Asymptotically Stable Adaptive–Optimal Control Algorithm With Saturating Actuators and Relaxed Persistence of Excitation

By Kyriakos G. Vamvoudakis, Member, IEEE, Marcio Fantini Miranda, and João P. Hespanha, Fellow, IEEE

Leshchev Dmitrii
Mezentsev Gleb

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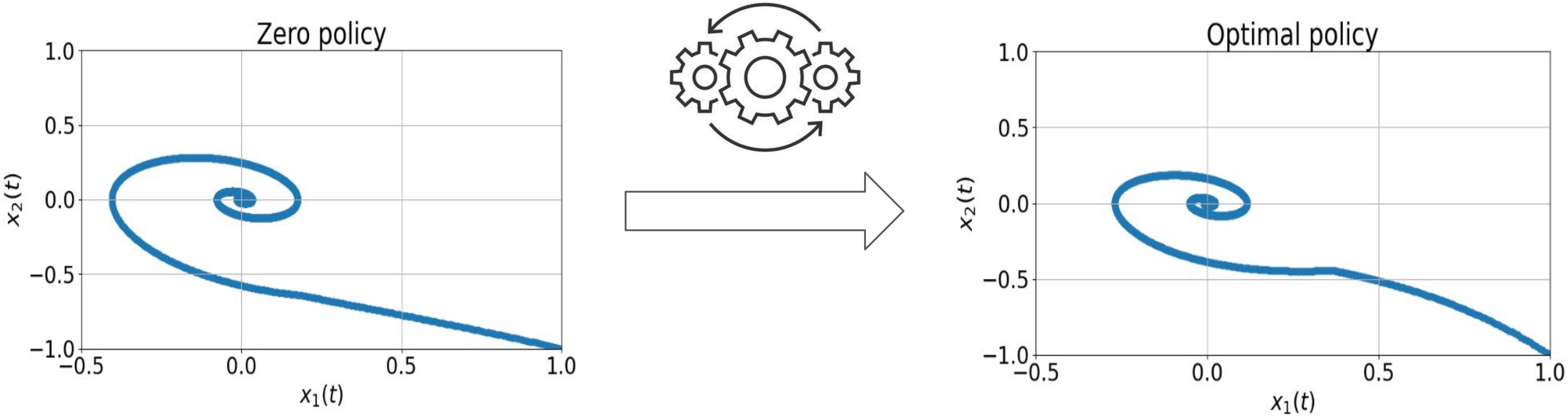
Summary of the project

The contributions of this project lie in the implementation of an adaptive learning algorithm to solve an infinite-horizon optimal control problem for known deterministic nonlinear systems, while considering symmetric input constraints



Problem studied and relevant theory

Nonlinear continuous-time system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t);$$
$$x(0) := x_0, \;\; t \geq 0$$

Infinite horizon loss-function

$$V(x(0)) = \int_0^{\infty} r\,(x(\tau), u(\tau))\, d\tau \quad \forall x(0)$$
$$r(x, u) = Q(x) + R_s(u) \quad \forall x, u$$

Penalty function (in matrix form)

$$\begin{aligned} R_s(u) \;&=\; 2\int_0^{u} (\theta^{-1}(v))^T R dv \\ &:= 2\int_0^{u} \left(\bar{u}\tanh^{-1}(v/\bar{u})\right)^T R dv > 0 \quad \forall u. \end{aligned}$$

Penalty function

$$R_s(u) = 2\sum_{i=1}^{m}\int_0^{u_i} (\theta^{-1}(v_i))^T \varrho_i dv_i \quad \forall u$$

Optimal value function

$$V^{\star}(x(t)) = \min_{u \in U}\int_t^{\infty} r(x, u) d\tau \quad \forall x, \;\; t \geq 0$$

