Assignment3

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Assignment 3 - 10/14 Questions Complete

Question 1

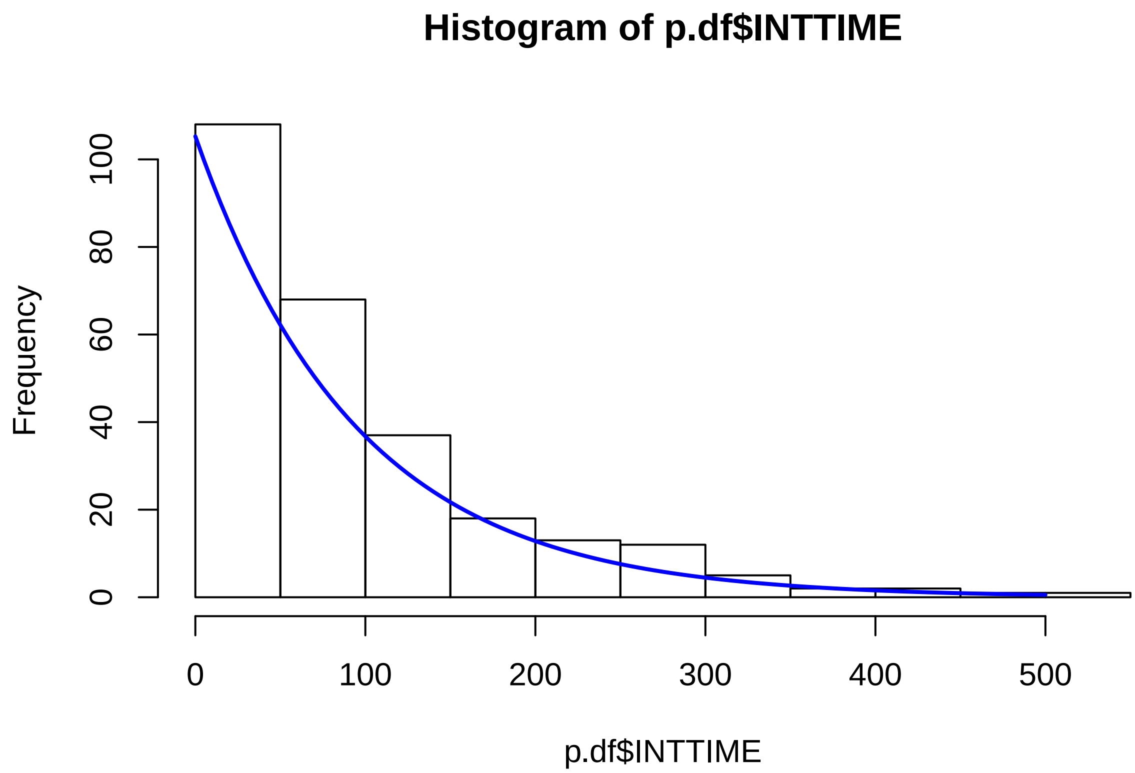
A

$$P(y \geq 120) = e^{-120/95}$$
$$P(y \geq 120) = e^{-1.263}$$
$$P(y \geq 120) = 0.283$$

B

```r
p.df = read.csv("PHISHING.csv")

hist(p.df$INTTIME)
myexp = function(x, a, b) {
  (a / b) * '^'(2.71828183,-(a * x / b))
}
curve(
  myexp(x, 1, 95)*10000, #Just for sizing to actual times. Could have went other way
  xlim = c(0, 500),
  ylim = c(0, 100),
  col = "Blue",
  lwd = 2,
  ylab = "Exponential Density",
  main = "Alpha = 1, Beta = 95",
  add=TRUE
)
```



Yes, you can clearly see that the data follow an exponential distribution of B=95

Question 2

A

$$Mean = \mu = \alpha\beta = 3(.07) = .21$$
$$Variance = \sigma^2 = \alpha\beta^2 = 3(.07)^2 = 0.0147$$

B

$$Standard\ Deviation = \sigma = \sqrt{\sigma^2} = \sqrt{0.0147} = 0.1212$$
$$\mu + \sigma + \sigma = .4524$$

.60 million cubic feet lies well out of 2 standard deviations of the mean, so no, I would not expect to observe .60 in a gamma distribution with the constraints as this one. The data seems insufficient.

