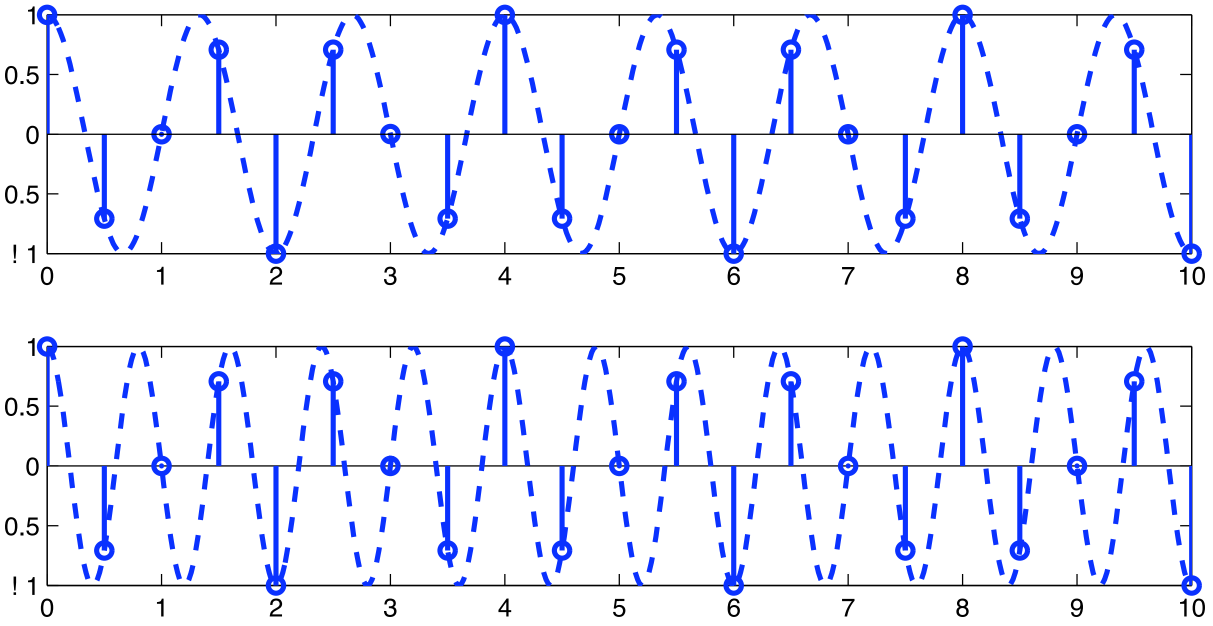
- The spectral overlap




- The shaded frequencies overlap and are ambiguous.
- High positive frequencies wrap around to high negative frequencies
- What signal would you reconstruct if you assumed the signal was actually band limited?

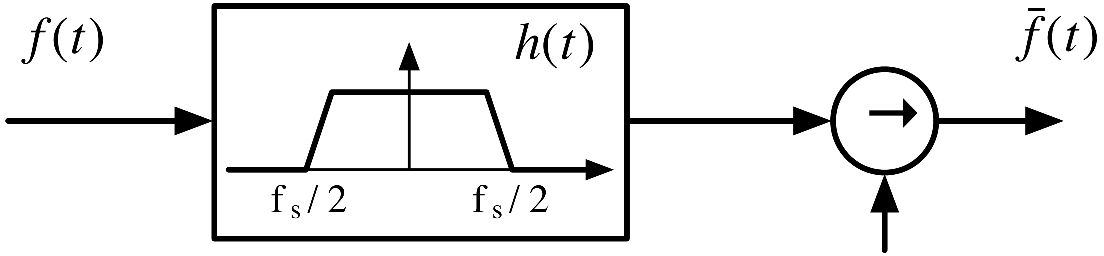
# Aliasing Contd.

- Cosines at frequencies of 0.75 Hz and 1.25 Hz produce exactly the same samples at a sampling rate of 1 Hz



# Anti-aliasing Filter

- In practice, a sampler is always preceded by a filter to limit the signal bandwidth to match the sampling rate
- This may delete part of the signal if it isn't bandlimited.
- It ensures that the signal that is sampled is bandlimited.



# Questions ❓

- You can ask on Menti

# Further Reading

- Section 5.1 - Sampling Theorem

  Modern Digital and Analog Communication Systems, $5^{th}$ Edition
- B P Lathi and Zhi Ding

# Get in touch

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