Problem studied and relevant theory

HJB equation with optimal cost and optimal control

$$H^\star(x, \bar{\mathcal{K}}^\star(x), \nabla V^\star(x)) := \nabla V^\star(x)^T (f(x) + g(x)\bar{\mathcal{K}}^\star(x)) + Q(x) + R_s(\bar{\mathcal{K}}^\star(x)) = 0 \quad \forall x.$$

$$\bar{\mathcal{K}}^\star(x) = {W_u^\star}^T \phi_u(x) + \epsilon_u(x) \quad \forall x$$

Optimal control policy

Optimal value function

$$V^\star(x) = {W^\star}^T \phi(x) + \epsilon(x) \quad \forall x$$

Methods: Critic

Optimal value function

$$\boxed{V^\star(x) = {W^\star}^T \phi(x) + \epsilon(x) \quad \forall x}$$

Gradient-descent-like rule for Critic NN tuning

$$\begin{aligned}
\dot{\hat{W}} &= -\alpha \frac{\partial E}{\partial \hat{W}} = -\alpha \frac{\omega(t)e(t)}{(\omega(t)^T \omega(t) + 1)^2} - \alpha \sum_{i=1}^{k} \frac{\omega(t_i)e_{\text{buff}_i}(t_i, t)}{(\omega(t_i)^T \omega(t_i) + 1)^2} \\
&= -\alpha \frac{\omega(t)(\omega(t)^T \hat{W}(t) + R_s(u(t)) + Q(x(t)))}{(\omega(t)^T \omega(t) + 1)^2} \\
&\quad - \alpha \sum_{i=1}^{k} \frac{\omega(t_i)(\omega(t_i)^T \hat{W}(t) + Q(x(t_i)) + R_s(u(t_i)))}{(\omega(t_i)^T \omega(t_i) + 1)^2}
\end{aligned}$$

where $\omega(t_i) := \nabla\phi(x(t_i))\,(f(x(t_i)) + g(x(t_i))u(t_i))$.

Methods: Actor

Optimal control policy

$$\bar{\mathcal{K}}^{\star}(x) = W_u^{\star T} \phi_u(x) + \epsilon_u(x) \quad \forall x$$

Gradient-descent-like rule for Critic NN tuning

$$\begin{aligned}\dot{\hat{W}}_u &= -\alpha_u \frac{\partial E_u}{\partial \hat{W}_u} = -\alpha_u \phi_u e_u \\ &= -\alpha_u \phi_u \left(\hat{W}_u^T \phi_u + \theta \left(\frac{1}{2} R^{-1} g^T(x) \nabla \phi^T \hat{W} \right) \right)^T\end{aligned}$$

