Daisyworld: A walk in Lovelock's shoes

John Wesley
Applied Computer Science and Cybernetics MEng
siu99jw2@reading.ac.uk

Abstract - This paper describes the work undertaken by the author to create a classical Daisyworld model and extend it by adding other, competing species, into the biosphere. This work is part of a larger project to compare and contrast two separate mathematical approaches to the parable of Daisyworld and population modelling as a whole.

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2. Introduction

The work in this paper is done as part of a third year project to create a simplistic model of the Earth and its biosphere. This model was first created by Dr. James Lovelock in support of his Gaia theory; he called it the Daisyworld model. Daisyworld was originally modelled using the 'black box' technique used heavily in Cybernetics. In this paper two models are to be created, the first being as Lovelock saw it, the second using an Cellular Automaton method [12].

2.1. Black Box Method

The black box method is used in Cybernetics to model systems. A hypothetical black box is placed over the system in question so that one may not see what is inside. The function of the system may then only be determined by monitoring the inputs and outputs of the box.
This technique can be used to great advantage when one is not interested in what is in the box, only what it does.

2.2. Cellular Automaton

The Cellular Automaton method is used to show that complex emergent behaviours may come from simple rules or conditions [12].

It models the elements of a system, i.e. it is primarily concerned with what is in the box.

These two methods will compared and contrasted throughout this paper, with the focus on the Black Box or classical method, and the results documented.

3. The History

In 1961 a letter was delivered to the house of Dr. James Lovelock, it was from the director of the NASA space flight operations and was an invitation to be an experimenter on its first lunar instrument mission.

He began work in NASA's Jet Propulsion laboratory but soon moved into designing sensitive instruments to analyse the atmosphere of other planets. With an academic background in biology and medicine, he began designing experiments to detect life.

During this period Dr. Lovelock was visited by a philosopher called Dian Hitchcock, and, after many talks on the best way to detect life on distant planets, they published a paper suggesting that life on other planets would be likely to use the atmosphere and oceans as conveyors of raw materials. This would result in a change for the atmosphere, thus leaving it very different from that of lifeless planets.

By using infra- red telescopes, the atmosphere of other planets may be analysed from the Earth, showing that none of the planets in the solar system are fit for life. But it was only when an imaginary one of these telescopes was trained on the Earth that Dr. Lovelock really started to think about Gaia's existence. [1]

"Viewed from the distance of the moon, the astonishing thing about the Earth, catching the breath, is that it is alive..." [2]

3.1. Gaia theory/hypothesis

The Gaia hypothesis began life 1965 as an idea by Dr. Lovelock while he was working in the Jet Propulsion Laboratory. Then after collaborating with Lynne Margulis, the idea developed and was published in the Tellus and Icarus journals.

The Gaia hypothesis (now Gaia theory) states that the Earth is a system able to regulate its temperature and chemistry as a consequence of the organisms upon it [3]. These organisms are actually microbes. They keep the Earth in a state that is fit for life.

Many Biologists and other scientists at the time were not so convinced about the Earth being considered 'alive'. But with the use of a computer simulation, later called Daisyworld, Lovelock showed that a simplistic view of the Earth's biosphere could be modelled using feedback loops, and that Gaia could indeed exist.

3.1.1. Daisyworld

The Daisyworld model is an attempt at showing how a system's temperature can be regulated from the organisms present upon it. It consists of a planet on which the only species are daisies of various colours (albedos), and a sun. The result of the model is that the two species of daisy prolong life past the point at which it would end if the Gaia theory were not true. That is to say the Daisies, by their own interaction, live past the point where, with no life, the temperature of the planet would exceed their maximum living temperature.

4. The Project

The following description is taken from [4]

James Lovelock produced his Daisyworld model to demonstrate that life and the plant could interact to their mutual advantage. The aim of this project is to produce a Windows based implementation of Daisyworld. The program will begin with the basic two daisy species model, will then introduce extra species, then spherical geometry, then plagues and finally add foxes and rabbits.

As this project is to be done by a group of two, the decision was made that one member would work on the mathematical model and one on the Cellular Automaton model. This paper will focus on the mathematical model.

4.1. Proposal

For this project we set out to first compare and contrast two techniques that can be used for population modelling and then extend whichever one was most suitable. From reading the above synopsis of the classical Daisyworld one may not at first see the limitations and problems. Many of these limitations lie within the premise that the simulation is not particularly representative of the Earth in its original state. The Earth is not a flat plane, therefore not a constant temperature over the entire surface, and more than two species of life exist.

This project then is more aimed at showing which model is the better, for simulating the Earths biosphere. By adding different species and more realistic physics into Daisyworld, and creating a Cellular Automaton based model to do the same, the hope is that a clear distinction will appear between the two.

4.1.1. The Mathematical model

The mathematical model follows directly in the footsteps of Daisyworld's creator James Lovelock. Using a simple differential equation, the population of a particular species of daisies, can be calculated purely from the energy given by the sun and the amount of fertile ground that is available. A simple quadratic equation defines a beta function (simple parabola), which is the fitness for growth of a daisy species (colour/albedo). This value eventually determines the sign of the change in population, and is used in the equation to calculate the new population.

4.1.1.1. What Happens

In the simplest case Daisyworld is a flat plane with a flat plane sun shining directly down on it. Two species of daisy exist on the planet, black and white, though they are not pure black and white. A fitness function defines how well daisies may grow between a certain upper and lower limit of temperature, Daisies grow best at the mid point. The sun is heating up over time, just like our sun, so the 'earth' is slowly getting hotter, and once the minimum temperature is reached daisies may begin to grow.

To understand this initial state one must partake in a thought experiment, assume for a moment, that a black and a white daisy grow. Now, as earlier stated the only energy source is temperature, and a daisies temperature will be dependant on its albedo (colour/reflectivity). The black daisy will absorb the light energy causing it to heat up, where as the white one will reflect the heat and cool down. The initial state for daisy world is therefore dominated by black daisies.

4.1.2. The Cellular Automaton model

The cellular automaton model uses very much the same physics as the mathematical one, but deals with the probability of a daisy growing dependant upon the temperature at its point on the plane (in the simple model) rather than how many daisies should exist for that temperature (as done in the mathematical model). These probabilities are based on the original differential equations.

4.1.2.1. What Happens

In the simplest model Daisyworld is represented as a grid (the sun is still planar), each element of the grid may grow one daisy, and the probability of growing a daisy is governed by temperature. Once a particular daisy has grown, a probability associated with the beta value determines the life time of that daisy. The presence of this daisy also increases the probability of the same colour daisy growing in the cell neighbourhood.

Once again if one partakes in the earlier thought experiment then the same initial state is reached.

4.2. Overall idea/end product

The overall idea was to create a population modelling language/tool for use by biologists or undergraduates. This could be implemented as a simple click and drag GUI where every element of the biosphere is an object that can be attached to the world, very much like simulink in Matlab. The creation of a library of functions or classes that could be used within C++, making it more of a programming language than a tool was another possibility.

5. Research

5.1. Block diagrams

As the mathematical model is so heavily based in Cybernetics it can easily be represented in a block diagram with a feedback loop. This also eases the implementation as each loop may be represented by a function, and a loops place in the diagram will determine the order to call the functions and the parameters to pass.

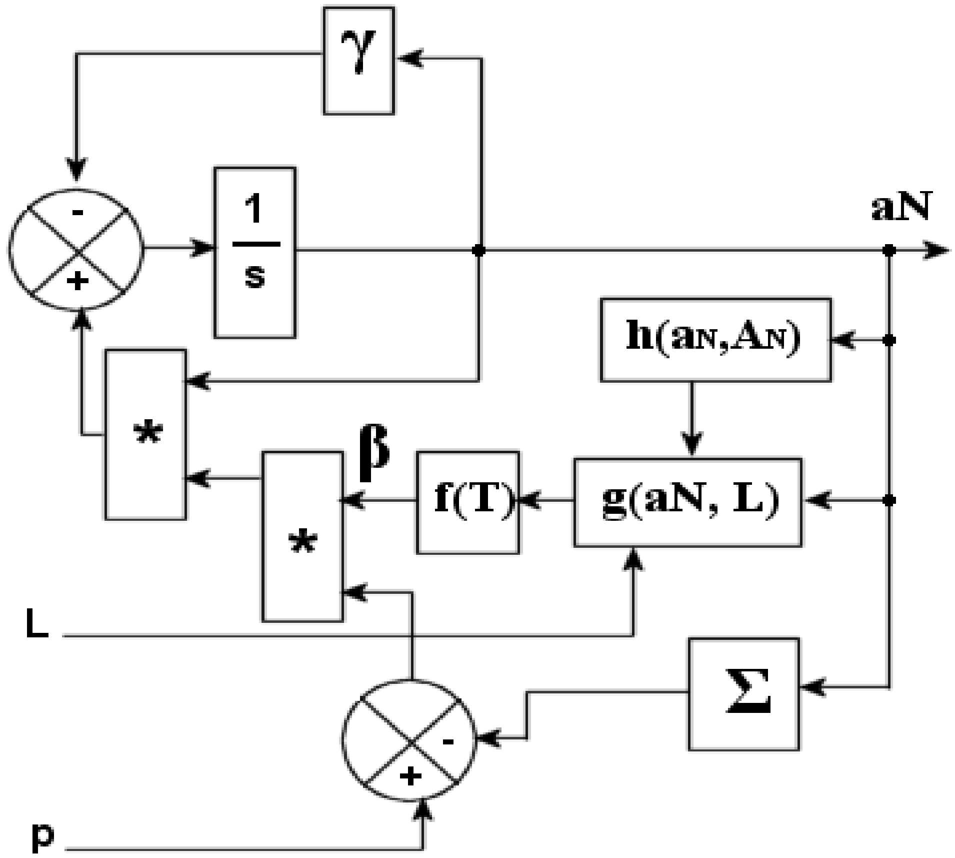


Figure 1 A block diagram of an N-Species Daisyworld adapted from a diagram taken from [5]

The following tables explain the functions and symbols, respectively, used in the above figure.

Table 1 Descriptions of the functions used in the block diagram of N-Species Daisyworld

Function	Meaning
$h(a_N,A_N)$	Calculating the average albedo of the planet (albedo * area).
$g(a_N, L)$	Calculating global and in turn local temperature for a daisy species.
f(T)	Beta function, calculating the fitness for growth
1/s	Laplacian integrator, to calculate the new population
Σ	Summation of all the daisy populations to calculate the remaining fertile ground

Table 2 Description of the symbols used in the block diagram of N-Species Daisyworld

Symbol	Meaning
L	Solar Luminosity
p	Proportion of fertile ground (usually 1)
β	Beta value (fitness for growth)
γ	Death rate
a_N	Area covered by daisy species N

6. Modelling

In this section the modelling techniques will be discussed in detail, including equations and results. Unexpected results and outcomes may also be discussed, as well as improvements on the original Daisyworld model.