Question 3

A

$$A(\alpha = 2, \beta = 2)$$

$$B(\alpha = 1, \beta = 4)$$

$$Expected\ length\ of\ time(A) = \frac{\alpha}{\beta} = 2/2 = 1$$

$$Expected\ length\ of\ time(B) = \frac{\alpha}{\beta} = 1/4 = 0.25$$

B

$$Variance\ length\ of\ time(A) = \frac{\alpha}{\beta^2} = 2/4 = 0.5$$

$$Variance\ length\ of\ time(B) = \frac{\alpha}{\beta^2} = 1/16 = 0.0625$$

C

$$P(A < 1) = \int_0^1 \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$P(A < 1) = \int_0^1 \frac{2^2}{\Gamma(2)} x^{2-1} e^{-2x}$$

$$\Gamma(2) = 1$$

$$P(A < 1) = 4\int_0^1 xe^{-2x}$$

$$P(A < 1) = 4[-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}|_0^1$$

$$P(A < 1) = 4[\frac{1}{4} - \frac{e^{-2}}{2} - \frac{e^{-2}}{4}]$$

$$P(A < 1) = 1 - 3e^{-2}$$

$$P(A < 1) = .594$$

$$P(B < 1) = \int_0^1 \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$P(B < 1) = \int_0^1 \frac{4^1}{\Gamma(1)} x^{1-1} e^{-4x}$$

$$\Gamma(1) = 1$$

$$P(B < 1) = 4\int_0^1 e^{-4x}$$

$$P(B < 1) = -1[e^{-4x}|_0^1$$

$$P(B < 1) = -[e^{-4} - e^0]$$

$$P(B < 1) = 1 - e^{-4}$$

$$P(B < 1) = .9817$$

Formula B has a higher probability of generating human reaction in less than 1 minute.

Question 4

A

$$F(x) = \frac{\alpha}{\beta} y^{\alpha-1} e^{-y^{\alpha}/\beta}$$

$$P(x < 2) = F(2) = \int_0^2 \frac{\alpha}{\beta} y^{\alpha-1} e^{-y^{\alpha}/\beta} dy$$

$$P(x < 2) = 1 - e^{-2^2/4}$$

$$P(x < 2) = 1 - e^{-1}$$

$$P(x < 2) = 0.63212$$

B

$$Mean = \mu = \beta^{1/\alpha} \Gamma(\frac{\alpha+1}{\alpha})$$

$$\mu = 2\Gamma(\frac{3}{2})$$

$$\mu = 1.7316$$

$$Variance = \sigma^2 = \beta^{2/\alpha} [\Gamma(\frac{\alpha+2}{\alpha}) - \Gamma^2(\frac{\alpha+1}{\alpha})]$$

$$\sigma^2 = 4[\Gamma(\frac{1}{2}) - \Gamma^2(\frac{3}{2})]$$

$$\sigma^2 = 4[0.886 - 0.8698]$$

$$\sigma^2 = 0.93$$

C

$$P(\mu - 2\sigma < X < \mu + 2\sigma) = P(-0.08 < X < 3.64)$$

$$P(-0.08 < X < 3.64) = P(x < 3.64) - P(x < -0.08)$$

$$= F(3.64) - F(-0.08)$$

$$P(-0.08 < X < 3.64) = e^{0.04^4} - e^{-(1.82)^4} = 0.99$$

D

$$P(X > 6) = 1 - F(6) = e^{-3^4} = 6X10^{-36}$$

It is very unlikely that the machine will exceed 6 years without repair.

Question 5

A

$$Mean = \mu = \frac{\alpha}{\alpha + \beta} = 2/11$$

$$Variance = \sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{18}{11^2(12)} = 18/1452$$

B

$$P(Y \geq 0.40) = 1 - \frac{1}{B(2,9)} \int_0^{0.40} y(1-y)^8 dy$$

$$P(Y \geq 0.40) = 1 - \frac{1}{B(2,9)}([\frac{-y(1-y)^9}{9}|_0^{0.40} - [\frac{(1-y)^{10}}{90}|_0^{0.40})$$

$$P(Y \geq 0.40) = 1 - \frac{1}{\frac{\Gamma(2)\Gamma(9)}{\Gamma(11)}}(0.0106) = .9998$$