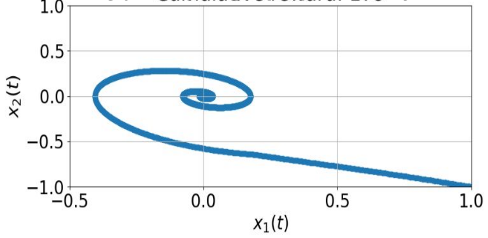
Methods. Algorithm

Adaptive-Optimal Control Algorithm With Relaxed PE

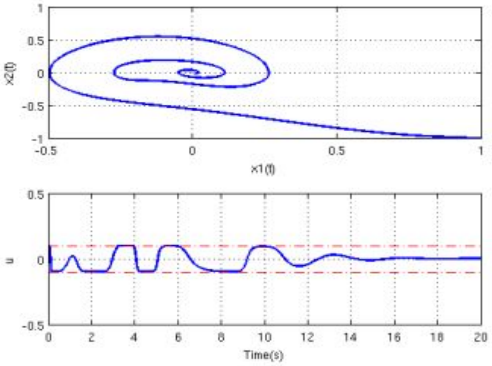
1. Start with initial state $x(0)$, random initial weights $\hat{W}_u(0)$, $\hat{W}(0)$ and $i = 1$
2. **procedure**
3. Propagate $t, x(t)$ using (1) and $u(t) := \hat{\hat{\mathcal{K}}}(x) \triangleright \{x(t)$ comes from integrating the nonlinear system (1) using any ordinary differential equation (ode) solver (e.g. Runge Kutta) while the time t comes from the Runge Kutta integration process, i.e. $[t_i, t_{i+1}]$, $i \in \mathcal{N}$ where $t_{i+1} := t_i + h$ with $h \in \mathbb{R}^+$ the step size$\}$
4. Propagate $\hat{W}_u(t)$, $\hat{W}(t) \triangleright \{$integrate $\dot{\hat{W}}_u$ as in (31) and $\dot{\hat{W}}$ as in (20) using any ode solver (e.g. Runge Kutta)$\}$
5. Compute $\hat{V}(x) = \hat{W}^T \phi(x) \triangleright$ output of the Critic NN,
6. Compute $\hat{\hat{\mathcal{K}}}(x) = \hat{W}_u^T \phi_u(x) \triangleright$ output of the Actor NN
7. **If** $i \neq k \triangleright \{\{\omega(t_1), \omega(t_2), \ldots, \omega(t_i)\}$ has N linearly independent elements and t_k is the time instant that this happens$\}$
8. Select an arbitrary data point to be included in the history stack (c.f. Remarks 1-2)
9. $i := i + 1$
10. **end if** $\triangleright$ when the history stack is full
11. **end procedure**