6.1. Mathematical model

In the mathematical model, differential equations are used to calculate the change in population of the daisy species.

The general equation for the change in population of a daisy species, taken from [3] is;

$$da_N / dt = a_N(x\beta - \gamma) \qquad (1)$$

The symbols are described in the table below

Table 3 Descriptions of the symbols used in eqn. 1

Symbol	Meaning
a_N	The albedo of a daisy species
x	The current percentage of fertile ground
β	The fitness for growth of a daisy species
γ	The death rate

The temperature of the world can be found by equating emitted and absorbed radiation, as in [3], giving;

$$\sigma(T_e + 273)^4 = SL(1 - A) \qquad (2)$$

Or

$$T_e = \sqrt[4]{\left(\frac{SL}{\sigma}(1 - A)\right)} - 273 \qquad (3)$$

The following table describes the symbols used.

Table 4 Descriptions of the symbols used in eqn. 2 and eqn. 3

Symbol	Meaning
T_e	Temperature of the planet
S	Constant having units of flux
L	Dimensionless Luminosity of the sun
A	Global Albedo
σ	Stefan's Constant

Global Albedo is the average albedo of the planet, defined in [3] as thus;

$$A = a_g * A_g + a_b * A_b + a_w * A_w \tag{4}$$

for the two species case, or more generally;

$$A = a_g * A_g + \sum_{i=1}^{N} a_i * A_i \tag{5}$$

The following table describes the symbols used.

Table 5 Descriptions of the symbols used in eqn. 4 and eqn. 5

Symbol	Meaning
A_g	Albedo of the Earth (ground)
a_g	Area of the Earth (fertile ground)
A_i	Albedo of a daisy species
a_i	Area covered by daisy species

All the energy for life comes from the sun as heat; the local temperature (temperature for a species of daisies) is given in [3] as;

$$(T_N + 273)^4 = q(A - A_N) + (T_e + 273)^4 \tag{6}$$

Or

$$T_N = \left[\sqrt[4]{q(A - A_N) + (T_e + 273)^4}\right] - 273 \tag{7}$$

The following table describes the symbols used.

Table 6 Descriptions of the symbols used in eqn. 6 and eqn. 7.

Symbol	Meaning
T_N	Local temperature of a daisy species
T_e	Temperature of the planet
A	Global Albedo
A_N	Albedo of a daisy species
q	Positive constant

The local temperature is then used to compute the fitness for growth, or beta value, from [3];

$$\beta_N = 1 - 0.003265(22.5 - T_N)^2 \tag{8}$$

Finally the resulting population change must be integrated to get the new population; this can be done via Euler integration;

$$A_N = A_{N-1} + {da_N}/{dt} * stepTime \tag{9}$$

Or, via fourth order Runge- Kutta integration,

$$K_1 = {da_N}/{dt} \tag{10}$$

$$K_2 = {da_N}/{dt} + (K_1 * {stepTime}/{2}) \tag{11}$$

$$K_3 = {da_N}/{dt} + (K_2 * {stepTime}/{2}) \tag{12}$$

$$K_4 = {da_N}/{dt} + (K_3 * stepTime) \tag{13}$$

$$A_N = ({stepTime}/{6}) * (K_1 + 2 * K_2 + 2 * K_3 + K_4) \tag{14}$$

6.1.1. Simple 2 species

As described in the introduction, in the simple two species case Daisyworld is a flat plane with a planar sun above it. This is found to be a good approximation for the relationship between the Earth and Sun, as, due to their size, they act like parallel planes for small areas.

The species of daisy are as follows;

Table 7 Descriptions of the species of daisy used in the two species model

Colour	Albedo
Black	0.25
White	0.75

Below are the resulting population and temperature graphs for this model.

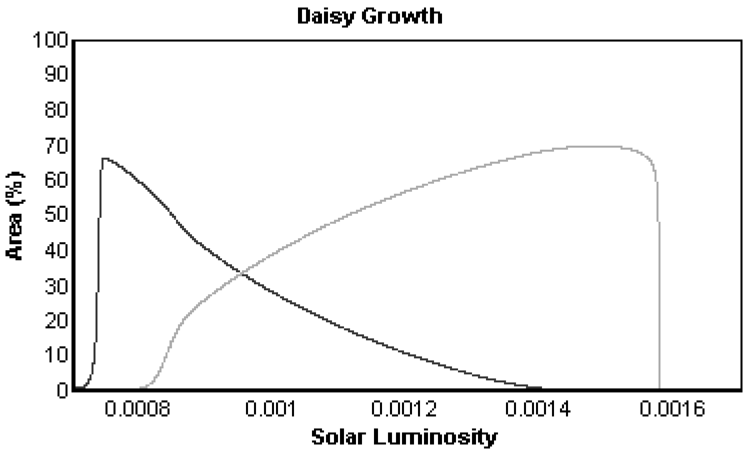


Figure 2 Population graph for two species Daisyworld

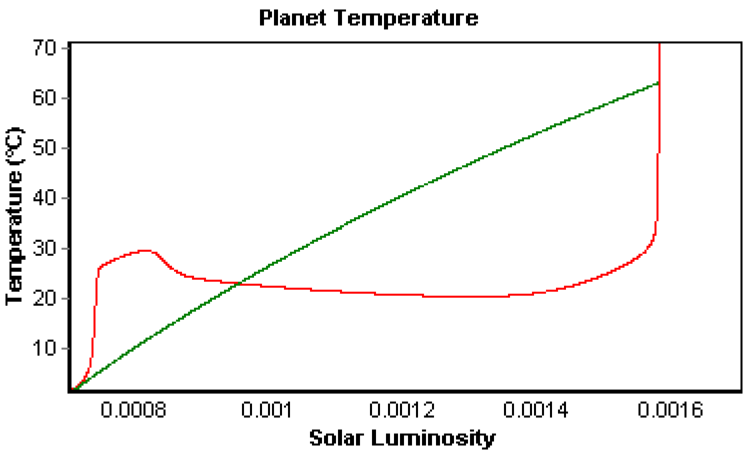


Figure 3 Temperature graph for two species Daisyworld

6.1.1.1. Pure black and white

The general two species model uses impure black and white daisies. This is because it is very unrealistic to have a perfect absorber and a perfect reflector. But for the sake of interest the simulation was run using these pure shades. One would assume that this would mean that the simulation would never die, a perfect reflector absorbs no heat, but one must remember that the daisies do not take hold of the entire planet (even if it is most of it) due to the death rate. This allows the planet to heat up and eventually die, though the temperature may be well in excess of two- hundred degrees by then.

The resulting population and temperature graphs for the pure species are shown below.

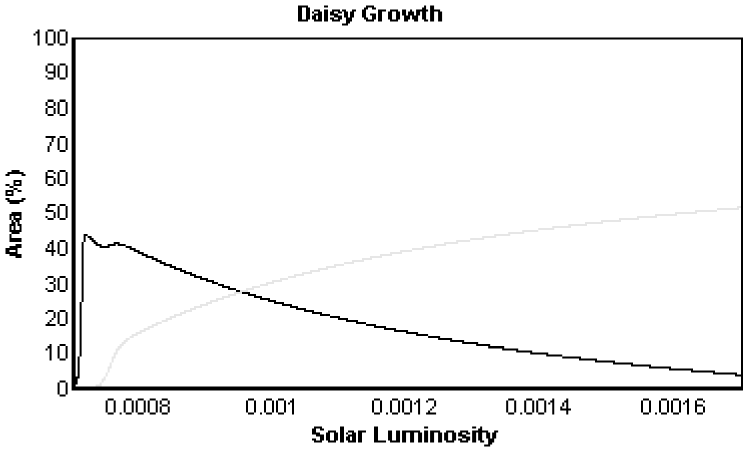


Figure 4 Population graph for pure two species Daisyworld

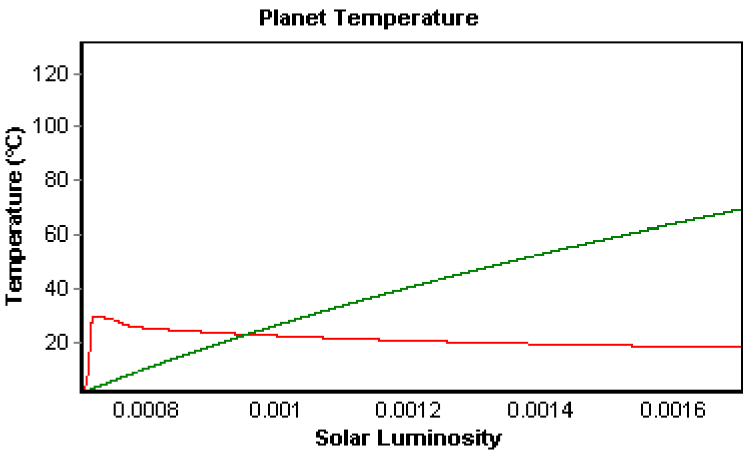


Figure 5 Temperature graph for pure two species Daisyworld

As is clear from Figure 5, perfect reflectors and absorbers allow Daisyworld to carry on functioning well past its previous impure maximum temperature.

6.1.2. N-species

The N-species simulation creates many daisy species each with a different albedo, these are usually limited between 0.25 (black) and 0.75 (white).

The algorithm used to spread the daisy albedos linearly between 0.75 and 0.25 is shown below.

$$A_N = N\left\{\left[\frac{1}{N-1}\right]\left[MAX - MIN\right]\right\} + MIN \qquad (15)$$

The following table explains the symbols used in eqn.15.

Table 8 Descriptions of symbols used in eqn. 15

Symbol	Meaning
A_N	Albedo of species N
MIN	Minimum value of albedo allowed
MAX	Maximum value of albedo allowed
N	Index of species

The temperature and population graphs for a 5 species world are shown below.

