Assignment_2

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17/17 questions complete

1. [3 marks] MS 3.36 - pg 105-106

A

$$P(Match \cap Experts) = 92.12\%$$

B

$$P(Match \cap Novice) = 74.55\%$$

C

Novice are more likely to fail the fingerprint match test with a lesser probability that the Novice will be right.

2. [3 marks] MS 3.52 - pg 111

A

$$P(U|+) = 50\%$$

B

$$P(N|-) = 99\%$$

C

$$P(+|U) = P(U \cap +)P(U) = \frac{0.059 \cdot 0.50}{1} = 29.5\%$$

3. [1 marks] MS Theorem 3.1 - pg 113 Prove the theorem in your own words.

Multiplicative Rule

$$n_1 n_2 n_3 n_4 \cdot \cdots \cdot n_k$$

This can be explained by simply having an x amount of sequences of y amount of numbers.

4. [1 marks] MS Theorem 3.2 - pg 114 Prove the theorem in your own words.

Permutations Rule

$$P_n^N = N(N-1)(N-2)\cdot\cdots\cdot(N-n-1) = \frac{N!}{(N-n)!}$$

This can be proved through the multiplication rule. After one element from a set of K elements has been placed in a position, the element is removed and the set of elements to be chosen from is K-1. After the next is pulled, then the set consists of K-2 elements and so on, until the nth position is filled which is written as (K-n-1) where n is the factorial. Applying the Multiplicative Rule will result in the above equation.

5. [1 marks] MS Theorem 3.3 - pg 116 Prove the theorem in your own words.

Partitions Rule

$$\frac{N!}{n_1!n_2!\cdots n_k!} \quad \text{where } n_1 + n_2 + \cdots + n_k = N$$

This is just a variation of the Permutations Rule where there are N elements partitioned into multiple sets. The sets together still contain N amount of elements cumulatively. Through the Multiplicative Rule, you can find the above equation by multiplying the Permutation of N elements by the factorial of cardinality of each set of numbers that you are trying to partition.

6. [1 marks] MS Theorem 3.4 - pg 117 Prove the theorem in your own words.

Combinations Rule

$$\binom{N}{n} = \frac{N!}{n!(N-n)!}$$

This rule is a variation of the Partitions Rule, where the result is just the number of ways you can take n elements from a Set of N elements. The “two” sets of numbers n + N-n = N so this still follows the Partition’s Rule.

7. [3 marks] MS 4.2 - pg 138

A

$$.09 + .3 + .37 + .2 + .04 = 1$$

B

$$P(3 \leq X \leq 4) = P(3) + P(4) = 24\%$$

C

$$P(X \leq 2) = P(0) + P(1) = 39\%$$

8. [4 marks] MS 4.12 - pg 143

A

$$P(0 \leq X \leq 20) = 1$$

B

$$P(10 \leq Y) = P(10) + P(11) + ... + P(20) = 14\%$$

C

$$\mu = \sum_{y=0}^{n} yP(y) = 0(0.17) + 1(0.1) + ... + 20(0.005) = 4.655$$

$$\sigma^2 = \sum_{y=0}^{n} (y-\mu)^2 P(y) = (0 - 4.655)2(.17) + ... + (20 - 4.655)2(0.005) = 19.856$$

D

$$P(0 \leq Y \leq 6)$$

9. [4 marks] MS 4.34 - pg 154

A

$$P(y) = \binom{n}{y} p^y q^{n-y} = \frac{n!}{y!(n-y)!} p^y q^{n-y} = \frac{25!}{10!15!} 0.70^{10} 0.31^5 = 0.13249\%$$

B

$$P(Y \leq 5) = \sum_{y=0}^{n} \binom{n}{y} p^y q^{n-y} = 0.0027726\%$$

C

$$Mean = \mu = np = 17.5$$

$$Variance = \sigma^2 = npq = 5.25$$

D

Most of the probability lies within 2 standard deviations of 17.5, which is

$$12.9174 \leq Y \leq 22.0826$$

10. [2 marks] MS 4.46 - pg 158

A

$$P(5:5) = \frac{50!}{5!^{10}} \cdot .1^{50} = 0.00004912\%$$

B

$$P(1, Yn) = \frac{50!}{1!49!}(.1)^{49+1} = 5X10^{-49}$$

11. [2 marks] MS 4.54 - pg 162

A

$$P(Y) = (1-p)^{y-1}p$$

B

$$E(y) = 1/p = 1/.4 = 2.5$$

C

$$P(1) = (1-0.4)^0.4 = 0.4$$

D

$$P(Y > 2) = 1 - P(Y \leq 2) = 1 - P(1) - P(2)$$

$$P(Y > 2) = 1 - .4 - .6 * .4 = 0.36$$

12. [2 marks] MS 4.66 - pg 168

A

$$r = 8 \quad n = 10 \quad N = 209$$

$$E(y) = \frac{rn}{N} = .38278$$

B

This means the mean is less than 1 facility treats hazardous waste on site, and the average is closer to zero for a sample of 10 factories.