Results.Van der Pol Oscillator

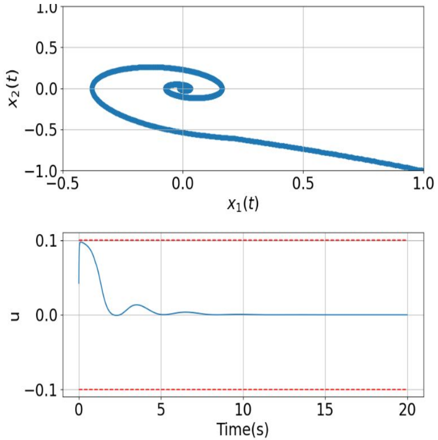
Zero policy
Cumulative reward = 179



Paper results

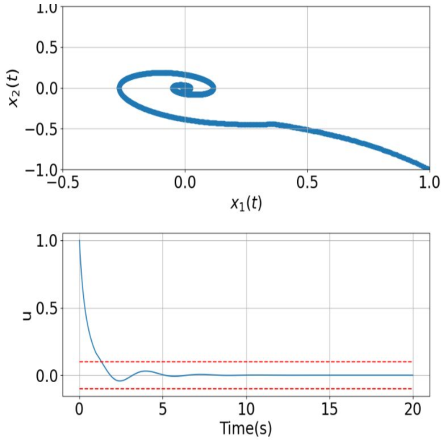


Adaptive policy
Cumulative reward = 168



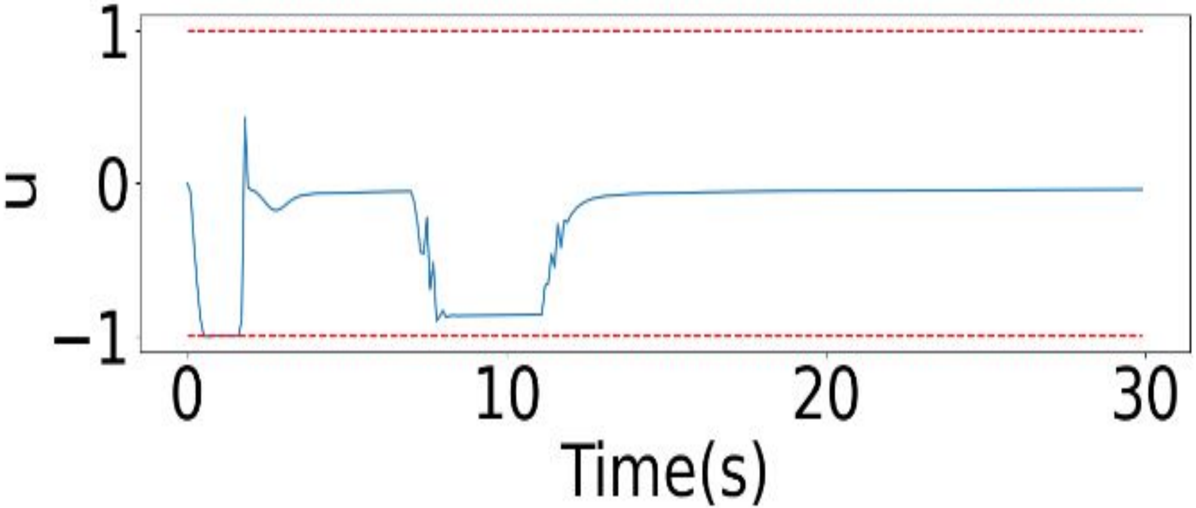
Our implementation of the algorithm

Optimal policy
Cumulative reward = 134

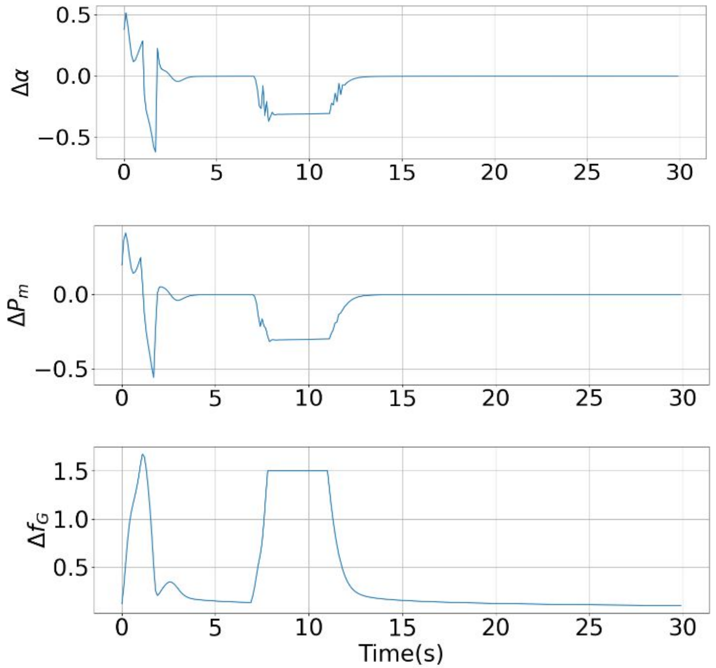


Theoretical optimal policy

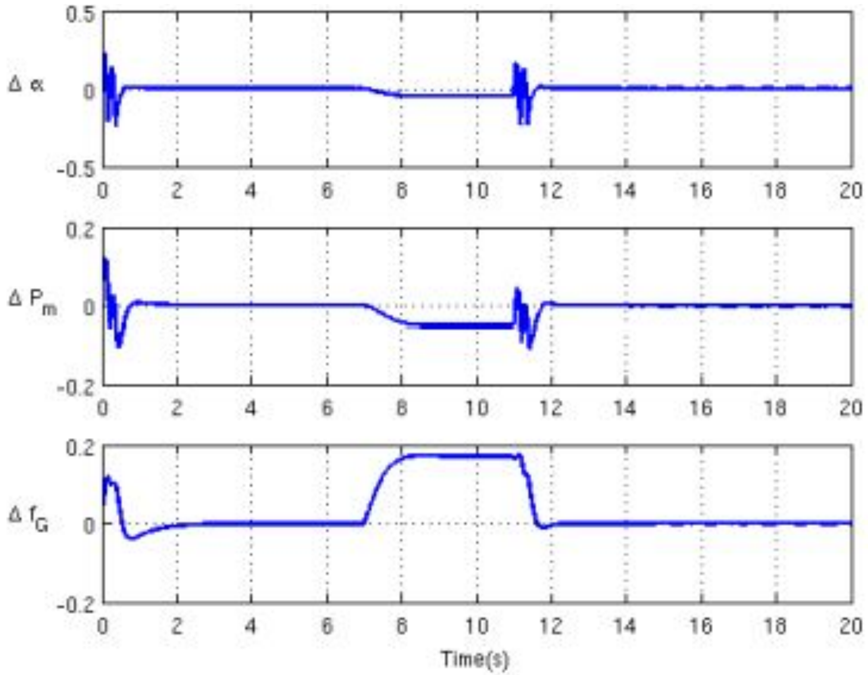
