

$$\text{The Pfund Mass } M_F = \sqrt{\frac{k_e^2 q_e^4 T}{c G M L^2}} = \sqrt{\frac{c \alpha^2 \hbar^2 T}{G M L^2}}$$

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Here we treat the Dalton not as a value to be "found" through physical artifacts, but as an algebraic identity emerging from constants of the universe. We transition from measuring string to using math to solve for the constant. It is possible to further reduce the number of constants in use  $M_F^W L_F^X T_F^Y Q_F^Z =$

$$\left( \sqrt{\frac{c \alpha^2 \hbar^2 T}{G M L^2}} \right)^W \left( \frac{G}{c^2} M_F \frac{M L^2}{\alpha^2 \hbar T} \right)^X \left( \frac{G}{c^3} M_F \frac{M L^2}{\alpha^2 \hbar T} \right)^Y (q_e)^Z$$

$$\begin{aligned} k_e &= \alpha M_F^1 L_F^3 T_F^{-2} Q_F^{-2} & \hbar &= 1 M_F^1 L_F^2 T_F^{-1} \\ \mu_0 &= \alpha 4 \pi M_F^1 L_F^1 Q_F^{-2} & c &= 1 L_F^1 T_F^{-1} \\ \epsilon_0 &= \frac{1}{\alpha 4 \pi} M_F^{-1} L_F^{-3} T_F^2 Q_F^2 & N_A &= 1 M_F^{**} \\ R_\infty &= \frac{m_e}{M_F} \frac{\alpha^2}{4 \pi} L_F^{-1} & M_F c^2 &= E_F = 1 M_F^1 L_F^2 T_F^{-2} \\ & & \frac{2 \pi \hbar}{c q_e} &= k_b = 2 \pi M_F^1 L_F^1 Q_F^{-1} \end{aligned}$$

The SI definition of  $\frac{M}{m} \frac{8 V}{\alpha^3} = N_A$  (Avogadro constant) and  $\frac{2 R_\infty \hbar}{c \alpha^2 A_r(e)} = m_u$  (the Dalton), remains experimental, relying on physical measurements to approximate a fundamental ratio rather than deriving it as an algebraic identity. Scientists must count atoms by measuring the sphere's mass, volume, and lattice parameter. Determining  $N_A$  via silicon sphere measurement resembles measuring string wrapped around a circle to calculate  $\pi$ .

## 1 Dimensionality

Stoney-Mass  $M_S = \sqrt{\frac{\hbar \alpha c}{G}}$  units are correct, but with  $M_F = M_S \sqrt{\hbar \alpha}$  alone gives correct arithmetic; for all the equations above to not only work arithmetic wise,  $\sqrt{\frac{T}{M^1 L^2}}$  is needed to cancel any wrong units put into  $M_F$  from  $\hbar$ . M, L, and T with no subscripts are meant to mean from whichever unit system you are converting from or currently using, in this case; SI units 1kg, 1m, and 1s respectively.  $\sqrt{\frac{T}{M^1 L^2}}$  reappears in the solution for  $G$ . However units for  $G$  are wrong because it's not a force, it is the curvature of space time.

$$\alpha^2 \hbar = \alpha^2 \frac{M_F L_F^2}{T_F} = \frac{G}{M_F^3 L_F^{-1} T_F^{-2}}$$

## 2 Time is On Our Side(& Distance)

$$c T_{SI} \left( \frac{G}{c^2} M_F \frac{M L^2}{\alpha^2 \hbar T} \right)^{-1} = D_C \quad \frac{2.267061 \text{ grams}}{D_C} \approx M_F$$

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With a Sympathetic Constant  $= D_C = \frac{c T_{SI}}{L_F} = \text{light-second} / L_F$  to move from the atomic scale to the macroscopic scale. Calibrating scales to  $D_C M_F \approx 2.267061$  grams and ammeter to  $D_C q_e \approx 222702.26$  Coulombs will result in  $M_F^W L_F^X T_F^Y Q_F^Z = \left( \frac{p = D_C M_F}{D_C} \right)^W \left( \frac{299792458 \text{ Meters}}{D_C} \right)^X \left( \frac{1 \text{ Second}}{D_C} \right)^Y \left( \frac{q = D_C q_e}{D_C} \right)^Z$

$$\frac{\hbar \alpha}{c q_e^2} \frac{2 c^2}{4 \pi} = \frac{\mu_0 c^2}{4 \pi} = \frac{1}{4 \pi \epsilon_0} = k_e = \alpha \left( c \frac{s}{m} \right)^3 p^1 m^3 s^{-2} q^{-2}$$

$$\begin{aligned} \mu_0 &= \alpha 4 \pi \left( c \frac{s}{m} \right) p^1 m^1 q^{-2} & k_b &= 2 \pi D_C^{-1} \left( c \frac{s}{m} \right) p^1 m^1 q^{-1} \\ \epsilon_0 &= \frac{1}{\alpha 4 \pi} \left( c \frac{s}{m} \right)^{-3} p^{-1} m^{-3} s^2 q^2 & c &= \left( c \frac{s}{m} \right) m^1 s^{-1} \\ R_\infty &= \frac{\alpha^2}{4 \pi} D_C^2 \left( c \frac{s}{m} \right)^{-1} \frac{m_e}{p} m^{-1} & \hbar &= D_C^{-2} \left( c \frac{s}{m} \right)^2 p^1 m^2 s^{-1} \\ G &= \frac{\hbar T}{M L^2} \alpha^2 \left( c \frac{s}{m} \right)^3 p^3 m^{-1} s^{-2} & N_A &= D_C \\ & * 0.002267061 & \alpha &= \frac{\mu_0}{2} \frac{c q_e^2}{2 \pi \hbar} \end{aligned}$$

$= 0.00729735257$  "It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it." [Feynman, 1985, p. 129].