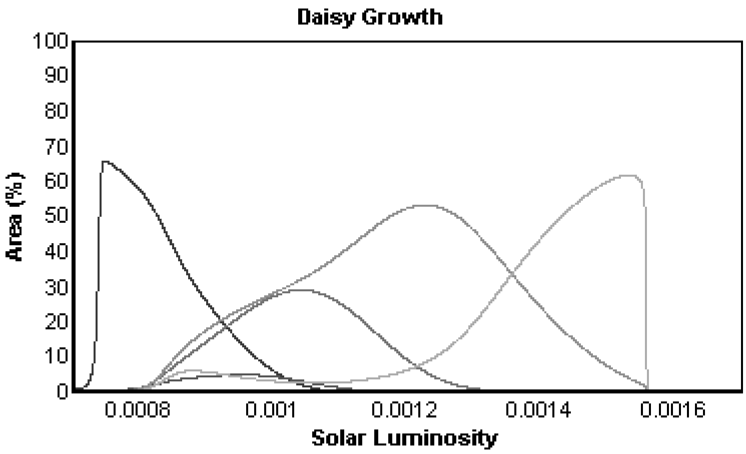


Figure 6 Population graph for 5 species Daisyworld

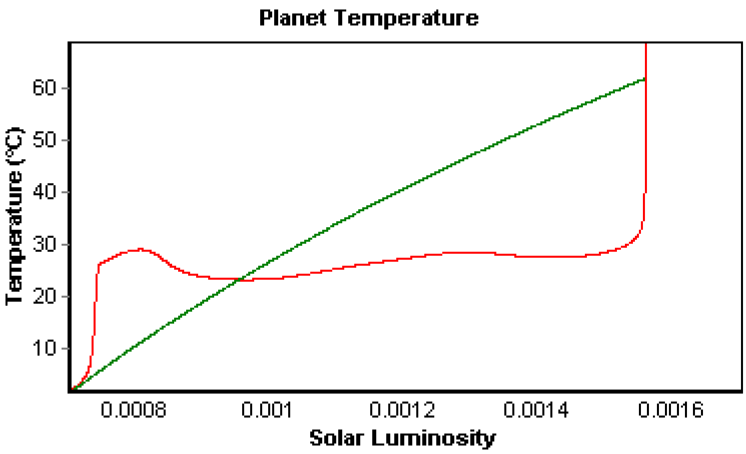


Figure 7 Temperature graph for 5 species Daisyworld

N-species was originally a test by Dr. Lovelock of Lotka's comment, that diversity augmented instability, taken from his observations about foxes and rabbits. Even from a non scientific viewpoint this statement seems flawed, why would the Earth be so diverse in species if it decreased the stability of the biota.

Daisyworld agreed, the more varied the species are the more stable the system became.

6.1.2.2. Plagues

To prove that more species increased stability (up to a point) Dr. Lovelock implemented plagues in Daisyworld to see how well they recovered [6]. A plague removes a certain percentage of the population of one species of daisy, or the total population. The following results show a percentage (plus a small random element) of the whole population being removed periodically.

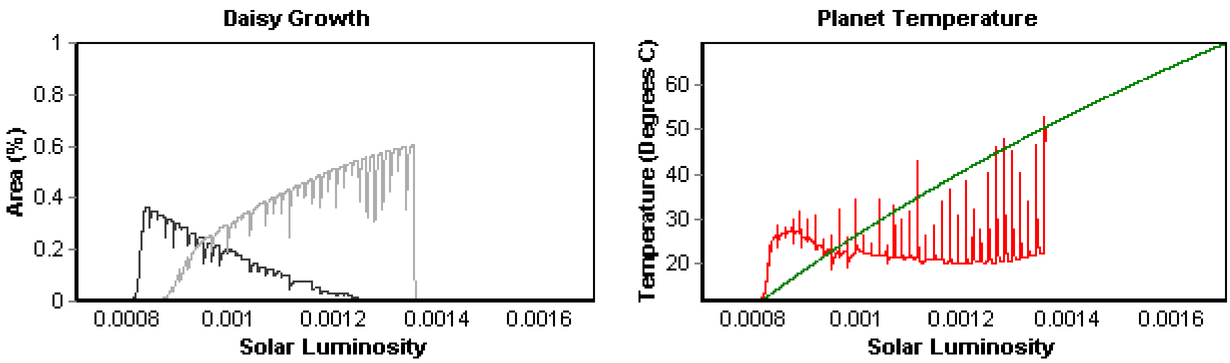


Figure 8 Population and temperature graphs for a 2 species Daisyworld with a plague that removed 20% of the total population

From the above figures it is obvious that a plague kills the simple two species system earlier, but in the following graphs of the three species system the period of life is slightly extended.

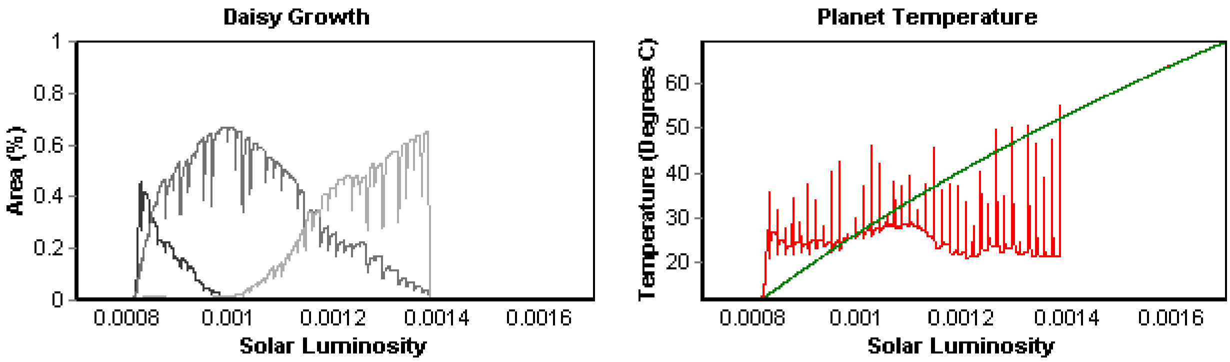


Figure 9 Population and temperature graphs for a 3 species Daisyworld with a plague that removed 20% of the total population.

6.1.2.3. Grey daisies

Grey daisies are an interesting issue when automatically spacing an unknown number of daisy albedos between 0.25 and 0.75. A 'grey daisy' has same albedo as the Earth (0.5), and acts almost like a plague, by preventing non grey daisy species from having some area, while absorbing/reflecting as much heat as the bare ground does.

In effect it dynamically reduces the amount of fertile ground, but not the overall amount of area.

After much deliberation it was decided that grey daisies would be allowed in Daisyworld, as equivalent situations may be found on the Earth.

6.1.3. Spherical Geometry

Daisyworld is meant to be a simplistic model of the Earth's biosphere. But with all the previous models it has been assumed that the Earth is a flat plane that absorbs sunlight equally over its surface. This is far from true, so a method was created for constructing a Daisyworld that behaved as if it were spherical.

First of all it is assumed that the Earth and the sun are perfectly aligned with one another, so that the centre of the equator is the hottest point on the Earth. This idea can then be extended into the concept of 'rings' of equal temperature circling this point. The following image taken from [7] shows the concept graphically.

Figure 10 Graphical representation of the 'ring' concept

Using the idea of each ring being an even temperature all over, one could see that a spherical Daisyworld could be modelled by many separate planar planets, shown in Figure 11.

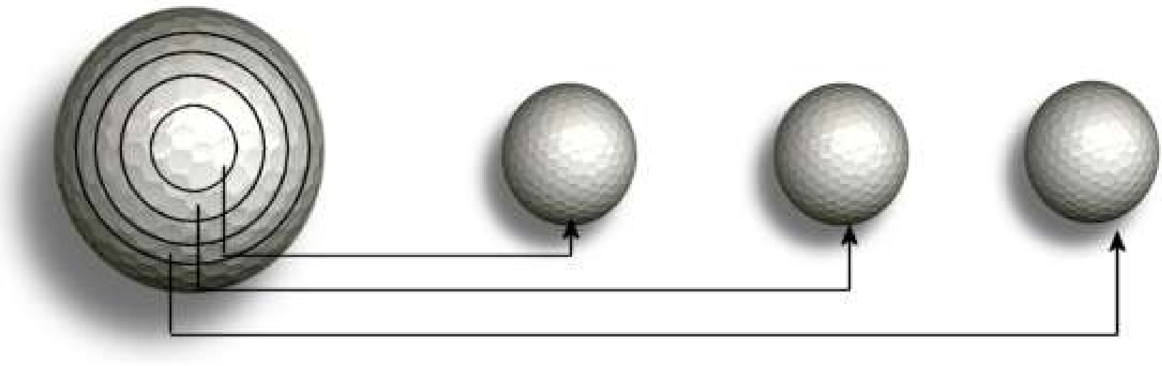


Figure 11 Graphical representation of each ring being a planet

Each of these planets acts as if it were a single Daisyworld, only the starting value of luminosity is changed to simulate the planets being at different distances from the sun. The number of planets may be chosen by the user at the beginning of the simulation.

The final stage to create a spherical Daisyworld is to allow the populations interact. This is done by summing the populations of each species of daisy and averaging them over the number of rings chosen. These are now the new populations of each ring (planet).

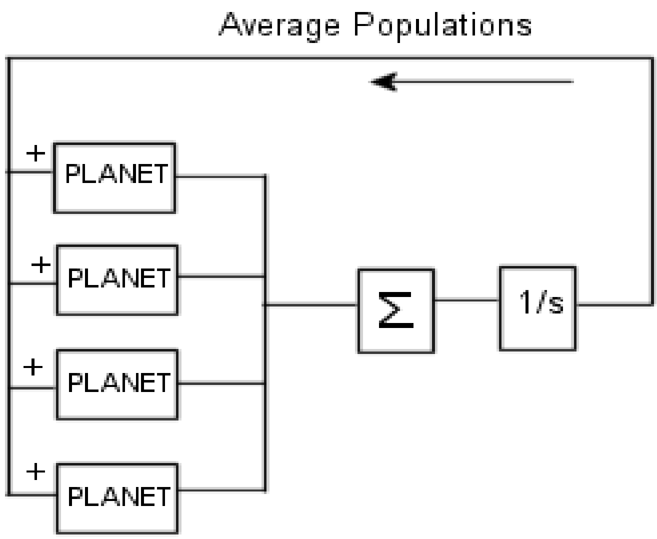


Figure 12 A simplified block diagram of feedback in the spherical model

Below are some figures showing Daisyworld being run with and without spherical geometry.