Results.Power Plant System



Results.Power Plant System

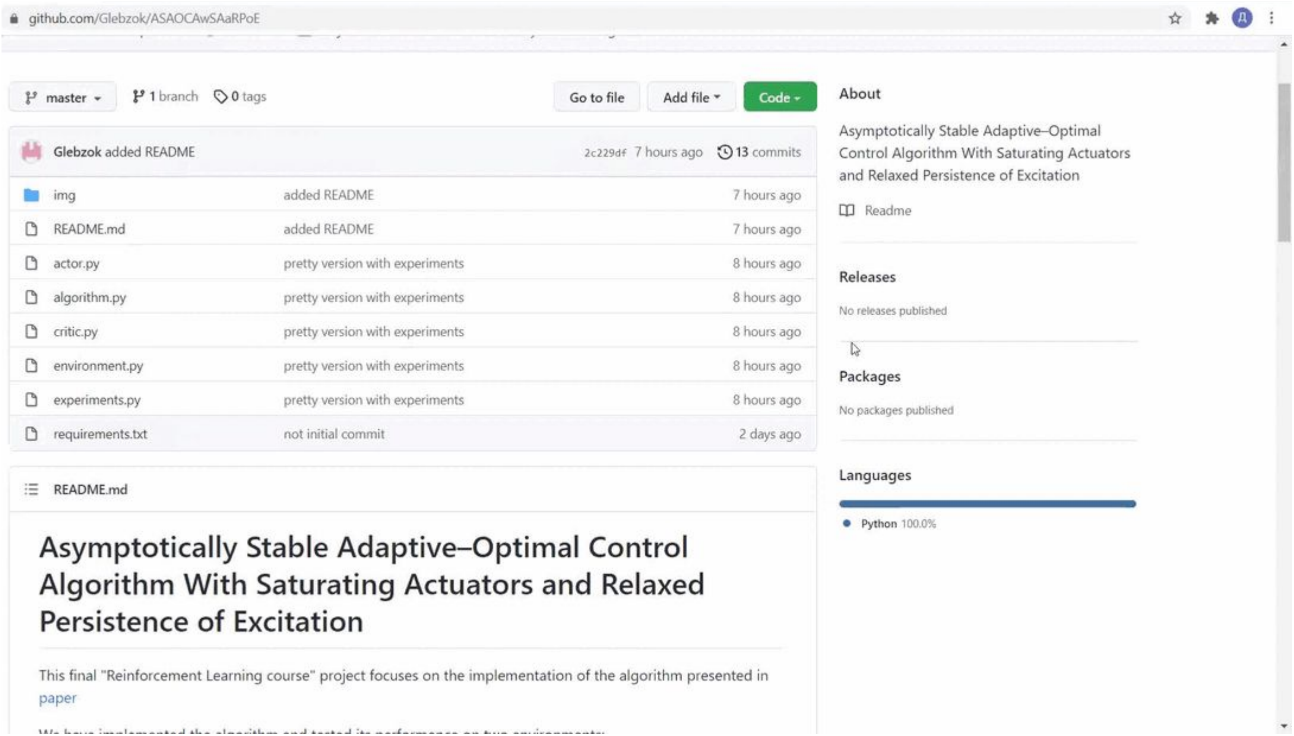


Our implementation



Paper results

Github



https://github.com/Glebzok/ASAOCAwSAaRPoE

Conclusion

- **Implemented the algorithm**
- **Checked its efficiency on two environments**
- **Achieved improvements in convergence in both cases**
- **Satisfied all initial constraints in both cases**

Thank you for your attention!