C

$$P(Y \leq 0.10) = \frac{1}{B(2,9)}([\frac{-y(1-y)^9}{9}|_0^{0.1}[\frac{(1-y)^10}{90}|_0^{0.1})$$

$$P(Y \leq 0.10) = \frac{1}{B(2,9)}(0.0029)$$

Question 6

A

$$f(y) = \alpha\beta y^{\beta-1}e^{-\alpha y^{\beta}}$$

$$\beta = 2,\ \alpha = 1/16$$

B

$$Mean = 1/8 \int_0^{\infty} y^2 e^{-y^2/16} dy == 2\sqrt{\pi} = 3.545$$

$$E(X^2) = 1/8 \int_0^{\infty} y^3 e^{-y^2/16} dy = 16$$

$$Variance = E(X^2) - E(X)^2 = 16 - 4pi = 3.4335$$

C

$$P(X \geq 6) = 1 - P(X < 6) = 1 - F(6) = 1 - 1/8 \int_0^{6} ye^{-y^2/16} = 1 - 0.8946 = 0.1054$$

Question 7

A

$$P(x,y) = 1/36,\ 1 \leq x \leq 36,\ 1 \leq y \leq 36$$

B

$$P_1(x) = \sum_{y=0}^{6} \frac{1}{36} = 6/36 = 1/6$$

$$P_2(y) = \sum_{y=0}^{6} \frac{1}{36} = 6/36 = 1/6$$

C

$$P_1(x|y) = \frac{P(x,y)}{P_2(y)} = \frac{1/36}{1/6} = 6/36 = 1/6$$

$$P_2(y|x) = \frac{P(x,y)}{P_1(x)} = \frac{1/36}{1/6} = 6/36 = 1/6$$

D

$$P(x,y) = P_1(x)P_2(y)$$

$$1/36 = 1/6 * 1/6$$

This shows X and Y are independent.

Question 8

A

P(EnergyLevel, TimePeriod)= {0.142857, 0.285714, 0.142857, 0, 0, 0.285714, 0, 0, 0.142857}

B

$$P_1(X = 1) = 0.142857$$

$$P_1(X = 2) = 0.285714$$

$$P_1(X = 3) = 0.571428$$

C

$$P_2(Y = 1) = 0.571428$$

$$P_2(Y = 2) = 0.285714$$

$$P_2(Y = 3) = 0.142857$$

D

$$P_2(y|x) = P(Y \cap X)/P(X)$$
$$P_2(y=1|x=1) = P(Y=1 \cap X=1)/0.143 = 1$$
$$P_2(y=2|x=1) = P(Y=2 \cap X=1)/0.143 = 0$$
$$P_2(y=3|x=1) = P(Y=3 \cap X=1)/0.143 = 0$$
$$P_2(y=1|x=2) = P(Y=1 \cap X=2)/0.285 = 1$$
$$P_2(y=2|x=2) = P(Y=2 \cap X=2)/0.285 = 0$$
$$P_2(y=3|x=2) = P(Y=3 \cap X=2)/0.285 = 0$$
$$P_2(y=1|x=3) = P(Y=1 \cap X=3)/0.571 = 1/3$$
$$P_2(y=2|x=3) = P(Y=2 \cap X=3)/0.571 = 2/3$$
$$P_2(y=3|x=3) = P(Y=3 \cap X=3)/0.571 = 1/3$$

Question 9

A

$$f(y) = \int_{-\infty}^{\infty} f(x,y)dx$$
$$f(y) = \int_{y}^{2y} \frac{e^{-y/10}}{10y}dx = \frac{e^{-y/10}}{10}$$

Exponential Distribution

B

$$Mean = \mu = \beta = 10$$

Question 10

A

$$1 = \int_0^{\infty}\int_0^{x} ce^{-x^2}dydx = \int_0^{\infty}[yce^{-x^2}|_0^x dx$$
$$= \int_0^{\infty} xce^{-x^2}dx = 1/2[-ce^{-2x}|_0^{\infty} = \frac{c}{2}$$
$$1 = C/2 \Rightarrow C = 2$$

B

$$f(x) = \int_0^{x} ce^{-x^2}dydx = [yce^{-x^2}|_0^x = 2xe^{-x^2} = 1$$
$$1/2 = xe^{-x^2}$$

C

$$f(y|x) = \frac{f(x,y)}{f(x)} = \frac{2e^{-x^2}}{2xe^{-x^2}} = 1/x$$

Question 11

Question 12

Question 13

Question 14