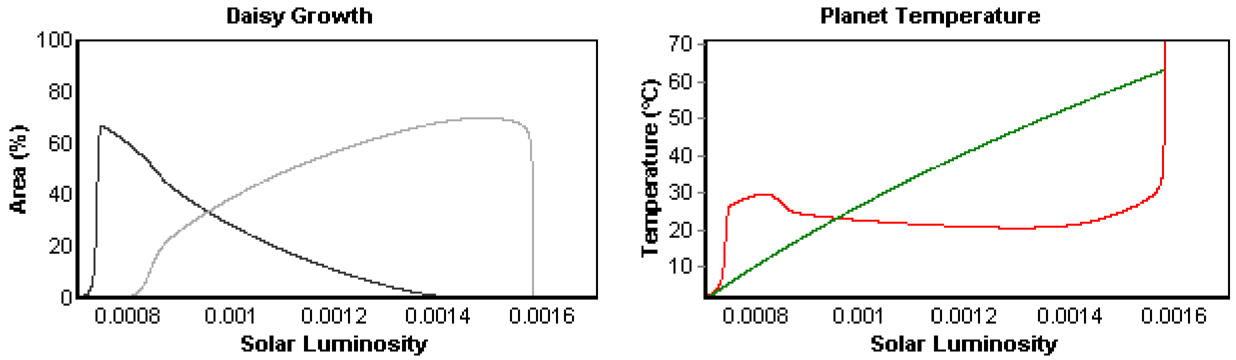


Figure 13 Population and temperature graphs for a simple 2 species planar Daisyworld

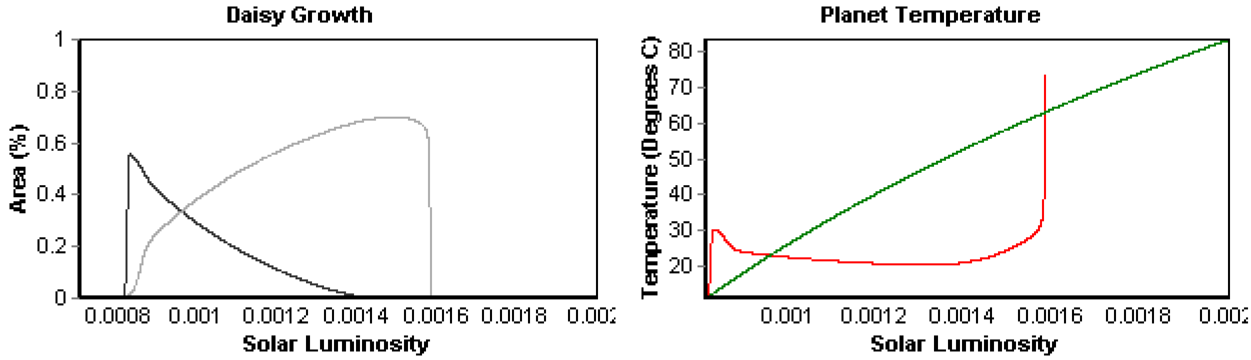


Figure 14 Population and temperature graphs for a 2 species Spherical Daisyworld using 100 planets

From these graphs it is clear that spherical geometry has very little effect on Daisyworld apart from to reduce the time period between black daisies growing and white daisies growing. This would seem a little counter intuitive, but it may be due to the fact that even though the outer most (coolest) planet may be teaming with life, that life is being averaged over all the planets and therefore being greatly reduced.

6.1.3.1. Correctness

Though the method described above is conceptually correct one might wish there was some weighting in the rings as to what the new population should be, rather than just and average. As, when it is hot, more white daisies will grow at the equator (inner rings) and more black daisies will grow at the poles (outer rings). This is more a problem of trying to split an 'analogue' simulation into discrete parts, and then recombine it to regain the analogue.

This problem has been tackled differently by using a function to weight the temperature as if the plane were a sphere [8]. The results of using this method are said to differ only subtly from the results of the simple planar model, which would seem a little counter intuitive, as the poles of our Earth differ massively in temperature to the equator.

6.1.4. Foxes and Rabbits

The most common example of a population model using interacting species is the famous Foxes and Rabbits example derived by Lotka and Volterra (Lotka- Volterra) [9] [10].

Rabbits eat the grass (of which there is assumed to be an infinite amount), and the foxes eat the rabbits. When the foxes eat too many rabbits, their population crashes, this causes a sharp rise in rabbit population, and in turn a rise in fox population. The model is generally chaotic but is stable in two cases that are inherent in the ratio of the initial conditions to one another.

That is to say, unless the model is started in one of the two stable conditions, it will become chaotic and crash (both the rabbit and fox population cycle out of control with increasing amplitude) as seen in Figure 15 and Figure 16 with the parameters listed in Table10.

The first stable condition is when the starting conditions are set to give no change in population for either species from the initial values. This is seen as a single point in Figure 17, and a line in Figure 18. Table 11 lists the initial values used.

The second stable state is a cyclic one where the two populations describe a sine wave, this can be seen in Figure 19 and Figure 20 with parameters listed in Table 12.

The differential equations used to model this relationship are shown below.

$$dR/dt = a * R - b * R * F \tag{16}$$

$$dF/dt = c * R * F - d * F \tag{17}$$

Table 9 defines the symbols used in eqn. 16 and eqn. 17.

Table 9 Descriptions of the symbols used in eqn 16 and eqn 17

Symbol	Meaning
R	Rabbit population
F	Fox population
a	Rabbit growth rate
b	Rabbit death rate
c	Fox growth rate
d	Fox death rate

A simple stand alone foxes and rabbits model was constructed before integration into Daisyworld.

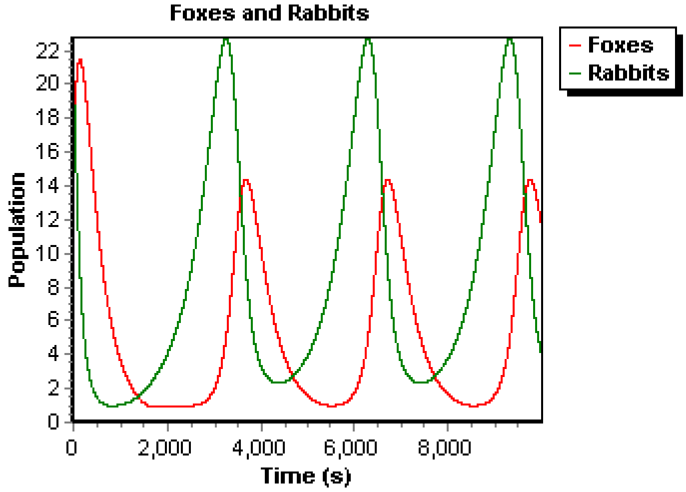


Figure 15 A plot of fox and rabbit populations vs. time for the chaotic case

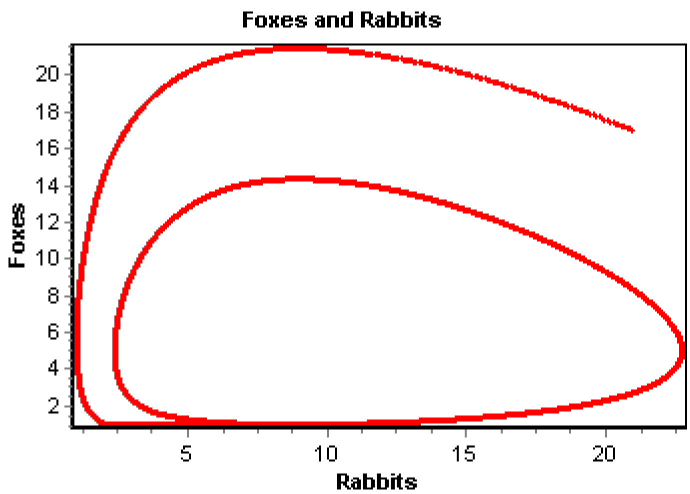


Figure 16 A plot of fox vs. rabbit populations for the chaotic case

Table 10 Parameters used to create Figure 15 and Figure 16

Parameter	Value
F	17
R	9
a	20
b	4
c	3
d	27

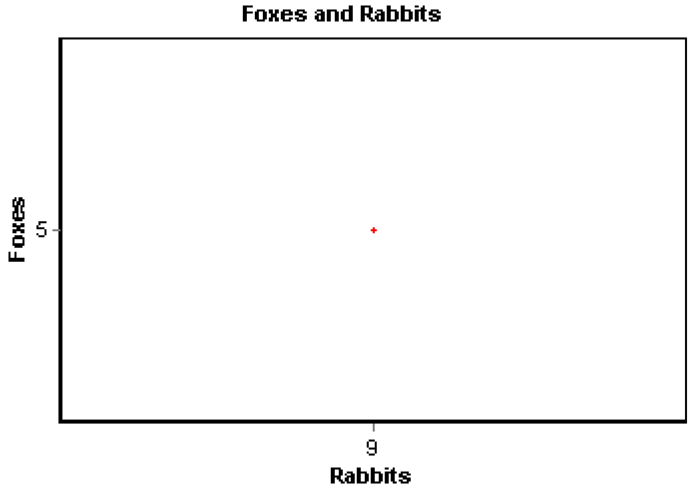


Figure 17 A graph showing fox population vs. rabbit population for the stable case

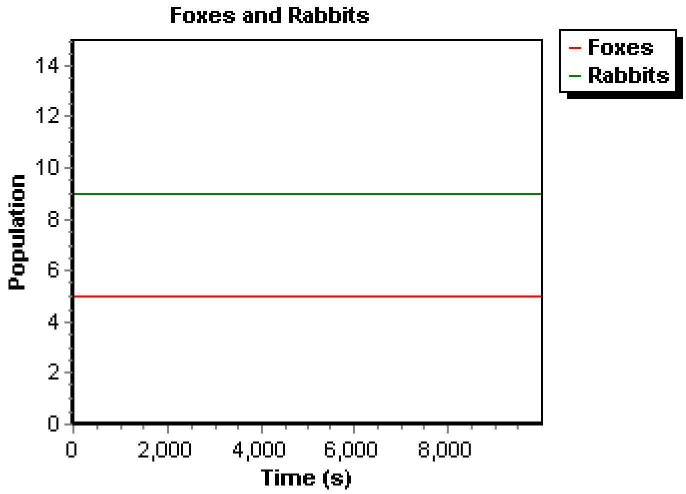


Figure 18 A plot of fox and rabbit populations against time for the stable case

Table 11 List of parameter values for the stable non cyclic state

Parameter	Value
F	5 (a/b)
R	9 (d/c)
a	20
b	4
c	3
d	27

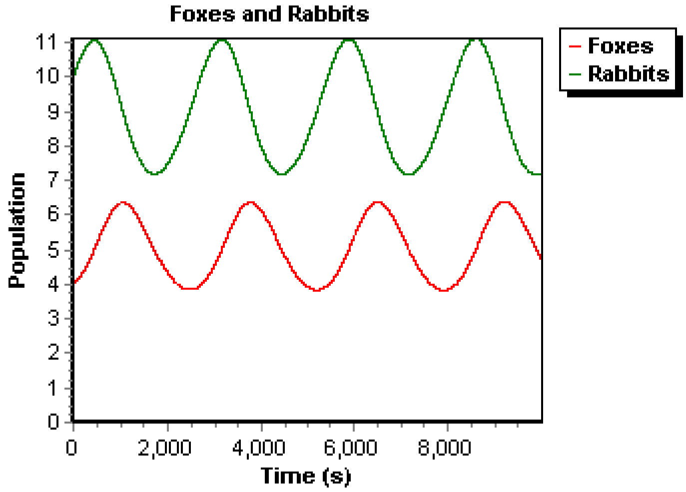


Figure 19 Population graph for foxes and rabbits

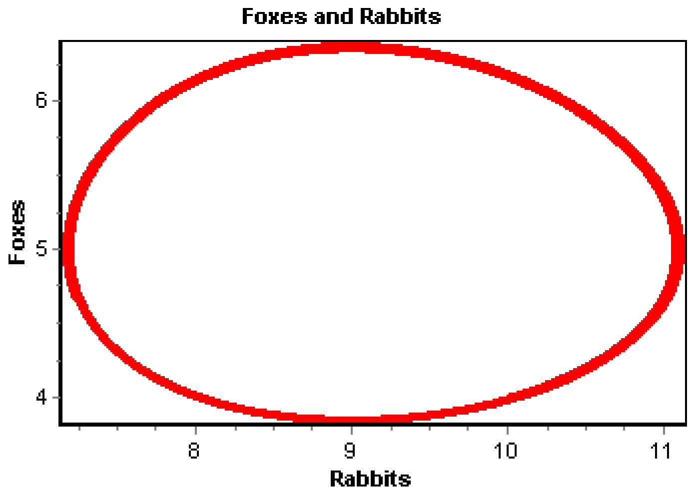


Figure 20 Graph showing fox population vs. rabbit population

Table 12 The parameters used to create Figure 19 and Figure 20

Parameter	Value
F	10
R	4
a	20
b	4
c	3
d	27

6.1.4.1. Integration Techniques

The fox and rabbit population model was originally implemented using Euler integration, for speed. It was later discovered that Euler integration is too inaccurate for the fox and rabbit simulation, and so fourth order Runge- Kutta was implemented in place of it.

