NAO6 Humanoid Left Arm Modelling

Masses of respective arm parts:

- $m_{shoulder} \approx 0.07504\ kg$
- $m_{bicep} \approx 0.15777\ kg$
- $m_{elbow} \approx 0.06483\ kg$
- $m_{forearm} \approx 0.07761\ kg$
- $m_{wrist} \approx 0.18533\ kg$
- $m_{gripper} \approx 0.000002\ kg$

Lengths of respective arm parts:

Upper = Shoulder + Bicep; Lower = Elbow + Forearm; Hand = Wrist + Gripper

- $L_{upper} \approx 0.105\ m$
- $L_{forearm} \approx 0.05595\ m$
- $L_{hand} \approx 0.05575\ m$
- $r_{upper} \approx r_{lower} \approx 0.02\ m$
- $r_{hand} \approx 0.025\ m$

Parameters:

- $\theta_{shoulder,\,pitch}$: $angle\ of\ shoulder\ rotation\ in\ Y - axis$
- $\theta_{shoulder,\,roll}$: $angle\ of\ shoulder\ rotation\ in\ Z - axis$
- $\theta_{elbow,\,yaw}$: $angle\ of\ elbow\ rotation\ in\ X - axis$
- $\theta_{elbow,\,roll}$: $angle\ of\ elbow\ rotation\ in\ Z - axis$
- $\theta_{wrist,\,yaw}$: $angle\ of\ wrist\ rotation\ in\ X - axis$
- Hand (Gripper) does not have an angle of rotation, since it only can open and close

General equation of motion:

$$\tau_{i,\,j} = I_i(\theta_i)\frac{d\theta_i^2}{d^2t} + b_i\frac{d\theta_i}{dt} + m_j g l_j \sin(\theta_i)$$

$where\ \tau_i\ is\ total\ torque\ acting\ on\ i^{th}\ joint,\ I_i - inertia\ matrix,\ b_i - damping\ coefficient,\ m_j - mass\ of\ j^{th}\ link,\ and\ l - length.$

General form of Inertia Matrix:

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

*where diagonal elements are known as **moments of inertia**, and off-diagonal components are known as **products of inertia**.*

How to calculate components of Inertia Matrix:

$$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + l^2)$$

$$I_{zz} = \frac{1}{2} mr^2$$

*where **r** and **l** are radius and length of the coressoping link, respectively.*

$$I_{xy} = -mx_{com}\, y_{com}$$

$$I_{xz} = -mx_{com}\, z_{com}$$

$$I_{yz} = -my_{com}\, z_{com}$$

where $\boldsymbol{x_{com}}$, $\boldsymbol{y_{com}}$ and $\boldsymbol{z_{com}}$ are the coordinates of the center of mass relative to the frame of reference (a joint connected to a link can be used as one).

Parameters for Denavit-Hartenberg (D-H) method:

- a_i : $link\ length\ (between\ two\ joints\ along\ X - axis)$
- α_i : $link\ twist,\ representing\ the\ angle\ between\ the\ two\ successive\ joint\ axes$
- d_i : $link\ offset\ between\ previous\ and\ current\ joint\ along\ Z - axis$
- θ_i : $joint\ angle$

General D-H Transformation Matrix:

$$T_i^{i+1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Particular Case Parameters for NAO6:

1. Shoulder Pitch (LShoulderPitch)

- **Rotation**: About the Y-axis (front and back motion).
- **D-H Parameters**:
  - θ_1: Variable (joint angle for shoulder pitch).
  - d_1: Length along Z-axis (distance between shoulder and elbow). This is typically fixed and represents the link length.
  - a_1: 0 (since this is the first link, there's no X-axis offset).
  - α_1: $\pi/2$ (90°) as the next axis (shoulder roll) is around the Z-axis.

2. Shoulder Roll (LShoulderRoll)

- **Rotation**: About the Z-axis (right and left motion).
- **D-H Parameters**:
  - θ_2: Variable (joint angle for shoulder roll).
  - d_2: 0 (the link doesn't move along the Z-axis).
  - a_2: This would be the distance between the shoulder joint centers (if any). If negligible, it can be 0.
  - α_2: $-\pi/2$ (-90°) since the next axis (elbow yaw) rotates around the X-axis.

3. Elbow Yaw (LElbowYaw)

- **Rotation**: About the X-axis (twist motion at the elbow).
- **D-H Parameters**:
  - θ_3: Variable (joint angle for elbow yaw).
  - d_3: The length of the upper arm (distance between shoulder and elbow). This is a fixed length.
  - a_3: 0 (no offset along the X-axis).
  - α_3: 0° (as both elbow roll and yaw have aligned Z-axes).

4. Elbow Roll (LElbowRoll)

- **Rotation**: About the Z-axis (elbow rotation).
- **D-H Parameters**:
    - θ_4: Variable (joint angle for elbow roll).
    - d_4: 0 (no movement along the Z-axis for this joint).
    - a_4: Fixed, representing the forearm length (distance between elbow and wrist).
    - α_4: $\pi/2$ (90°) as the next axis (wrist yaw) rotates around the X-axis.

5. Wrist Yaw (LWristYaw)

- **Rotation**: About the X-axis (wrist twist).
- **D-H Parameters**:
    - θ_5: Variable (joint angle for wrist yaw).
    - d_5: 0 (no displacement along the Z-axis).
    - a_5: Length from wrist to hand (small, if considered).
    - α_5: 0° (end effector's frame is aligned with the wrist).

Summary of Limits for DH Parameters:

- $\theta_1 \in [-2.0857, 2.0857]$
- $\theta_2 \in [-0.3142, 1.3265]$
- $\theta_3 \in [-2.0857, 2.0857]$
- $\theta_4 \in [-1.5446, -0.0349]$
- $\theta_5 \in [-1.8238, 1.8238]$

Calculated Transition Matrices for NAO6 Humanoid Left Arm Joints:

1. T_1 (Shoulder Pitch, Y-axis):

$$T_1 = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. T_2 (Shoulder Roll, Z-axis):

$$T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0.105 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. T_3 (Elbow Yaw, X-axis):

$$T_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. T_4 (Elbow Roll, Z-axis):

$$T_4 = \begin{bmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & -\cos(\theta_4) & 0 \\ 0 & 1 & 0 & 0.05595 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. T_5 (Wrist Yaw, X-axis):

$$T_5 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0.05575 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Full Transformation T:

The full transformation matrix T is:

$$T = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$$