C

$$P(y) = \frac{E(y)n}{N} = \frac{.38278 \cdot 4}{10} = 0.15311$$

13. [3 marks] MS 4.78 - pg 173

A

$$Var(y) = \lambda = 0.03$$

B

Y=1, the probability is 2.911% and at Y=2, the probability drops exponentially. This shows, the distribution is very steep and is greater than 0.

C

$$P(y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

$$P(0) = \frac{.03^0 e^{-.03}}{0!} = e^{-.03} = 97.045\%$$

14. [4 marks] MS 5.2 - pg 191

A

$$\int_x f(x)dx = 1$$

$$\int_1^0 c(2-y)dy = 1 \Rightarrow c[2y - y^2)^2|_0^1 = 1$$

$$= c(3/2) = 1 \Rightarrow c = 2/3$$

B

$$F(x) = 2/3(2y - \frac{y^2}{2}) \quad 0 \le Y \le 1$$

C

$$F(0.4) = 2/3(.8 - .08) = 0.48$$

D

$$P(.1 \leq Y \leq .6) = 2/3(1.2 - .18) - 2/3(.2 - 0.05) = .58$$

15. [3 marks] MS 5.10 - pg 196

A

$$Mean = E(y) = \int_{-\infty}^{\infty} yf(y)dy = \int_{-5}^{5} \frac{3}{500}(25y - y^3)dy = 3500[252(y2|_{-5}^{5}) - 14(y4|_{-5}^{5})] = \frac{3}{500}\left[\frac{25}{2}(0) - \frac{1}{4}(0)\right] = 0 \quad minutes$$

$$Variance = E(y^2) - (E(y))^2 \text{ where E(y)=0} \Rightarrow E(y^2) Variance = \frac{3}{500}\int_{-5}^{5}(25y^2 - y^4)dy = \frac{3}{500}[25/3(y^3|_{-5}^{5}) - 1/5(y^5|_{-5}^{5}] = 75/1$$

B

$$-1/12 < Y < 1/12$$

The mean does not change and stays Zero because the limits are circular.

The variance does not change, but is converted to hours, which is 1/12 hours.

C

$$-300 < Y < 300$$

The mean does not change and stays Zero because the limits are circular.

The variance does not change, but is converted to seconds, which is 300 seconds.

16. [3 marks] MS 5.36 - pg 205

A

$$\mu = 50 \text{ milligrams} \quad \sigma = 3.2 \text{ milligrams}$$

$$P(X > 45)$$

$$P(z > \frac{45 - 50}{3.2}) = 1 - P(z \leq \frac{45 - 50}{3.2})$$

$$= 1 - P(z \leq -1.56) \Rightarrow 1 - 0.0594 = 94.06\%$$

B

$$P(X < 55)$$

$$P(z < \frac{55-50}{3.2}) = z < 1.56 = 94.06\%$$

C

$$P(51 < X < 52) = P(x < 52) - P(x < 51) = P(z < \frac{52-50}{3.2}) - P(z < \frac{51-50}{3.2}) = (z < .63) - (z < .31) = .7357 - .6217 = 11.4$$

17. [5 marks] MS 5.38- pg 205

A

$$z = \frac{x-\mu}{\sigma}$$

$$z(500) = \frac{500-605}{185} = -.57 \qquad z(700) = \frac{700-605}{185} = .51$$

$$P(-.57 < Z < .51) = .695 - .2843 = 41.07\%$$

B

$$z = \frac{x-\mu}{\sigma}$$

$$z(400) = \frac{400-605}{185} = -1.11 \quad z(500) = \frac{500-605}{185} = -.57$$

$$P(-1.11 < Z < -.57) = .2843 - .1335 = 15.08\%$$

C

$$z = \frac{x-\mu}{\sigma} = \frac{850-605}{185} = 1.32$$

$$P(z < 1.32) = 90.66\%$$

D

$$z = \frac{x-\mu}{\sigma} = \frac{1000-605}{185} = 2.14$$

$$P(z < 2.14) = 1.62\%$$

E

$$P(X > x_0) = 0.1 \Rightarrow P(X < x_0) = 0.9 P(\frac{X-\mu}{\sigma} < \frac{x_0-\mu}{\sigma}) = 0.9 P(Z < \frac{x_0-\mu}{\sigma}) = 0.9 x_0 = 605 + 185 \cdot 1.282 = 842.17$$