6.1.5. Foxes and Rabbits in Daisyworld

Integrating foxes and rabbits into Daisyworld is considered a non-trivial task by many of the people working on Daisyworld based population models, but has been accomplished before by Lovelock and others.

To start with one must devise two equations that relate rabbits and daisies, like those below taken from [11]. The symbols used are explained in Table 13.

$$dH/dt = aH - \lambda HP \tag{18}$$

$$dP/dt = \mu HP - \beta P \tag{19}$$

Table 13 Description of the symbols used in eqn. 18 and eqn. 19

Symbol	**Meaning**
dH/dt	Change in prey population
dP/dt	Change in predator population
H	Current prey population
P	Current predator population
a	Fitness value for prey
μ	Fitness value for predator
λ	Death rate for prey
β	Death rate for predator

The equations look similar to those used to relate foxes to rabbits, and so they should as this is another predator prey model. Unfortunately one does not wish to complicate Daisyworld further than necessary to accomplish the integration, so more work must be done now to allow less work to be done later. This involves modifying the original equations, used in the Daisyworld and in the Foxes and Rabbits simulation, to include the important parts from the previous equations.

The assumption made by the following equations is that, though the predator and prey may affect one another's 'fitness for growth', this is either too small an effect, or can be taken into account by altering the death rate. This simplifies the equations and results in the following. Table 14 explains the symbols used.

$$da_N/dt = a_N\left(x\beta - (\gamma + R*\lambda)\right) \tag{20}$$

$$dR/dt = a * R - [(b * R * F) + ((1 - a_N) * \mu)] \quad (21)$$

Table 14 Descriptions of the symbols used in eqn. 20 and eqn. 21

Symbol	**Meaning**
λ	Constant factor determining how many daisies a rabbit eats.
μ	Constant factor determining how the population of daisies affects the population of rabbits

The following figures depict the results using different values of μ and λ to find a stable equilibrium; each set of results has a table of the values used.

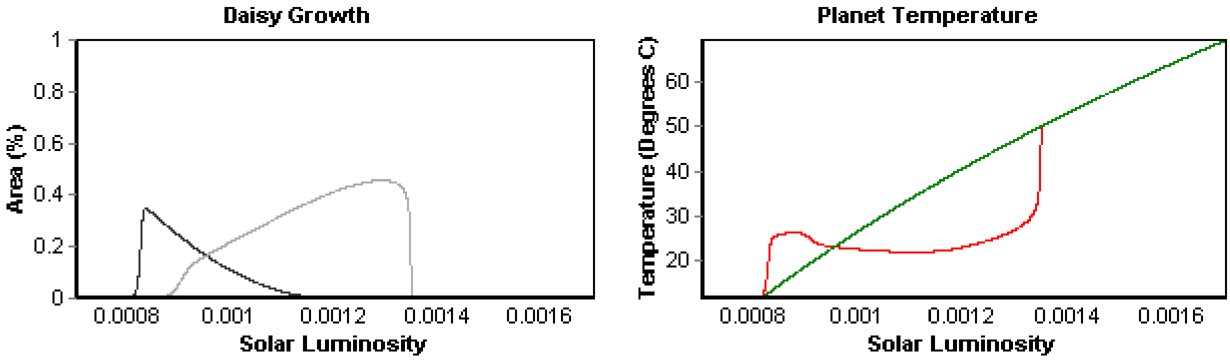


Figure 21 Daisy population graph and daisy temperature graph with foxes and rabbits

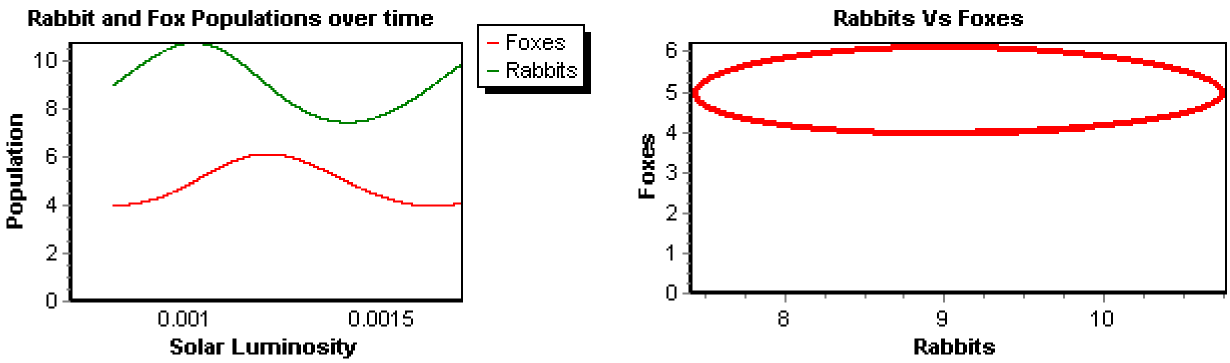


Figure 22 Fox and Rabbit population graph and fox vs. rabbit population graph with daisies

Table 15 Parameters used in the above figures.

Parameter	**Value**
Initial Fox population	4
Initial Rabbit population	9
λ	0.03
μ	1

These results indicate good temperature regulation by the daisies and a stable population of foxes and rabbits.

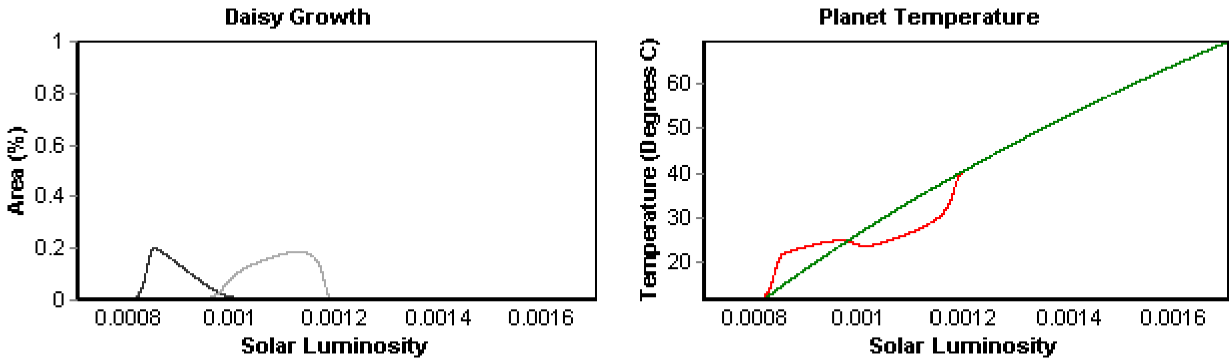


Figure 18 Daisy population graph and daisy temperature graph with foxes and rabbits

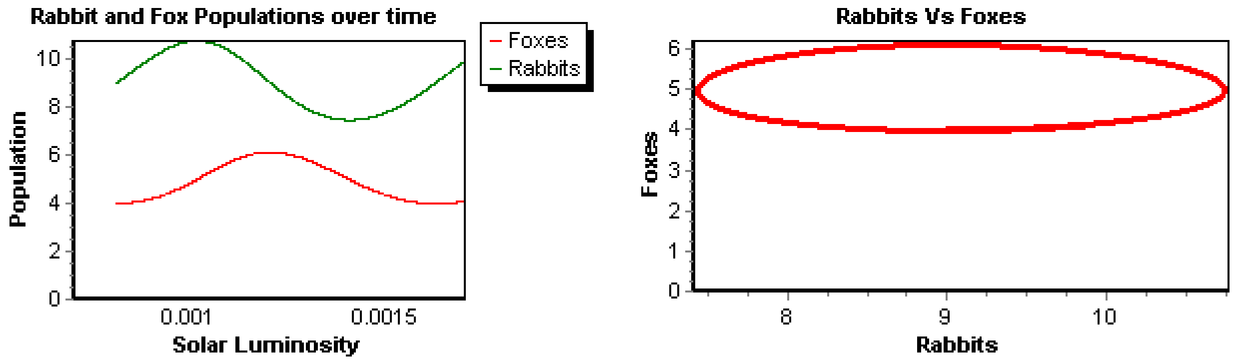


Figure 19 Fox and Rabbit population graph and fox vs. rabbit population graph with daisies

Table 16 Parameters used in the above figures.

Parameter	Value
Initial Fox population	4
Initial Rabbit population	9
λ	0.05
μ	1

These results indicate that the rabbits have eaten too many daisies; this in turn reduces the ability to regulated temperature causing an early end to life on the planet.

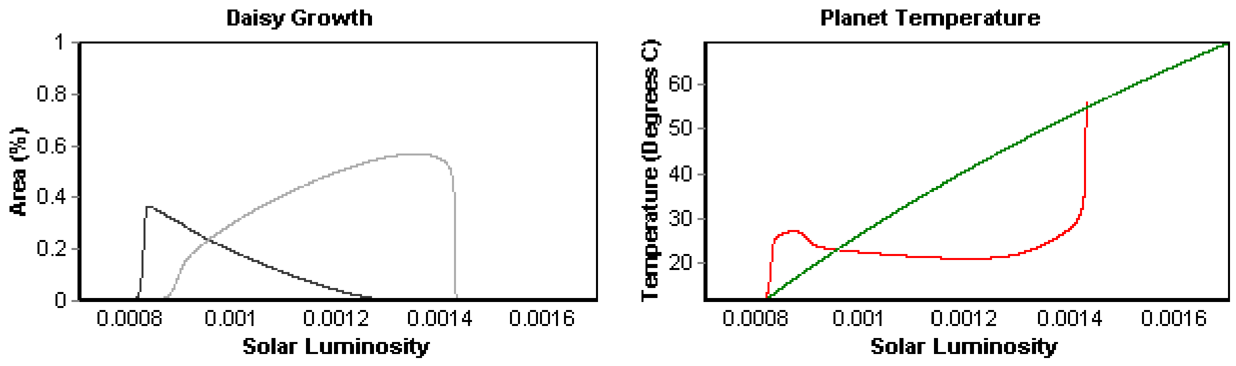


Figure 23 Daisy population graph and daisy temperature graph with foxes and rabbits

