Problem 3

$MA(1): X_t = W_t + \lambda W_{t-1}$

$n = 50$ $AR(1): X_t = \lambda X_{t-1} + W_t$

$ARMA(1,2):$ $(1 - 0.5B) y_t = (1 + 0.2B - 0.15B^2) w_t$

$y_{48} = 1$ $\hat{y}_{48} = 1.5$

$y_{49} = 2$ $\hat{y}_{49} = 1.2$

$y_{50} = 1$

$(1 - 0.5B) y_t = (1 + 0.2B - 0.15B^2) w_t$

$y_t - 0.5 y_{t-1} = W_t + 0.2 W_{t-1} - 0.15 W_{t-2}$

$\boxed{y_t = 0.5 y_{t-1} + W_t + 0.2 W_{t-1} - 0.15 W_{t-2}}$ ← — our model

$\hat{y}_{50} = 0.5 \cdot 2 + 0 + 0.2(2 - 1.2) - 0.15(1 - 1.5) = 1.275$

$\hat{y}_{51} = 0.5 \cdot 1 + 0 + 0.2(1 - 1.275) - 0.15(2 - 1.2) = 0.325$

$\hat{y}_{52} = 0.5 \cdot 0.325 + 0 + 0.2(0) - 0.15(1 - 1.275) = 0.20375$

$\hat{y}_{53} = 0.5 \cdot 0.203 + 0 + 0.2(0) - 0.15(0) = 0.1015$