## 3 Speed of Sound = $c_0$

Given  $c_0 = \sqrt{\frac{\gamma_0 N_A k_B K}{M_{\text{molar Mass}}}}$  and  $39.947 \text{ g mol}^{-1} = \text{molar mass of the argon gas from the experiment measuring } c_0 \text{ in a purified isotope of argon gas at the Triple-Point of water} = 273.16 \text{ K}$  [de-Podesta et al., 2013] Where  $U_F = \frac{M L^2}{k_b T^2}$ ,  $K = \frac{\text{Kelvin}}{U_F} U_F$ , and  $\gamma_0 = 5/3$  for monotonic gases. Let's see how this matches up with the  $c_0^2 = 94756.245 \text{ m}^2 \text{ s}^{-2}$  from the experiment in 2013. e.g. on how to correct  $N_A$  being based on the Dalton  $= \frac{1}{12}$  the mass of Carbon isotope  $C^{12}$  over to  $M_F$

$$**\text{adjusted argon gas molar mass} = 40.671 M_F = \frac{39.947 \text{ kg}}{M_F N_A}$$

$$c_0 = \sqrt{\frac{\frac{5}{3} N_A k_B \frac{273.16 \text{ Kelvin}}{U_F} U_F}{40.671 M_F}} = 307.701 \text{ ms}^{-1} \approx \sqrt{c_0^2}$$

## 4 Constants

SI Avogadro constant  $N_A = 6.02214076 \times 10^{23} \approx \frac{0.012 \text{ kg}}{\text{mole } ^{12}\text{C}}$

Planck constant  $h = 6.62607015 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$

lightspeed constant  $c = 299792458 \text{ m s}^{-1}$

electron charge  $q_e = 1.602176634 \times 10^{-19} \text{ Coulombs}$

gravity constant  $G = 6.67430 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$s$  = “duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom”[SI, 1968] to finish the definition of  $c$ , An international agreement in Paris on Oct. 20 1983 defines the meter as  $\frac{1}{299792458}$  the distance light travels in a vacuum in 1 second[Times, 1983], 2018 Codata values of  $q_e$ , and  $G$ , come from [Tiesinga et al., 2021]. On May 20, 2019 the values of  $\hbar = \frac{h}{2\pi}$ , and  $h$ , were fixed to their current value along with  $N_A$  to the Dalton =  $\frac{1}{12}$  the mass  $^{12}\text{C}$  isotope of Carbon[Bettin].

### 4.1 How the Avogadro constant was measured for the last time

This section is paraphrased almost verbatim from [Bettin].  $N_A$  and  $h$  were measured using a incredibly round & pure ball of  $Si^{28}$  and a Kibble balance. Where  $a^2 m_e c / 2 h = R_\infty$  is the Rydberg constant,  $\sum_{i=28}^{30} x_i A_r(^iSi) = A_r(Si)$  average molar mass of a silicon atom in the crystal is calculated using the proportions  $x_i$  of the various isotopes  $^iSi$ ,  $V$  is Volume of Silicon Sphere,  $a$  Lattice parameter of the silicon crystal, 8 is the number of atoms in an elementary cell of the lattice(cube with edge length  $a$ ).  $M$  Molar mass of silicon contained in sphere.  $m$  mass of sphere.

$$N = \frac{8 V}{a^3} = \text{Number of atoms in silicon sphere}$$

$$N_A = \frac{M 8 V}{m a^3} ; \quad m_u = \frac{2 R_\infty h}{c a^2 A_r(e)}$$

$$m(Si) = \frac{m}{N} = \frac{m a^3}{8 V} = m(e) \frac{A_r(Si)}{A_r(e)}$$

$$m(e) = \frac{2 h R_\infty}{c a^2} = \frac{2 (2\pi \hbar) R_\infty}{c a^2}$$

$$h = \frac{c a^2 m a^3}{2 R_\infty 8 V} \frac{A_r(e)}{\sum_{i=28}^{30} x_i A_r(^iSi)}$$

## References

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## Supplementary Materials

Only reason I found this was for some reason or another I had just looked at  $\sqrt{\hbar \alpha}$ , and I had Avogadro constant for Stoney-Mass  $M_S = \sqrt{\frac{\hbar \alpha c}{G}}$  mass popping up in a script I had running with some notes to see how the Planck-Units, and the Stoney-Units[Stoney, 1883], baked out the need for certain constants. But when I had left the speed of sound calc to see how the planck units canceled out the Boltzman constant, and the Avogadro constant for  $M_S$  also popped up, not exact match but so close I had to do more tests.