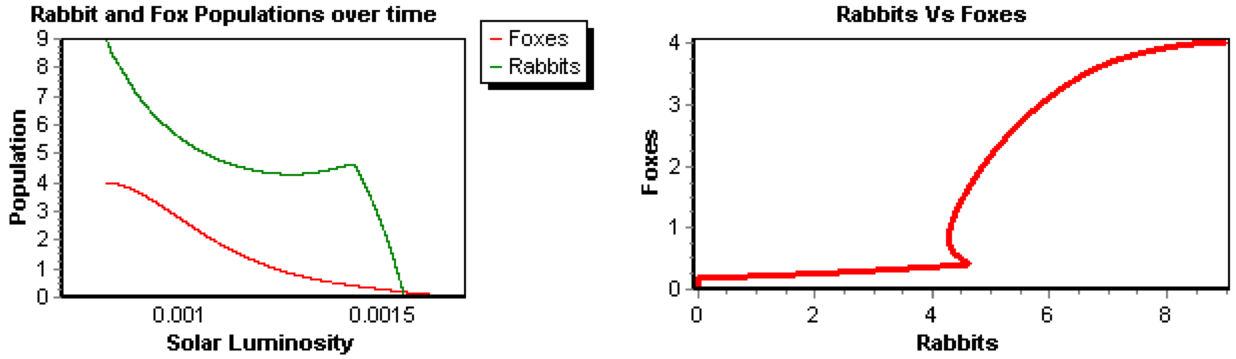


Figure 24 Fox and Rabbit population graph and fox vs. rabbit population graph with daisies

Table 17 Parameter values used in the above figures

Parameter	Value
Initial Fox population	4
Initial Rabbit population	9
λ	0.03
μ	160

These results show that the population of rabbits is too high for the daisies to sustain. The figure below shows the resulting Fox and Rabbit population graph with a smaller initial population of foxes and rabbits and slightly reduced μ value.

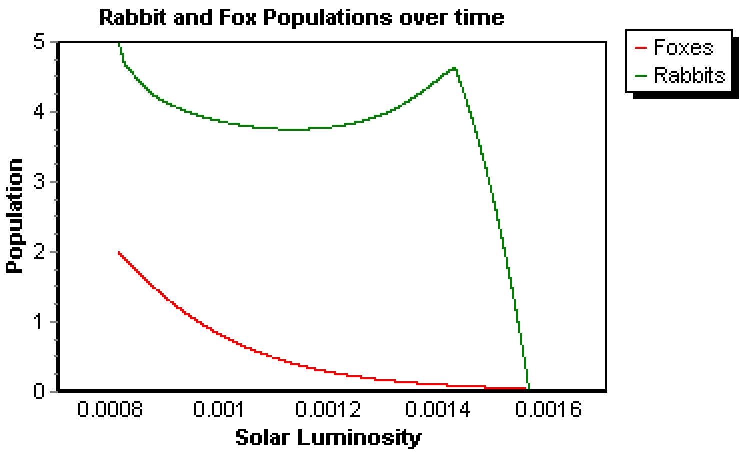


Figure 25 Fox and Rabbit population graph with daisies

All the source code and class diagrams for the above work can be found in appendix A and appendix B at the end of this report.

6.2. Cellular Automaton

In the Cellular model, the differential equations used for the mathematical model are converted into probability functions. All work in this section is taken from [5].

The simplest form for this purpose is shown below

$$a_{new} = a_{old} + G(a_{old}) - D(a_{old}) \tag{22}$$

Where G is a growth function, and D a death function.

The original equation (1) for the change in population can be rearranged like so.

$$a_{new} = a_{old} + x\beta a_{old} - \gamma a_{old} \tag{23}$$

Giving.

$$G(a_{old}) = x\beta a_{old} \tag{24}$$

$$D(a_{old}) = \gamma a_{old} \tag{25}$$

Now that probability equations have been derived, the task of determining the probability of events comes to pass. To accomplish this it is useful to understand the elements of each equation, these are documented in the table below.

Table 18 Descriptions of the symbols from the above equations, and their meaning in the classical model

Symbol	Meaning

x	The proportion of un- colonised ground available on the planet. It prevents a daisy species from growing on top of other daisies. That is to say x prevents growth when there is insufficient space.
β	The fitness value of a daisy species. This prevents the daisies from growing when conditions are unsuitable.
γ	The death rate of a species, a percentage dictating how many existing daisies die at each iteration.

For the Cellular Automaton model the meanings of the symbols need to be altered slightly, but their use will remain the same. The following table documents the new meanings.

Table 19 Descriptions of the new meanings of the symbols in the above equations

Symbol	New Meaning
x	x will now become a Boolean value indicating whether the cell in question is empty(1) or not(0).
β	Beta will still be a fitness function, but rather than being used for an entire species of daisy, it will only be used for the cell in question.
a_{old}	This is now a percentage of the neighbouring cells with species N. There is an imposed lower limit of 0.01 to allow growth to start on an unpopulated planet.
γ	This is simply the probability of death, but it must only be used on existing daisies so as to have no effect on new daisies.

This results in the following equations for the Cellular Automaton model.

$$P(\text{Growth of species n}) = x\beta a_n \quad (26)$$

$$P(\mathit{Death}\text{ of species n}) = \gamma \quad (27)$$

In the classical Daisyworld conduction between daisies happens only with the same species of daisy, in the Cellular Automaton model however conduction happens only between the neighbours of the cell in question (called the cells neighbourhood). So, in order to calculate the local temperature, a cell must look at the daisies around it.

6.2.1. Simple 2 species

For the simple two species case a square grid of cells (50 x 50) was set up, and the simulation run. A simple 4 x 4 neighbourhood was used and is shown in the figure below taken from [7].

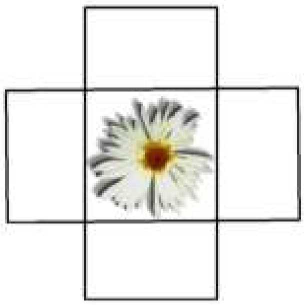
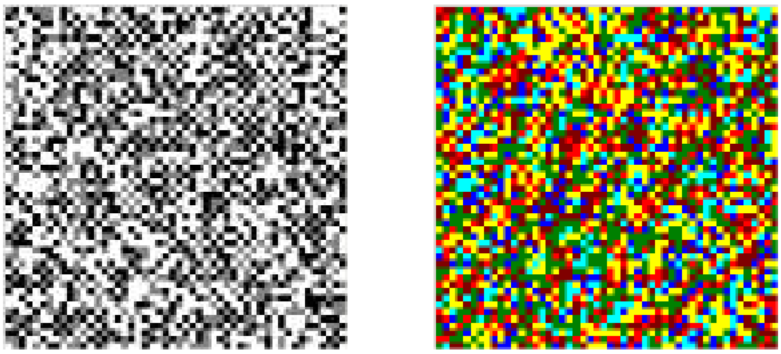


Figure 26 Neighbourhood used in simple 2-species Cellular Automaton

The resulting graphs are shown in the figures below.



(a) **(b)**

Figure 27 The population of the Cellular Automaton model in the visible (a) and the thermal (b) spectrums.

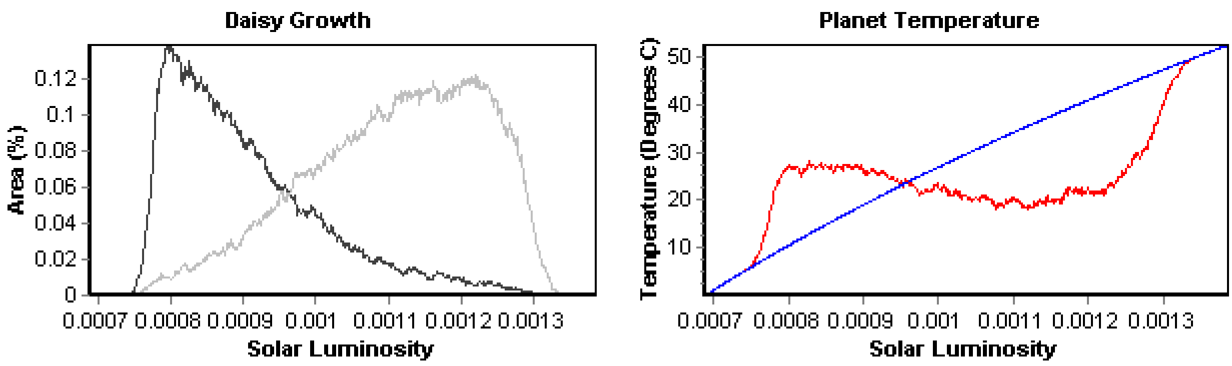
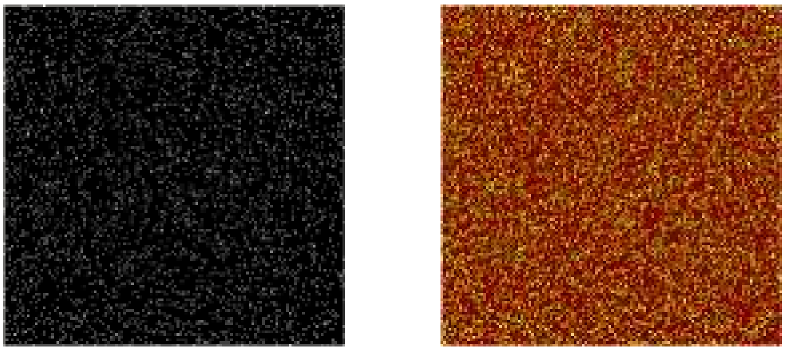


Figure 28 Population and temperature graphs for the Cellular Automaton model in the two species case with a 50 x 50 grid

The figures below are of the same simulation setup using a 100 x 100 grid; this new density removes a lot of the noise from the temperature and population graphs. The downside of this is that the computation time is increased by a factor of one hundred.



(a) (b)

Figure 29 The population of the Cellular Automaton model in the visible (a) and the thermal (b) spectrum using a finer grid.

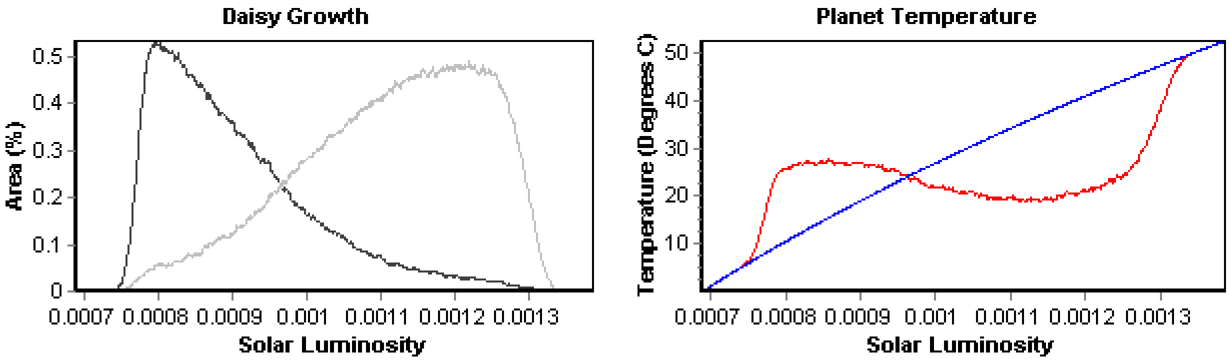


Figure 30 Population and temperature graphs for the Cellular Automaton model in the 2 species case with a 100 x 100 grid

6.2.1.1. Emergent Behaviours

As previously discussed Cellular Automaton models can be used to show that complex behaviours may come from a set of simple rules. On the following page are two examples of such complex behaviour and the rules that brought it about taken from ideas in [12].

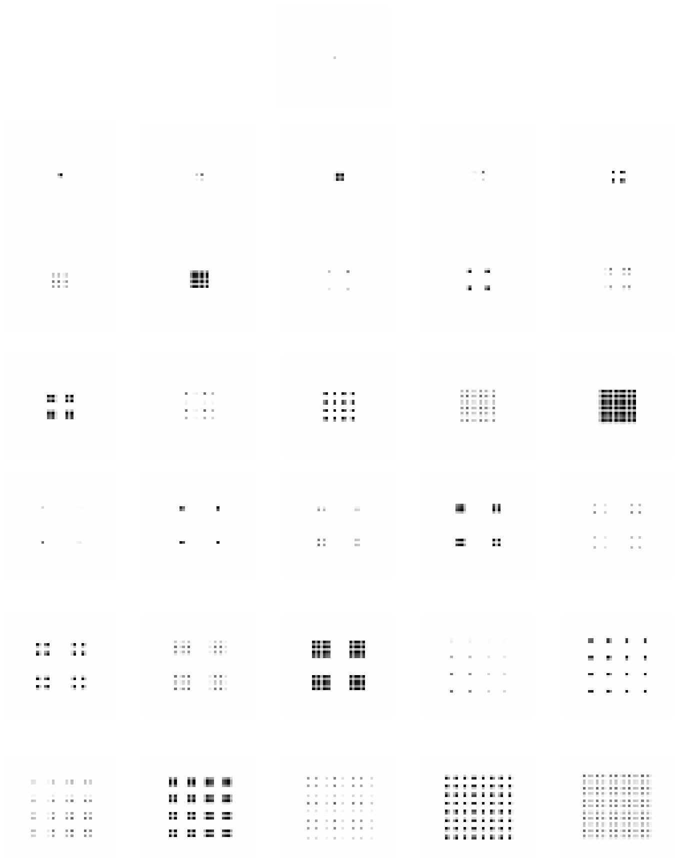
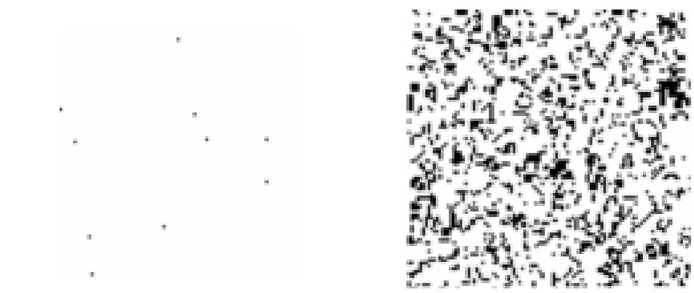


Figure 31 The 1st 30 iterations of a Cellular Automaton, initialised with a single black cell in the centre .

The rules used to create this are shown in the following table.

Table 20 Table of rules used to show emergent behaviour

Situation	Rule
If you have no black neighbours	stay as you are
If you have one black neighbour	turn black
If you have more than one black neighbour	turn white



(a) **(b)**

Figure 32 The same model as previously but initialised randomly (a), and ending with (b)

The simple 2-Species model showed some interesting emergent behaviour which

was completely unplanned. Due to the nature of conduction through a neighbourhood of cells, and the shape of the original neighbourhood, the daisies found a way to exist well past the usual end of the simulation.
If one looks at the neighbourhood once again it can be seen that the only cells considered when calculating the local temperature are those directly above, below, left and right to the cell in question. So if an occupied cell were to be in any of the diagonally available cells, they would not contribute to the temperature. This meant that the daisies could form a checkerboard like pattern with no conduction between themselves and keep the planet cool for a great deal of time.

6.2.2. *N-species*

The N-Species model used a new neighbourhood that removed the problems with conduction; the new neighbourhood is shown in the figure below. The species were distributed using the same method as the mathematical model as shown in eqn. 15.

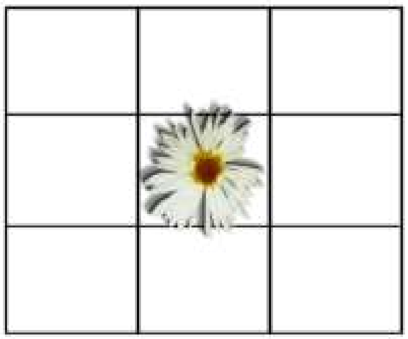


Figure 33 Neighbourhood used in the N-Species Cellular Automaton

The following figure shows the output of a 5 species Cellular Automaton with the grid size set at 100 x 100.

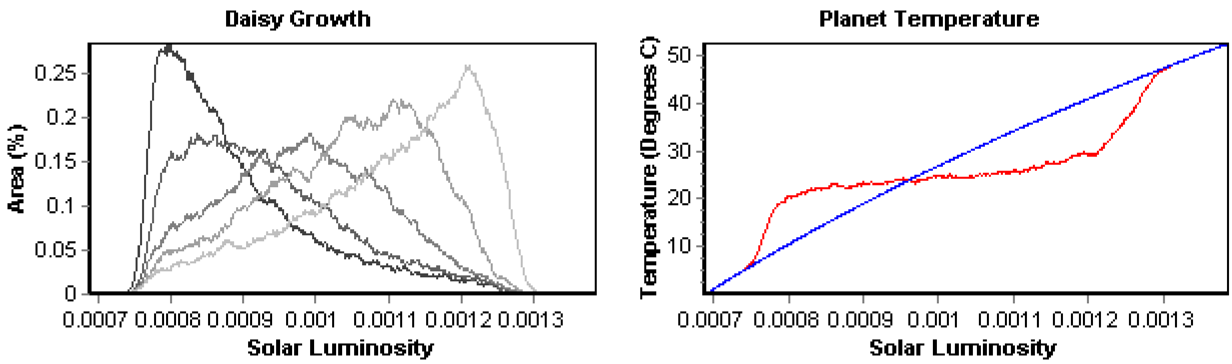


Figure 34 Population and temperature graphs for the Cellular Automaton model for the 5 species case using a 100 x 100 grid

6.2.3. *Spherical Geometry*

Spherical Geometry for the Cellular Automaton is considerably more realistic than for the classical case. With the cellular model each cell is mapped on to a polygon based sphere, the number of polygons being the number of cells required. Then using a technique called surface refinement, the polygons are made of equal area, thus giving a direct mapping between the triangles and cells.

For every cell, the angle between the corresponding polygon on the sphere and the direction of light is calculated and used to calculate the amount of area 'visible' to the light source for that cell. This dictates the amount of light falling on it, and therefore its temperature. The rest of the simulation may then continue as before.

Unfortunately spherical geometry causes a problem with the calculation of the actual planet temperature, causing it to be approximately 20 degrees below the expected value.

The following figures show the resulting graphs from the spherical simulation.

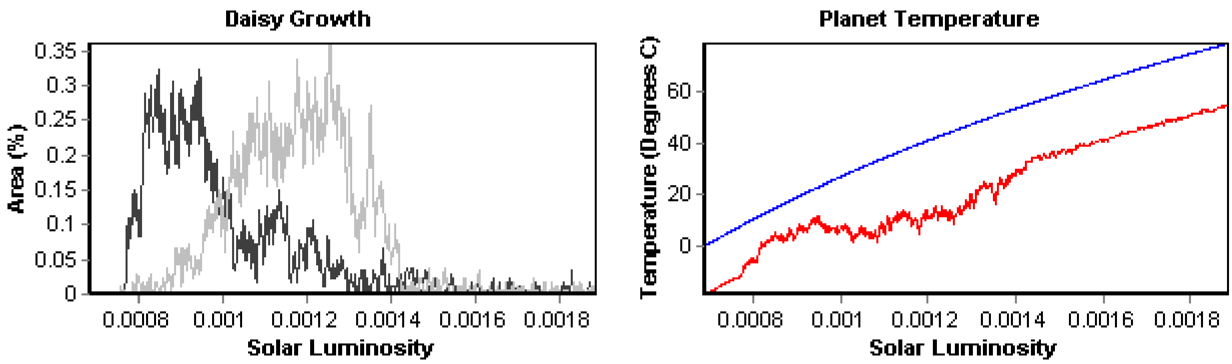


Figure 35 Population and temperature graphs for the Cellular Automaton model with 2 species on a simple sphere

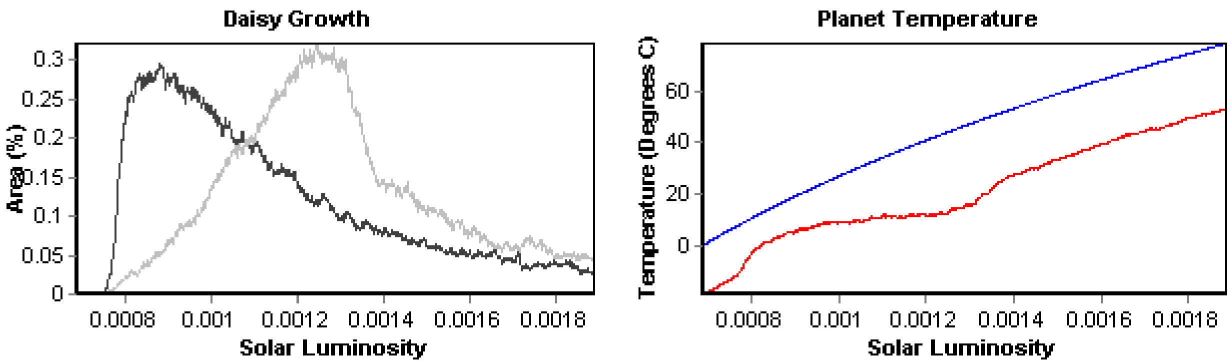


Figure 36 Population and temperature graphs for the Cellular Automaton model with 2 species on a complex sphere

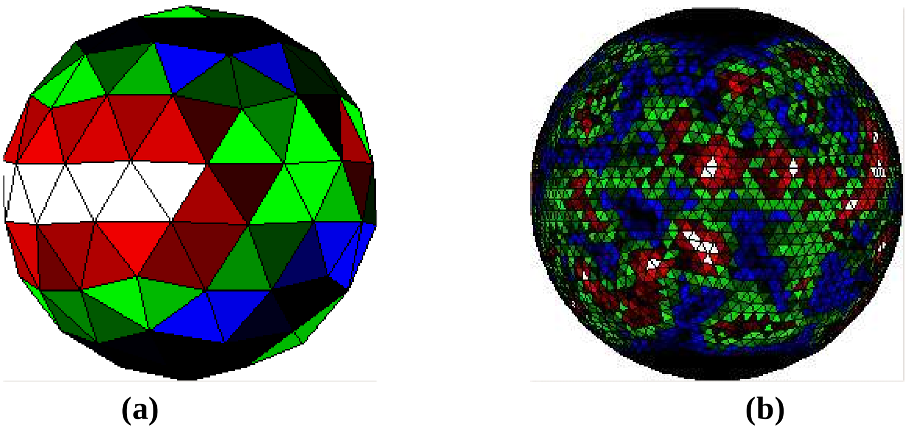


Figure 37 The simple sphere used (a) and the complex sphere used (b) for producing the above results

6.2.4. *Plagues*

Plagues are implemented in the Cellular model just as they are in the classical one, by removing a certain percentage of the total population periodically. The following graphs were created using the complex sphere.

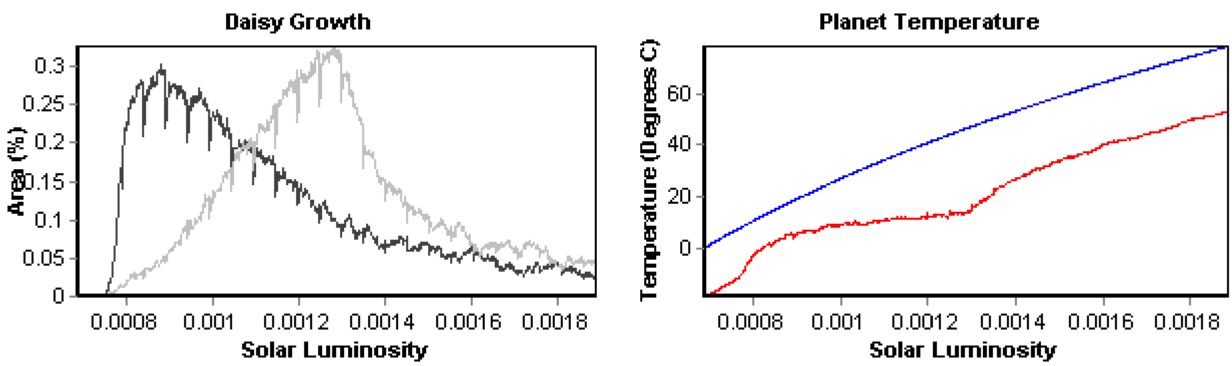


Figure 38 Population and temperature graphs for the Cellular Automaton model with 2 species on the complex sphere with a plague of 20%

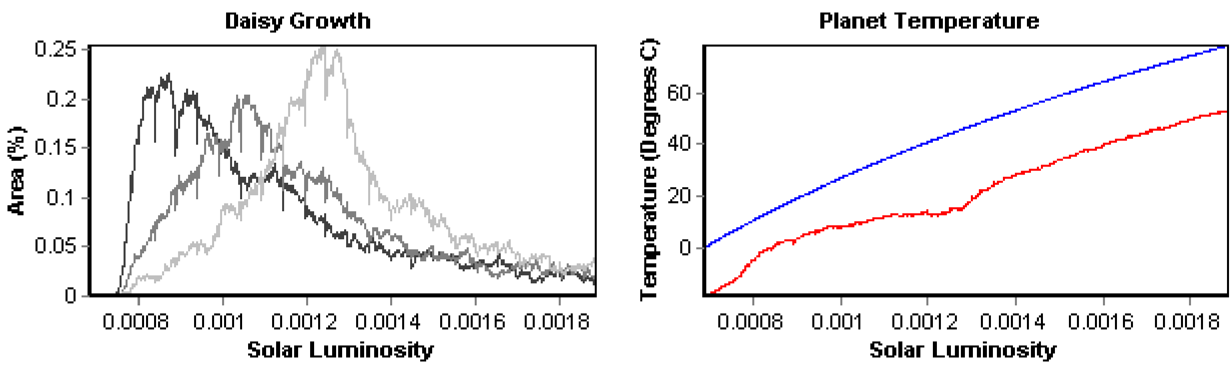


Figure 39 Population and temperature graphs for the Cellular Automaton model with 3 species on the complex sphere with a plague of 20%

As plagues were added to the Cellular Automaton model after spherical geometry it is difficult to draw any conclusions from these results as the planet lives much longer anyway.

7. Future Work

In this section the various improvements/work to be done of the Classical and Cellular models (as they stand at the time of writing) will be discussed in terms of that model. At the end of this section a short list of ideas for simple extensions can be found which try to augment the realism of the simulation.

7.1. Foxes and Rabbits

The next stage for the Cellular Automaton model would be to introduce foxes and rabbits in to the system, but unfortunately this was not possible.

It was theorised that fox and rabbit objects could be added and would monitor their own food-consumption rate then decide where to go next on a random-walk across the planet. Collision between a fox and rabbit would allow the fox to eat the rabbit, and collision between a rabbit and a daisy would allow the rabbit to eat the daisy. After consuming a given amount of food a rabbit/fox could spawn a new creature of its kind, failure to eat the correct amount of food in a given time period would result in the death of a rabbit or fox.

Another implementation would involve much of the above but also giving a small amount of intelligence to each species, this would allow a fox to follow a rabbit once it was 'spotted', and allow a rabbit to evade a fox in the same case. But this is not really a Cellular Automaton implementation, more of an A-Life one.

It was decided within the group that continued research and development into this area of the project was in danger of having a detrimental effect on the project as a whole. Instead it is left as further work for future investigations.

7.2. Growth Tax

Growth tax was added to Daisyworld by Dr. Lovelock in response to a biologists comment about the daisies needing no energy to grow pigment, and therefore making it unrealistic. This was implemented by reducing the beta value by 1%.

This also means that grey daisies have an advantage over all the other species, and though Lovelock has claimed that grey daisies did not out compete the other species, others have claimed that the opposite is true [13]

7.3. Extensions

Whilst writing the Daisyworld simulation, many ideas for simple extensions were generated, the resulting list can be found below

- Soil nutrients – The amount of fertile ground changes as parts may only be occupied for a finite period of time before there are no nutrients left.

- Water/Oceans – The water could be simulated using a daisy whose initial population is set as a constant (i.e. it may not grow), the water would absorb heat a great deal of heat while radiating very little. This would also reduce the amount of fertile ground; but may in turn assist temperature regulation.

- Clouds – Clouds would cause a reduction in solar luminosity for a certain percentage of the total population.

- Mountains – Mountains would be areas of increased solar luminosity for a certain percentage of the total population.

7.4. Classical Spherical Geometry

As previously mentioned work still needs to be done on the spherical model for the classical simulation. This would be based on allowing a ring only to interact with its neighbouring rings rather than all of them, hopefully removing the averaging problem.

8. Conclusions

8.1. Mathematical model

8.1.1. Criticisms

One of the main criticisms of the mathematical model is that it deals only in global averages; this means that even small disturbances (i.e. plagues) cause large fluctuations in temperature and population. It also leaves no room for local activity. For example if one glances at the graphs for the Cellular Automaton, it can be seen that the two daisy species begin life at the same time. This is due to the white daisies living in symbiosis with clumps of blacks. The mathematical model shows the species growing at different points in time, as only the global temperature is considered when deciding whether a daisy may grow.

Another criticism is that there is no positional data about the daisies; this is most obvious when looking at the conduction between daisies. Where one daisy may warm another (no matter how far away) instantly, the dynamics (heat conducted slowly over distance) of the Cellular Automaton mean that the conduction physics are much more realistic.

8.1.2. Benefits

Whilst doing this project a clear benefit of the mathematical model became clear, pure speed. The classical simulation using five species can be completed in the blink of an eye, even using more advanced integration techniques. The Cellular Automaton, on the other hand, takes at a minimum a couple of minutes to complete as the calculations involved with the simulation are much more complex.

As seen in the section concerning the formulation of the probability equations, most of the work for the classical model has already done mainly by biologists. It is not difficult to adapt the sorts of simplistic population equations found in Daisyworld into a probabilistic form, but more complex systems will inevitably involve more complex equations, the makes the Cellular technique increasingly more difficult to use as the complexity of the system increases.

8.2. Cellular Automaton model

8.2.1. Criticisms

The main drawback of Cellular Automaton modelling (as mentioned earlier) is that it is computationally expensive. Many more calculations need to be performed and many (pseudo) random numbers need to be generated for the simulation to work. But for the results and insight gained this cost seems menial, also with the speed of processing doubling every twelve months or so, very soon the Cellular technique will be just as fast.

The second drawback, also mentioned earlier, is the adequate formation of the probability equations either from analysis of the system to be modelled or by adapting the differential equations that may or may not exist for that system.

8.2.2. Benefits

The main benefit of the Cellular Automaton is the introduction of positional data. Where as the classical model uses mainly global data (such as global temperature) within the equations, the Cellular model uses the data gained from the neighbours of the cell in question. This means that more realistic behaviour can be observed. For instance a white daisy may live in a clump of black daisies during the initial stages of the Cellular model, whereas the global temperature would say that it was unfit to do so, as in the classical model.

This is also prudent when applying plagues. Due to the classical model averaging the population and using instant conduction, a reduction in the population causes a large spike in temperature. The Cellular model on the other hand uses local data and dynamic conduction and so only shows a small rise in temperature.

The Cellular model also makes adding other species easier and more realistic as they may be designed to look after themselves, i.e. to be more like agents. For example the rabbits, as previously discussed, may travel on a random walk, eating daisies as they land upon them and multiplying accordingly. In the classical model however, the rabbits always eat a certain percentage of the daisies, with no chance of not finding them.

8.3. Outcomes of the project

Due to time constraints and rather over ambitious target setting the final goal of creating a population modelling tool (in whatever form) has not been met, this was addressed in the new version of the project specification that was submitted six months or so after the initial specification. The code already written, certainly for the mathematical simulation, is done so in a modular object oriented way, this would enable a programmer to use the parts of the simulation for a larger project, so maybe in that sense the goal has been met.

The secondary goal however, to compare and contrast the two simulation techniques, has been fully met and addressed in this paper and in [5]. It has been made clear by the authors of this paper and [ref] that the Cellular Automaton gives many advantages over the original mathematical Daisyworld model, and this would fit with the general theme of [12].

Along the way, it should also be mentioned, it was discovered that a spherical Cellular Automaton had never before been used to simulate Daisyworld, which was described in this paper and in [5].

Acknowledgements: Many of the novel ideas in this paper came from discussions with the other member of the project R. Oates and with the project supervisor Dr. R. Mitchell of the University of Reading. The author also wishes to thank Emma Conein, Marc Fox, Gareth Scott and Mark Basham for assistance in bouncing thoughts around and for ideas about simple extensions.

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